WFS is tolerable to DC mis-centering

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This short memo is to briefly show mathematically the wave-front sensing (WFS) can work without so-called DC centering against the relevant RF QPD. Assume a usual Fabry-Perot cavity, the length of which is being locked with the Pound-Drever-Hall (PDH) method; the carrier is on resonance with the cavity, while the RF sideband is not. Then, the WFS signal can be extracted by beating HG00 of RF sideband vs HG10 of carrier.

Consider a tilt error case, which corresponds to the one in Eq.(10) of ref [1]. It correspondingly says the tilted carrier field can be

$$E(x, \alpha) \propto U_0(x) + i\frac{\alpha}{\alpha_0}U_1(x), \tag{1}$$

where α is a tilt angle of the beam, α_0 is the divergence angle, and x is a lateral spacial coordinate. Also,

$$U_0(x) \propto \exp(-x^2/\omega_0^2), \tag{2}$$

$$U_1(x) \propto x\exp(-x^2/\omega_0^2), \tag{3}$$

where ω_0 is the beam waist size. The beat signal at x will be roughly[*1]

$$I(x, \alpha) \propto U_0(x)U_0^*(x) + i\frac{\alpha}{\alpha_0}U_1(x)U_0^*(x), \tag{4}$$

where the first term corresponds to the usual length error signal, and to be suppressed to zero due to the length control, so let's ignore it.

If this is a single RF PD, the demodulate signal would be

$$V_1(\alpha) \propto \frac{\alpha}{\alpha_0}\int_{-\infty}^{\infty} f(x)dx, \tag{5}$$

$$f(x) \equiv x\exp(-2x^2/\omega_0^2), \tag{6}$$

and unfortunately and well-known, the resultant integral is zero, as the integrand is an odd function. More directly,

$$\int f(x)dx = -\frac{1}{4}\omega_0^2\exp(-2x^2/\omega_0^2) + \text{const.} \tag{7}$$

Using a divided PD, and taking a differential of the two outputs, one obtains a resultant signal

$$V_2(\alpha) \propto \frac{\alpha}{\alpha_0}\left(\int_{\delta}^{\infty} f(x)dx - \int_{-\infty}^{\delta} f(x)dx\right) \tag{8}$$

$$= \frac{\alpha}{\alpha_0}\frac{\omega_0^2}{2}\exp(-2\delta^2/\omega_0^2). \tag{9}$$

If we are so lucky to keep $\delta = 0$, the proportional coefficient takes the maximum. Even if not, you can still obtain[*2] a certain amount of signal proportional to α without adding any offset component, as you can see. In the realistic case, you need to consider signal-to-noise ratio, but anyway it seems tolerable against the DC mis-centering to the RF QPD.

References

[1] E. Morrison, B. J. Meers, D. I. Robertson, and H. Ward, "Automatic Alignment of optical interferometers", Appl. Opt. **33**, 5041 (1994)

[*1] Escape: $i\frac{\alpha}{\alpha_0}U_1^*U_0$ should be included as well, but the main conclusion of this memo won't be changed.

[*2] Note that even if the integral is within a finite region, V_2 dose not have an offset.