Study of birefringence effects with realistic mirror maps

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KAGRA F2F meeting, 28 Aug, 2021
JGW-G2113262

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Birefringence issues

- A portion of input light in s-pol is converted to p-pol, counted as loss, leading to a reduction in sensitivity [1, 2].
- The p-pol beam has an ugly shape. The birefringence is inhomogeneous [3, 4].
- The amount of birefringence is not balanced between the arms, which leads to additional laser intensity/frequency noises
- The existence of p-pol can have a bad consequence to LSC and ASC signal [1, 5].
- Scattering of s/p fundamental model to higher order mode (HOM). If p-HOM is resonant, it may pump energy from s-pol to p-pol.
- The resonant condition of p-pol can hardly be controlled. Thus the amount of p-pol energy pumped from s-pol can hardly be controlled.

P-pol image observed

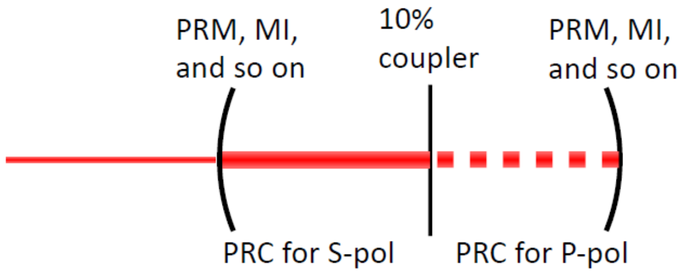
Motivation of simulation:

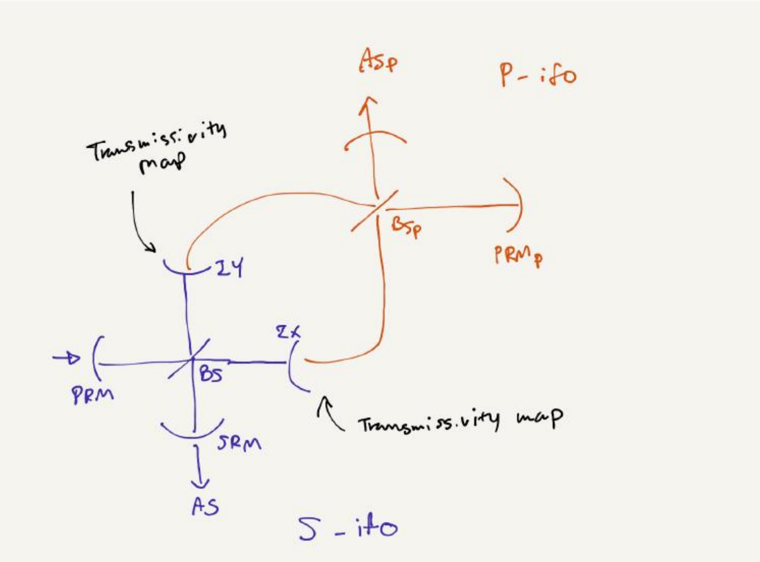
- To understand the influence of the birefringence of sapphire mirrors in the cryogenic interferometer.
- To study scattering effects of both beams in s-polarization and p-polarization and the influence of them to the control system.
- To include birefringence in laser intensity/frequency noise coupling in the simulation.
- To find ways of mitigating the birefringence effect.
- To help improve the polishing of the mirror.

[1] Yutaro Enomoto, Polarization issue in the central IFO, JGW-G1910388.
[2] Matteo Leonardi, et al, Polarization issue in PRC, JGW-G1910369.
[3] Kentaro Somiya, FINESSE simulation for the ITM inhomogeneity problem, JGW-G1809362.
[4] Yutaro Enomoto, et al, Arm cavity round-trip loss measurement with ITM inhomogeneity and birefringence, JGW-T2011633.
[5] External Review for interferometer alignment controls, August 2021.

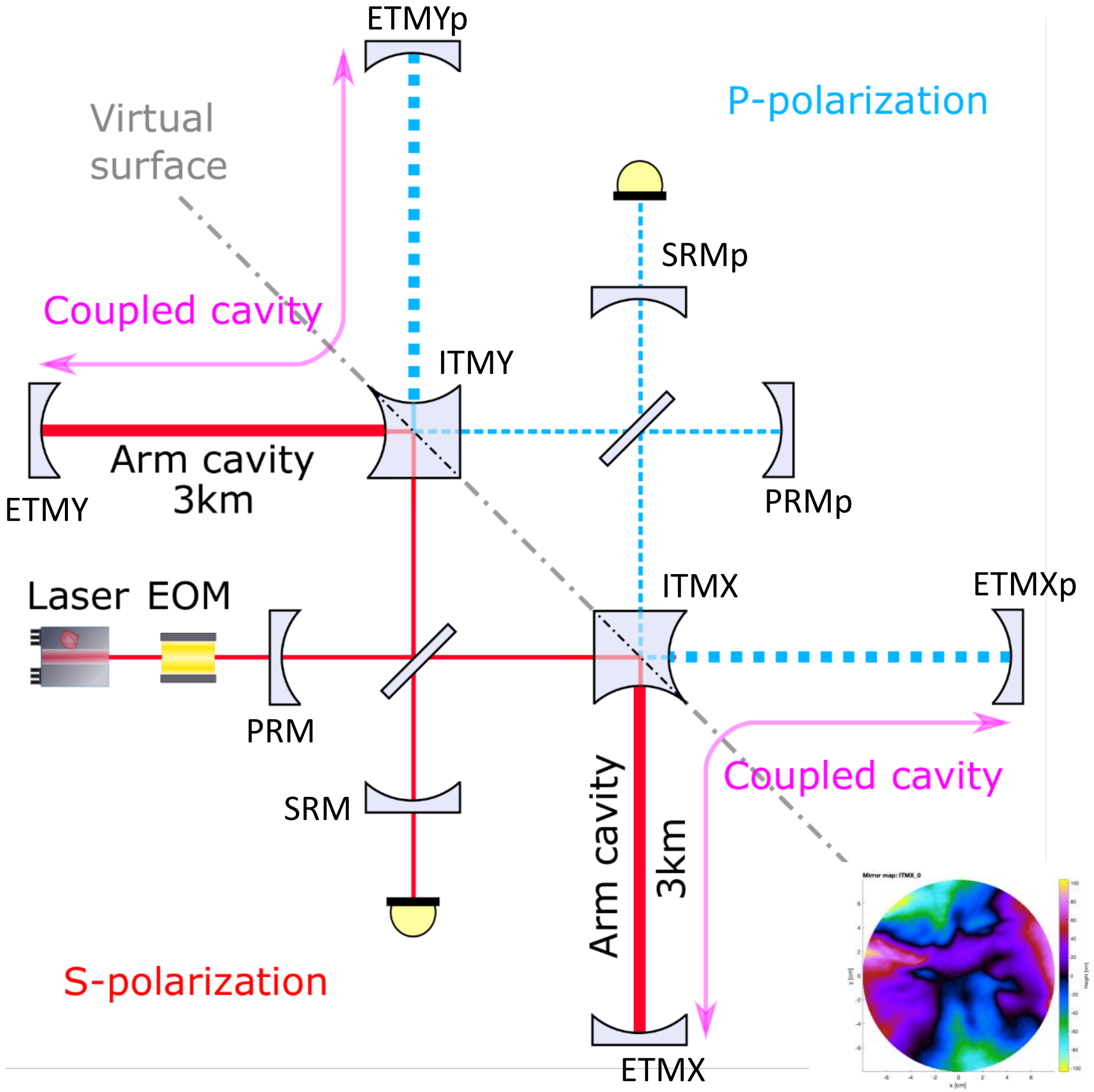
Two-world approach + Mirror maps

S/P coupled cavity [6]





Proposed by Enomoto, Aso, Kokeyama [7]

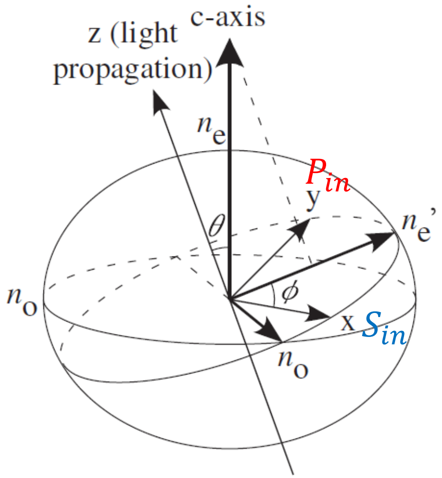


We proposed a two-world model using realistic birefringence maps [8].

The key is to create a “magic” surface that defines the coupling between s-pol and p-pol light.

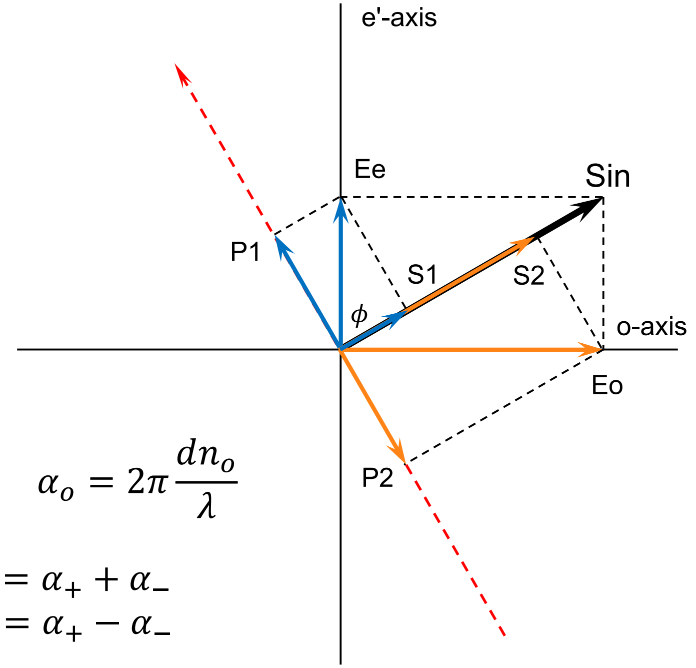
[6] Yutaro Enomoto, S-P coupled PRC, JGW-G1910373.
[7] Masayuki Nakano, KAGRA birefringent Cavity, JGW-G1910461.
[8] Haoyu Wang, et al, Finesse simulation for birefringence, JGW-T2011792.

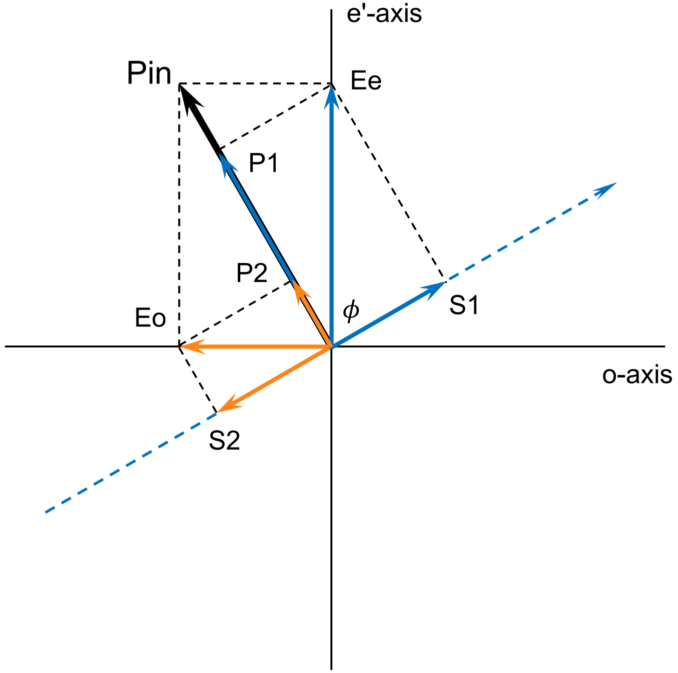
S/P Coupling matrix [9, 10]

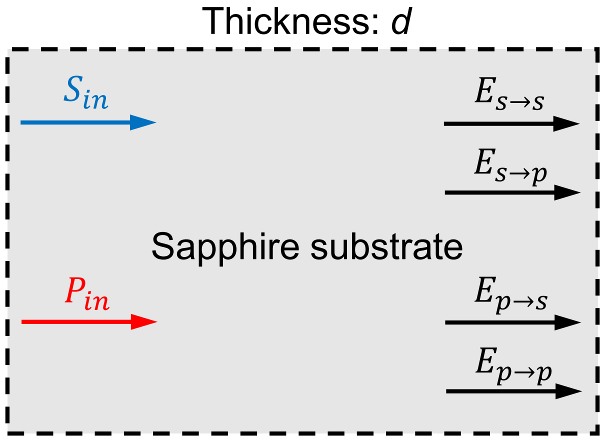


$$\alpha_e = 2\pi \frac{dn_e'}{\lambda} \qquad \alpha_o = 2\pi \frac{dn_o}{\lambda}$$

Note $\alpha_e = \alpha_+ + \alpha_-$
$\alpha_o = \alpha_+ - \alpha_-$







One-way phase difference α_-

$$\alpha_- = \frac{\alpha_e - \alpha_o}{2} = \frac{\pi d}{\lambda}(n_e' - n_o)$$

The four fields are

$$E_{s\to s} = S_{in} e^{i\alpha_+}(\cos\alpha_- + i\cos 2\phi \sin\alpha_-)$$
$$E_{s\to p} = S_{in} e^{i\alpha_+} \cdot i \sin 2\phi \sin\alpha_-$$
$$E_{p\to s} = P_{in} e^{i\alpha_+} \cdot i \sin 2\phi \sin\alpha_-$$
$$E_{p\to p} = P_{in} e^{i\alpha_+}(\cos\alpha_- - i\cos 2\phi \sin\alpha_-)$$

The common phase term $e^{i\alpha_+}$ can be omitted.

The output fields can be written as

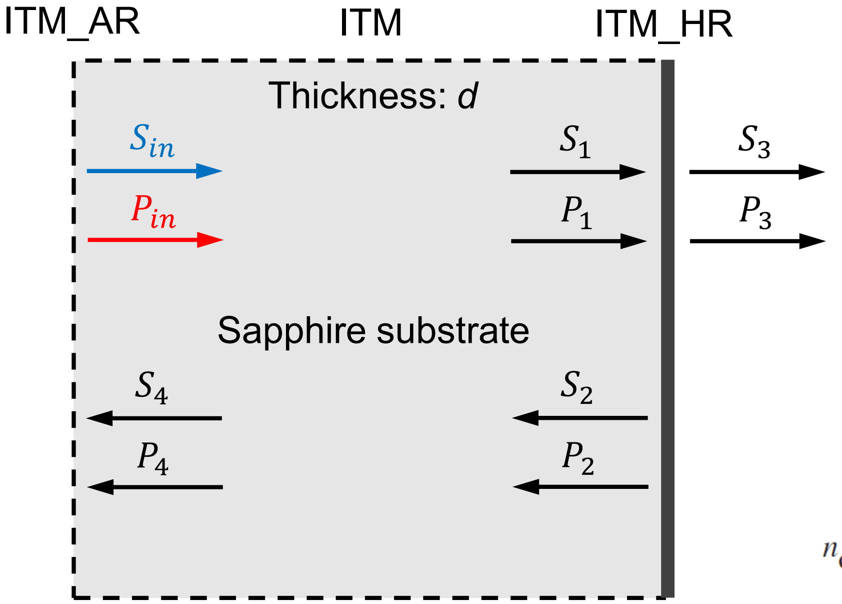
$$\begin{bmatrix} S_{out} \\ P_{out} \end{bmatrix} = \begin{bmatrix} r_{s\to s} & t_{p\to s} \\ t_{s\to p} & r_{p\to p} \end{bmatrix} \times \begin{bmatrix} S_{in} \\ P_{in} \end{bmatrix} \quad \text{or} \quad \boldsymbol{E_{out}} = \boldsymbol{G} \times \boldsymbol{E_{in}}$$

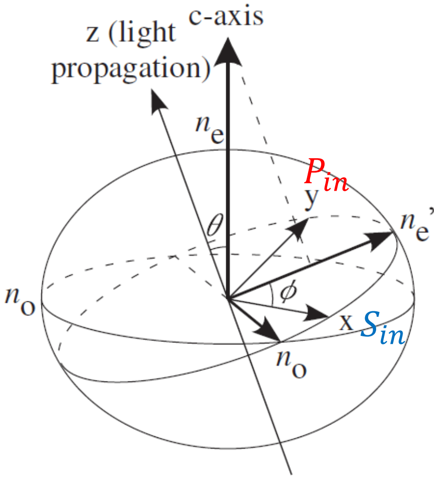
The one way coupling matrix is

$$\boldsymbol{G(\phi, \alpha_-)} = \begin{bmatrix} r_{s\to s} & t_{p\to s} \\ t_{s\to p} & r_{p\to p} \end{bmatrix} = \begin{bmatrix} \cos\alpha_- + i\cos 2\phi \sin\alpha_- & i\sin 2\phi \sin\alpha_- \\ i\sin 2\phi \sin\alpha_- & \cos\alpha_- - i\cos 2\phi \sin\alpha_- \end{bmatrix}$$

[9] Yoichi Aso, How to compute birefringence map from TWE maps, JGW-T1910380.
[10] Haoyu Wang, Phase relation of polarization couplings in birefringence, JGW-T2012048.

A model representing birefringence couplings





One-way phase difference α_-

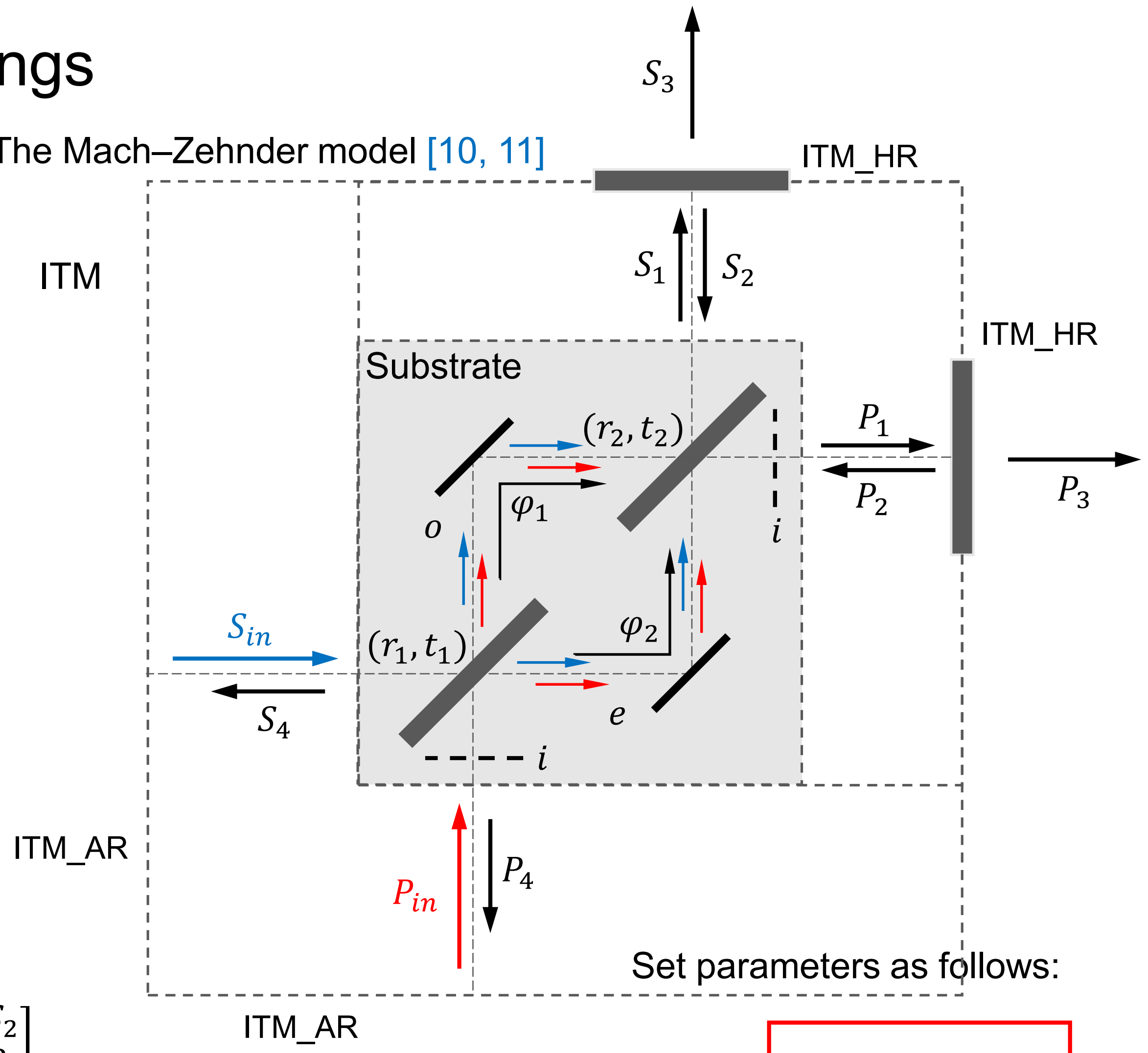
$$\alpha_- = \frac{\alpha_e - \alpha_o}{2} = \frac{\pi d}{\lambda}(n_e' - n_o)$$

In both cases, we have fields as follows

$$\begin{bmatrix} S_1 \\ P_1 \end{bmatrix} = \boldsymbol{G} \times \begin{bmatrix} S_{in} \\ P_{in} \end{bmatrix} \quad \begin{bmatrix} S_2 \\ P_2 \end{bmatrix} = r_{HR} \begin{bmatrix} S_1 \\ P_1 \end{bmatrix} \quad \begin{bmatrix} S_3 \\ P_3 \end{bmatrix} = i t_{HR} \begin{bmatrix} S_1 \\ P_1 \end{bmatrix} \quad \begin{bmatrix} S_4 \\ P_4 \end{bmatrix} = \boldsymbol{G} \times \begin{bmatrix} S_2 \\ P_2 \end{bmatrix}$$

$$\boldsymbol{G} = \begin{bmatrix} r_{s\to s} & t_{p\to s} \\ t_{s\to p} & r_{p\to p} \end{bmatrix} = \begin{bmatrix} \cos\alpha_- + i\cos 2\phi \sin\alpha_- & i\sin 2\phi \sin\alpha_- \\ i\sin 2\phi\sin\alpha_- & \cos\alpha_- - i\cos 2\phi \sin\alpha_- \end{bmatrix}$$

The Mach–Zehnder model [10, 11]



Set parameters as follows:

$$r_1 = r_2 = \sin\phi$$
$$t_1 = t_2 = \cos\phi$$
$$\varphi_1 = -\alpha_-$$
$$\varphi_2 = \pi + \alpha_-$$

We need to know ϕ and α_- .

[10] Haoyu Wang, Phase relation of polarization couplings in birefringence, JGW-T2012048.
[11] Haoyu Wang, Update of simulation for birefringence in KAGRA, JGW-G2012222.

Two modelling strategies:

- Treat p-pol as loss. $loss = |r_{s\to p}|^2 = \sin^2 2\phi \sin^2 \alpha_-$

  We apply the loss map to ITMs.

  Advantages:

  - Fast.
  - Use current interferometer configuration. No s/p coupled cavities.
  - The back-coupling from p-pol to s-pol is omitted because it is small.

  Disadvantages:

  - Maybe not accurate enough.

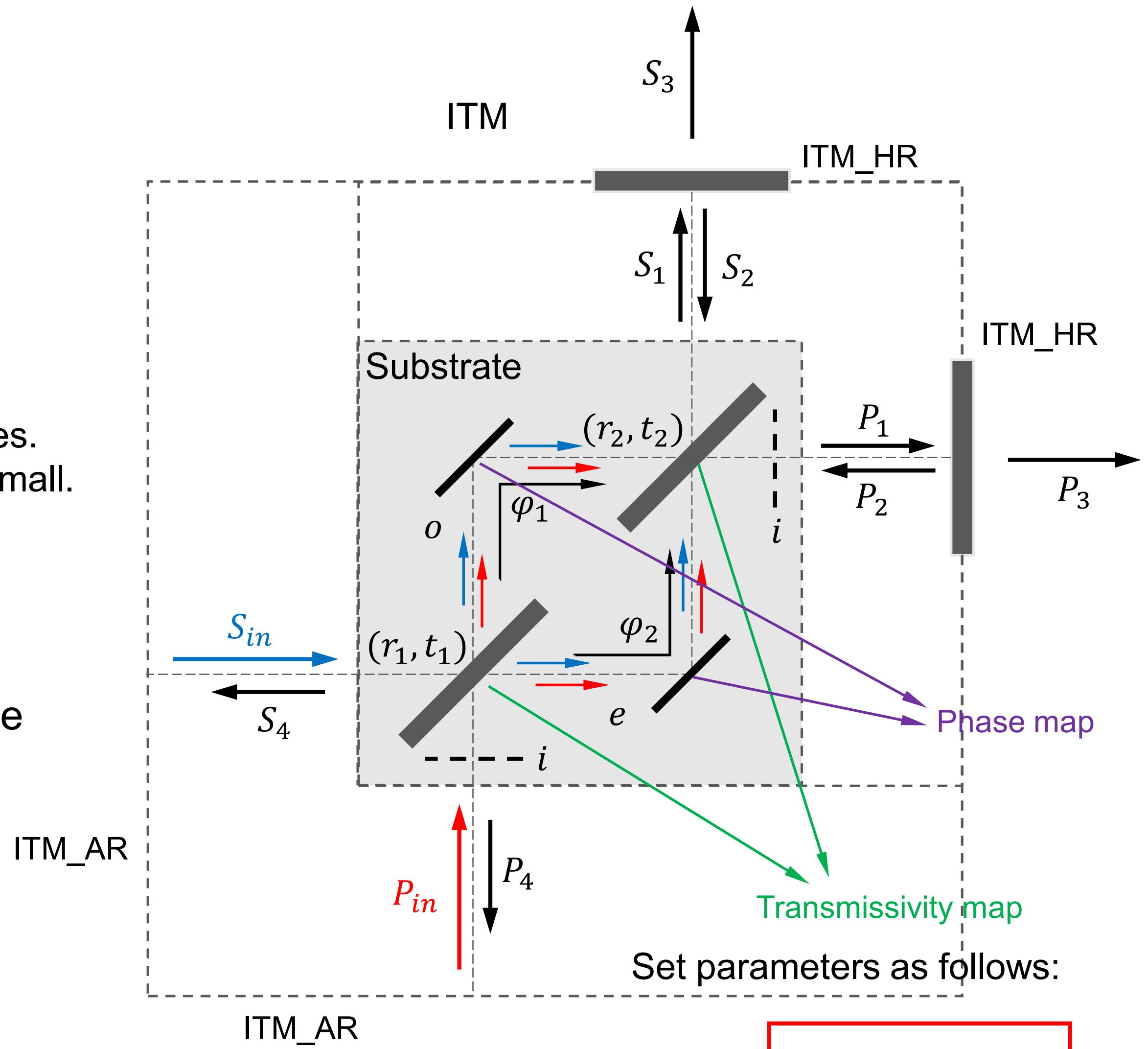
- Simulate s-pol and p-pol in the two-world approach with the Mach–Zehnder model.

  Advantages:

  - Precise and more practical.
  - Can help us fully understand birefringence effect.

  Disadvantages:

  - Complex model. We need to use two Mach–Zehnder modules for two ITMs.
  - Complex interferometer configuration with s/p coupled cavities.
  - A lot of mirror maps to apply. Take long time to calculate.



Set parameters as follows:

$$r_1 = r_2 = \sin\phi$$
$$t_1 = t_2 = \cos\phi$$
$$\varphi_1 = -\alpha_-$$
$$\varphi_2 = \pi + \alpha_-$$

TWE map measurements [9]

For either modelling strategy, we need to derive maps of ϕ and α_-.

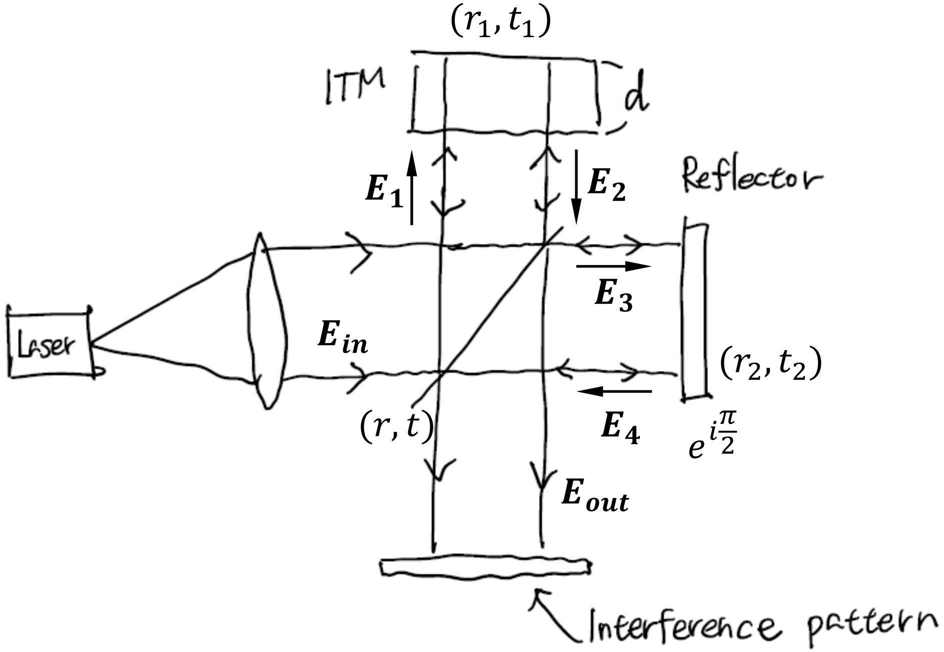
$$\boldsymbol{E_{in}} = \begin{bmatrix} S_{in} \\ 0 \end{bmatrix} \quad \boldsymbol{E_1} = r\boldsymbol{E_{in}} \quad \boldsymbol{E_2} = \boldsymbol{G}(\boldsymbol{2d}) \times r_1 \boldsymbol{E_1} \quad \boldsymbol{E_3} = it\boldsymbol{E_{in}} \quad \boldsymbol{E_4} = r_2 e^{i\frac{\pi}{2}} \boldsymbol{E_3} = ir_2 \boldsymbol{E_3}$$

$$\boldsymbol{E_{out}} = it\boldsymbol{E_2} + r\boldsymbol{E_4} = irt(\boldsymbol{G}(\boldsymbol{2d}) \times r_1 \boldsymbol{E_{in}} + ir_2 \boldsymbol{E_{in}})$$

Here we omit the common amplitude factor irt.

$$\begin{aligned}
\boldsymbol{E_{out}} &= \boldsymbol{G}(\boldsymbol{2d}) \times r_1 \boldsymbol{E_{in}} + ir_2 \boldsymbol{E_{in}} \\
&= r_1 \begin{bmatrix} \cos 2\alpha_- + i\cos 2\phi \sin 2\alpha_- & i \sin 2\phi \sin 2\alpha_- \\ i \sin 2\phi \sin 2\alpha_- & \cos 2\alpha_- - i \cos 2\phi \sin 2\alpha_- \end{bmatrix} \begin{bmatrix} S_{in} \\ 0 \end{bmatrix} + ir_2 \begin{bmatrix} S_{in} \\ 0 \end{bmatrix} \\
&= r_1 S_{in} \begin{bmatrix} \cos 2\alpha_- + i\cos 2\phi \sin 2\alpha_- \\ i \sin 2\phi \sin 2\alpha_- \end{bmatrix} + ir_2 S_{in} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
&= r_1 S_{in} \begin{bmatrix} \cos 2\alpha_- + i(\cos 2\phi \sin 2\alpha_- + r_2/r_1) \\ i \sin 2\phi \sin 2\alpha_- \end{bmatrix} \qquad A = r_2/r_1
\end{aligned}$$

$$P_{out} = \cos^2 2\alpha_- + (\cos 2\phi \sin 2\alpha_- + A)^2 + \sin^2 2\phi \sin^2 2\alpha_- = 1 + A^2 + 2A\cos 2\phi \sin 2\alpha_-$$



If we take 4 measurements by rotating the polarization of the input beam

$$\begin{aligned}
P_{out}(\phi + 0) &= 1 + A^2 + 2A\cos 2\phi \sin 2\alpha_- \\
P_{out}(\phi + 45) &= 1 + A^2 - 2A\sin 2\phi \sin 2\alpha_- \\
P_{out}(\phi + 90) &= 1 + A^2 - 2A\cos 2\phi \sin 2\alpha_- \\
P_{out}(\phi + 135) &= 1 + A^2 + 2A\sin 2\phi \sin 2\alpha_-
\end{aligned}$$

Here, θ in $P_{out}(\phi + \theta)$ is the rotation angle of the polarization of the input beam.

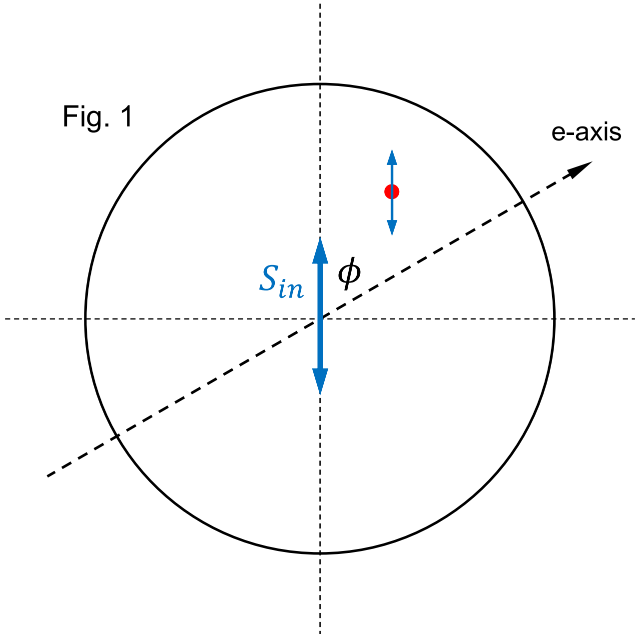
So, the birefringence map can be constructed from TWE maps!

$$\phi = -\frac{1}{2}\tan^{-1}\frac{TWE(\phi + 45) - TWE(\phi + 135)}{TWE(\phi + 0) - TWE(\phi + 90)}$$

$$\alpha_- = \frac{\alpha_e - \alpha_o}{2} = \frac{2\pi}{\lambda} \cdot \frac{TWE(\phi + 0) - TWE(\phi + 90)}{\cos 2\phi}$$

[9] Yoichi Aso, How to compute birefringence map from TWE maps, JGW-T1910380.

Fig. 1



Clockwise rotate the mirror by 45 degrees
and measure the map

Fig. 2

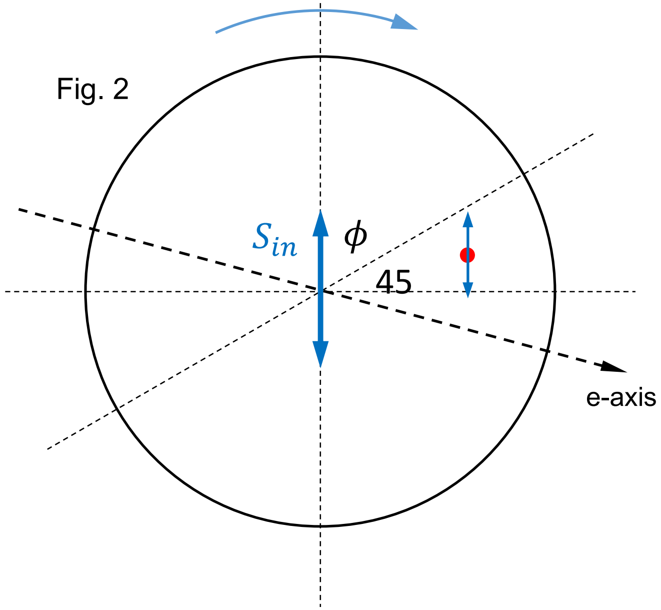
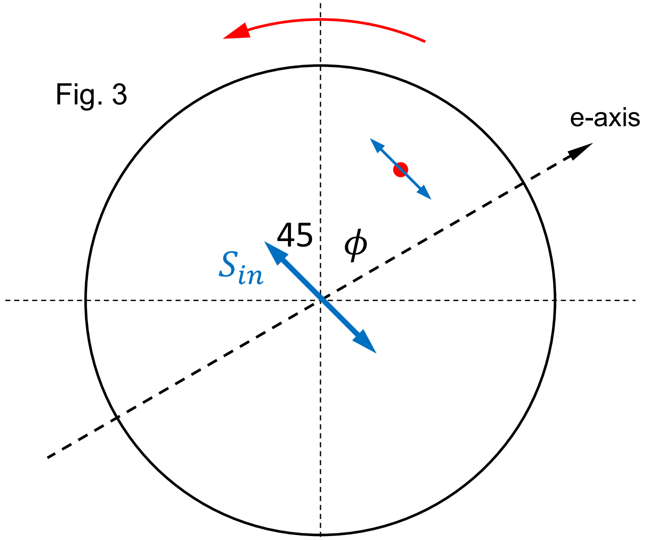
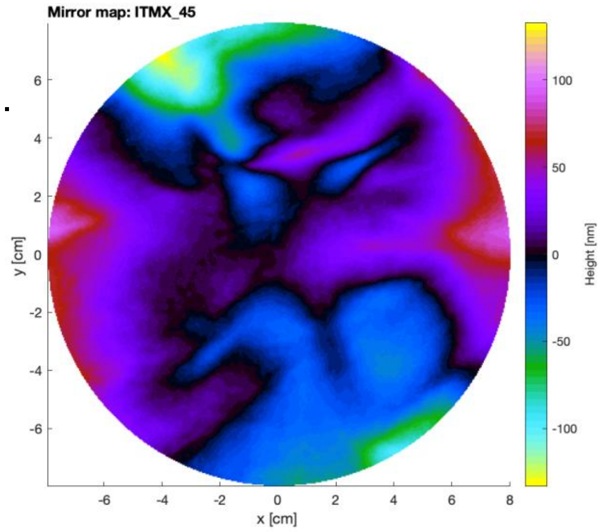


Fig. 3

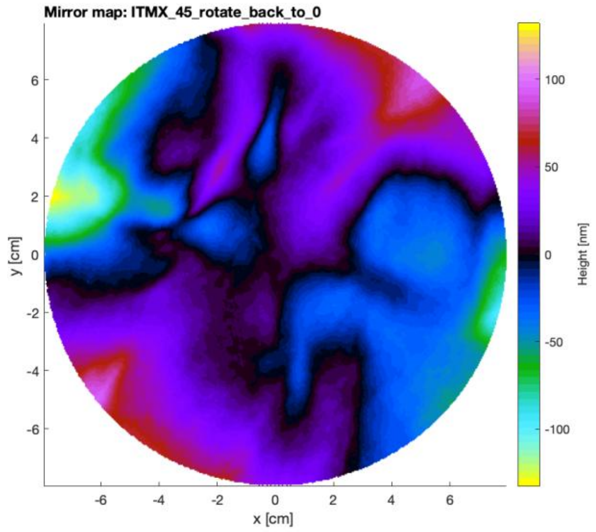


The measured map is anticlockwise
rotated back by 45 degrees.

$TWE'(45)$



$TWE'(45 \to 0) = TWE(\phi + 45)$



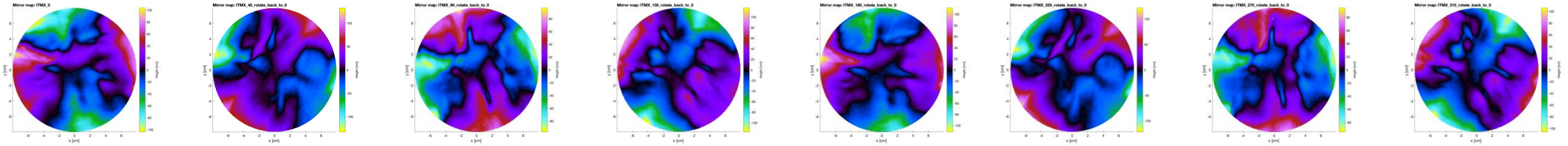
In TWE measurements, the mirror is rotated [12, 13].
So we need to manually rotate the map back.

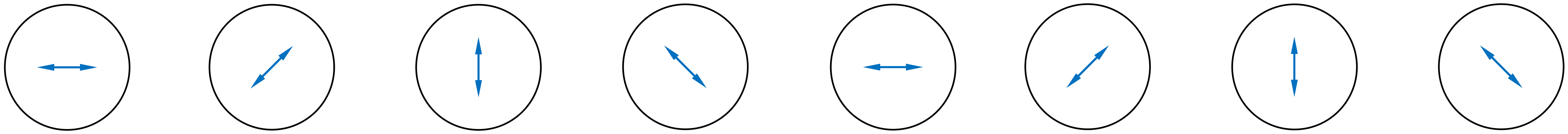
After rotating the map back, it is equivalent that we change the polarization rotation of the input beam and take the measurement, while the mirror keeps still. (Compare Fig. 1 and Fig. 3.)

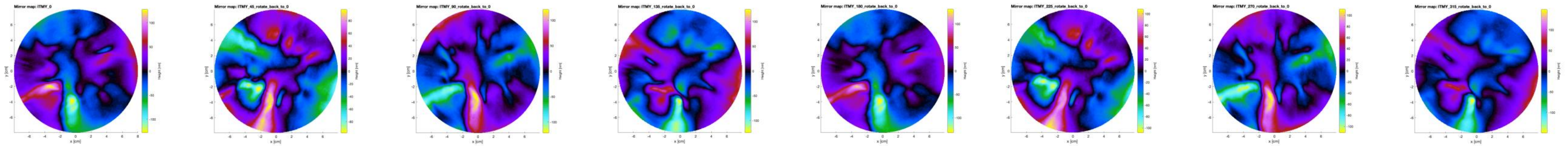
[12] Eiichi Hirose, HR through AR of ITMY, JGW-T1808715.
[13] Eiichi Hirose, HR through AR of the KAGRA sapphire test masses, JGW-T1910386.

TWE maps for ITMX [14, 15, 16]

$TWE(\phi+0)$ $TWE(\phi+45)$ $TWE(\phi+90)$ $TWE(\phi+135)$ $TWE(\phi+180)$ $TWE(\phi+225)$ $TWE(\phi+270)$ $TWE(\phi+315)$



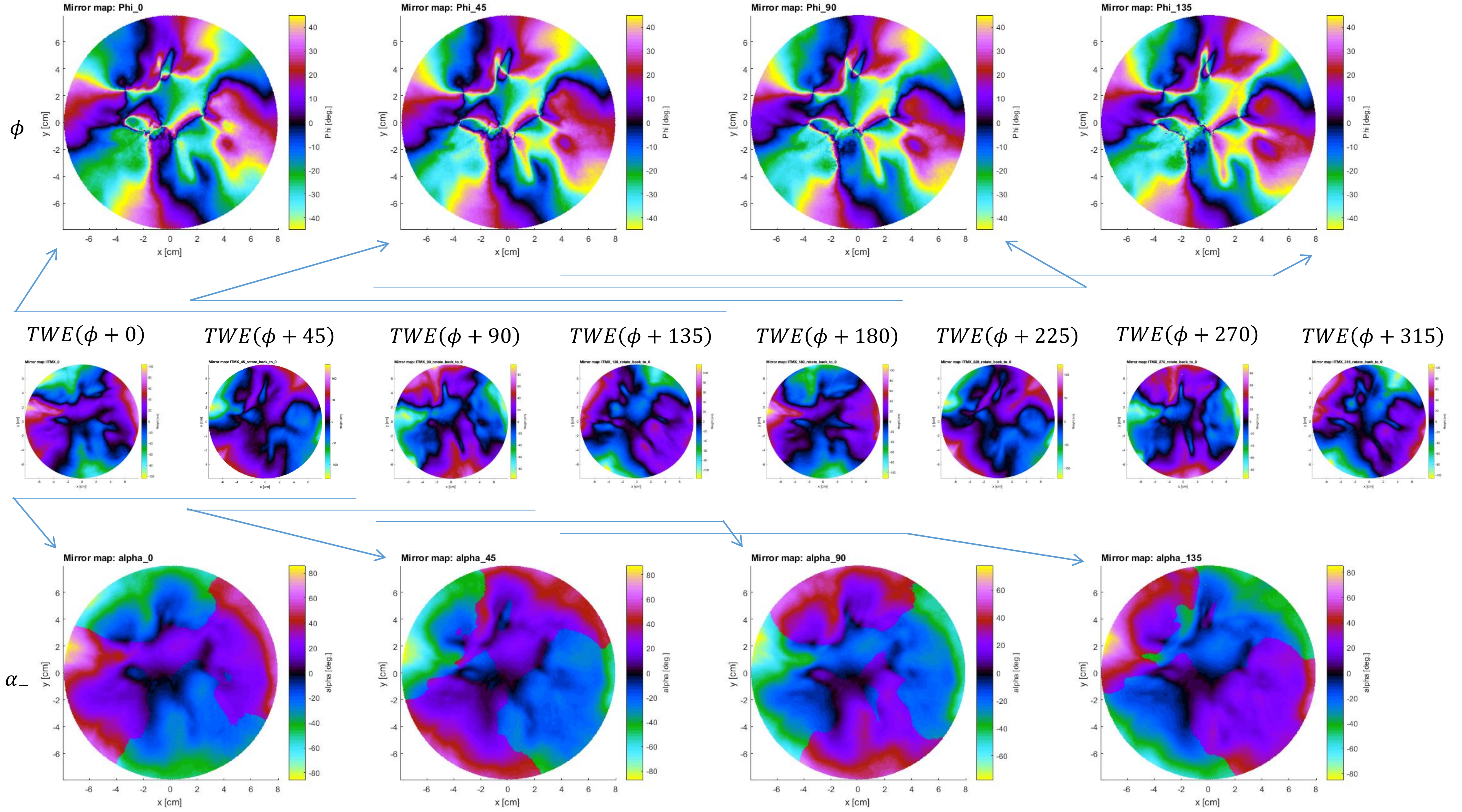




$TWE(\phi+0)$ $TWE(\phi+45)$ $TWE(\phi+90)$ $TWE(\phi+135)$ $TWE(\phi+180)$ $TWE(\phi+225)$ $TWE(\phi+270)$ $TWE(\phi+315)$

TWE maps for ITMY

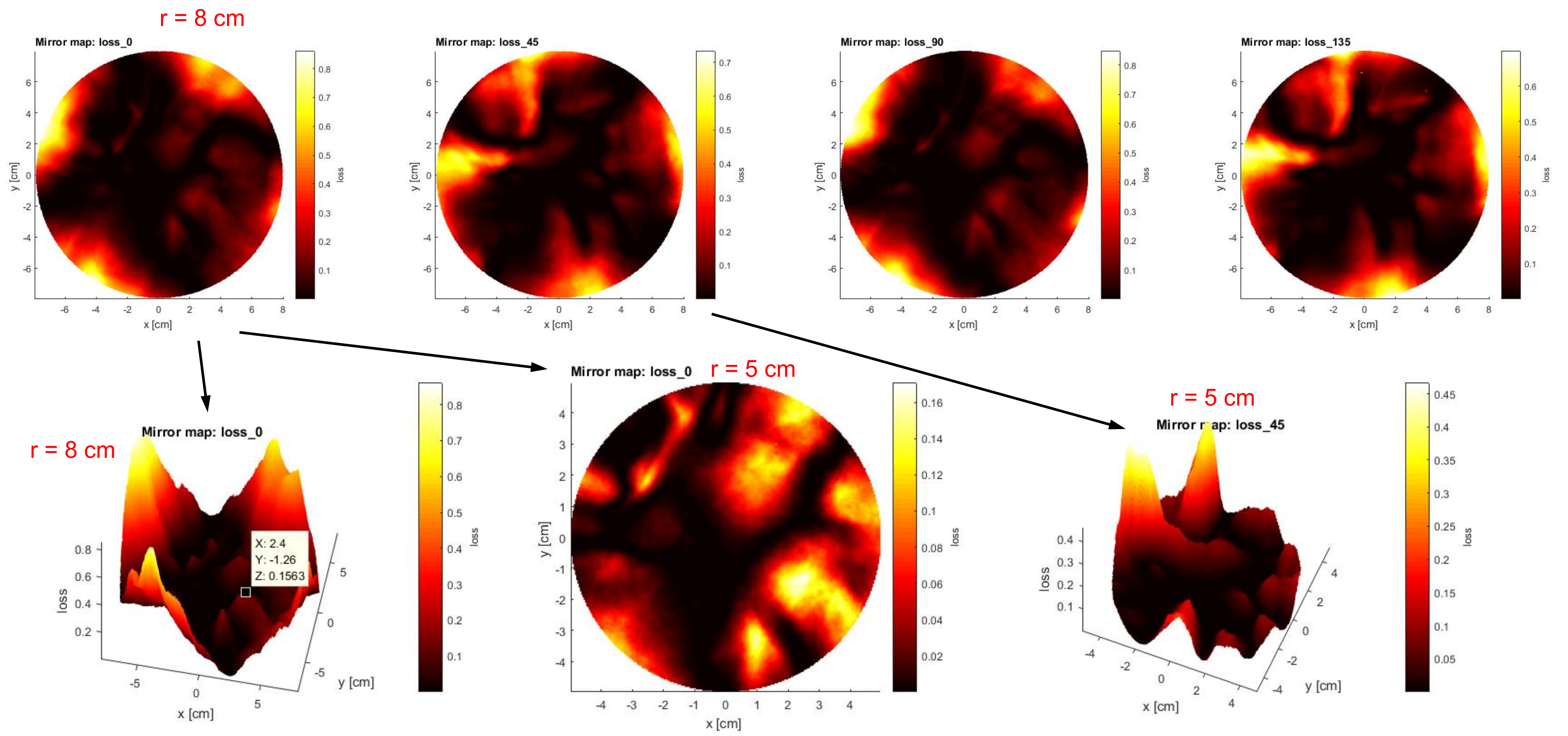
[14] Eiichi Hirose, Characterization of the coated ITMs, JGW-T1809173.
[15] Haoyu Wang, Phi map in Birefringence, JGW-T2012208.
[16] Haoyu Wang, Calculation of birefringence maps in KAGRA, JGW-T2012416.



Mirror map: Phi_0
Mirror map: Phi_45
Mirror map: Phi_90
Mirror map: Phi_135
Phi [deg.]
x [cm]
y [cm]
ϕ
$TWE(\phi + 0)$
$TWE(\phi + 45)$
$TWE(\phi + 90)$
$TWE(\phi + 135)$
$TWE(\phi + 180)$
$TWE(\phi + 225)$
$TWE(\phi + 270)$
$TWE(\phi + 315)$
Mirror map: ITMX_0
Mirror map: ITMX_45_rotate_back_to_0
Mirror map: ITMX_90_rotate_back_to_0
Mirror map: ITMX_135_rotate_back_to_0
Mirror map: ITMX_180_rotate_back_to_0
Mirror map: ITMX_225_rotate_back_to_0
Mirror map: ITMX_270_rotate_back_to_0
Mirror map: ITMX_315_rotate_back_to_0
Mirror map: alpha_0
Mirror map: alpha_45
Mirror map: alpha_90
Mirror map: alpha_135
alpha [deg.]
α_-

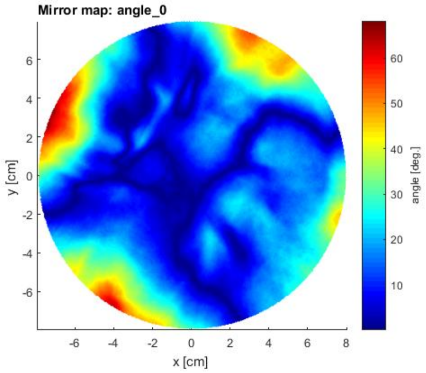
The loss map for ITMX is calculated by: $loss = |r_{s\to p}|^2 = \sin^2 2\phi \sin^2 \alpha_-$

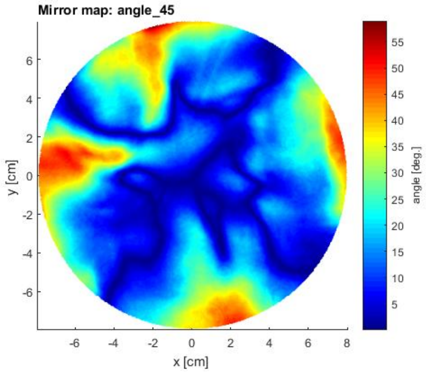
$$\boldsymbol{G} = \begin{bmatrix} r_{s\to s} & t_{p\to s} \\ t_{s\to p} & r_{p\to p} \end{bmatrix} = \begin{bmatrix} \cos\alpha_- + i\cos 2\phi \sin\alpha_- & i \sin 2\phi \sin\alpha_- \\ i \sin 2\phi \sin\alpha_- & \cos\alpha_- - i\cos 2\phi\sin\alpha_- \end{bmatrix}$$



The single path polarization rotation angle for ITMX is calculated by: $angle = \tan^{-1}\frac{\sqrt{P_P}}{\sqrt{P_S}}$

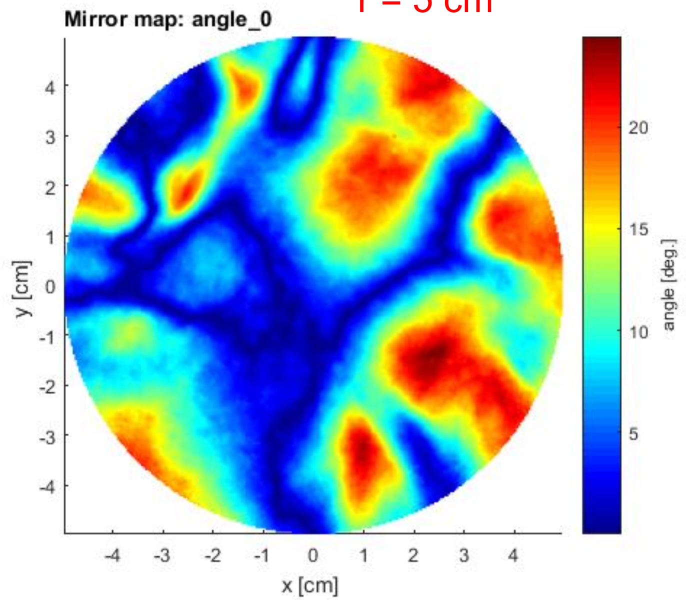
r = 8 cm





room in

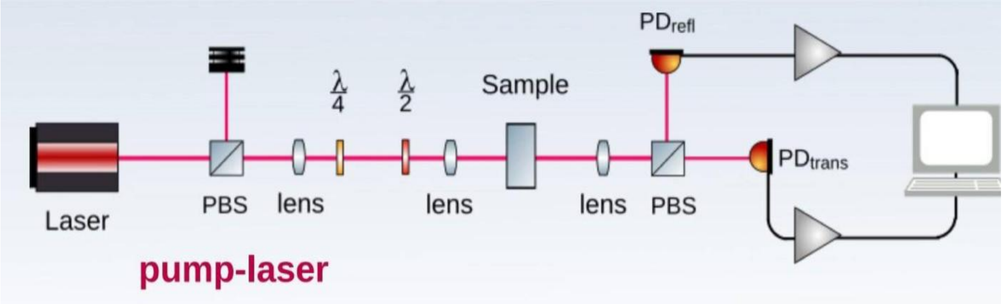
r = 5 cm

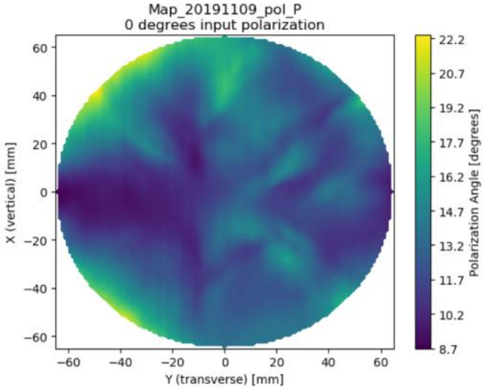


ITMX

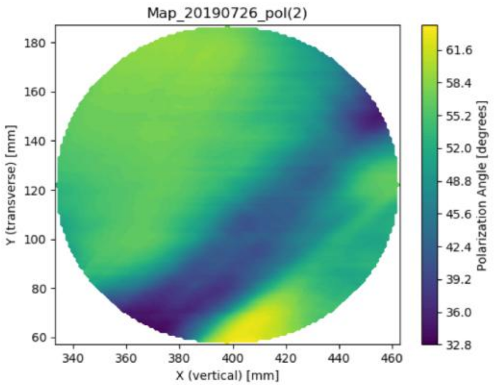
Verifying the model

Measurements at NAOJ [17, 18]





Discarded ITM



Spare ETM

Also, phase camera experiment is proposed and will be set up in Cardiff, by Keiko Kokeyama, *et al*.

[17] Matteo Leonardi, Introduction to Mirror (MIR) subsystem, JGW-G2112906.
[18] Homare Abe, et al, Birefringence measurement of a sapphire mirror for KAGRA, JGW-G2112887.

Birefringence model in FINESSE

Currently, we are building the IFO models in FINESSE, tensting them to gain understanding of the two-world s/p-coupled model.

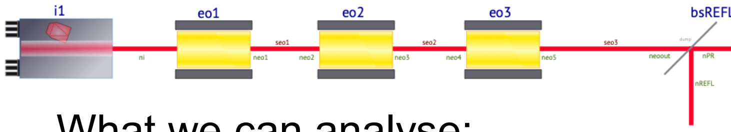
Models include

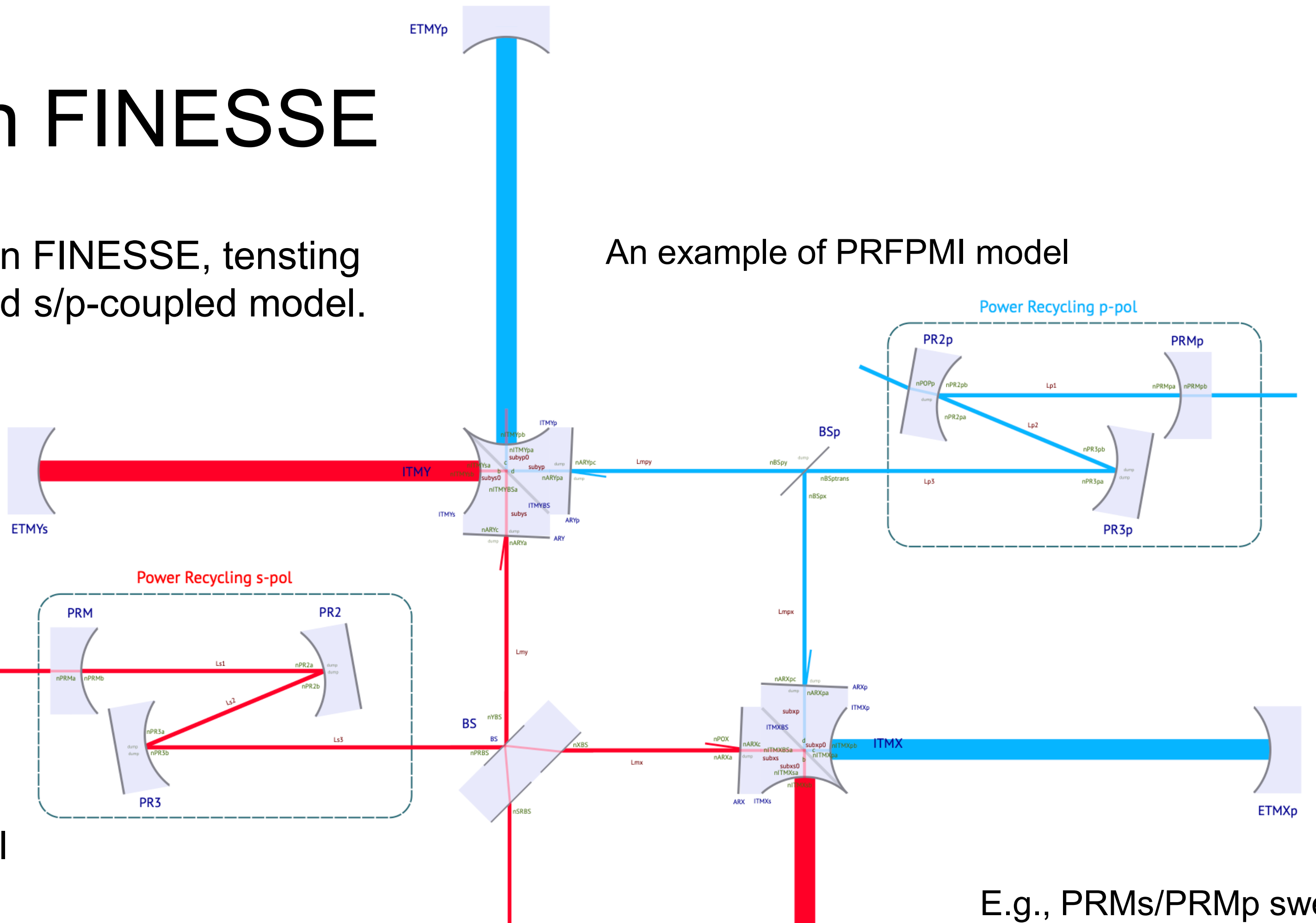
- single ITM, single arm, MI
- PRMI, PRFPMI
- DRMI, DRFPMI

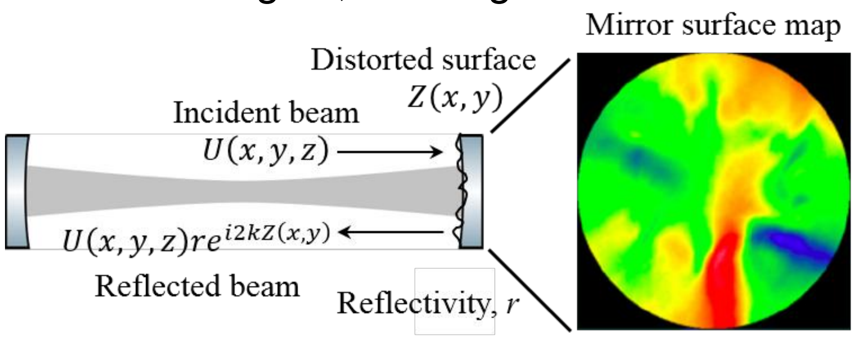
What we can analyse:

- beam shape
- cavity gains and power loss in s-pol
- scattering of higher order mode
- LSC signal, ASC signal

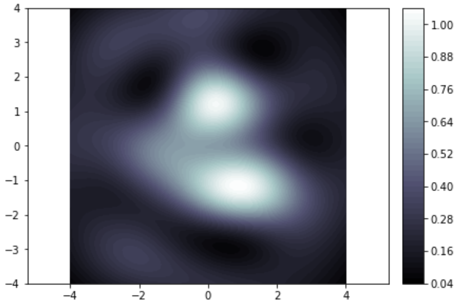
An example of PRFPMI model



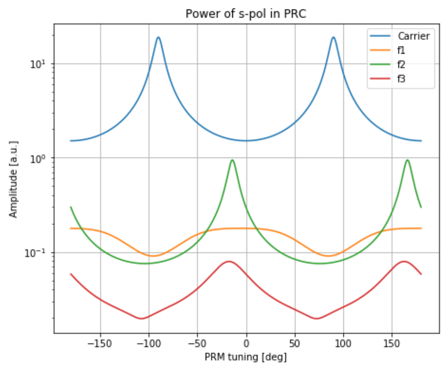




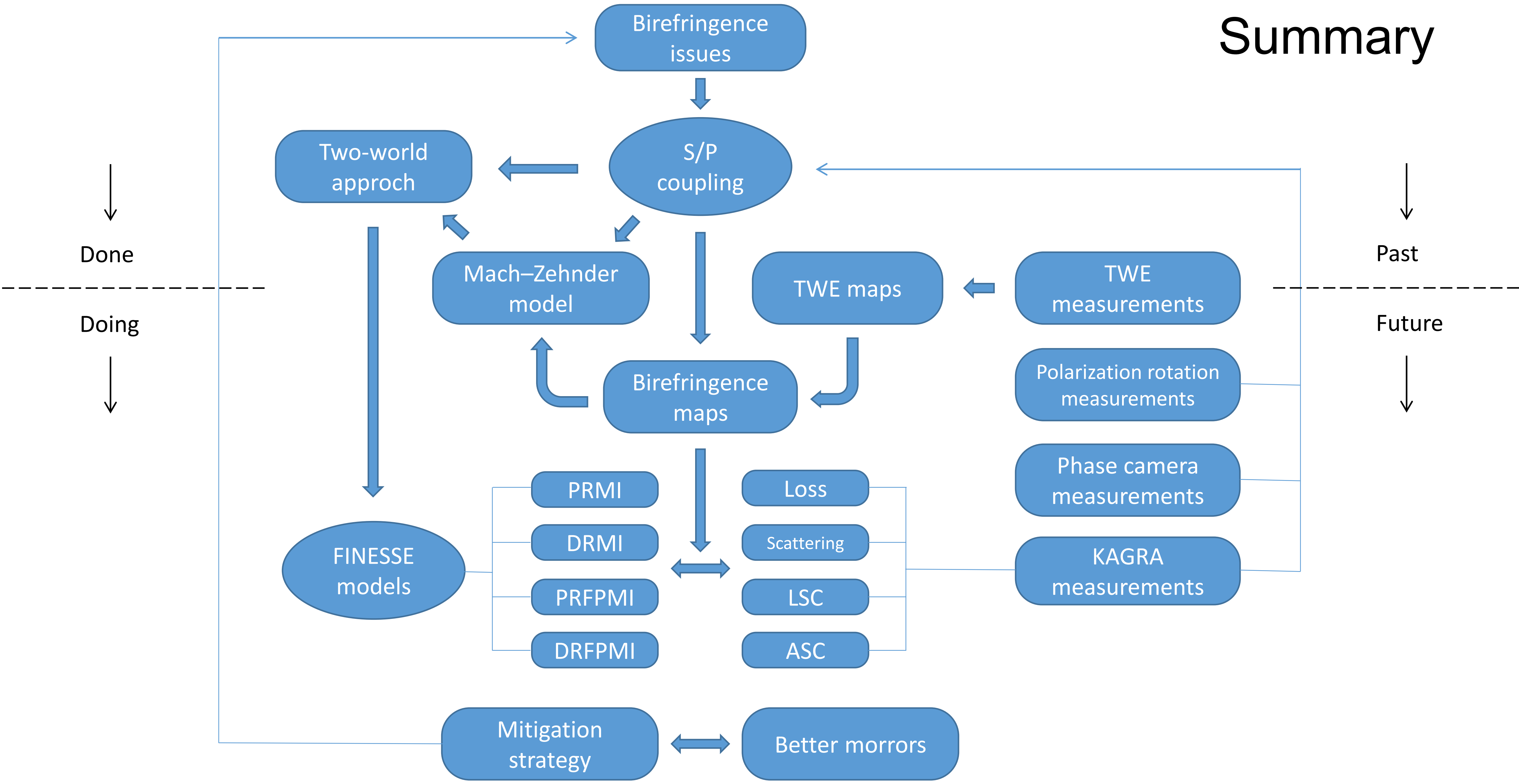
E.g., single reflection beam shape



E.g., PRMs/PRMp sweep



Summary


Birefringence issues
S/P coupling
Two-world approch
Mach–Zehnder model
TWE maps
TWE measurements
Birefringence maps
Polarization rotation measurements
Phase camera measurements
KAGRA measurements
FINESSE models
PRMI
DRMI
PRFPMI
DRFPMI
Loss
Scattering
LSC
ASC
Mitigation strategy
Better morrors
Done
Doing
Past
Future

List of References

[1] Yutaro Enomoto, Polarization issue in the central IFO, JGW-G1910388.

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