Optimal Control and Optimization for KAGRA Vibration Isolation system

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Overview

Part I: Introduction, Definitions, and Problem Formulation

Today
2020/08/19

Part II: Optimal Control Using H_2 and H_∞ Synthesis

Part III: Sensor Correction Filter Optimization, and Inter-calibration Minimization.

Part IV: Optimal Complementary Filters

Next meeting

Part V: Model Reference Diagonalization

Next Next meeting

Part VI: File Management System/Data Pipeline for Suspension Models, both Simulation and regressed.

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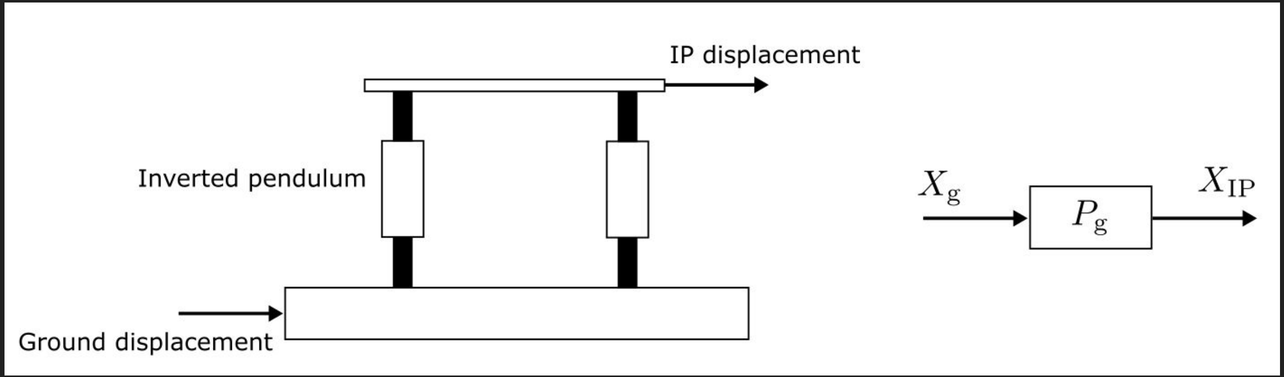
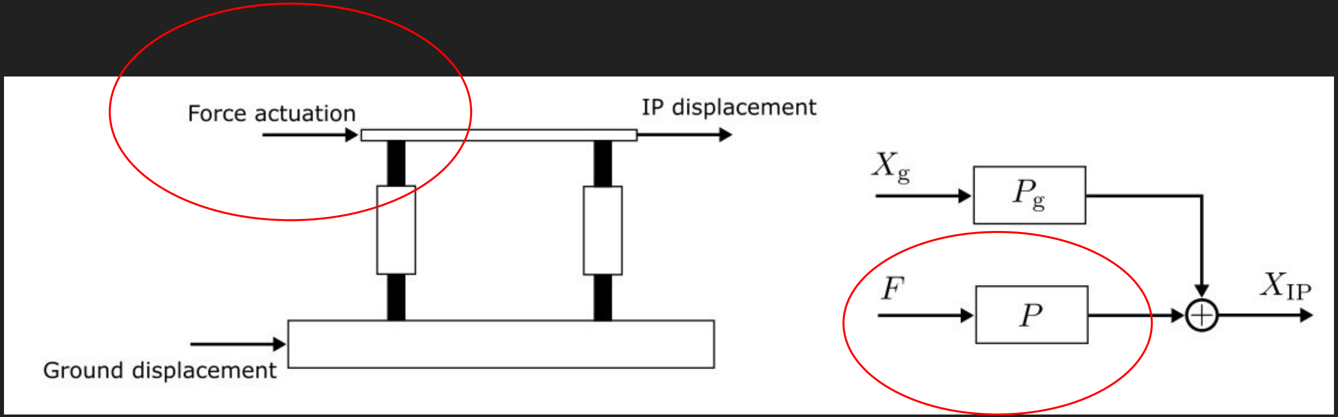
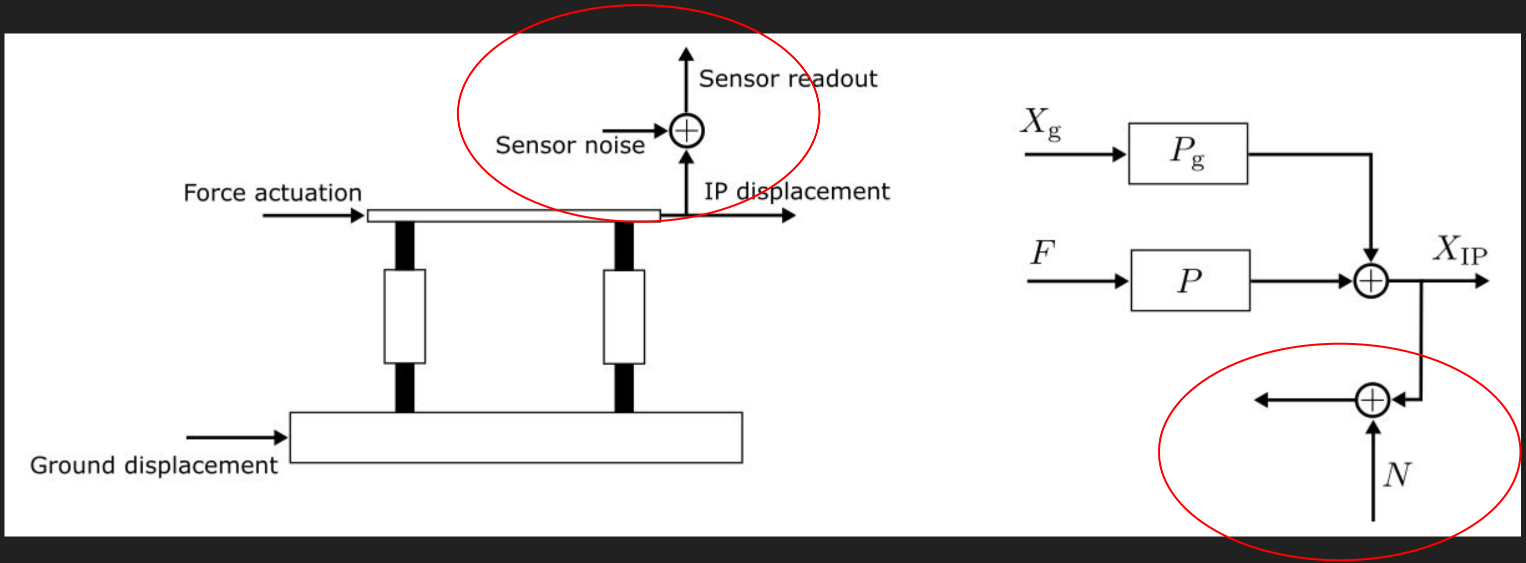
Pre-isolator



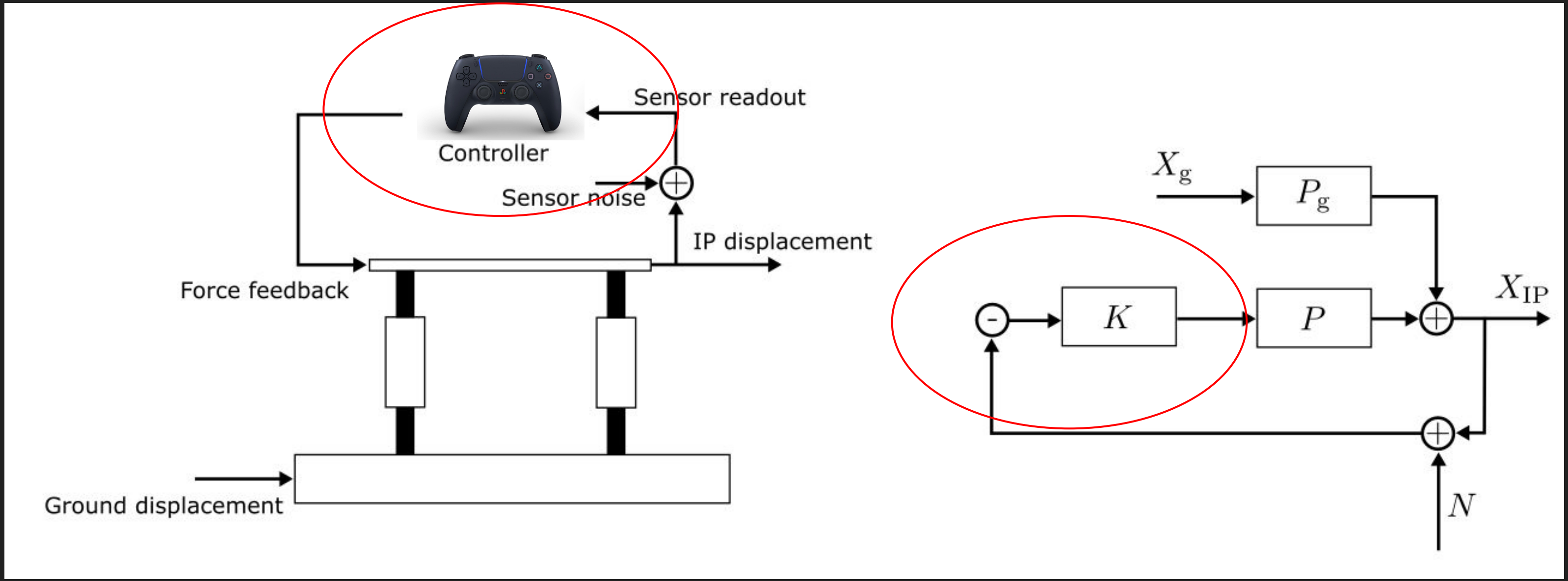
Pre-isolator



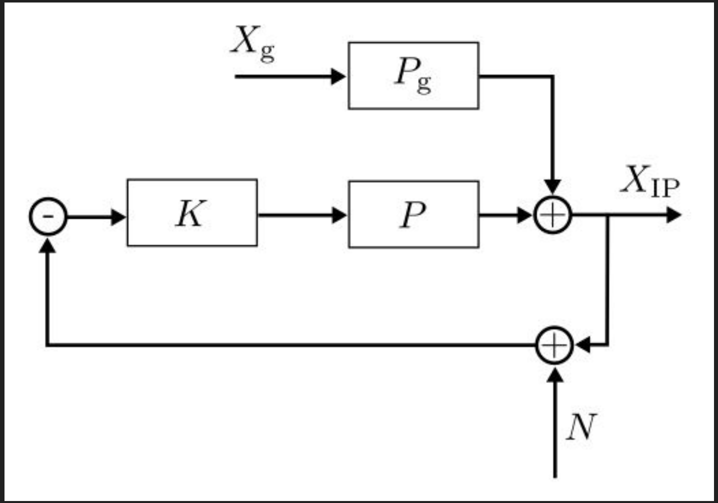
Pre-isolator

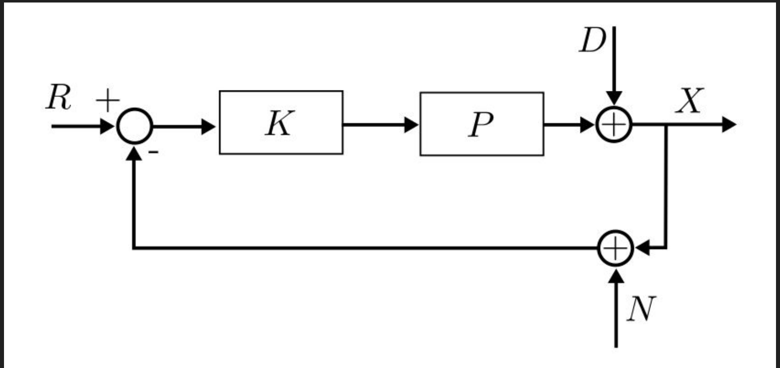


Pre-isolator



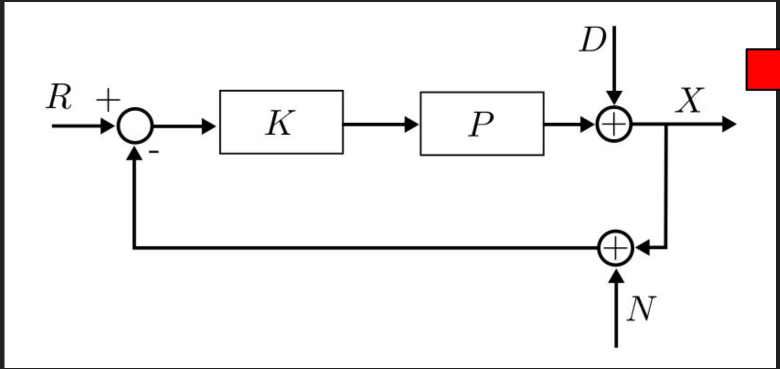
Generalization





General system for any DoF.

Displacement



$$X = \frac{KP}{1+KP}R + \frac{1}{1+KP}D - \frac{KP}{1+KP}N$$

PS/ PSD:

$$\langle X^2 \rangle = \left| \frac{1}{1+KP} \right|^2 \langle D^2 \rangle + \left| \frac{KP}{1+KP} \right|^2 \langle N^2 \rangle$$

Problem statement

Disturbance, external, cannot be reduced, but can be induced.

Displacement that we want to minimize (Goal)

$$\langle X^2 \rangle = \left| \frac{1}{1+KP} \right|^2 \langle D^2 \rangle + \left| \frac{KP}{1+KP} \right|^2 \langle N^2 \rangle$$

Noise, can be reduced, but exist limitations.

Plant, mechanical, fixed

Control filter, can be whatever we want (almost)
→ Into optimal control

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Fundamental Limitation in Control System

Minimizable, $K \rightarrow \infty$

Minimizable, $K \rightarrow 0$

$$\langle X^2 \rangle = \left| \frac{1}{1+KP} \right|^2 \langle D^2 \rangle + \left| \frac{KP}{1+KP} \right|^2 \langle N^2 \rangle$$

Not simultaneously minimizable
Coupling terms are complementary

$$\frac{1}{1+KP} + \frac{KP}{1+KP} = 1$$

Optimization

$$\langle X^2 \rangle = \left| \frac{1}{1+KP} \right|^2 \langle D^2 \rangle + \left| \frac{KP}{1+KP} \right|^2 \langle N^2 \rangle$$

Simple observation: It is a positive definite function.

→ It must be minimizable by some optimal controller *K*, given some disturbance *D* and noise *N*.

→→ Optimization

Optimization Interlude

I mean mathematical optimization.

→ Minimization of a cost function by choosing the critical parameters within an allowed set.

For example,

Cost function: $J(x) = x^2$

Allowed set: $x \in \mathbb{R}$

Minimization: $\min J(x) = 0, \quad x^* = 0$

In a compact way

$$x^* = \arg\min_{x \in \mathbb{R}} (J(x)) = 0$$

Optimal Control

→ Find optimal controller that minimizes a cost function.
Some cost functions to be minimized:
E.g.,

The integrated RMS/expected RMS: $$J_2 = \int_0^\infty \langle X^2 \rangle \, df$$ → 2-norm of the system

The maximum of the displacement spectrum: $$J_\infty = \max \langle X^2 \rangle$$ → ∞-norm of the system

We can choose controllers however we like.
But, the system has to be stable.
I.e. The controllers/system must be within some mathematical set.

H_2/H_∞ Optimal Controller

Letter "H" comes from the mathematical space the optimization takes place, namely, Hardy space.

Hardy space contains all possible stable systems.

In a nutshell, H_2/H_∞ optimal controllers are:

H_2 optimal controller: $$K_{\mathcal{H}_2} = \underset{K \in S}{\arg\min} \int_0^\infty \langle X^2 \rangle \, df$$

H_∞ optimal controller: $$K_{\mathcal{H}_\infty} = \underset{K \in S}{\arg\min} \left(\max \langle X^2 \rangle\right)$$

$$S : \{\text{All possible controllers such that the system is stable}\}$$

Why H∞?

Some weighting filter/function according to requirements

$$J_\infty = \max \langle X^2 \rangle$$

$$J_\infty = \max \left(\langle X^2 \rangle \left| W_X \right|^2 \right)$$

Only minimizes the dominating peaks, e.g. resonances.

→ Trade-off between seismic noise suppression and control noise attenuation.

→→ Maximizing hardware potential to suppress seismic noise while meeting noise requirement.

Some Benefits of Optimization-based Approaches

Form of cost function is not limited, we can even add actuation signal as part of the cost function so it doesn't saturate.

E.g.

$$J_\infty = \max\left(\langle X^2\rangle \left|W_X\right|^2 + \langle F^2\rangle \left|W_F\right|^2\right)$$

Displacement spectrum

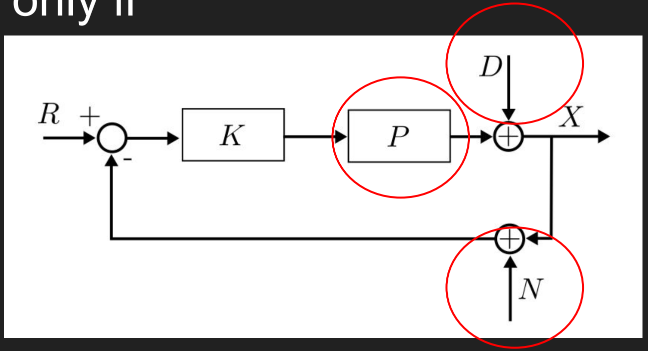
Actuation signal spectrum

→ Trade-off between suppression and actuation signal

Things to Do

Generating H_2 and H_∞ optimal filters are extremely easy, only if

1. We have precisely modelled the plant,
2. We have precisely modelled the disturbance, and
3. We have precisely modelled the noise.



But, those were never done in a systematic manner.

Of course, I can do those stages by stage, suspension by suspension.

But, wouldn't it be nice if we can have a data analysis pipeline that automates the workflow? → Part VI of my presentation.

Fundamental Limitations

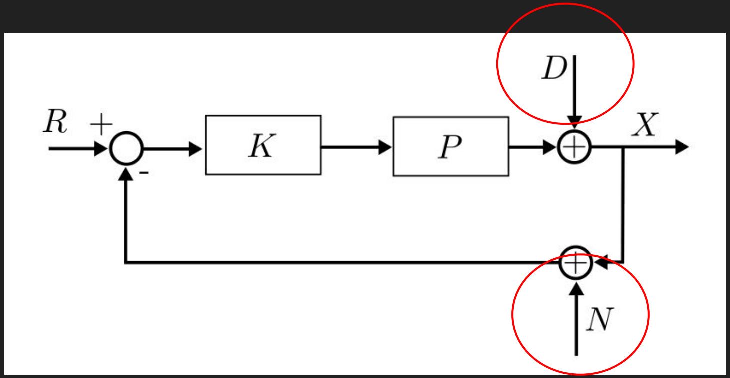
Having optimal controllers is only minimizing the disturbance and noise coupling to the displacement.

True limitations are the disturbances and noises.

Therefore, before doing optimal control, it is necessary to reduce the disturbance and noise level as much as possible.

Remains to be another discussion

→ Part III, IV, V of my presentation.



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