Intracavity squeezing for a gravitational-wave detector

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Injection of a squeezed vacuum into a gravitational-wave detector is a way to improve its shot-noise limited sensitivity without increasing the laser power. The squeezer is placed outside the detector and decreases the vacuum fluctuation in the phase quadrature. Here we propose a new scheme to utilize the squeezer. The squeezer is placed inside a cavity and decreases the vacuum fluctuation in the phase quadrature just slightly so as to realize the impedance matching, with which the shot-noise limited sensitivity can be improved much better than the conventional scheme with the squeezer outside the detector.

1. INTRODUCTION

Injection of squeezed vacuum is a way to improve the sensitivity of a gravitational-wave detector and the utility has been demonstrated in some of the detectors in the operation [1][2]. A zero-point energy fluctuation of the electric field makes sensing noise (shot noise) and limits the sensitivity of the detector at high frequencies. The squeezed vacuum is produced by a non-linear crystal with an arbitrary vacuum field as the seed and the second harmonic of the carrier light of the detector as the pump beam [3]. The squeezed vacuum is injected to the detector from its anti-symmetric port. In the case of the GEO600 interferometer, for example, there is a signal recycling mirror at the anti-symmetric port and thens there is a squeezer outside the signal-recycling cavity (Fig. 1, left panel). A Faraday isolator and a polarized beam splitter are placed after the signal recycling mirror to let the squeezed field get in to and the signal field comes out from the interferometer, but they are omitted here.

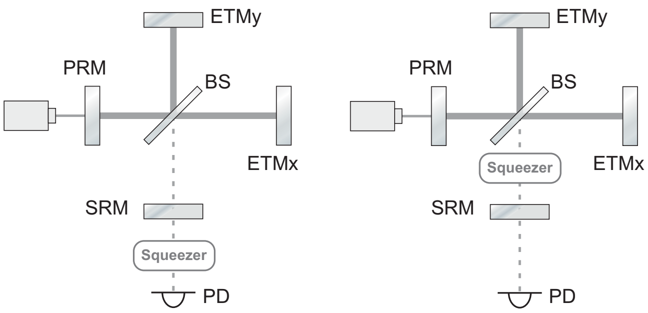

FIG. 1: *Right panel*: Conventional way to use squeezing in an interferometer. *Left panel*: Intracavity squeezing proposed in this paper.

Here we introduce a different configuration with the optical squeezing in the gravitational-wave detector. The squeezer is installed inside the signal-recycling cavity (Fig. 1, right panel). With this configuration, one can realize the sensitivity identical to that of a conventional interferometer with remarkably lower incident power. Figure 2 shows couple of examples. The power at the beamsplitter is 10 kW in either case. In the conventional configuration with a 10 dB squeezing, the reflectivity of the signal-recycling mirror is set 95 %. Using the intra-cavity scheme, we can realize the same sensitivity curve with the reflectivity of the signal-recycling mirror set 10 %. Another example, an extreme one, is the sensitivity of a 99.99 % signal-recycling mirror and the 30000 times higher laser power with the conventional interferometer, which can be realized with the intra cavity interferometer using the same parameters shown above except for a slight change of the squeezing.

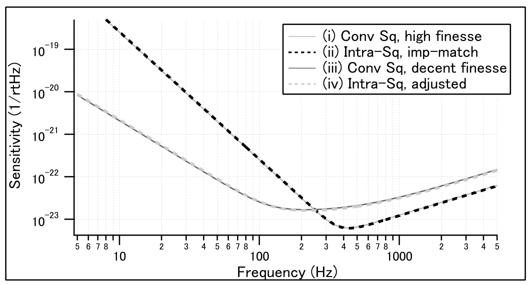

FIG. 2: Quantum noise spectra with different configurations. (i) Conventional interferometer with squeezing outside; the incident power and the finesse of the signal-recycling cavity are both infinitely high. (ii) Interferometer with intracavity squeezing where the squeeze factor is tuned to realize the impedance matching of the signal-recycling cavity for one quadrature; the noise curve is overlapped with the one for (i). (iii) Conventional interferometer with squeezing outside, a decent incident power, and a decent finesse. (iv) Interferometer with intracavity squeezing; the squeeze factor is tuned to realize the same noise spectrum as (iii).

One problem of the intracavity squeezing is optical losses in the squeezer. Even with an optical loss of 10^{-4}, the sensitivity degrades remarkably.

This paper is organized as follows: In Sec. 2, we explain the concept of the intracavity squeezing based on the impedance matching of the interferometer. In Sec. 3,

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we write down the input-output relations of the interferometer with the intracavity squeezer. In Sec. 4, we calculate the influence of optical losses in the squeezer. In Sec. 5, we discuss how we implement a squeezer in the signal-recycling cavity. In Sec. 6, we summarize our discussions.

2. IMPEDANCE MATCHING OF A CAVITY

In a Fabry-Perot cavity with the input reflectivity r_1 and the output reflectivity $r_2 = 1$, the reflected field is described as

$$E_{\rm out} \;=\; -r_1 E_{\rm in} + \frac{t_1^2}{1-r_1} E_{\rm in} \;, \tag{1}$$

where the input field is $E_{\rm in}$. The first term of the right-hand side (RHS) of Eq. (1) represents the field that is directly reflected by the input mirror and the second term represents the field that comes from the inside the cavity after some circulations. The amplitude of the reflected field is equal to the input field in the absence of optical losses.

The same rule is applied to the vacuum field in the gravitational-wave detector. Let us consider a dual-recycling configuration. The vacuum field entering from the anti-symmetric port is the source of quantum noise of the detector. It enters the interferometer through the signal-recycling mirror (SRM) and comes back after some circulations together with a gravitational-wave signal. In the absence of optical losses in the interferometer, the vacuum field will be completely reflected back. With some optical losses, a fraction of the input vacuum field will be lost but another vacuum field enters from the lossy optics and the output vacuum field ends up to have the same amplitude as the input vacuum field.

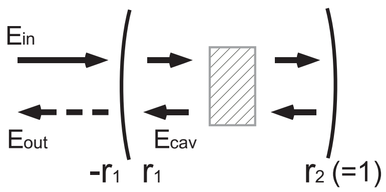

FIG. 3: Input and output fields of a cavity. The reflectivity of the end mirror (r_2) is set unity. Without anything in the shaded area, the amplitude of the reflected field is equal to that of the incident field. With a squeezer in the shaded area the amplitude of the reflected field at one quadrature decreases while the other quadrature increases.

If we could find a way somehow to decrease the second term of the RHS of Eq. (1) by $r_1(1-r_1)/t_1^2$ for the signal recycling cavity without imposing additional loss vacuums, the output field would be zero and there would be no shot noise. The key to realize the idea is the squeezing. Since the vacuum field has two quadrature components, namely the amplitude and phase quadratures, we can decrease one of them (phase) with increasing the other (amplitude) just by a small factor to cancel out the output field in the phase quadrature so that there is no shot noise.

In fact, a phase difference between the directly reflected field and the intracavity field prevents the complete cancellation at more than one frequency. In the end, the shot-noise spectrum of the impedance-matched intracavity-squeezing interferometer is equivalent to the one of the conventional configuration with the infinitely high-finesse signal-recycling cavity (shown in Fig. 2 as "high finesse" and "imp-match"). The slope of the shot-noise spectrum keeps decreasing linearly with f to lower frequencies. The spectrum is dominated by quantum radiation pressure noise at low frequencies. The slope of radiation pressure noise is not $1/f^2$ as a usual configuration with a decent finesse but $1/f^3$ due to the infinitely high finesse or the impedance matching of the signal-recycling cavity.

Such an infinitely high-finesse interferometer is not useful in practice. The finesse of the signal-recycling cavity shall be appropriately chosen to realize a good sensitivity and a decent bandwidth. A little adjustment from the impedance-matched signal-recycling cavity will decrease the effective finesse and broaden the bandwidth (shown in Fig. 2 with "decent finesse" and "adjusted").

In the next section, we see how one can realize the two cases.

3. INPUT-OUTPUT RELATION

Figure 4 shows the input and output vacuum fields from the anti-symmetric port of a dual-recycled interferometer. Each field consists of two quadrature components: the amplitude quadrature represented by a subscript 1 and the phase quadrature represented by a subscript 2. The input $\boldsymbol{a} = (a_1,\ a_2)$ is a coherent vacuum field. The two quadratures are given by the annihilation and creation operators at frequencies around the carrier frequency ω_0:

$$a_1 = \frac{a_+ + a_-^\dagger}{\sqrt{2}}, \quad a_2 = \frac{a_+ - a_-^\dagger}{\sqrt{2}i} \;. \tag{2}$$

Here $a_\pm$ and $a_\pm^\dagger$ are the annihilation and creation operators at $\omega_0 \pm \Omega$. The commutators of the vacuum fields satisfy

$$[a_1,\ a_{2'}^\dagger] = [a_2,\ a_{1'}^\dagger] = i2\pi\delta(\Omega - \Omega') \;, \tag{3}$$

while all other commutators of this kind vanish. Here the operator without the prime ($'$) is at frequency Ω and the one with prime is at frequency Ω'.

The input and output relations of the intracavity in-

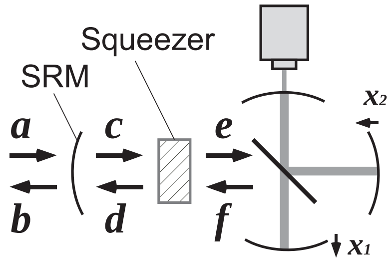

FIG. 4: Input and output fields of a dual-recycled interferometer with intracavity squeezing.

terferometer are given as

$$\begin{aligned} \boldsymbol{e} &= (1-\mathcal{E}/2)\boldsymbol{S}\boldsymbol{c} + \sqrt{\mathcal{E}}\boldsymbol{p} \,, &(4)\\ \boldsymbol{d} &= (1-\mathcal{E}/2)\boldsymbol{S}\boldsymbol{f} + \sqrt{\mathcal{E}}\boldsymbol{q} \,, &(5)\\ \boldsymbol{b} &= -r\boldsymbol{a} + t\boldsymbol{d} \,, &(6)\\ \boldsymbol{c} &= r\boldsymbol{d} + t\boldsymbol{a} \,, &(7) \end{aligned}$$

and

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = \left(1-\frac{\tilde{\mathcal{E}}}{2}\right)\begin{pmatrix} 1 & 0 \\ -\mathcal{K} & 1 \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} e^{2i\beta} + \sqrt{\tilde{\mathcal{E}}}\begin{pmatrix} 1 & 0 \\ J & 1 \end{pmatrix}\begin{pmatrix} n_1 \\ n_2 \end{pmatrix} e^{i\beta} + \left(1-\frac{\tilde{\mathcal{E}}}{2}\right)\begin{pmatrix} 0 \\ \alpha x \end{pmatrix} e^{i\beta} \,, \quad (8)$$

where $\boldsymbol{S}$ represents the squeezer:

$$\boldsymbol{S} = \begin{pmatrix} 1/s & 0 \\ 0 & s \end{pmatrix} \,, \quad (9)$$

and $\mathcal{E}$ and $\tilde{\mathcal{E}}$ represent the optical losses in the squeezer and the interferometer, respectively. $\mathcal{K}$ represents the opto-mechanical coupling [4]:

$$\mathcal{K} = \frac{20 I_0 \omega_0}{m c^2 \Omega^2} \,, \quad (10)$$

where m, I_0, and Ω are test mass weight, power in the power-recycling cavity, and the signal frequency, respectively, α represents the coupling to the signal:

$$\alpha = \frac{\sqrt{2\mathcal{K}}}{h_{\rm SQL} L} \,, \quad (11)$$

with $h_{\rm SQL}$, the standard quantum limit:

$$h_{\rm SQL} = \sqrt{\frac{20\hbar}{m\Omega^2 L^2}} \,, \quad (12)$$

β the phase shift during the one-way trip from the signal-recycling mirror to the end test mass:

$$\beta = \frac{\Omega L}{c} \,, \quad (13)$$

and J the coupling coefficient of radiation pressure noise from the loss vacuum:

$$J = \frac{1}{2}\left(1-\frac{\tilde{\mathcal{E}}}{2}\right)\mathcal{K} e^{i\beta} \,. \quad (14)$$

Let us first ignore the optical losses and the radiation pressure effects. Solving the equations above, we get

$$b_2 = \frac{1}{1-re^{2i\beta}}\left[(-r+s^2 e^{-2i\beta})a_2 + t\alpha x\right] \,. \quad (15)$$

In the absence of the squeezer ($s = 1$), the shot-noise limited displacement sensitivity is given by

$$h_x^{\rm conv} = \frac{1+r^2-2r\cos 2\beta}{(1-r^2)\alpha^2} \,, \quad (16)$$

while that with the squeezer is given by

$$h_x^{\rm intra} = \frac{s^4+\tilde{r}^2-2\tilde{r}s^2\cos 2\beta}{(1-\tilde{r}^2)\tilde{\alpha}^2} \,. \quad (17)$$

Here the reflectivity of the signal-recycling mirror and the incident laser power are different so that they are described with a tilde. Since β depends on frequency, the perfect impedance matching can be realized only at one frequency. If we choose

$$s = \sqrt{r} \,, \quad (18)$$

for example, the shot noise makes zero at DC ($f = 0$). This is equivalent to the situation without the squeezing and with the infinitely high finesse, which is not so useful in practice. In order to realize the same shot-noise limited sensitivity as that of a conventional configuration, we shall set the bandwidth equal. The numerator of Eq. (16) makes zero when $\cos 2\beta = (1+r^2)/(2r)$ and the numerator of Eq. (17) makes zero when $\cos 2\beta = (s^4+\tilde{r}^2)/(2\tilde{r}s^2)$, so the bandwidths will be the same if the following equation is satisfied:

$$\frac{1+r^2}{2r} = \frac{s^4+\tilde{r}^2}{2\tilde{r}s^2} \,, \quad (19)$$

that is,

$$s = \sqrt{\tilde{r}/r} \ \text{ or } \ \sqrt{\tilde{r}r} \,. \quad (20)$$

Compared with Eq. (18), the squeezing factor is slightly tuned to make a room of the flat frequency band. In addition to the bandwidth, the floor levels will also be the same if the following equation is satisfied:

$$\frac{\tilde{\alpha}}{\alpha} = \frac{t}{\tilde{t}r} \ \text{ or } \ \frac{t\tilde{r}}{\tilde{t}} \,, \quad (21)$$

when $s = \sqrt{\tilde{r}/r}$ or $\sqrt{\tilde{r}r}$, respectively. The square of the RHS of Eq. (21) represents the ratio of the required power to realize the same shot-noise limited sensitivity curve. In other words, if we use the input squeezing technique, the RHS of Eq. (21) represents the ratio of the required squeeze factor to realize the same sensitivity. It would be fair to compare this ratio not with s but with s^2 as we use squeezer twice for the round-trip in the intracavity scheme. One can see that the intracavity scheme requires less input power than the conventional scheme with the double-pass input squeezing by $(t/\tilde{t})^2$ or $(t/\tilde{t}r)^2$ for $s = \sqrt{\tilde{r}/r}$ or $\sqrt{\tilde{r}r}$, respectively. In the example for the decent finesse case shown in Fig. 2, where we take $s = \sqrt{\tilde{r}/r}$, this factor reads 18 times less power with the same squeezing used. Compared with the case without the squeezer, the required laser power is 171 times less. The squeezing used in the intracavity scheme here is only 4.9 dB.

4. OPTICAL LOSSES IN THE SQUEEZER

The intracavity scheme is, however, weak against optical losses in the squeezer. To realize the compensation of the vacuum with the impedance matching technique, the vacuum field directly reflected by the signal-recycling mirror and that coming out from the cavity should keep the coherence. The optical loss deteriorates the coherence and degrades the shot-noise limited sensitivity.

Solving Eqs. (4) to (8), one obtains the output field in the phase quadrature as follows:

$$\begin{aligned} b_2 \;=\; & -\frac{t\mathcal{K}}{M}\left\{\Gamma t e^{-2i\beta} a_1 + \left(\Gamma + \frac{\mathcal{E}}{2}\right)\sqrt{\mathcal{E}} s p_1 \right. \\ & \left. + \frac{\gamma}{2}\frac{\sqrt{\tilde{\mathcal{E}}} e^{-2i\beta} s}{1 - \Gamma r s^2 e^{-2i\beta}} n_1 + \Gamma\sqrt{\mathcal{E}} r e^{-2i\beta} q_1 \right\} \\ & + \frac{1}{1 - \Gamma r e^{2i\beta} s^2}\left\{\left(-r + \Gamma e^{-2i\beta} s^2\right) a_2 + \left(\Gamma + \frac{\mathcal{E}}{2}\right) t\sqrt{\mathcal{E}} s p_2 \right. \\ & \left. + \left(1 - \frac{\mathcal{E}}{2}\right) t\sqrt{\mathcal{E}} s e^{-i\beta} n_2 + \sqrt{\mathcal{E}} t q_2 + \left(1 - \frac{\mathcal{E}}{2}\right) t\alpha x \right\} \end{aligned} \tag{22}$$

with

$$\Gamma = \left(1 - \mathcal{E} - \frac{\tilde{\mathcal{E}}}{2}\right) , \tag{23}$$

$$\begin{aligned} \gamma = & \left(\Gamma + \frac{\mathcal{E}}{2}\right)\left\{1 - \Gamma r e^{-2i\beta} + (2\Gamma - 1) r^2 e^{-4i\beta}\right\} \\ & + \left(\Gamma - \frac{\mathcal{E}}{2}\right)\left(1 - \Gamma r e^{-2i\beta} s^2\right) 2 r e^{-i\beta} . \end{aligned} \tag{24}$$

Figure 5 shows the spectra with the intracavity squeezing with optical losses. Compared with the conventional configuration with the same squeezer outside the interferometer and with the higher incident power, the intracavity configuration is slightly weaker against the optical losses in the arm cavities and significantly weaker against the optical losses in the squeezer. Here we put 100 ppm losses per roundtrip for the arm cavity and 1000 ppm losses for the squeezer. The gray dashed curve shows the spectrum of the intracavity configuration with the incident power as high as the conventional configuration and with optical losses both in the arms and the squeezer.

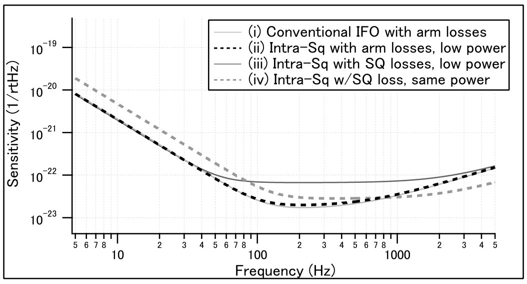

FIG. 5: Quantum noise spectra with losses. (i) Conventional interferometer with squeezing outside and with optical losses in the arm cavities. (ii) Low-power interferometer with intracavity squeezing with optical losses in the arm cavities. (iii) Low-power interferometer with intracavity squeezing with optical losses in the squeezer. (iv) Interferometer with intracavity squeezing with optical losses in the squeezer; the incident power is the same as (i).

5. IMPLEMENTATION OF THE SQUEEZER INSIDE THE CAVITY

So far we have treated the squeezer as if it is a two-way device with two input and two output ports, but the actual squeezer is usually a one-way device where an input field goes out from the same port. We shall make it clear that there is a way to realize the intracavity squeezing with the one-way squeezer.

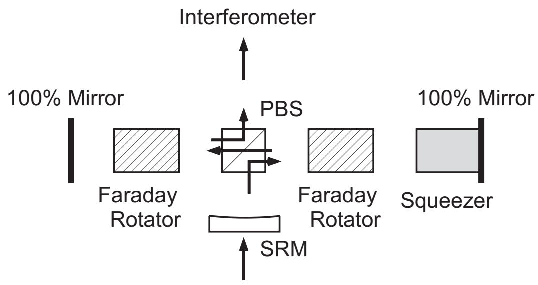

FIG. 6: One example to realize the two-way squeezer out of a one-way squeezer.

Figure 6 shows one example to realize the intracavity squeezing with the one-way squeezer. A field with a linear polarization enters the system through the signal-recycling mirror and first reflected by the polarization beam splitter (PBS). It goes through a Faraday rotator and then a squeezer, reflected back to the same Faraday rotator to change the polarization by 90 degrees in total. This time thee field then transmits through the PBS and then goes to the second Faraday rotator, reflected by a mirror back to the Faraday rotator to change the polarization by 90 degrees. The field is then reflected by the PBS and goes in to the interferometer. There can be a simpler way to realize the two-way squeezer, but this is at least one way to realize it.

6. SUMMARY

We proposed a new configuration for the next-generation gravitational-wave detector. Implementing a squeezer inside the signal-recycling cavity, one can realize the impedance matching of vacuum fluctuation in the phase quadrature. In the absence of optical losses, the same sensitivity can be achieved by much lower incident power even compared with the conventional configuration with two of the same squeezer used outside the interferometer. Although this new configuration is quite weak against optical losses in the squeezer, the sensitivity at high frequencies can be improved by using the same incident power as the conventional configuration.

Acknowledgments

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