Optimal Control and Optimization for KAGRA Vibration Isolation system

Tsang Terrence Tak Lun

Overview

Part I: Introduction, Definitions, and Problem Formulation

Part II: Optimal Control Using H_2 and H_∞ Synthesis

Today
2020/08/19

Part III: Sensor Correction Filter Optimization, and Inter-calibration Minimization.

Part IV: Optimal Complementary Filters

Next meeting

Part V: Model Reference Diagonalization

Next Next meeting

Part VI: File Management System/Data Pipeline for Suspension Models, both simulated and regressed.

Progress

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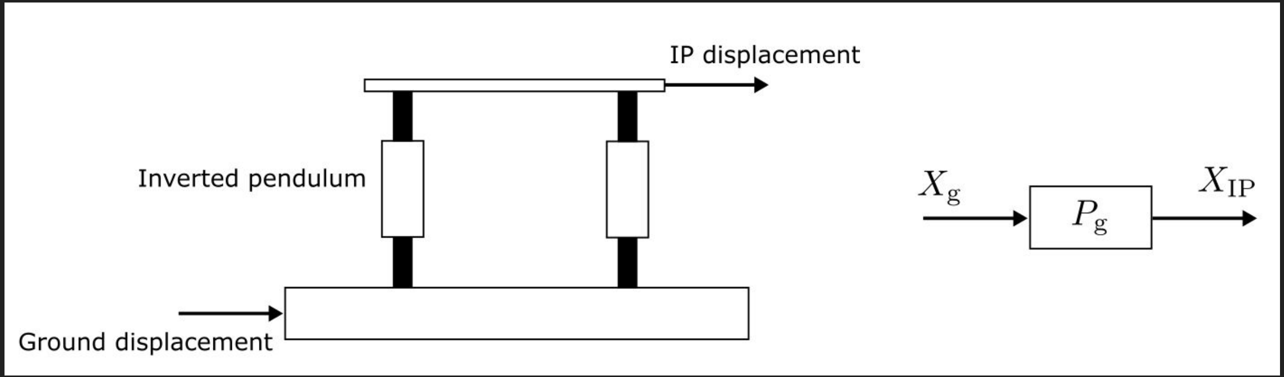
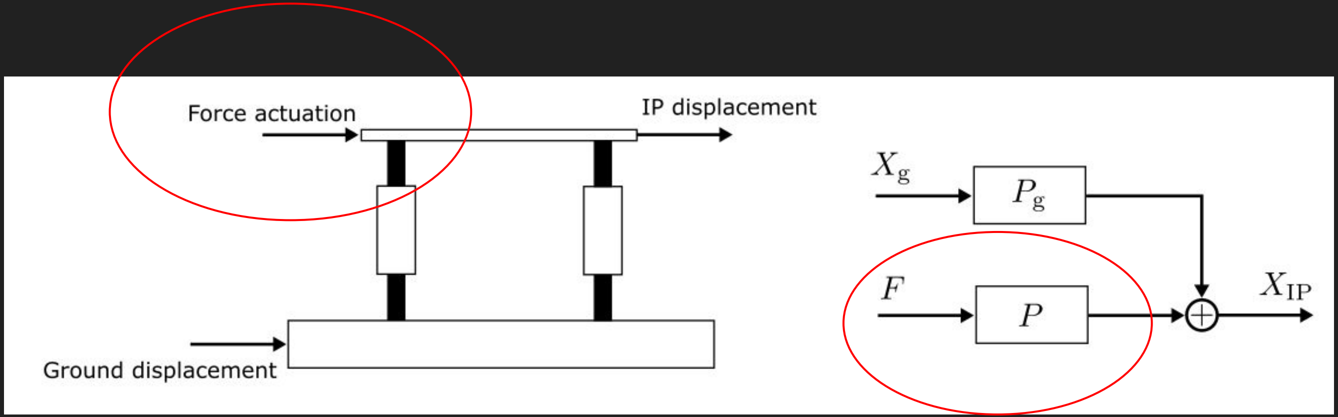
Pre-isolator



Pre-isolator



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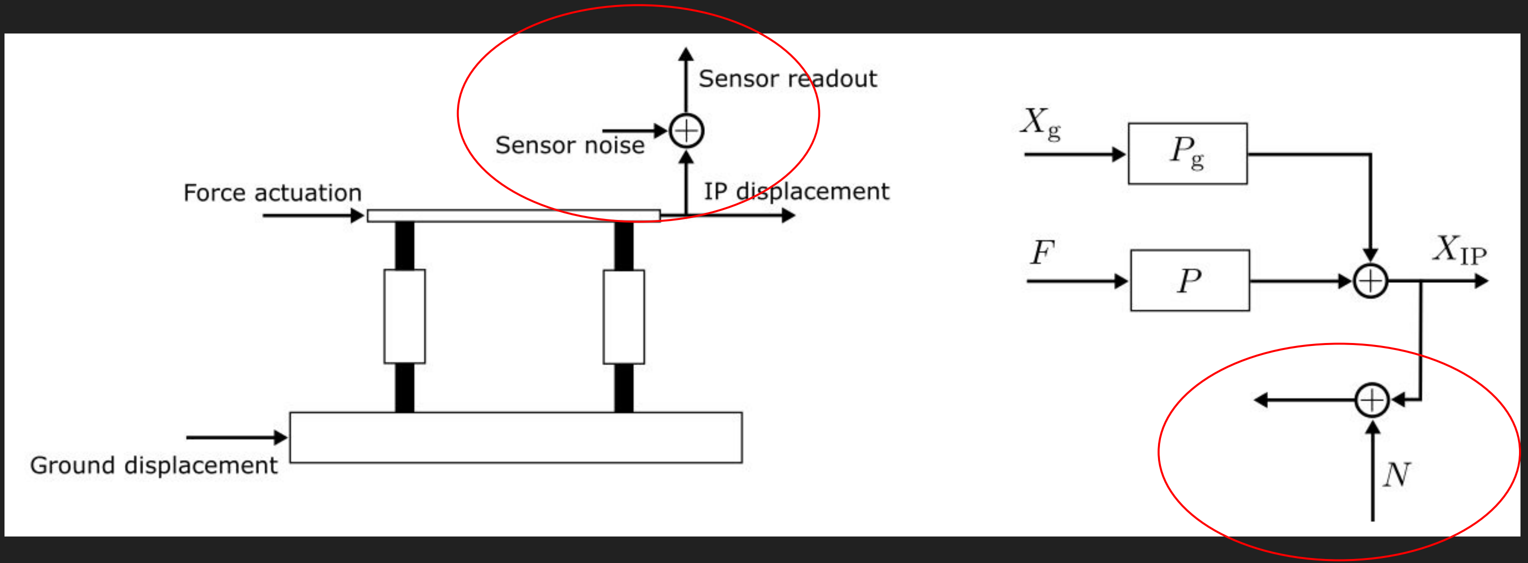

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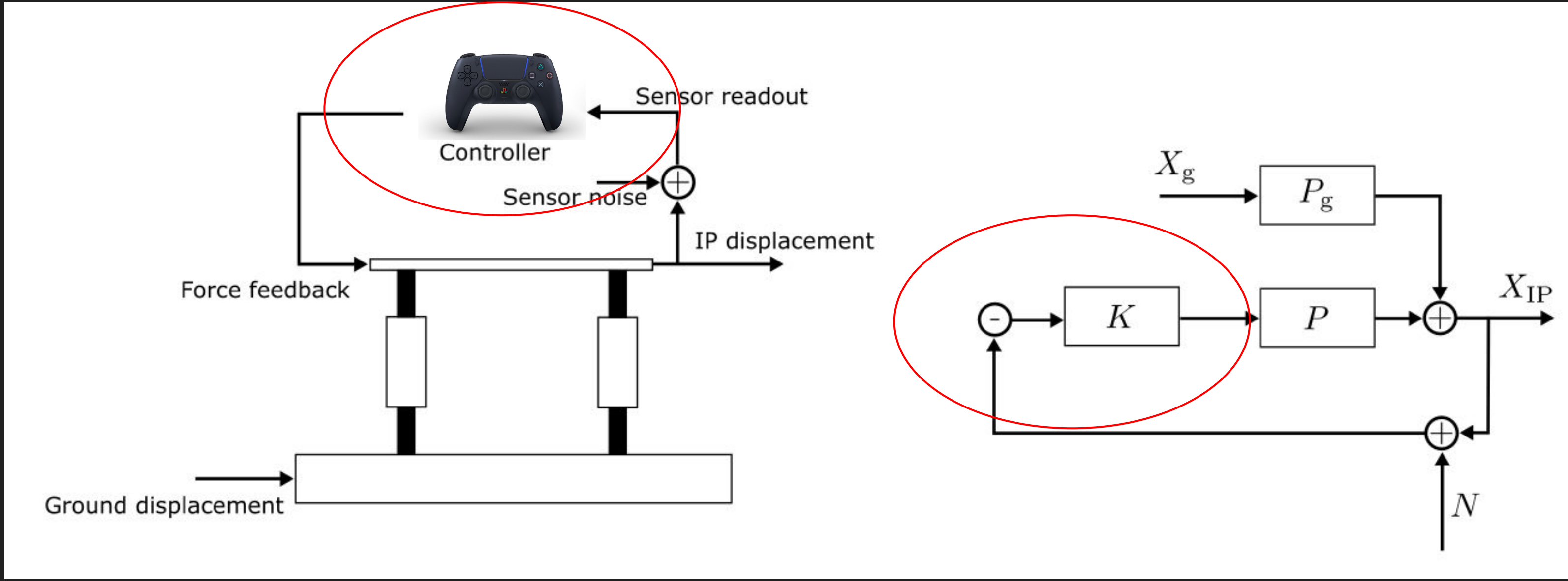
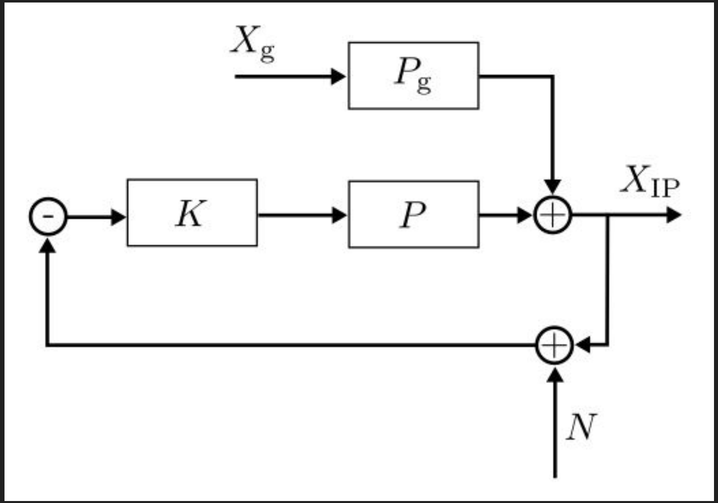
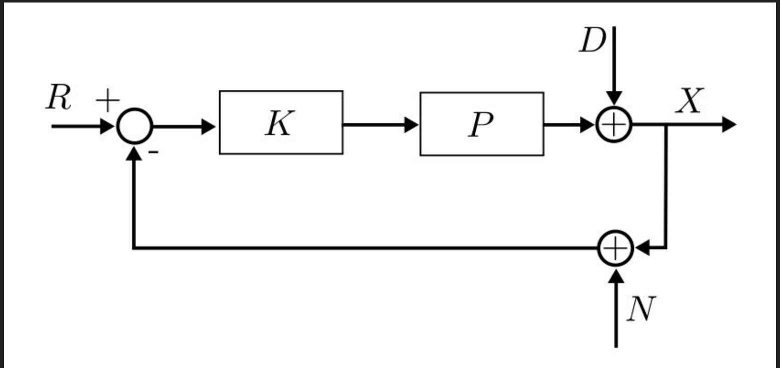


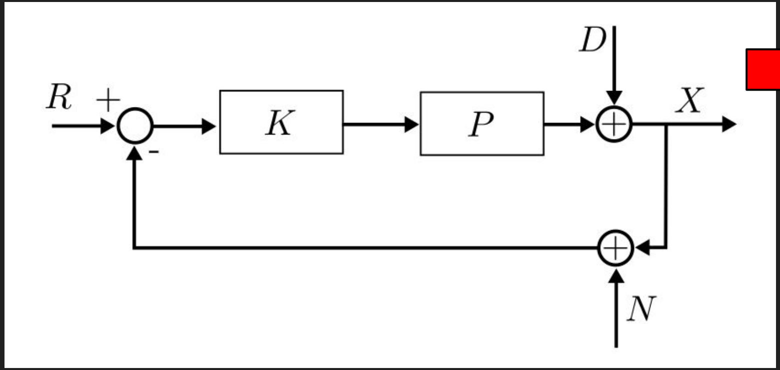
Generalization





General system for any DoF.

Displacement



$$X = \frac{KP}{1+KP}R + \frac{1}{1+KP}D - \frac{KP}{1+KP}N$$

PS/ PSD:

$$\langle X^2 \rangle = \left| \frac{1}{1+KP} \right|^2 \langle D^2 \rangle + \left| \frac{KP}{1+KP} \right|^2 \langle N^2 \rangle$$

Problem statement

Disturbance, external, cannot be reduced, but can be induced.

Displacement that we want to minimize (Goal)

$$\langle X^2 \rangle = \left| \frac{1}{1+KP} \right|^2 \langle D^2 \rangle + \left| \frac{KP}{1+KP} \right|^2 \langle N^2 \rangle$$

Noise, can be reduced, but exist limitations.

Plant, mechanical, fixed

Control filter, can be whatever we want (almost)
→ Into optimal control

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Fundamental Limitation in Control System

Minimizable, $K \to \infty$

Minimizable, $K \to 0$

$$\langle X^2 \rangle = \left| \frac{1}{1+KP} \right|^2 \langle D^2 \rangle + \left| \frac{KP}{1+KP} \right|^2 \langle N^2 \rangle$$

Not simultaneously minimizable
Coupling terms are complementary

$$\frac{1}{1+KP} + \frac{KP}{1+KP} = 1$$

Optimization

$$\langle X^2 \rangle = \left| \frac{1}{1+KP} \right|^2 \langle D^2 \rangle + \left| \frac{KP}{1+KP} \right|^2 \langle N^2 \rangle$$

Simple observation: It is a positive definite function.

→ It must be minimizable by some optimal controller *K*, given some disturbance *D* and noise *N*.

→→ Optimization

Optimization Interlude

I mean mathematical optimization.

→ Minimization of a cost function by choosing the critical parameters within an allowed set.

For example,

Cost function: $J(x) = x^2$

Allowed set: $x \in \mathbb{R}$

Minimization: $\min J(x) = 0, \quad x^* = 0$

In a compact way

$$x^* = \underset{x \in \mathbb{R}}{\arg\min}\,(J(x)) = 0$$

Optimal Control

→ Find optimal controller that minimizes a cost function.
Some cost functions to be minimized:
E.g.,

The integrated RMS/expected RMS: $$J_2 = \int_0^\infty \langle X^2 \rangle \, df$$ → 2-norm of the system

The maximum of the displacement spectrum: $$J_\infty = \max \langle X^2 \rangle$$ → ∞-norm of the system

We can choose controllers however we like.
But, the system has to be <u>stable</u>.
I.e. The controllers/system must be within some mathematical set.

H_2/H_∞ Optimal Controller

Letter "H" comes from the mathematical space the optimization takes place, namely, Hardy space.

Hardy space contains all possible stable systems.

In a nutshell, H_2/H_∞ optimal controllers are:

H_2 optimal controller: $$K_{\mathcal{H}_2} = \arg\min_{K \in S} \int_0^\infty \langle X^2 \rangle \, df$$

H_∞ optimal controller: $$K_{\mathcal{H}_\infty} = \arg\min_{K \in S} \left(\max \langle X^2 \rangle\right)$$

$$S : \{\text{All possible controllers such that the system is stable}\}$$

Why H∞?

Some weighting filter/function according to requirements

$$J_\infty = \max \left\langle X^2 \right\rangle \quad \Rightarrow \quad J_\infty = \max \left(\left\langle X^2 \right\rangle \left| W_X \right|^2 \right)$$

Only minimizes the dominating peaks, e.g. resonances.

→ Trade-off between seismic noise suppression and control noise attenuation.

→→ Maximizing hardware potential to suppress seismic noise while meeting noise requirement.

Some Benefits of Optimization-based Approaches

Form of cost function is not limited, we can even add actuation signal as part of the cost function so it doesn't saturate.

E.g.

$$J_\infty = \max\left(\langle X^2\rangle \left|W_X\right|^2 + \langle F^2\rangle \left|W_F\right|^2\right)$$

Displacement spectrum

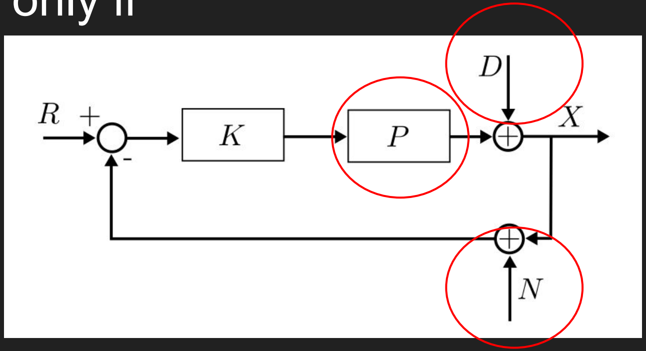
Actuation signal spectrum

→ Trade-off between suppression and actuation signal

Things to Do

Generating H_2 and H_∞ optimal filters are extremely easy, only if

1. We have precisely modelled the plant,
2. We have precisely modelled the disturbance, and
3. We have precisely modelled the noise.



But, those were never done in a systematic manner.

Of course, I can do those stages by stage, suspension by suspension.

But, wouldn't it be nice if we can have a data analysis pipeline that automates the workflow? → Part VI of my presentation.

Fundamental Limitations

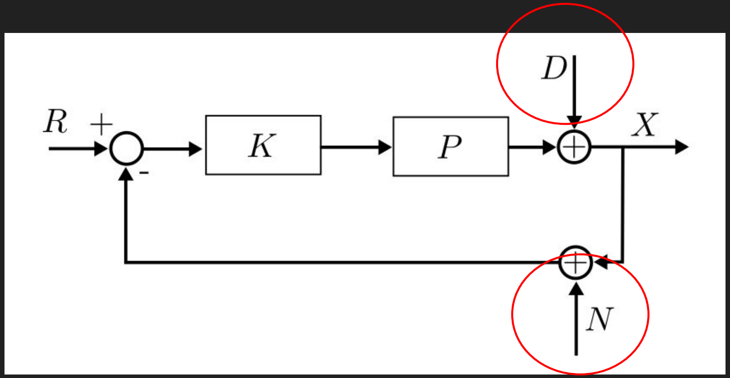
Having optimal controllers is only minimizing the disturbance and noise coupling to the displacement.

True limitations are the disturbances and noises.

Therefore, before doing optimal control, it is necessary to reduce the disturbance and noise level as much as possible.

Remains to be another discussion

→ Part III, IV, V of my presentation.



2020/08/28

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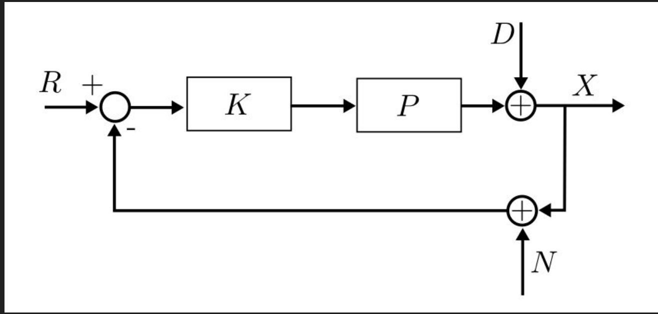
Done
2020/08/19

Today
2020/08/28

Next meeting
Or Next Next meeting

Recap

General model for a DoF



Displacement PSD,
The quantity that we want to minimize

$$\langle X^2 \rangle = \left| \frac{1}{1+KP} \right|^2 \langle D^2 \rangle + \left| \frac{KP}{1+KP} \right|^2 \langle N^2 \rangle$$

Cost function and optimization

$$J(x) = x^2$$

$$x^* = \arg\min_{x \in \mathbb{R}} \left(J(x) \right) = 0$$

H2 and H∞ optimal controller

$$K_{\mathcal{H}_2} = \arg\min_{K \in S} \int_0^\infty \langle X^2 \rangle \, df$$

$$K_{\mathcal{H}_\infty} = \arg\min_{K \in S} \left(\max \langle X^2 \rangle \right)$$

$$S : \{\text{All possible controllers such that the system is stable}\}$$

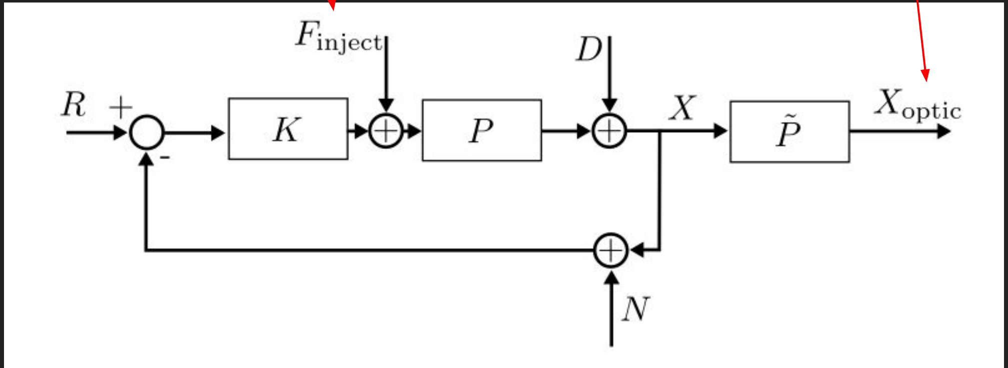
Testing and Evaluating Optimal Controllers

1. Wait when actual disturbance is small.
2. Pick a disturbance model, e.g. 90th percentile seismic noise.
3. Synthesize controller accordingly.
4. Inject the modeled disturbance to the system, which mimics the actual disturbance.
5. Measure $\frac{X_{\text{optic}}}{F_{\text{inject}}}$

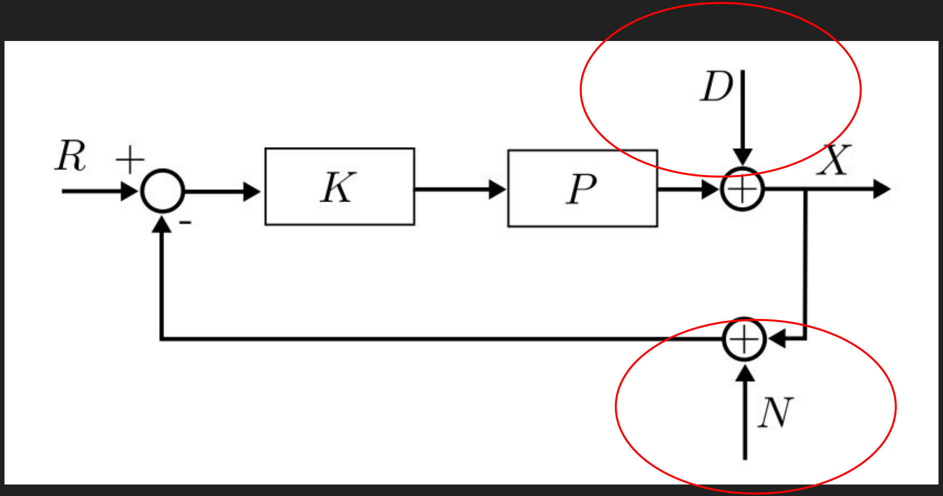
With/without control and with previous controller.

Simulated equivalent disturbance
E.g. simulated seismic noise

Oplev as an out-of-loop sensor.
(May need sensor correction)



Disturbance and Noise Limitation



$$\langle X^2 \rangle = \left| \frac{1}{1+KP} \right|^2 \langle D^2 \rangle + \left| \frac{KP}{1+KP} \right|^2 \langle N^2 \rangle$$

Limitations

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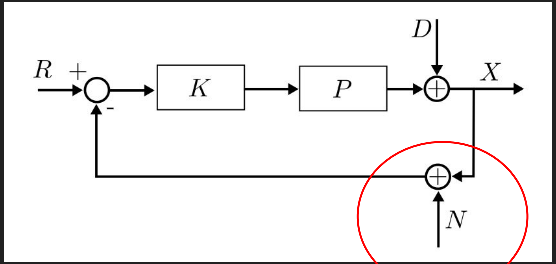
Problem with LVDT

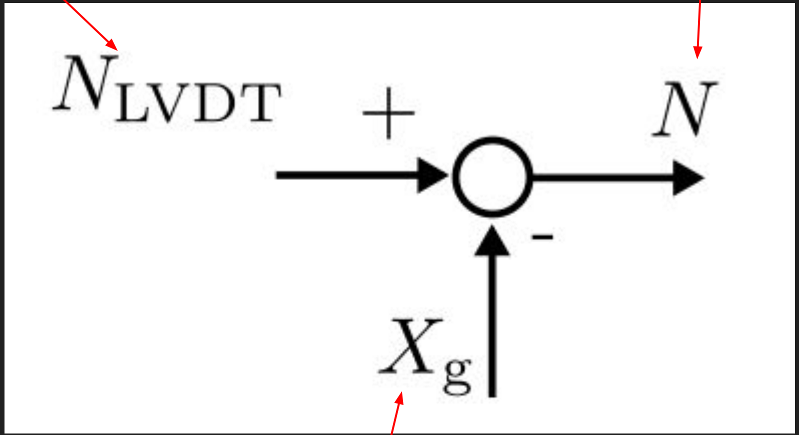
Senses relative displacement

→ Coupled with ground motion.

→→ Cannot actively suppress seismic noise without injecting it back to the system.

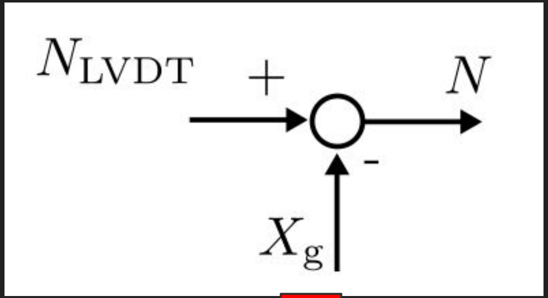
LVDT noise





Seismic noise

Sensor Correction

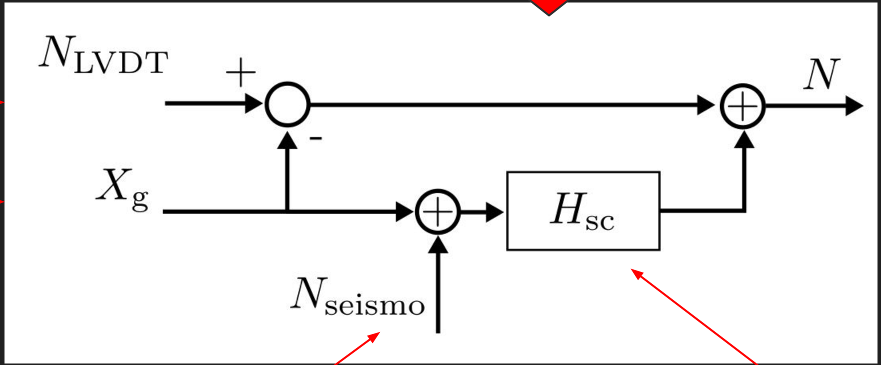


$$N = N_{\mathrm{LVDT}} - X_{\mathrm{g}}$$

Sensor correction

LVDT path

Seismometer path



Seismic noise removed

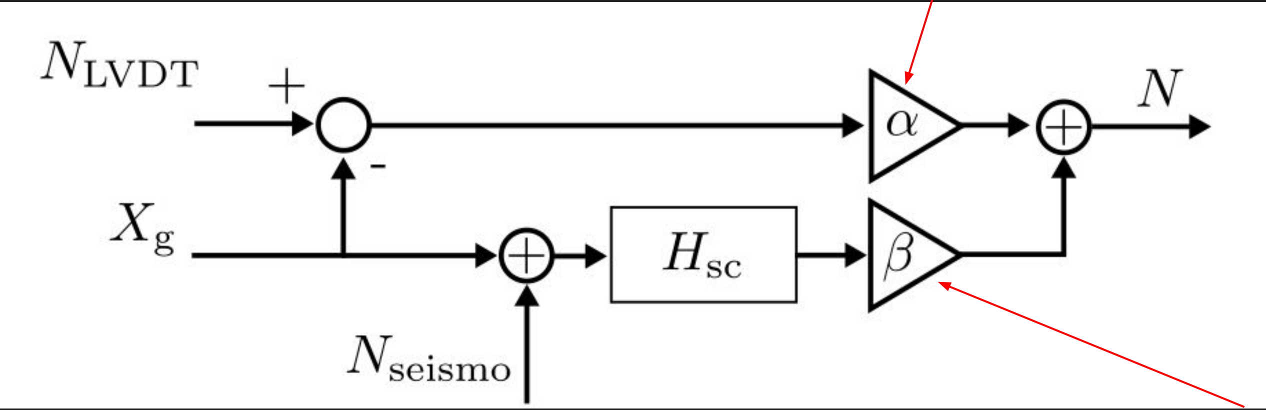
$$N = N_{\mathrm{LVDT}} + N_{\mathrm{seismo}} + (X_{\mathrm{g}} - X_{\mathrm{g}})$$

Seismometer noise

Sensor correction filter, =1 for now

Inter-calibration Mismatch

Unknown calibration mismatch,
fixed by LVDT calibration factor



Sensor correction gain.

$$N = N_{\mathrm{LVDT}} + N_{\mathrm{seismo}} + (\beta X_{\mathrm{g}} - \alpha X_{\mathrm{g}})$$

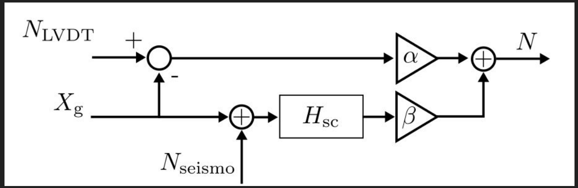
Measuring Alpha (A suboptimal way)

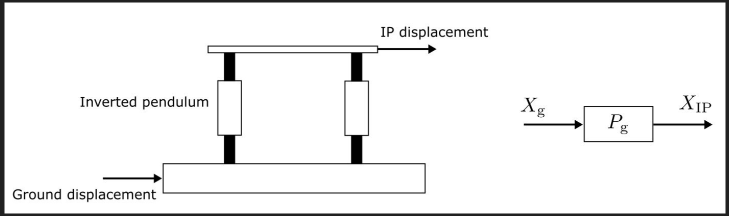
LVDT readout: $X_{\mathrm{IP}} + N_{\mathrm{LVDT}} - \alpha X_{\mathrm{g}}$

$$X_{\mathrm{IP}} = P_{\mathrm{g}} X_{\mathrm{g}}$$

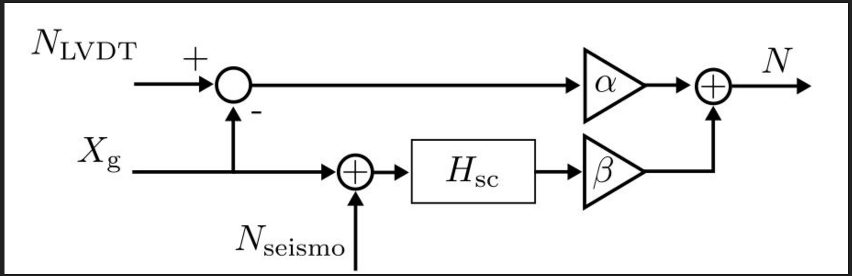
$$\beta \approx \alpha \pm |P_{\mathrm{g}}|$$

Unwanted bias





Find Sensor Correction Gain using Optimization

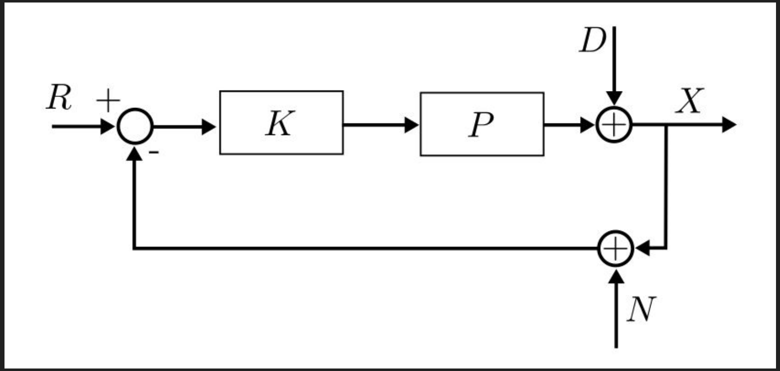


If we can measure *N*

$$J(\beta) = \langle N^2 \rangle = (\beta - \alpha)^2 \langle X_{\mathrm{g}}^2 \rangle + \dots$$

$$\beta^* = \underset{\beta \in \mathbb{R}}{\arg\min}\, J(\beta) = \alpha$$

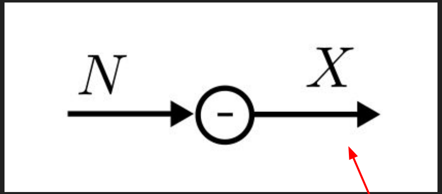
Measuring Sensor Noise



$$X = \frac{1}{1+KP} D - \frac{KP}{1+KP} N$$

High gain feedback

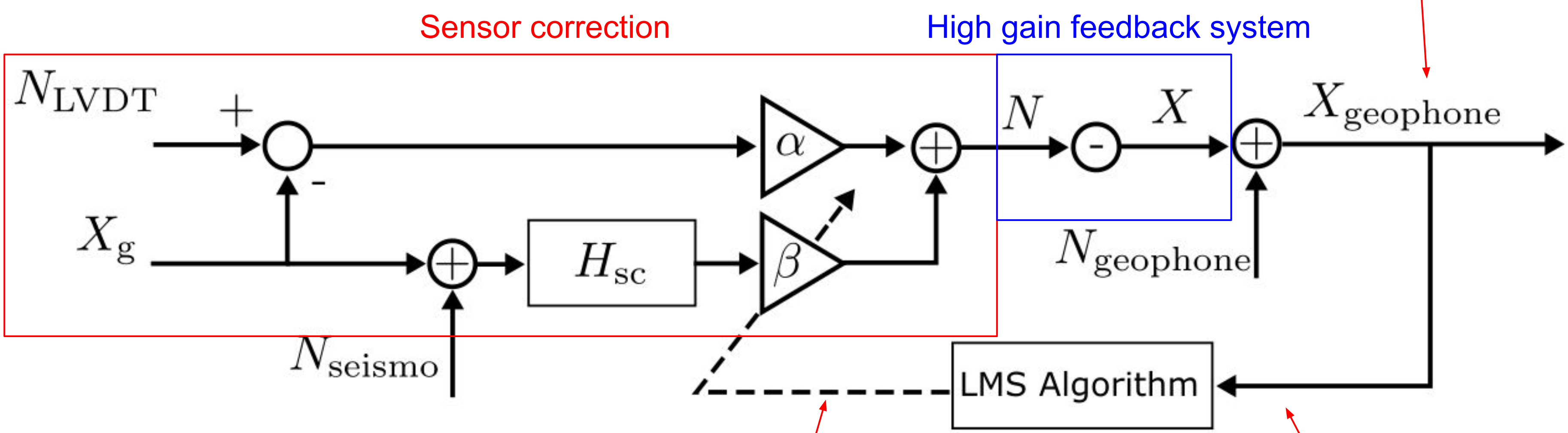
$$\lim_{K \to \infty} X = -N$$



Can be measured by geophones!

Inter-calibration minimization

Geophone readout



Update sensor correction gain

error

$$\text{mean-square error} = (\beta - \alpha)^2 \langle X_{\mathrm{g}}^2 \rangle + \ldots$$

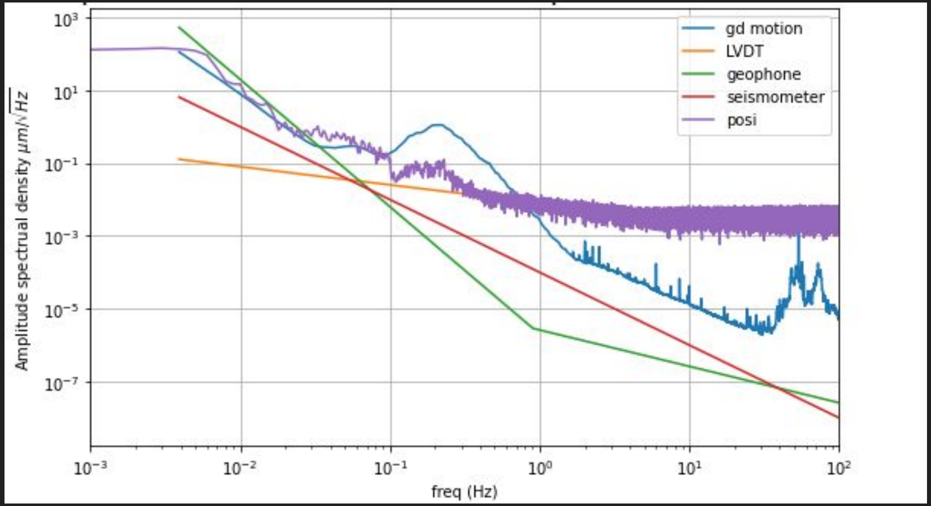
Simulation Condition

Controller $K = 1/P$ * (lowpass) * 1000

In time domain, simulate inverted pendulum displacement with typical noises.

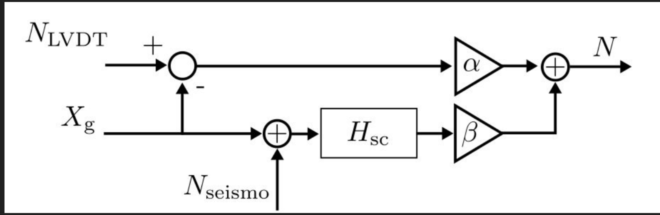
At each time step, we update the sensor correction gain using LMS algorithm

Typical noises

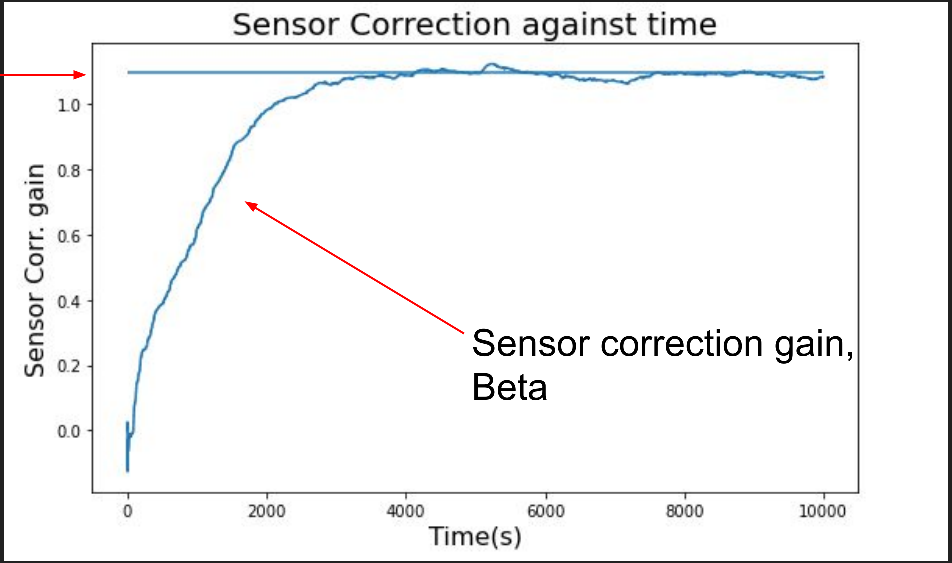


Credits: Lam Yee Ching (Jason)
Undergraduate at my university (CUHK)

Simulation Result



LVDT calibration,
Alpha



Credits: Lam Yee Ching (Jason)
Undergraduate at my university (CUHK)

Discussion and limitations

- I don’t know if this works with the real suspension
  - Is actuation going to saturate? What would happen?
  - Only works when seismic noise dominates other noises.
  - Many things to tweak, e.g update rate of LMS algorithm.
  - Original sensor correction filter with 3x peak noise amplification didn’t work.
  - Need to shape very good highpass for geophone.

2020/09/04

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Done
2020/08/19

Done
2020/08/28

Today
2020/09/04

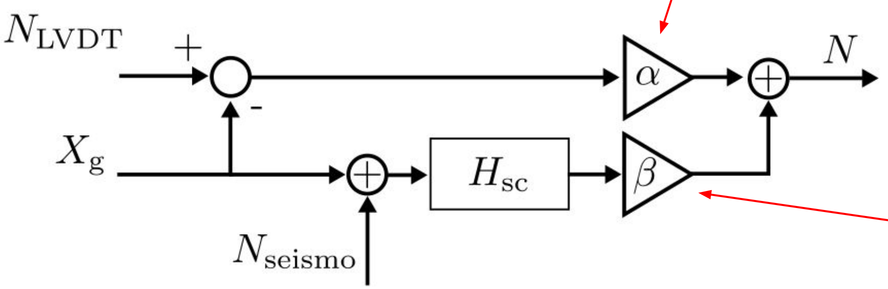
Cancel, trivial. Ask me if interested

Recap

Inter-calibration mismatch

LVDT

Seismometer



Sensor correction gain

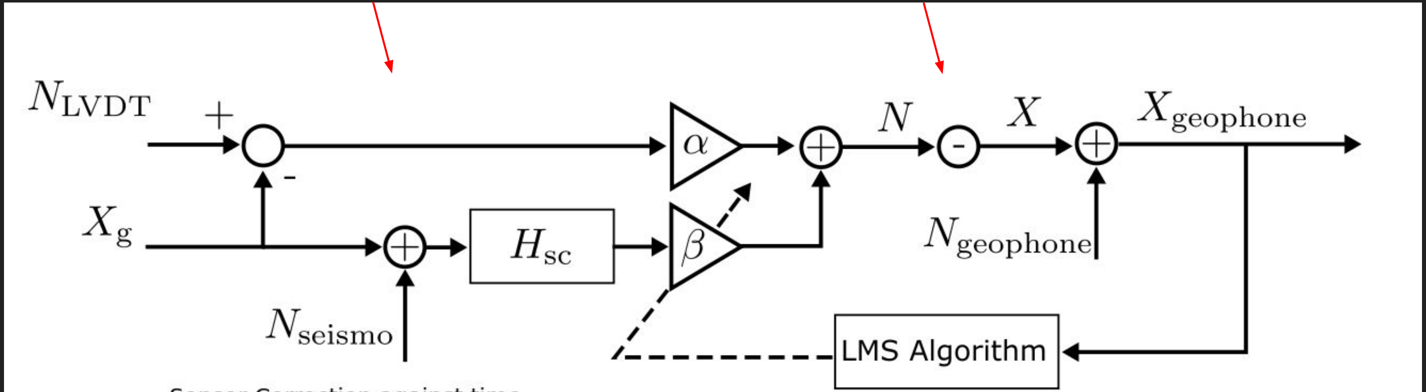
$$J(\beta) = \langle N^2 \rangle = (\beta - \alpha)^2 \langle X_{\mathrm{g}}^2 \rangle + \ldots$$

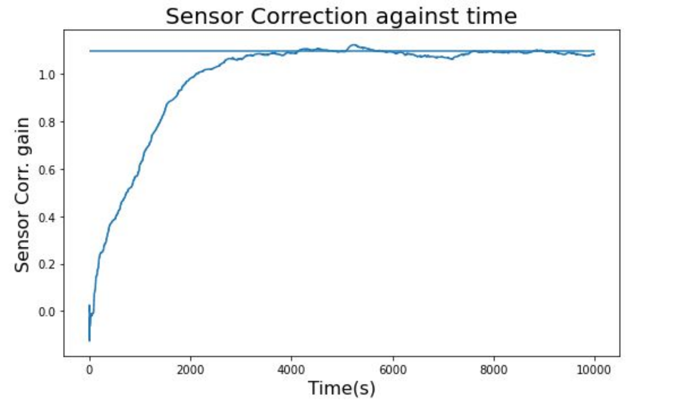
$$\beta^* = \underset{\beta \in \mathbb{R}}{\arg\min}\, J(\beta) = \alpha$$

Recap 2

Sensor correction scheme

High-gain feedback using LVDT+correction





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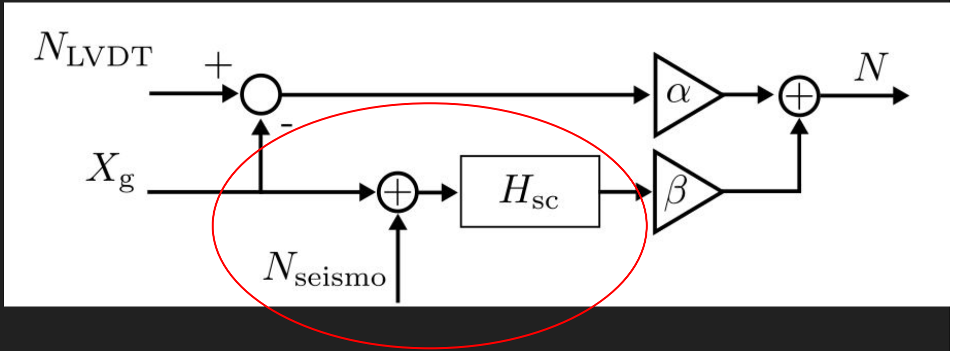
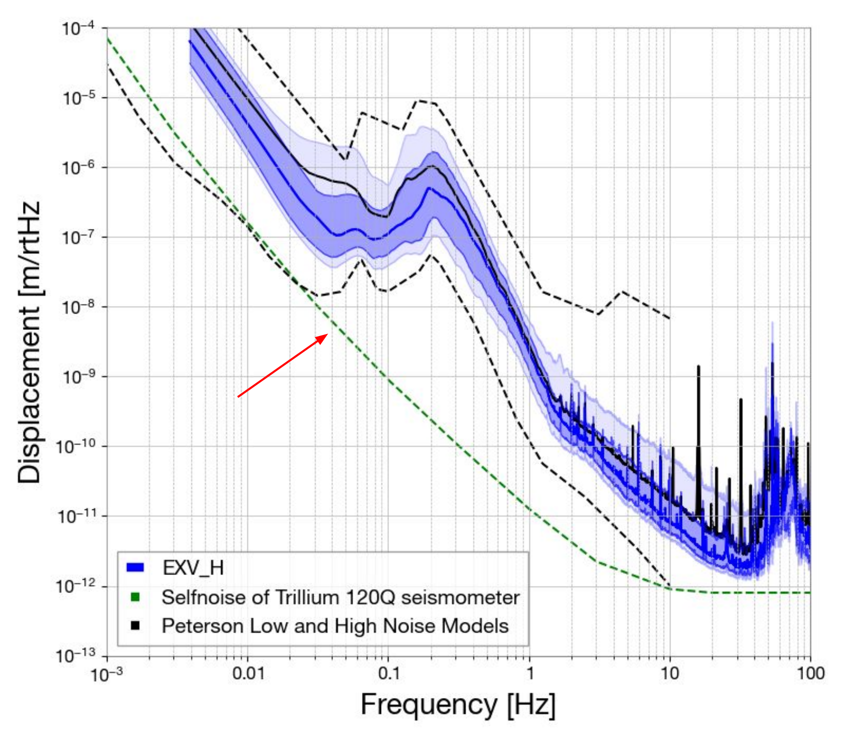
Sensor Correction Filter

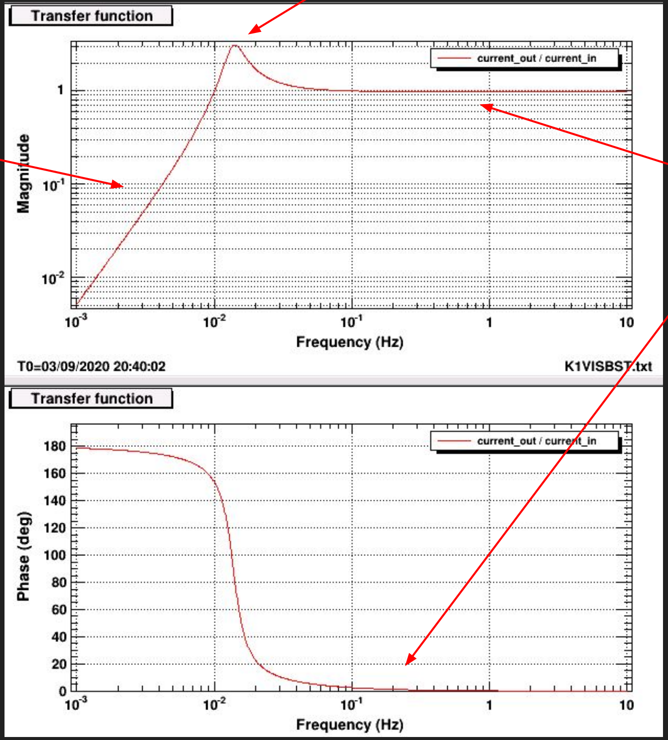




Credits: Miyo

Sensor Correction Filter In KAGRA

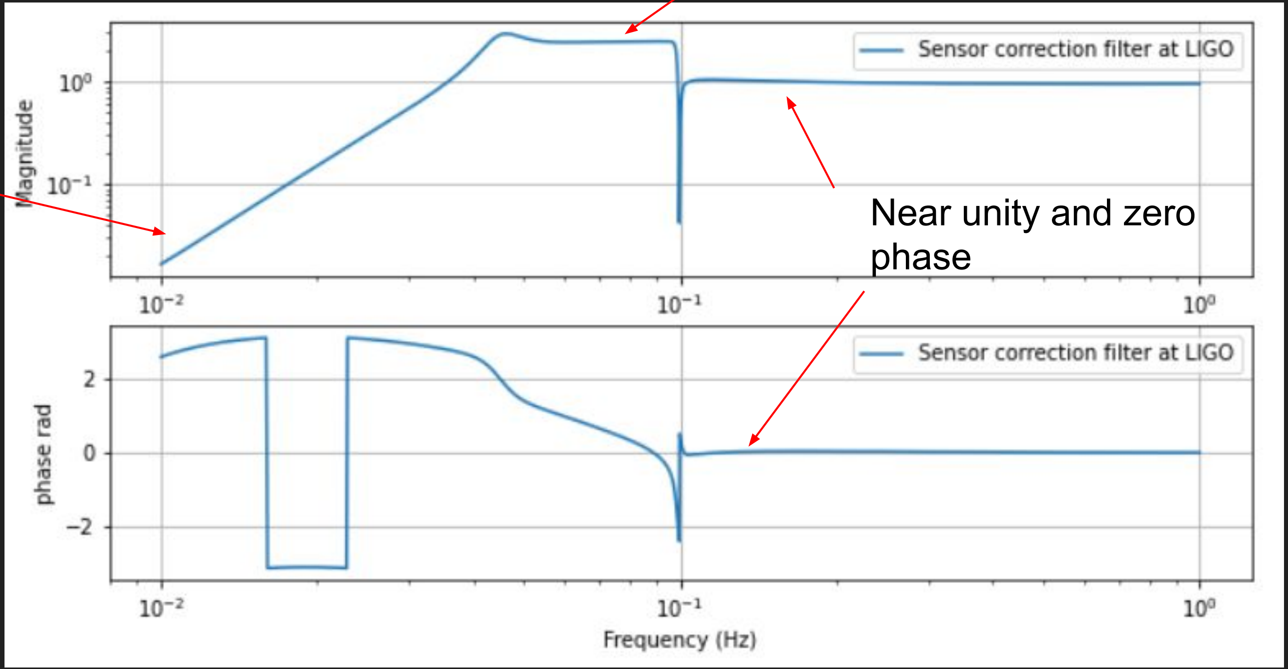
1. Particular order of attenuation (2)
2. Allowable noise amplification
3. Above certain frequency
   a. Close to 1
   b. Zero phase



Sensor Correction Filter (LIGO)

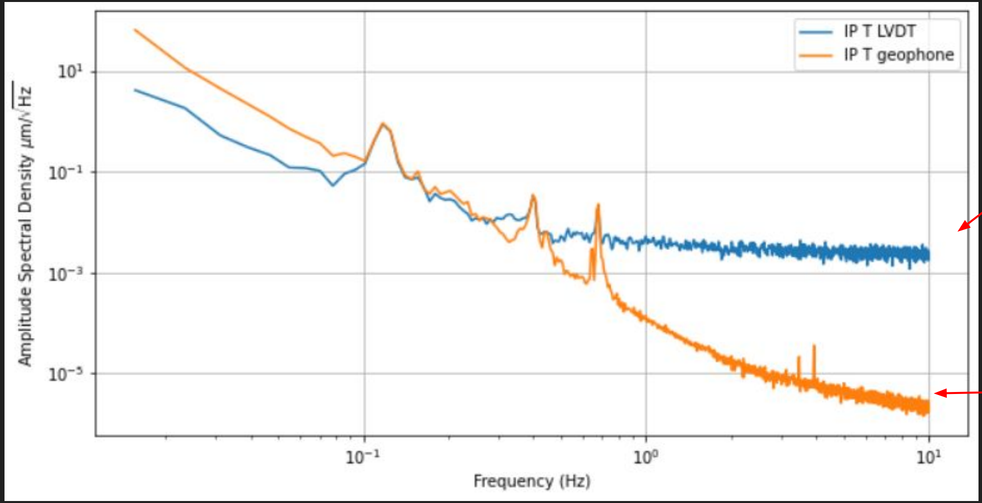
Less than 3

3rd-order



Near unity and zero phase

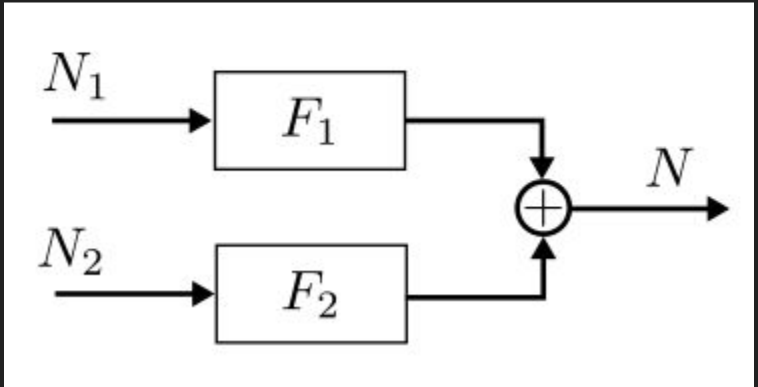
Complementary Filters



Sensor corrected LVDT

Geophone

Complementary Filter



$$N = F_1 N_1 + F_2 N_2$$

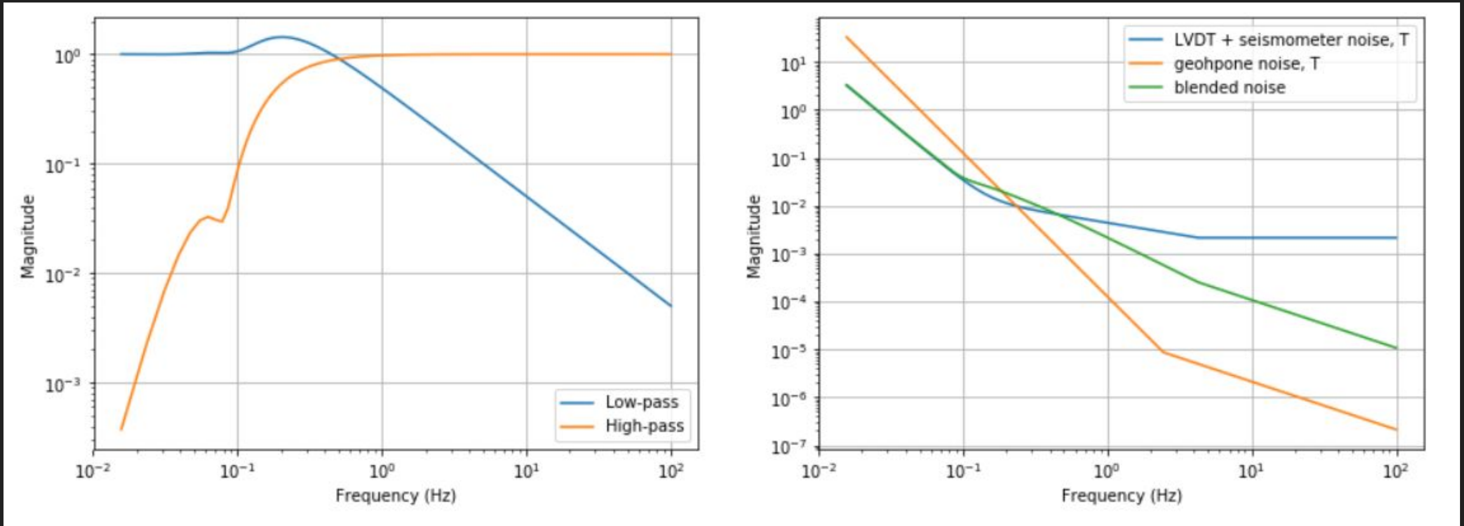
$$\left\langle N^2 \right\rangle = |F_1|^2 \left\langle N_1^2 \right\rangle + |F_2|^2 \left\langle N_2^2 \right\rangle$$

$$F_1^* = \underset{F1+F2=1}{\arg\min} \int \left\langle N^2 \right\rangle \, df$$

$$F_2^* = 1 - F_1^*$$

Complementary Filter Optimization (old)

$$F_2 = \frac{s^7 + a_1 7 s^6 + a_2 21 s^5 + a_3 35 s^4}{(s + a_4)^7}$$



Complementary Filter Optimization (old)

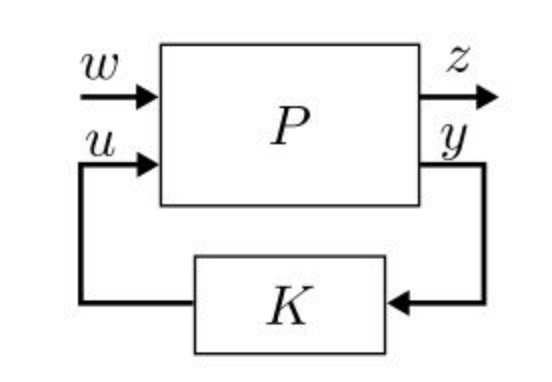
$$F_2 = \frac{s^7 + a_1 7s^6 + a_2 21s^5 + a_3 35s^4}{(s+a_4)^7}$$

- Predefined filter
- We don’t really know if this can best blend the sensors.

→ Limiting the performance. Could have been better.

→→ Second approach: H_2 and H_∞ Synthesis (Again!)

Interlude: Formalism of Robust control



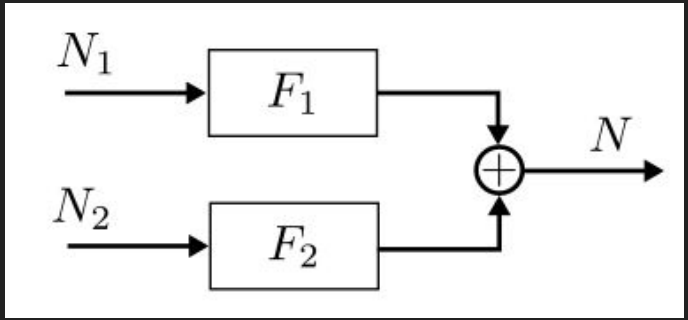
If we can model the system such that G=N, then H_2 synthesis automatically designs the complementary filter for us.

$$z = \left[P_{11} + P_{12}K\left(I - P_{22}K\right)^{-1}P_{21}\right]w$$
$$z = Gw$$

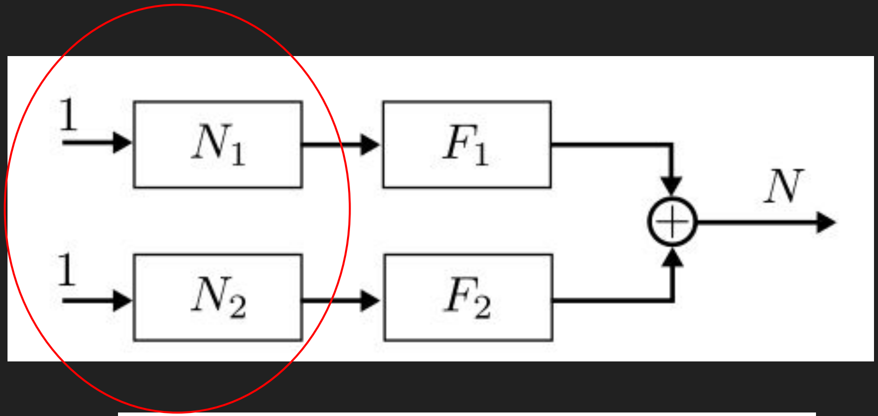
H_2 minimizes $\|G\|_2$

H_∞ minimizes $\|G\|_\infty$

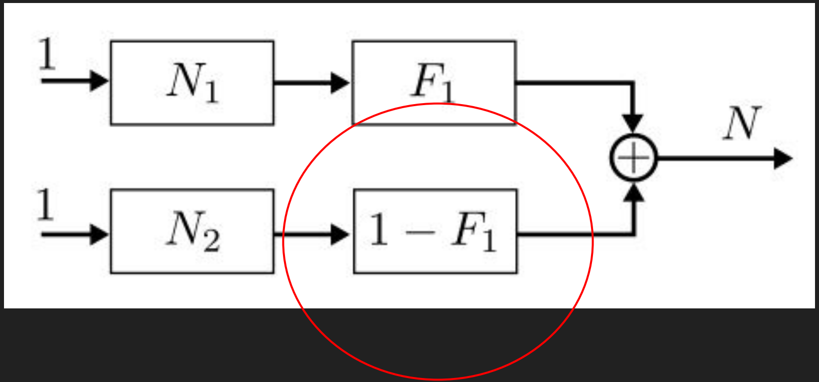
Converting to H_2 problem



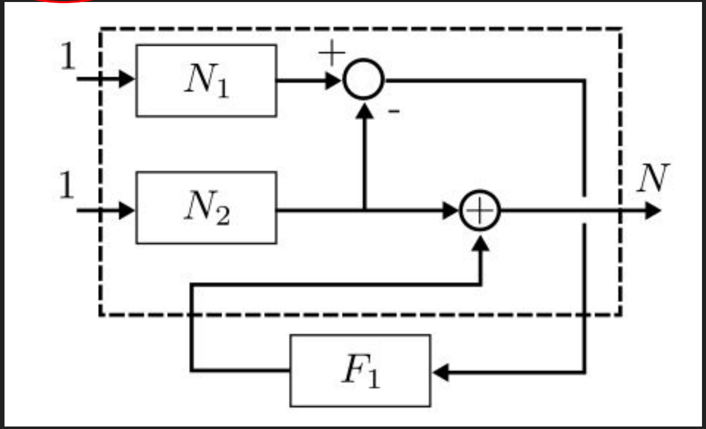
Modeling



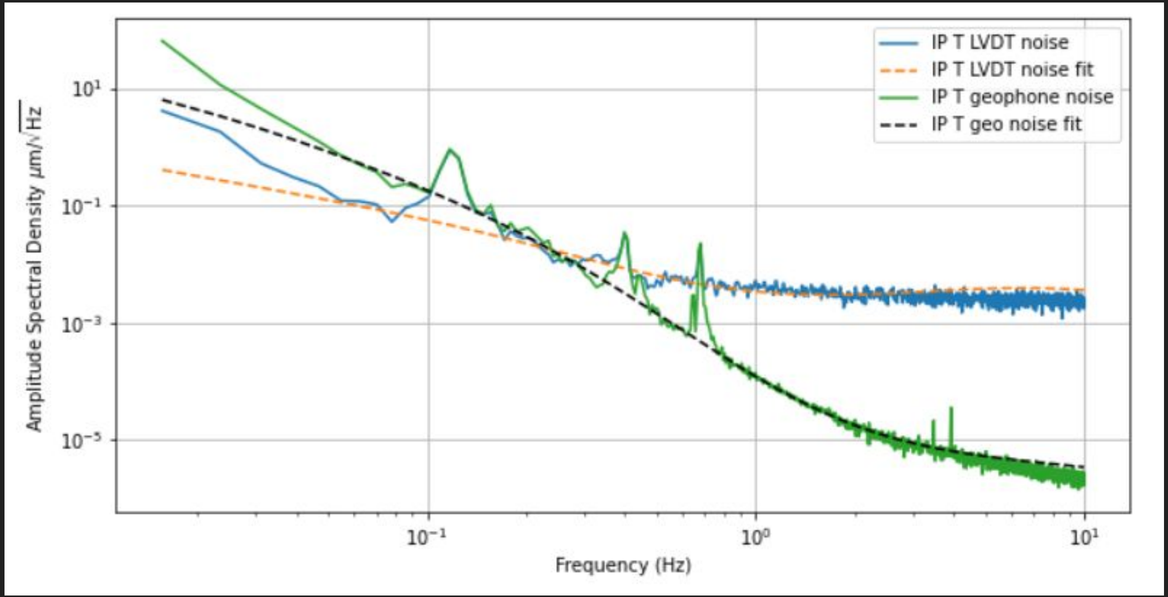
Substituting F_2



Rearranging

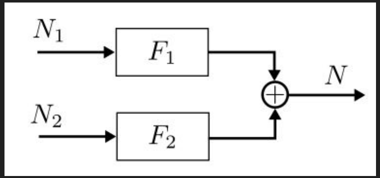


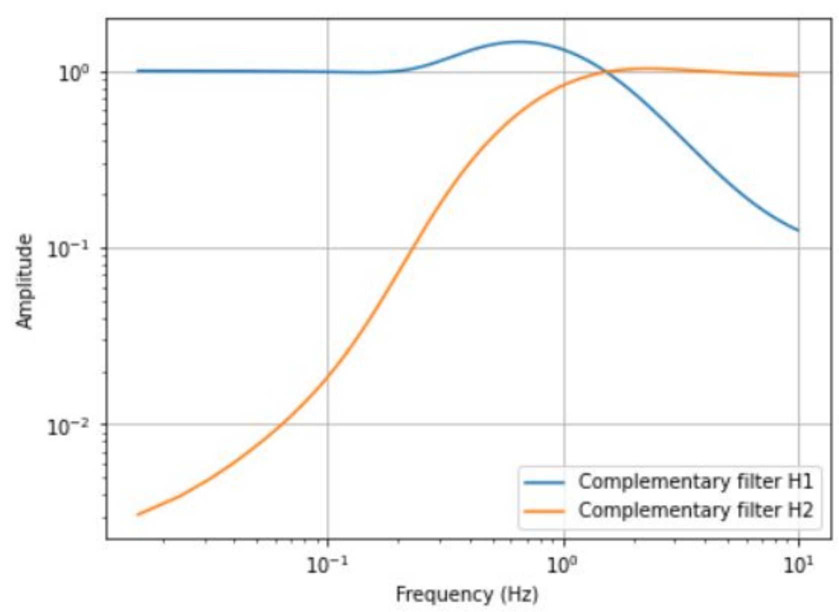
Results

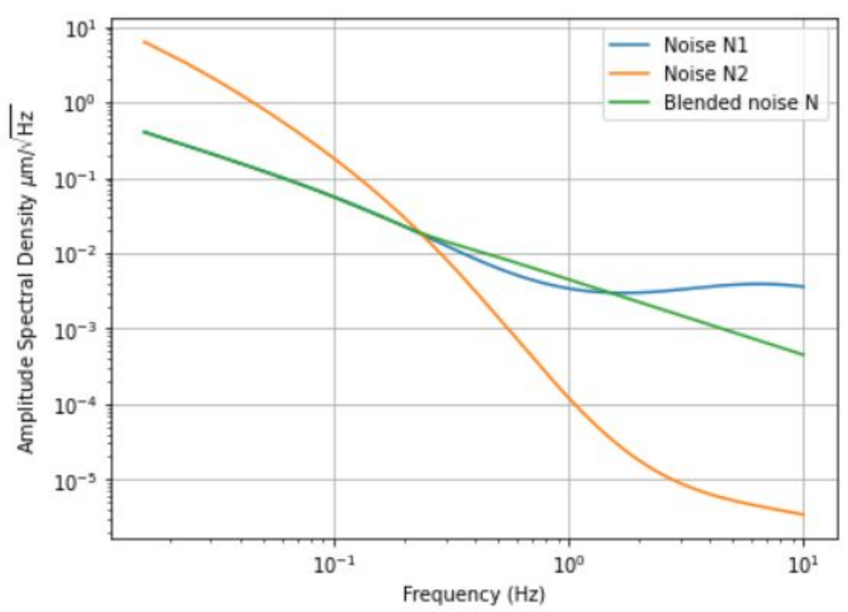


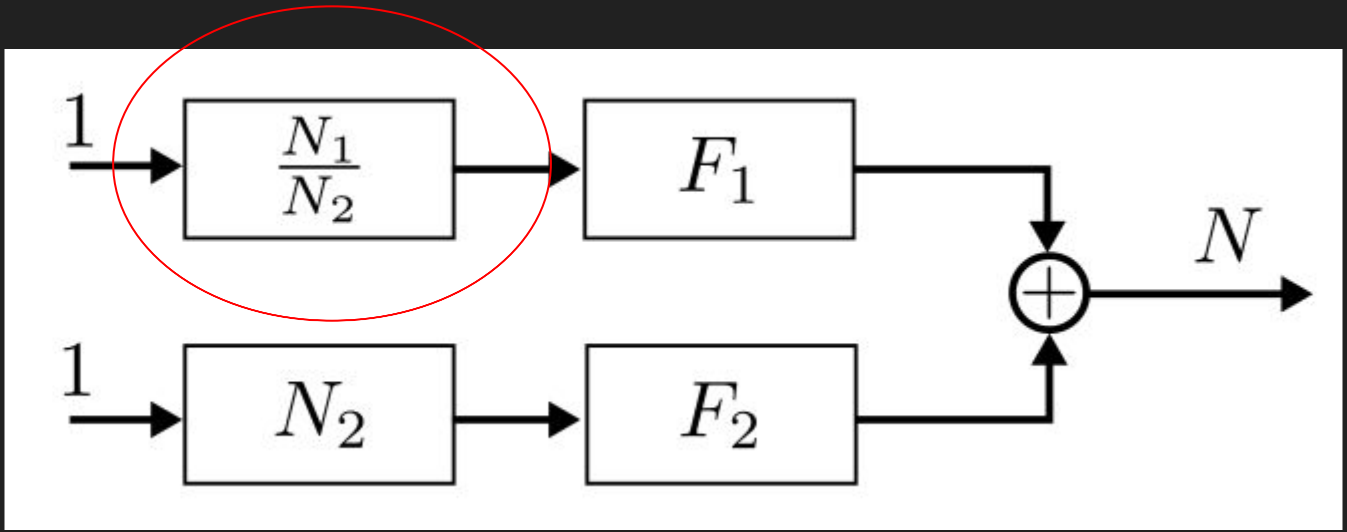
IP T LVDT noise
IP T LVDT noise fit
IP T geophone noise
IP T geo noise fit
Amplitude Spectral Density $\mu m/\sqrt{Hz}$
10^{1}
10^{-1}
10^{-3}
10^{-5}
10^{-1}
10^{0}
10^{1}
Frequency (Hz)

Results

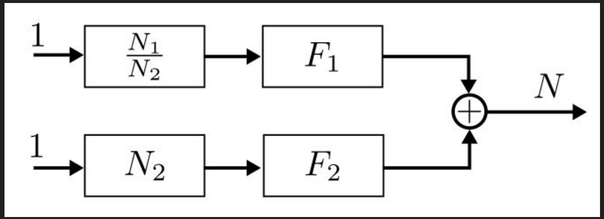

N_1
F_1
N
N_2
F_2

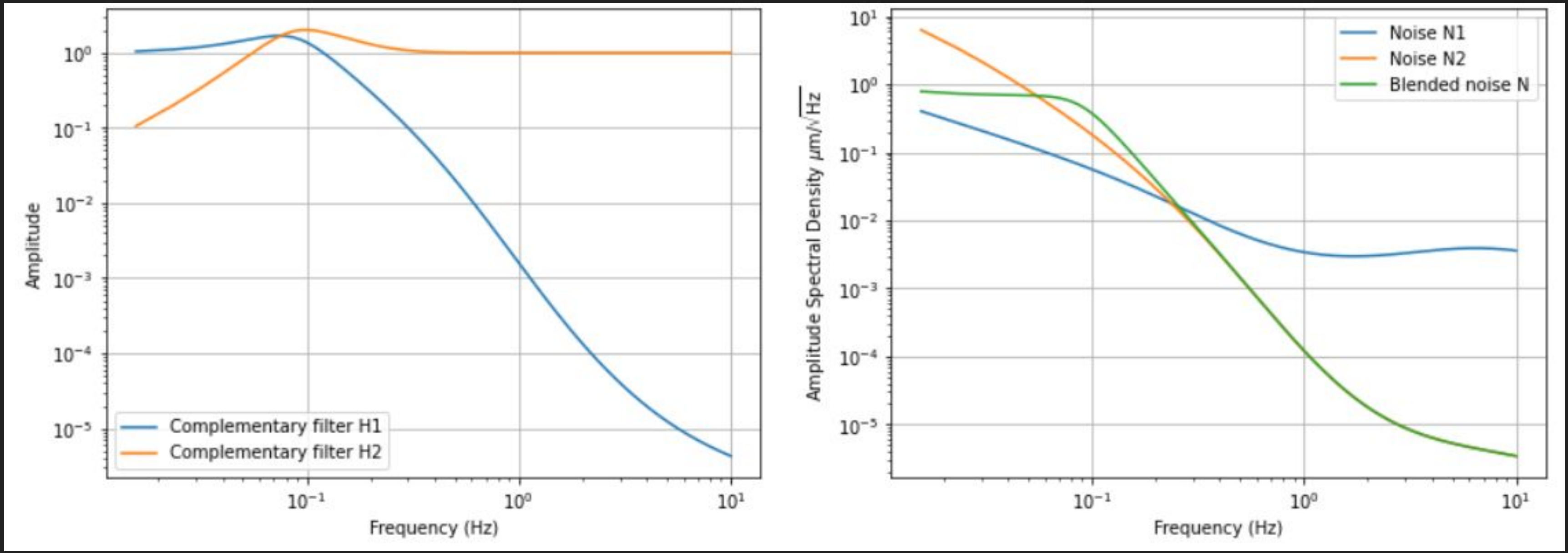

Amplitude
Frequency (Hz)
Complementary filter H1
Complementary filter H2


Amplitude Spectral Density $\mu m/\sqrt{Hz}$
Frequency (Hz)
Noise N1
Noise N2
Blended noise N

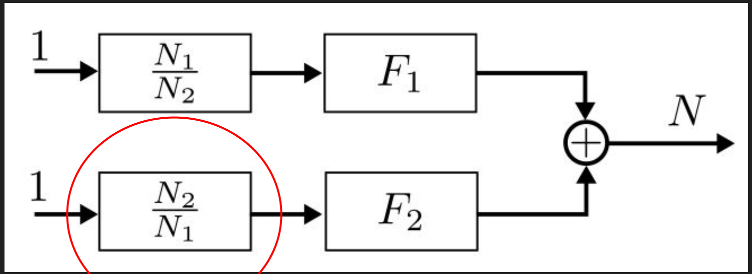

1
$\frac{N_1}{N_2}$
F_1
N
1
N_2
F_2

Results

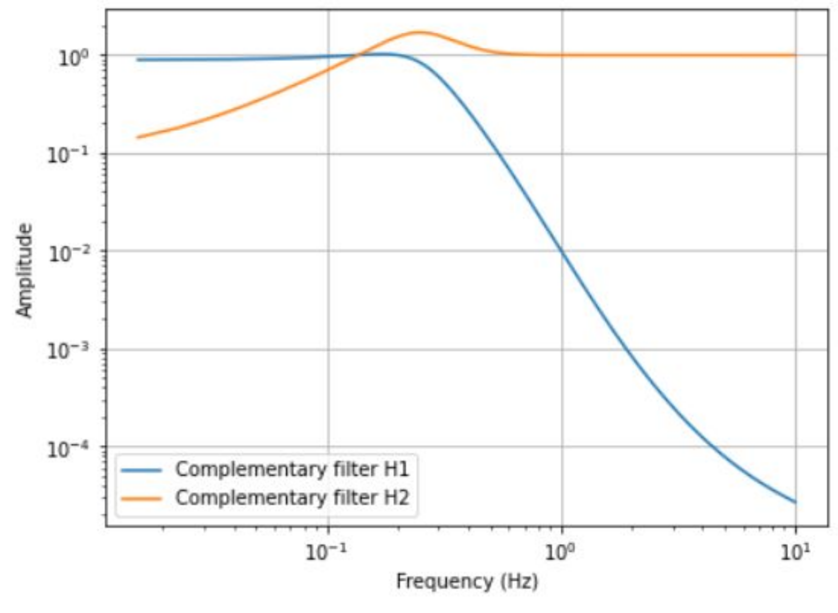

1
$\frac{N_1}{N_2}$
F_1
N
1
N_2
F_2


Amplitude
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
Complementary filter H1
Complementary filter H2
10^{-1}
10^0
10^1
Frequency (Hz)
Amplitude Spectral Density $\mu m/\sqrt{Hz}$
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
Noise N1
Noise N2
Blended noise N
10^{-1}
10^0
10^1
Frequency (Hz)

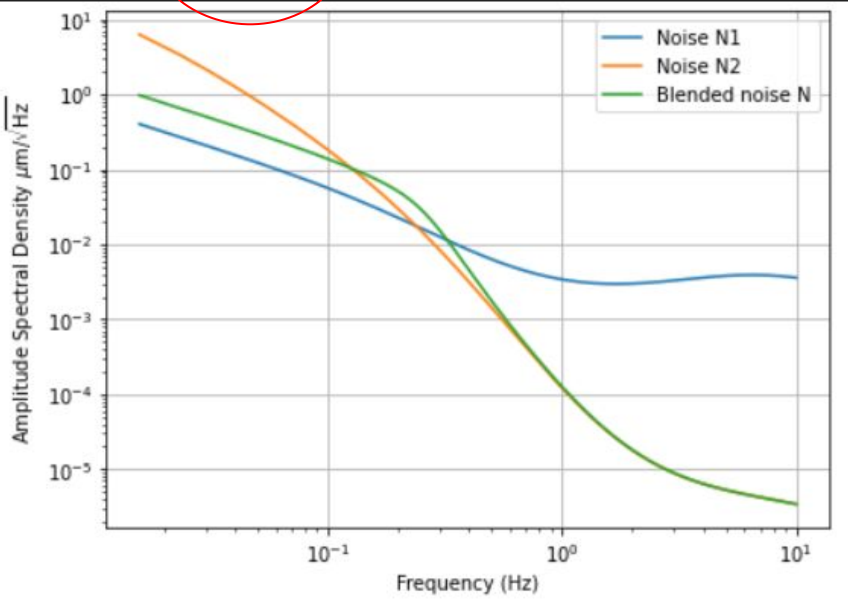
Results



1
$\frac{N_1}{N_2}$
F_1
1
$\frac{N_2}{N_1}$
F_2
N

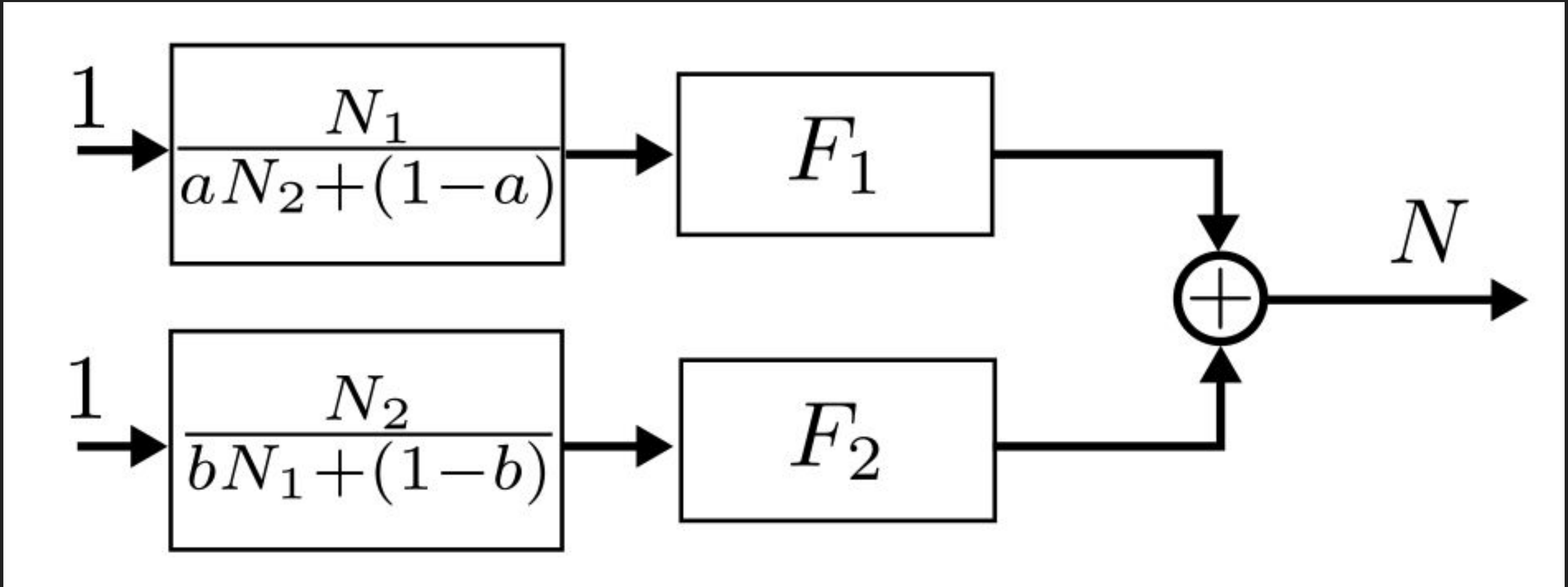


Amplitude
Frequency (Hz)
Complementary filter H1
Complementary filter H2



Amplitude Spectral Density $\mu m/\sqrt{Hz}$
Frequency (Hz)
Noise N1
Noise N2
Blended noise N

Generalization



Sensor Correction

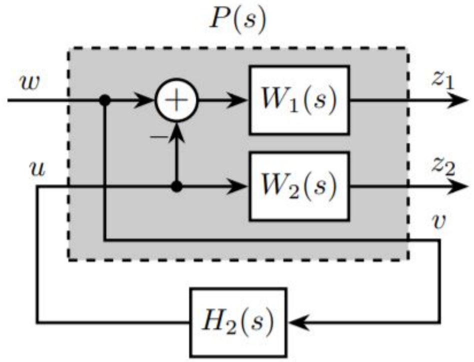


Fig. 4. Architecture used for $\mathcal{H}_\infty$ synthesis of complementary filters

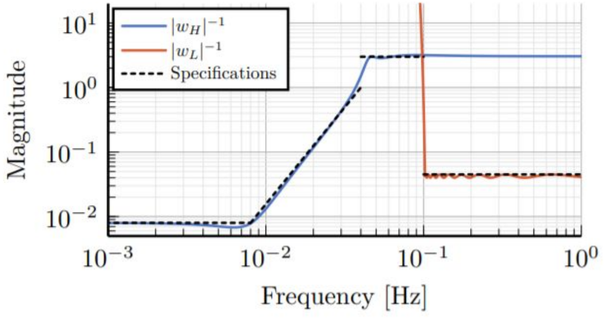
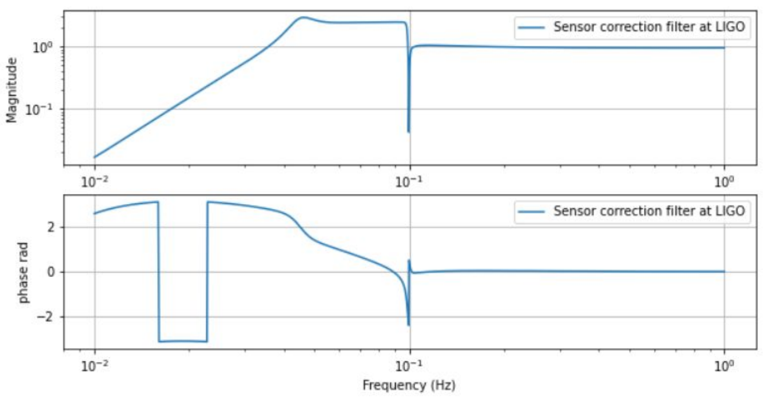


Fig. 9. Specifications and weighting functions magnitudes

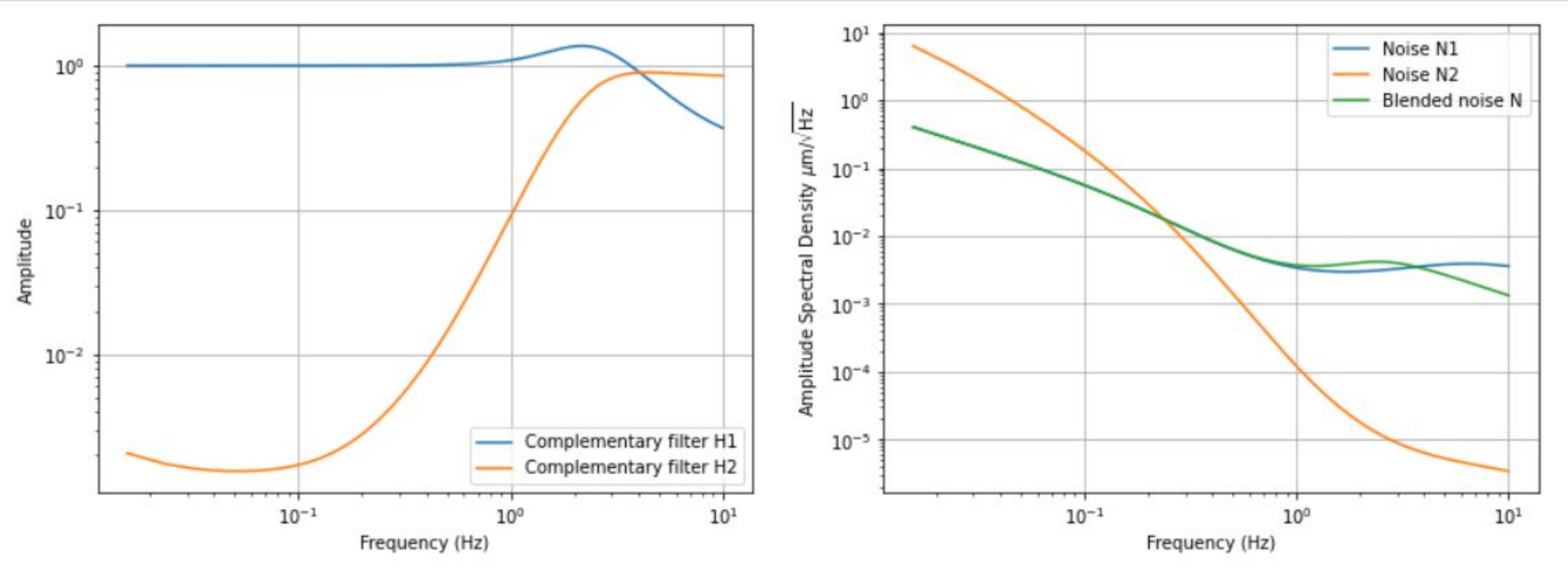


Credits: Thomas Dehaeze
https://orbi.uliege.be/bitstream/2268/241299/1/paper.pdf

Q&A

Appendix

a=0, b=1



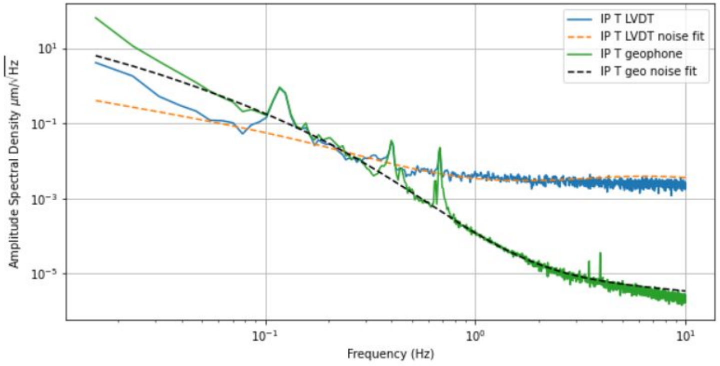
10^0
10^{-1}
10^{-2}
Amplitude
Complementary filter H1
Complementary filter H2
10^{-1}
10^0
10^1
Frequency (Hz)
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
Amplitude Spectral Density $\mu m/\sqrt{Hz}$
Noise N1
Noise N2
Blended noise N
10^{-1}
10^0
10^1
Frequency (Hz)

In [3]:
```python
from kontrol.model import noise2zpk
```

In [4]:
```python
weight = np.exp(-f)
# weight = None
# weight = 1/f
noise_lvdtt_fit=noise2zpk(f, noise_lvdtt_data, max_order=6, weight=weight)
noise_geot_fit=noise2zpk(f, noise_geot_data, max_order=6, weight=weight)
```

```
Optimization terminated successfully.
         Current function value: 9.706337
         Iterations: 18
         Function evaluations: 5197
Optimization terminated successfully.
         Current function value: 9.297857
         Iterations: 10
         Function evaluations: 2760
```

In [8]:
```python
plt.figure(figsize=(10,5))
plt.loglog(f, noise_lvdtt_data, label='IP T LVDT')
plt.loglog(f, abs(noise_lvdtt_fit.horner(2*np.pi*1j*f)[0][0]), '--', label='IP T LVDT noise fit')
plt.loglog(f, noise_geot_data, label='IP T geophone')
plt.loglog(f, abs(noise_geot_fit.horner(2*np.pi*1j*f)[0][0]), 'k--', label='IP T geo noise fit')
plt.ylabel('Amplitude Spectral Density $\mu\mathrm{m}/\sqrt{\mathrm{Hz}}$')
plt.xlabel('Frequency (Hz)')
plt.legend(loc=0)
plt.grid()
```



Sensor Correction filter?

Progress

Part I: Introduction, Definitions, and Problem Formulation

Part II: Optimal Control Using H_2 and H_∞ Synthesis

Part III: Sensor Correction Filter Optimization, and Inter-calibration Minimization.

Part IV: Optimal Complementary Filters

Part V: Model Reference Diagonalization

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