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Arm cavity round-trip loss measurement with ITM inhomogeneity and birefringence

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Background

- Arm cavity round-trip loss (RTL) is usually estimated from the power reduction in the arm reflection when the arm cavity is locked. But this method can be wrong when there is significant ITM birefringence and when you are detecting s-pol and p-pol with some bias.
- Correct way of measuring RTL is discussed.
- Related documents and klogs:
  - JGW-G1910369 (calculations to explain ~10% loss to p-pol)
  - JGW-G1910388 (summary including the effect to PRG for PRMI)
  - JGW-T1910380 (how to compute birefringence map from TWE maps)
  - klog #7307 (Xarm round-trip loss measurement before we noticed the birefringence issue)
  - klog #9393 (confirmed reduction in p-pol when Xarm is locked)

Naming The Effects

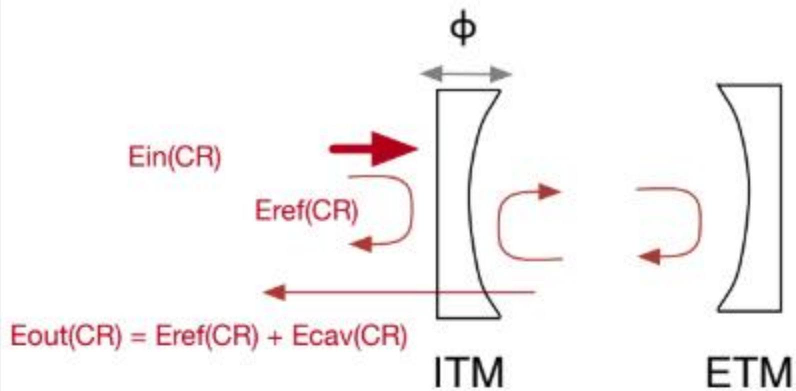
- Lawrence effect (conjugation effect)
  - the carrier field is insensitive to transmission wavefront error to the first-order when the cavity is locked to the carrier
  - known effect for thermal lensing and distortion, but also true for birefringence
- PRC/SRC mode healing
  - non-resonant mode is suppressed when PRC or SRC is locked

- These two are completely different effects!
  Please use the correct names!

Lawrence effect

- Probably first discussed in [Ph.D. thesis](#) by Ryan Lawrence (Section 2.1.1)

Hiro's note



$$\mathrm{Eref} = r_{ITM}\,\mathrm{Exp}[2\,i\,\phi]\,E_{in};$$

$$\mathrm{Ecav} = \frac{-t_{ITM}^2}{1-r_{ITM}}\,\mathrm{Exp}[i\,\phi]\,E_{in};$$

$$E_{out}[CR] = \left(r_{ITM}\,\mathrm{Exp}[2\,i\,\phi] - \frac{t_{ITM}^2}{1-r_{ITM}}\,\mathrm{Exp}[i\,\phi]\right)E_{in}$$

$$= \left(r_{ITM}\,\mathrm{Exp}[2\,i\,\phi] - (1+r_{ITM})\,\mathrm{Exp}[i\,\phi]\right)E_{in}$$

$$= \left(r_{ITM}\left(1+2\,i\,\phi+\frac{1}{2}(2\,i\,\phi)^2\right) - (1+r_{ITM})\left(1+i\,\phi+\frac{1}{2}(i\,\phi)^2\right)\right)E_{in}$$

$$= -\left(1+\phi^2\right)E_{in}\ \ [\text{with } r_{ITM} = 1]$$

$$E_{out}[SB] = E_{ref} = r_{ITM}\,\mathrm{Exp}[2\,i\,\phi]\,E_{in}$$

$$= (1+2\,i\,\phi)\,E_{in}$$

The effect of TWE (φ) is cancelled for carrier field since prompt reflection and cavity transmission has an opposite sign. The prompt reflection feels φ twice, while the cavity transmission feels φ once but has twice the amplitude

ITM Reflection (unlocked)

- ITM reflection when the cavity is not locked has

  s-pol TEM00: $r_1^2(1-\gamma^2-\beta^2-\alpha^2-m_\mathrm{s}^2-m_\mathrm{p}^2)$

  s-pol HOM: $r_1^2\gamma^2$ (from TWE for s-pol)

  s-pol HOM: $r_1^2 m_\mathrm{s}^2$ (from mode-mismatch)

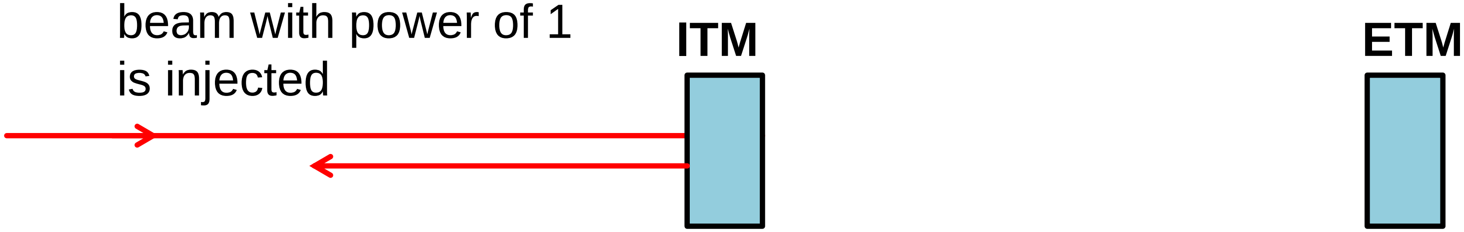
  p-pol TEM00: $r_1^2\beta^2$ (from birefringence)

  p-pol HOM: $r_1^2\alpha^2$ (from TWE for p-pol)

  p-pol HOM: $r_1^2 m_\mathrm{p}^2$ (from mode-mismatch)

Assume purely s-pol beam with power of 1 is injected

$r_1,\ t_1$ $\qquad$ $r_2=\sqrt{1-t_\mathrm{E}^2-t_\mathrm{loss}^2}$



Assume mirror reflectivity and losses are the same for s-pol and p-pol

ITM Reflection (locked)

- ITM reflection when the cavity is locked to TEM00 has
  - s-pol TEM00: $r_{\rm FP}^2(1-\beta^2-m_{\rm s}^2-m_{\rm p}^2)-r_{\rm LE}^2\gamma^2-r_{\rm LE}^2\alpha^2$
  - s-pol HOM: $r_{\rm LE}^2\gamma^2$ (reduced due to Lawrence effect)
  - s-pol HOM: $r_1^2 m_{\rm s}^2$ (unchanged)
  - p-pol TEM00: $r_{\rm FP}^2\beta^2$ (slightly reduced due to arm loss)
  - p-pol HOM: $r_{\rm LE}^2\alpha^2$ (reduced due to Lawrence effect)
  - p-pol HOM: $r_1^2 m_{\rm p}^2$ (unchanged)

  Here,

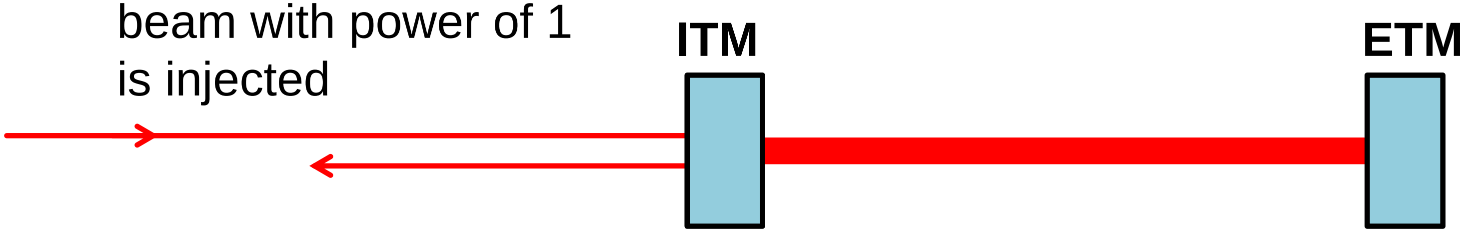
$$r_{\rm FP}=r_1-\frac{t_1^2 r_2}{1-r_1 r_2}\sim-\sqrt{1-N_{\rm rt}(T_{\rm E}+T_{\rm loss})}\qquad N_{\rm rt}=\frac{t_1^2 r_2}{(1-r_1 r_2)^2}$$

$$r_{\rm LE}=r_1-\frac{1}{2}\frac{t_1^2 r_2}{1-r_1 r_2}=\frac{1}{2}(r_1+r_{\rm FP})\sim\frac{1}{4}N_{\rm rt}(T_{\rm E}+T_{\rm loss})$$

$$r_2=\sqrt{1-t_E^2-t_{\rm loss}^2}$$

Assume purely s-pol beam with power of 1 is injected

$r_1,\ t_1$



Assume mirror reflectivity and losses are the same for s-pol and p-pol

ITM Reflection Comparison

- ITM reflection in total will be

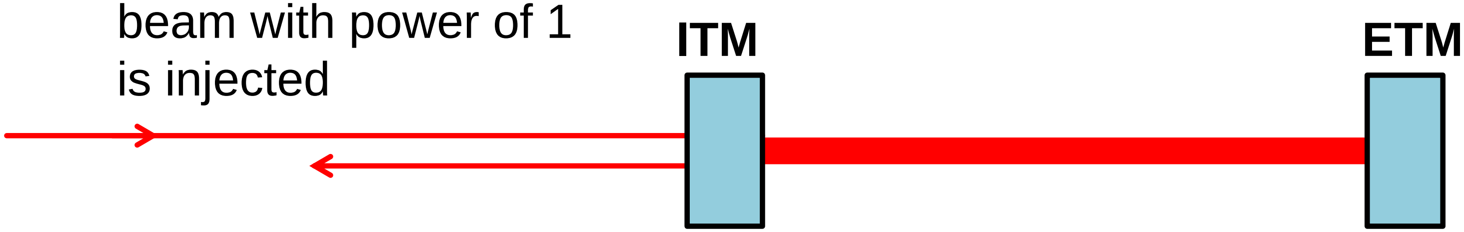
	unlocked	locked
total:	r_1^2	$r_{\mathrm{FP}}^2(1-m_{\mathrm{s}}^2-m_{\mathrm{p}}^2)+r_1^2(m_{\mathrm{s}}^2+m_{\mathrm{p}}^2)$
s-pol total:	$r_1^2(1-\beta^2-\alpha^2-m_{\mathrm{p}}^2)$	$r_{\mathrm{FP}}^2(1-\beta^2-m_{\mathrm{s}}^2-m_{\mathrm{p}}^2)-r_{\mathrm{LE}}^2\alpha^2+r_1^2 m_{\mathrm{s}}^2$
p-pol total:	$r_1^2(\beta^2+\alpha^2+m_{\mathrm{p}}^2)$	$r_{\mathrm{FP}}^2\beta^2+r_{\mathrm{LE}}^2\alpha^2+r_1^2 m_{\mathrm{p}}^2$

- If mode mismatch $m_{\mathrm{s}}^2+m_{\mathrm{p}}^2$ can be correctly estimated, round trip loss can be estimated from change in the total reflection
- If you are only monitoring s-pol component of reflection, you cannot estimate r_{FP}^2 correctly

Assume purely s-pol beam with power of 1 is injected

$r_1,\ t_1$

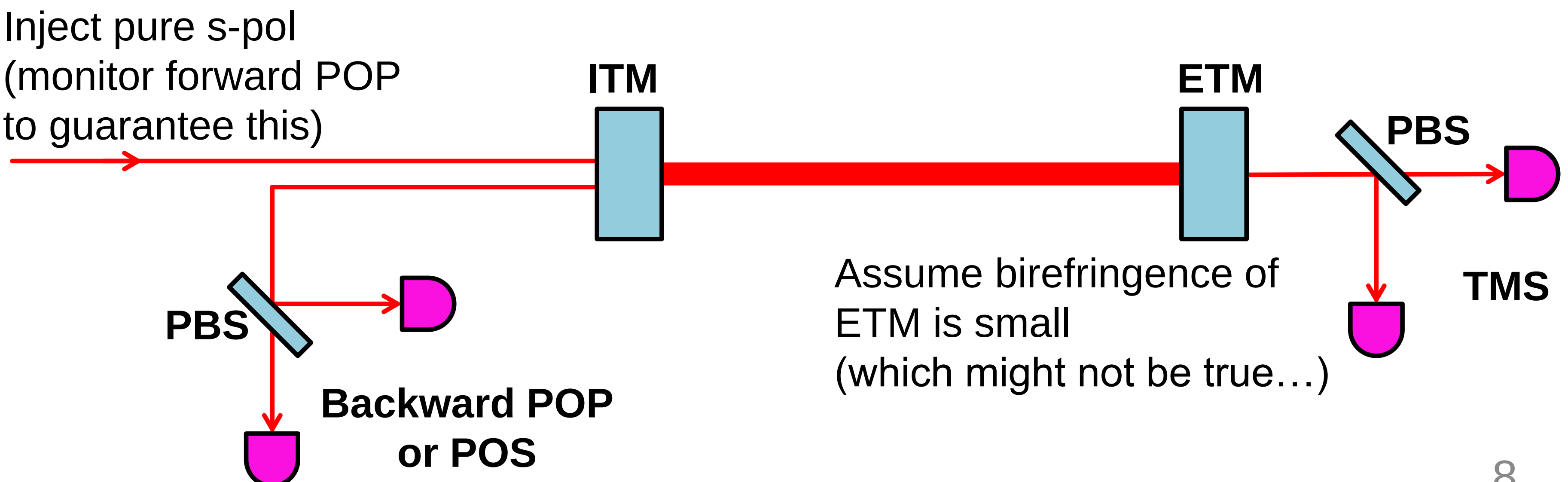
$r_2=\sqrt{1-t_E^2-t_{\mathrm{loss}}^2}$



Assume mirror reflectivity and losses are the same for s-pol and p-pol

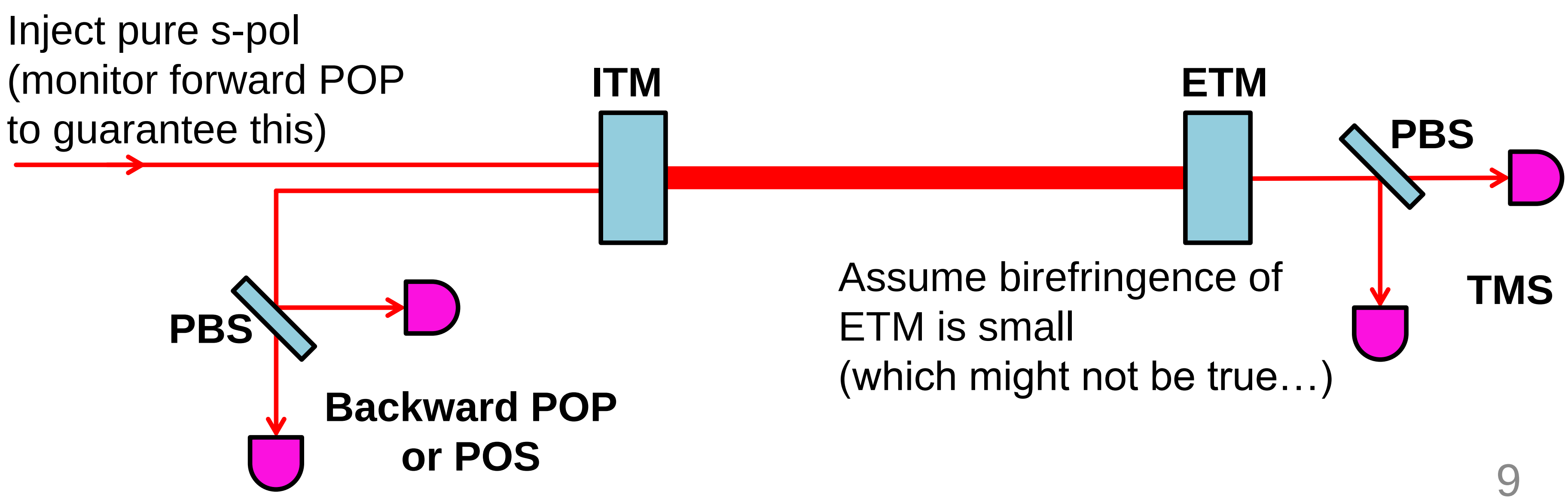
How to correctly estimate RTL

- Since there are polarization dependent components in TMS and central area (including BS), it is better to detect s-pol and p-pol separately, and sum them up after correcting the bias to estimate the total reflection and total mode-mismatch ($m_{\rm s}^2 + m_{\rm p}^2$)
- s-pol and p-pol throughput from ITM to POP (or POS) PDs and ETM to TMS PDs should be estimated separately to correct the bias (may be it is hard to measure directly in situ; discussed in next page)
- Finesse measurement for s-pol and p-pol at TMS can be useful to check if mirror reflectivity and RTL are the same for s-pol and p-pol



How to correctly add s-pol and p-pol

- As for s-pol and p-pol throughput from ITM to POP (or POS), put as little polarization dependent as possible between ITM and PBS, and assume BS is the only one which has polarization dependent reflectivity.
- As for s-pol and p-pol throughput from ETM to TMS, use TMS throughput measurement if there exists. You can also safely assume $m_{\rm s}^2 + m_{\rm p}^2 \simeq m_{\rm s}^2$ (if the mode matching is 90% and p-pol generation is 10%, m_s^2 is 10% and m_p^2 is 10%*10%=1%)
- I think you anyway have to assume that birefringence of ETM is small



What we know so far

- The latest measurement on arm cavity finesse at ~250 K (klog #14258)
  Xarm: 1456(21)　　　Yarm: 1312(26)
- ITM transmission by Hirose-san (JGW-T1809173)
  ITMX: 0.44%　　　ITMY: 0.48%
- ETM transmission by Hirose-san (JGW-T1807981)
  ETMX: 6.8(4) ppm　　　ETMY: 6.9(5.2) ppm
- p-pol from single bounce reflection of ITM when arm cavity is unlocked (after correcting the measurement at POP with different BS reflectivity for s- and p-pol; JGW-G1910388)
  ITMX: 6.1%　　　ITMY: 10.8%
- Assuming RTL of O(100ppm),

$$N_{\mathrm{rt}} \sim 10^3 \qquad r_{\mathrm{FP}}^2 \sim 1 - 10^3 T_{\mathrm{loss}}$$

Revisiting klog #9393

- p-pol power in PRX measured at forward POP was reduced by a factor of 3 when Xarm is locked
- This is a mixture of p-pol TEM00 reduction by $N_{rt}*T_{loss}$~O(10%), p-pol HOM reduction from the Lawrence effect by 1- $(N_{rt}*T_{loss}/4)^2$~O(1-0.06%), and PRX mode healing effect
- Finesse of PRX for p-pol is ~20 or so (BS act as a loss of ~20% for p-pol). Resonant condition in PRX for p-pol is not controlled and which mode will be suppressed with PRX mode healing is not controlled
- Assuming mode-mismatch m_p^2 is small and PRX mode healing effect is small, a factor of 3 reduction mostly comes from the Lawrence effect and the amount of p-pol HOM is larger by a factor of 2 compared with p-pol TEM00

Revisiting klog #7307

- Power at REFL (only s-pol component) is reduced by ~10% when the arm cavity is locked, and derived the round-trip loss to be 86(3) ppm (~10%/N_{rt})
- When unlocked, 6.1% of p-pol was created at ITMX reflection and rejected at IFI (roughly 2% is p-pol TEM00 and 4% is p-pol HOM, from the discussion in the previous page)
- Therefore, total power reduction when cavity is locked was actually 6.1%+~10% = ~16%
- Assuming $\beta^2 << 1$ (which is justified since ~2% is p-pol TEM00) and mode-mismatch to be small (mode matching ratio measured to be 91(1)%), the actual round-trip loss was ~16%/10^3 = ~160 ppm (the original estimate was an underestimate)