Adaptive Robust Control Systems for KAGRA Vibration Isolation System

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Motivation

I need a topic for my PhD.

Motivation

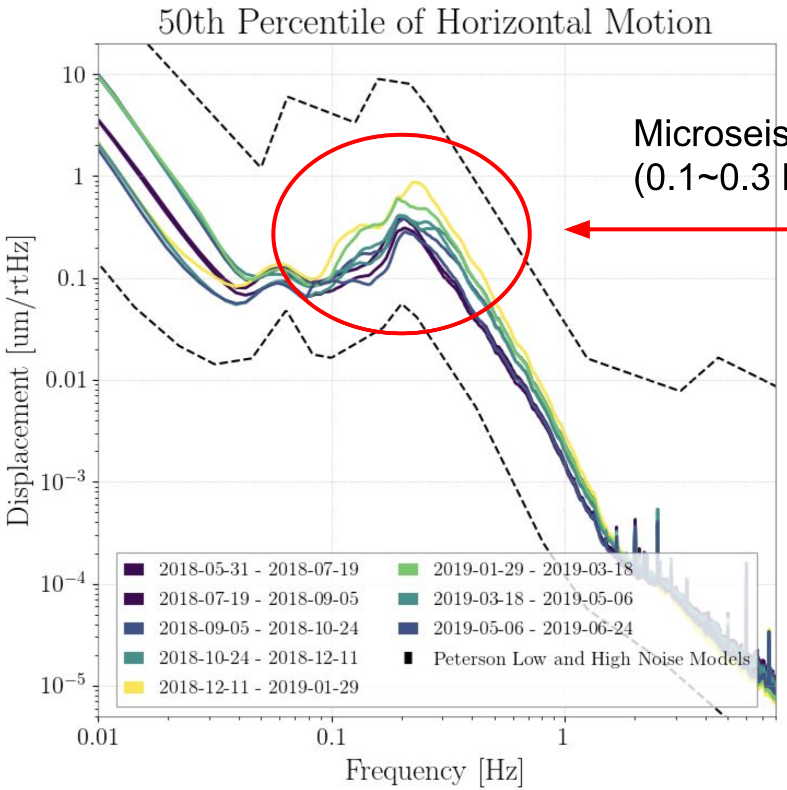
Suppressing microseismic noise was almost impossible before because

1. the geophones have poor sensitivity at low frequencies,
2. while the LVDTs are coupled to seismic noise.

After sensor correction, this becomes possible because the LVDT signals are “corrected” by the seismometers so they are not coupled to seismic noise.

So, I was asked to reshape the control filter for the preisolator to suppress the microseismic noise.

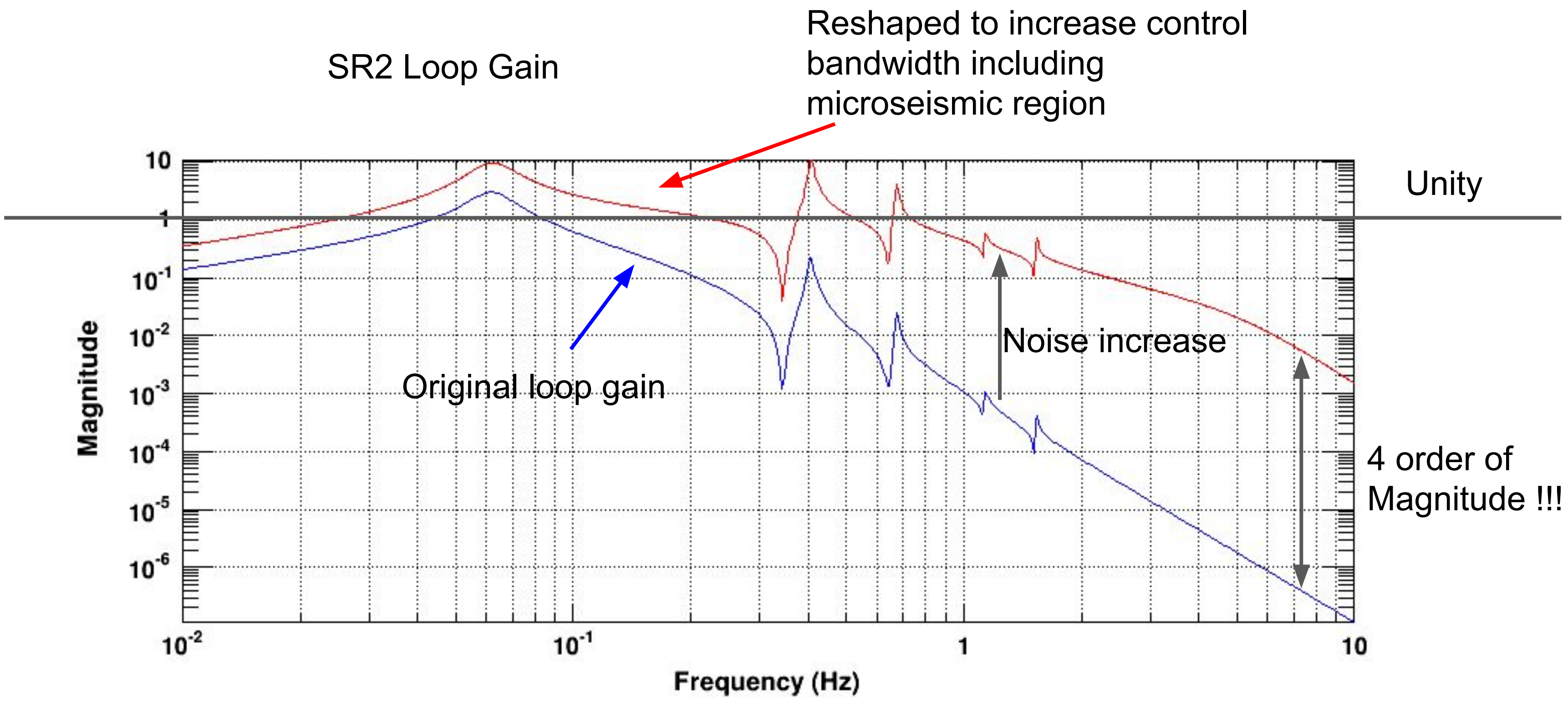
Seismic Noise ASD



Microseismic noise (0.1~0.3 Hz)

Credits: Miyo, Kouseki

Motivation


SR2 Loop Gain
Reshaped to increase control bandwidth including microseismic region
Unity
Noise increase
Original loop gain
4 order of Magnitude !!!
Magnitude
Frequency (Hz)
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-2}
10^{-1}
1
10

Questions...

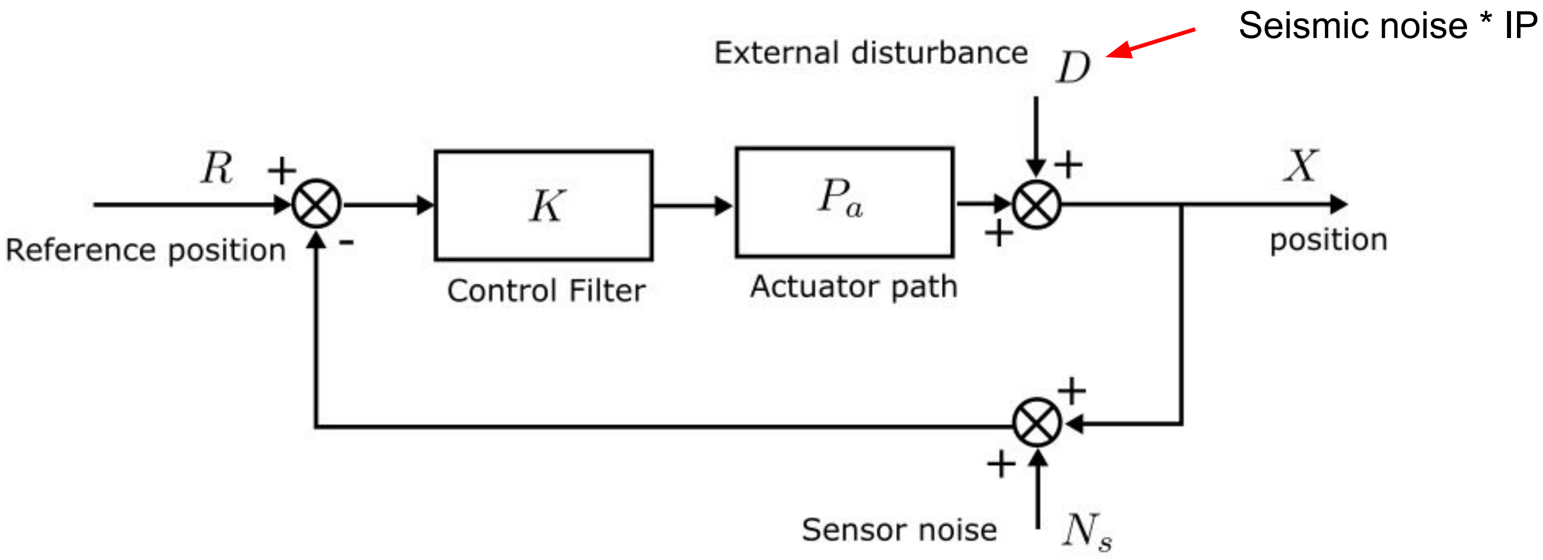
Do we really need additional gain?

At which frequencies do we need gain?

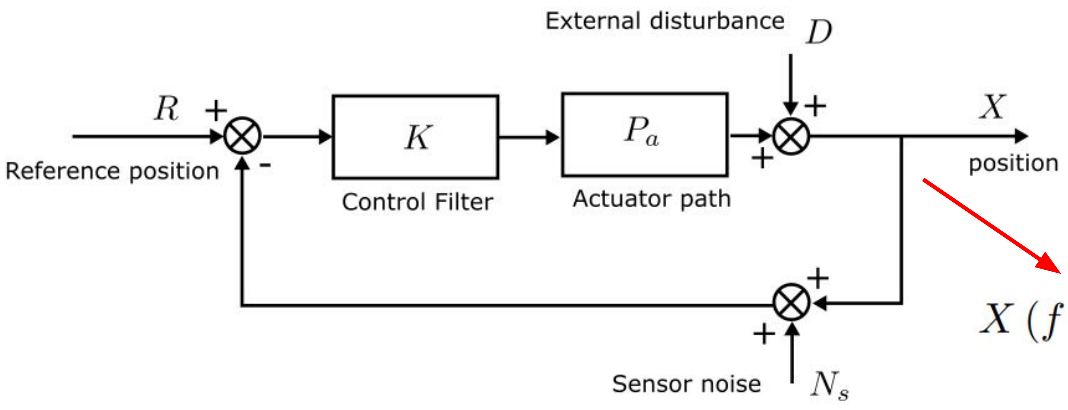
More importantly, at which other frequencies do we want NO gain?

More more importantly, how much gain?

Problem definition (Perfect sensor correction)



Problem definition



$$X\,(f>0) = \left(\frac{1}{1+P_aK}\right)D - \left(\frac{P_aK}{1+P_aK}\right)N_s,$$

$$\left(\frac{1}{1+P_aK}\right) + \left(\frac{P_aK}{1+P_aK}\right) = 1.$$

Simultaneous minimization is impossible.

Problem solving (Naively)

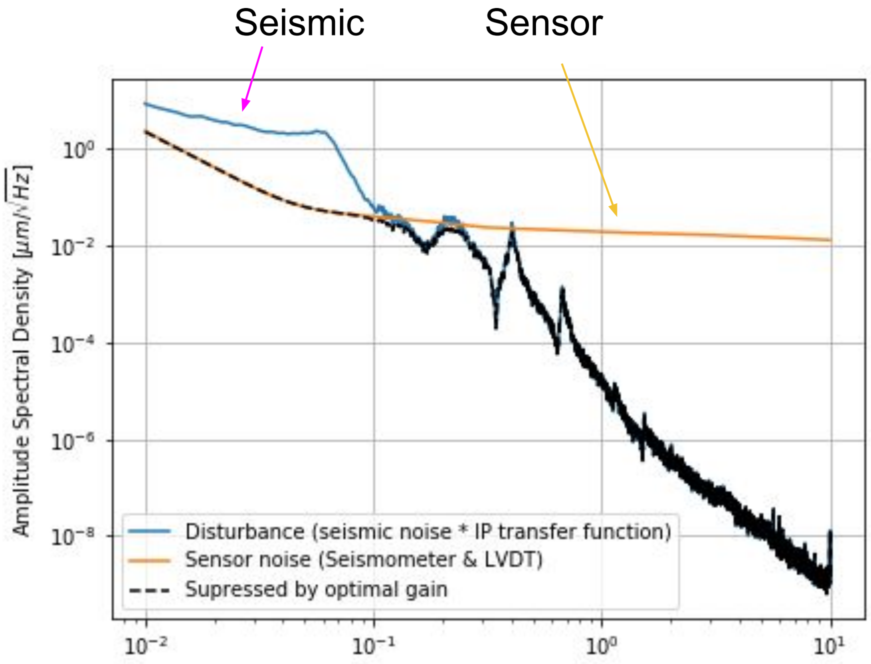
$$X\left(f>0\right)=\left(\frac{1}{1+P_aK}\right)D-\left(\frac{P_aK}{1+P_aK}\right)N_s,$$

Power spectral density (PSD)

$$\langle X^2\rangle=\left|\frac{1}{1+P_aK}\right|^2\langle D^2\rangle+\left|\frac{P_aK}{1+P_aK}\right|^2\langle N_s^2\rangle.$$

Find critical point

$$\frac{d}{dK}\langle X^2\rangle=0, \qquad K=K_{crit}=\frac{\langle D^2\rangle}{\langle N_s^2\rangle}\frac{1}{P_a},$$



Black line: Not min{*D,Ns*}, it's lower than that

Theoretical Optimal Gain

$$K = K_{crit} = \frac{\langle D^2 \rangle}{\langle N_s^2 \rangle} \frac{1}{P_a},$$

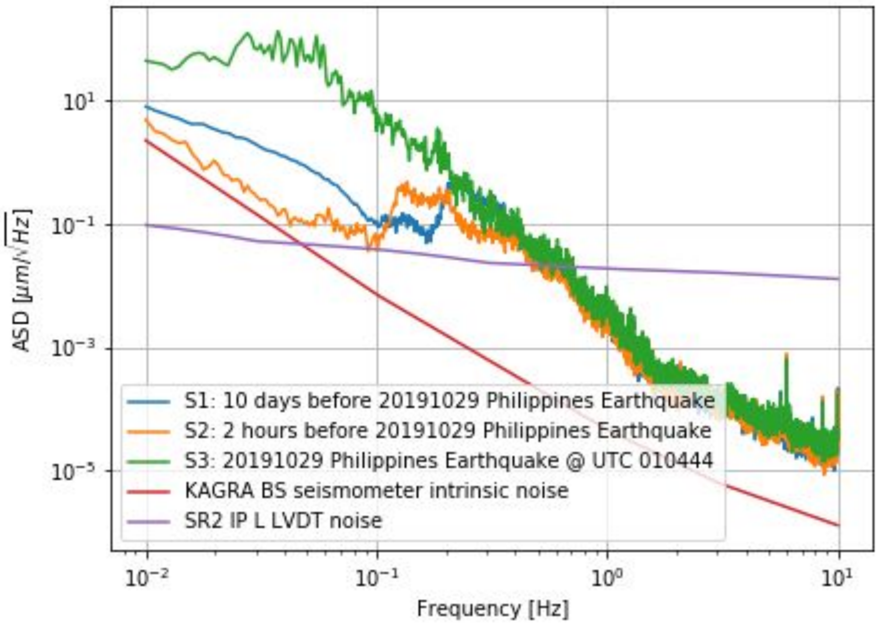
Loop gain has zero phase, but varies in magnitude.
It’s a zero phase filter, which is non-casual.

Evaluate the optimal gain at critical frequencies then use them as a reference to design a suboptimal controller.

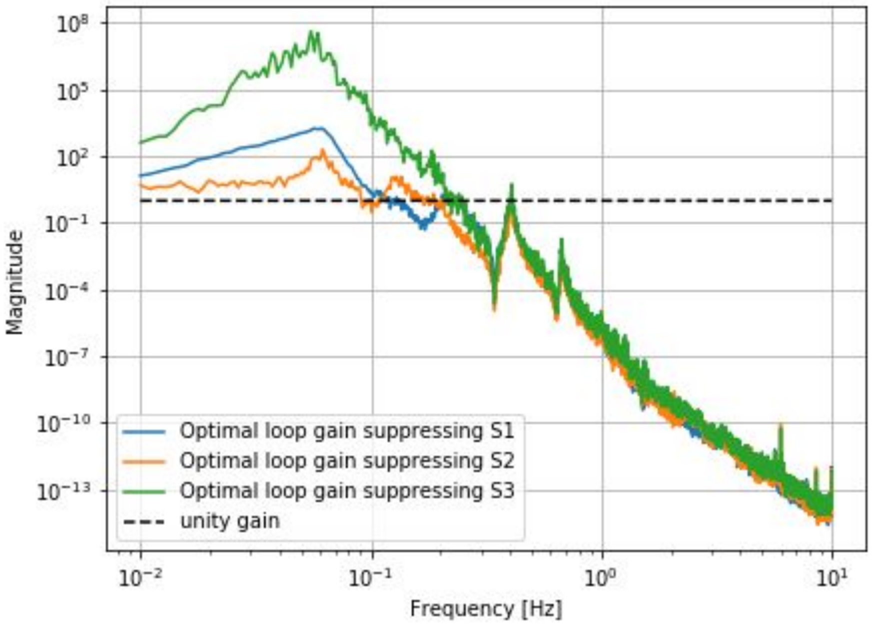
Probably work.
But, the disturbance level is not static.
→ Optimal gain is not static!

Seismic noise is not static

Seismic noise and sensor noise



Optimal loop gains



Solutions

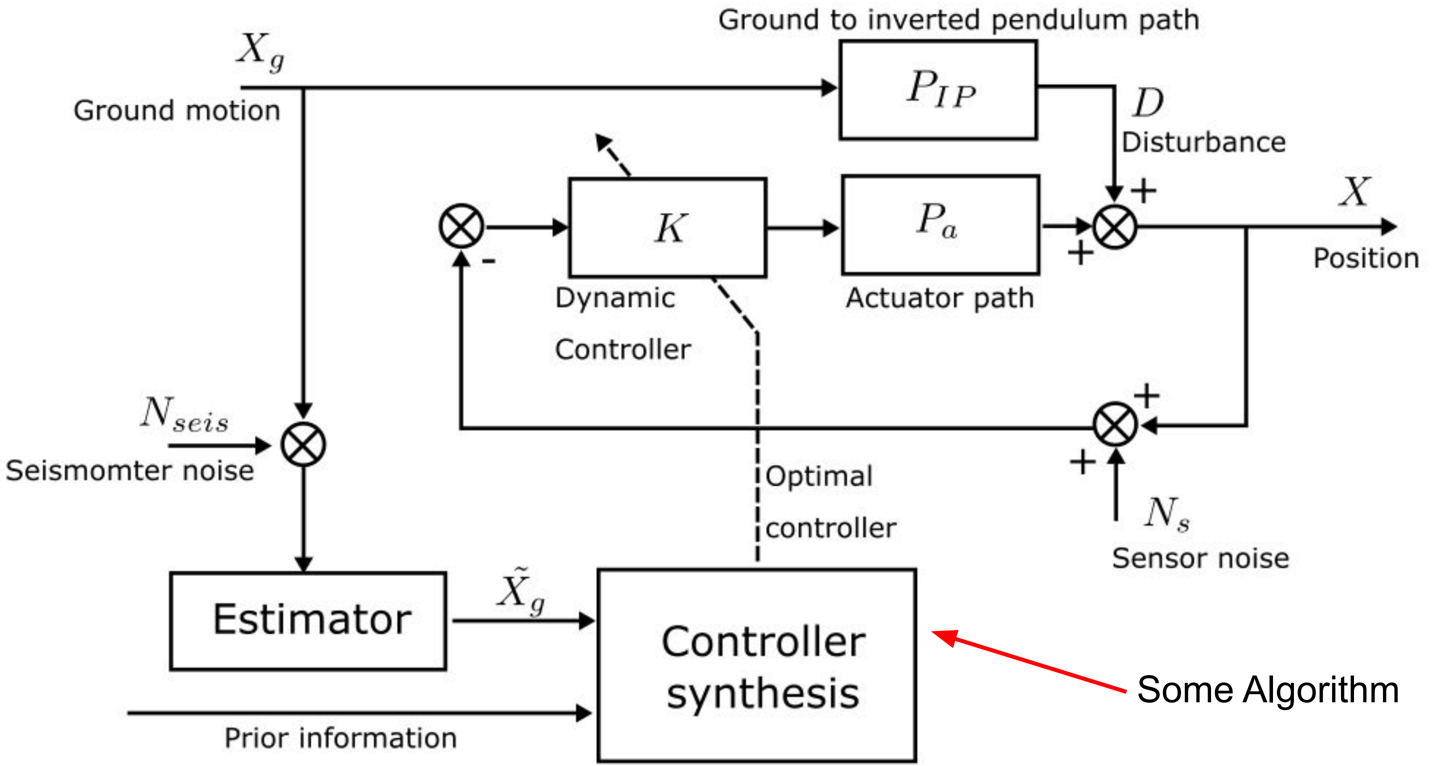
1. We can have Lucia, Fabian and Shoda-san keep updating the controllers according to the real-time seismic noise level during observation (Labor-intensive).*

Or,

2. We can use an algorithm to keep generating optimal controllers which can be automatically updated (Computationally costly).

*Presentation ends here if everyone picks 1.

Only if we pick the second solution



Controller Synthesis

In principle, we can simply preshape a filter and then apply an optimization algorithm to automatically tune the coefficients so that the residual motion is minimized.

However, stability is not guaranteed.

Robust control methods provides solution for synthesizing controllers that has guaranteed performance (stability) even if there is slight uncertainty in the system.

Let’s take a look to H_2 and H_∞ methods which are some well established, readily available and easy to implement methods. And later we shall see how we can use those methods to minimize residual motion.

Robust control methods: H_2 and H_∞ methods

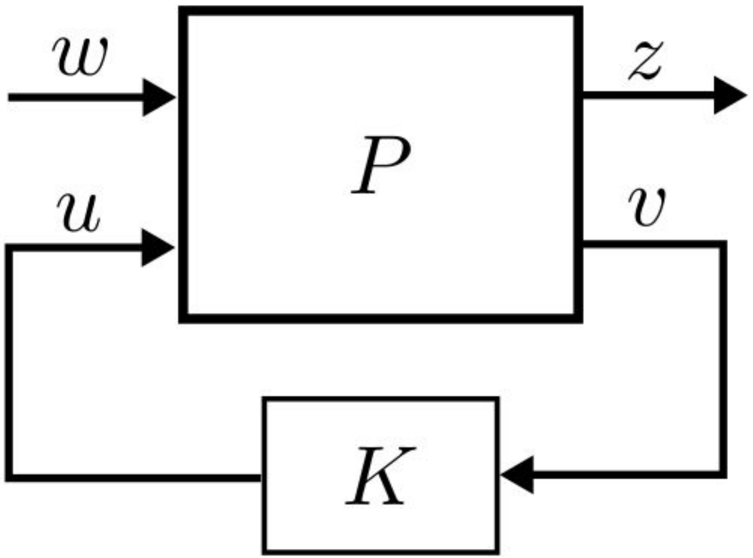
w: External inputs (setpoint, disturbance, noise,etc) (Normalized)

z: Error signals that we want to minimize (error signals, controller output, residual motion, etc)

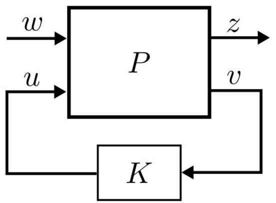
v: Input signals to the controller (error signal, measurements, etc)

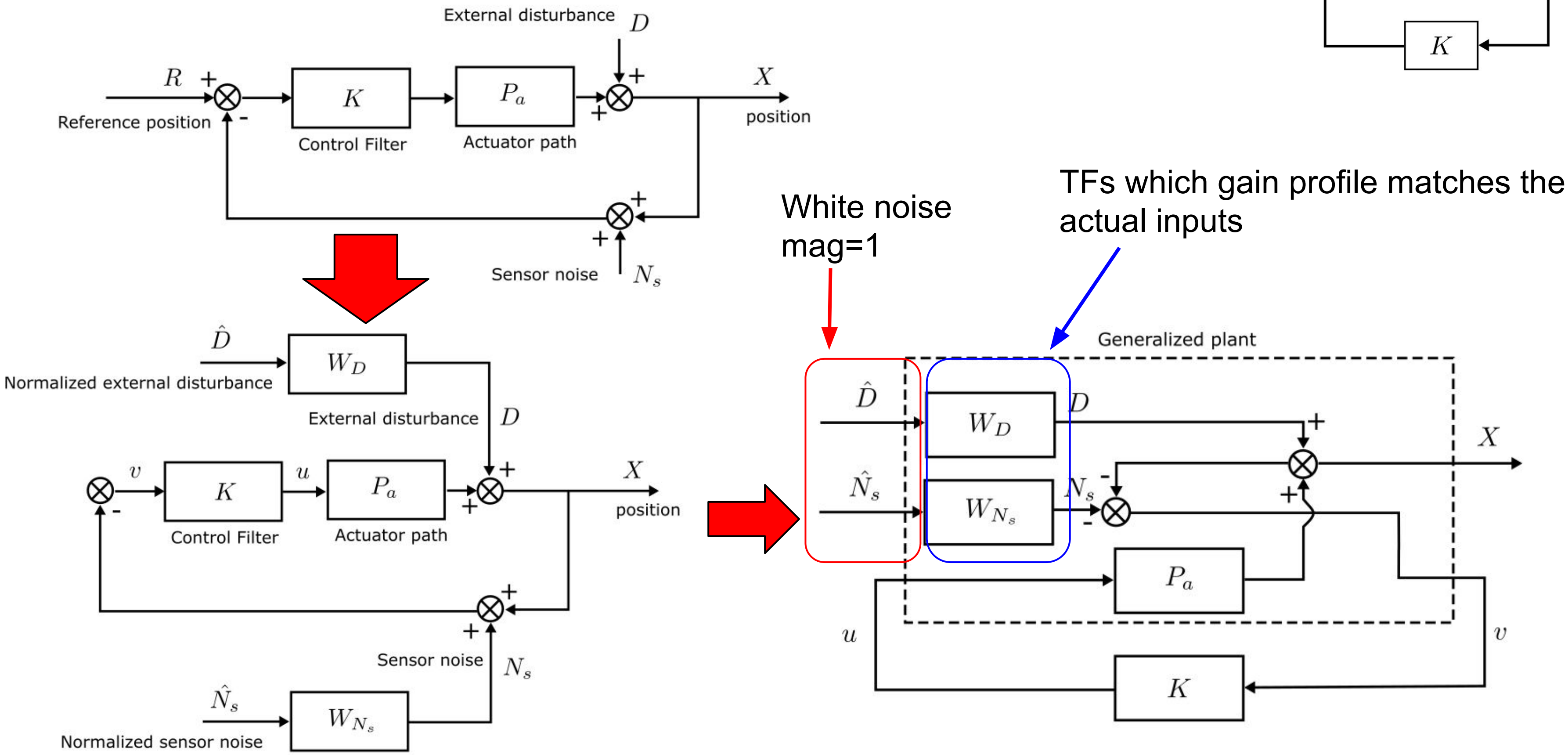
u: Manipulating variables (actuator signals, etc)

Generalized Plant Representation



Robust control methods: H_2 and H_∞ methods



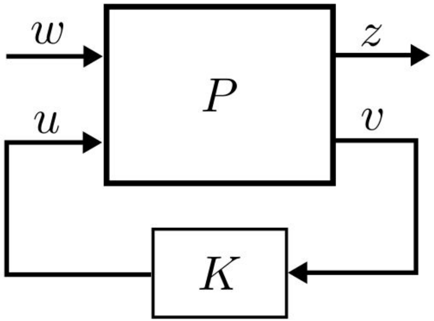


H_2 and H_∞ Optimal Controller

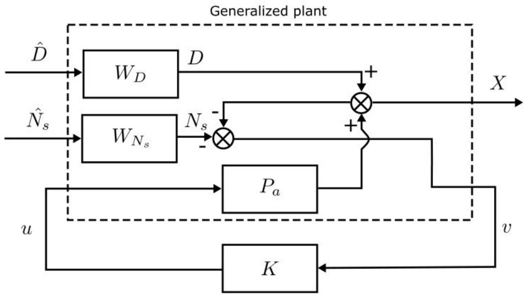
If *G* is the closed-loop transfer function matrix such that $z = G(P, K)\, w$

then, an H_2 optimal controller is a stable controller that will minimize the 2-norm of *G*. $||G||_2$

Likewise, an H_∞ optimal controller is a stable controller that will minimize the infinity norm $||G||_\infty$



Interpretation of H_2 and H_∞ Norms



H2 Norm:

$$||G||_2 = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{Tr}\left[G^{\mathrm{H}}G\right] d\omega}$$
$$= \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty} \left|\frac{W_D}{1+P_aK}\right|^2 + \left|\frac{W_{N_s}P_aK}{1+P_aK}\right|^2 d\omega},$$

Which happens to be the integrated RMS of the residual motion.

H∞ Norm:

$$||G||_\infty = \sup_{\omega\in\mathbb{R}} \bar{\sigma}\left(G\right),$$

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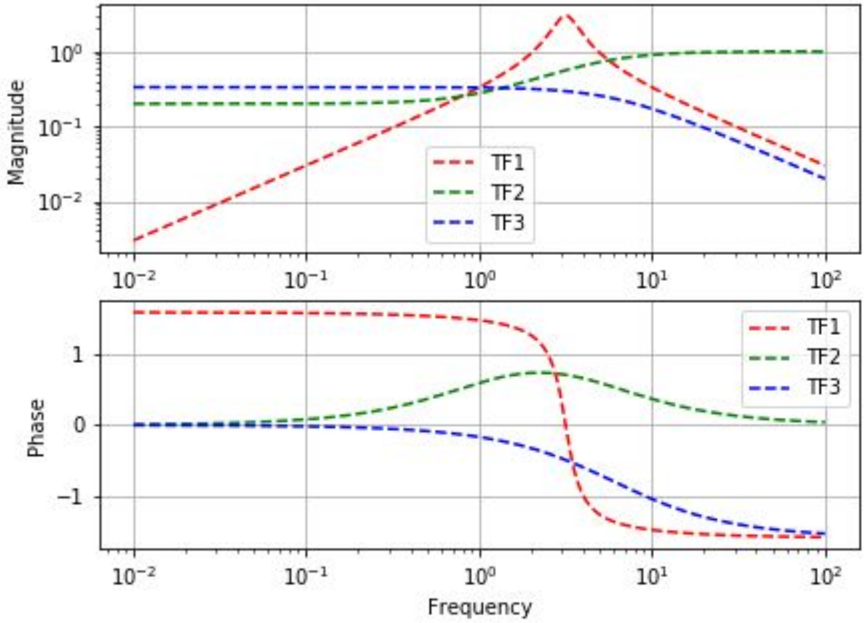
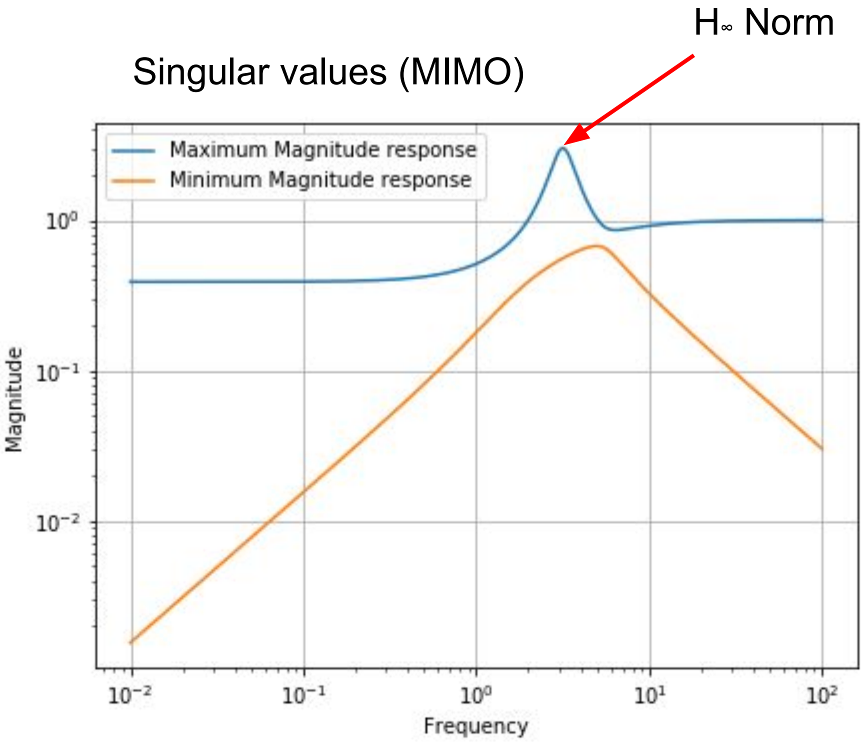
Which is (sort of) the maximum of the maximum magnitude response, i.e the maximum residual motion (resonance peaks, etc).

More on H_∞ Norm

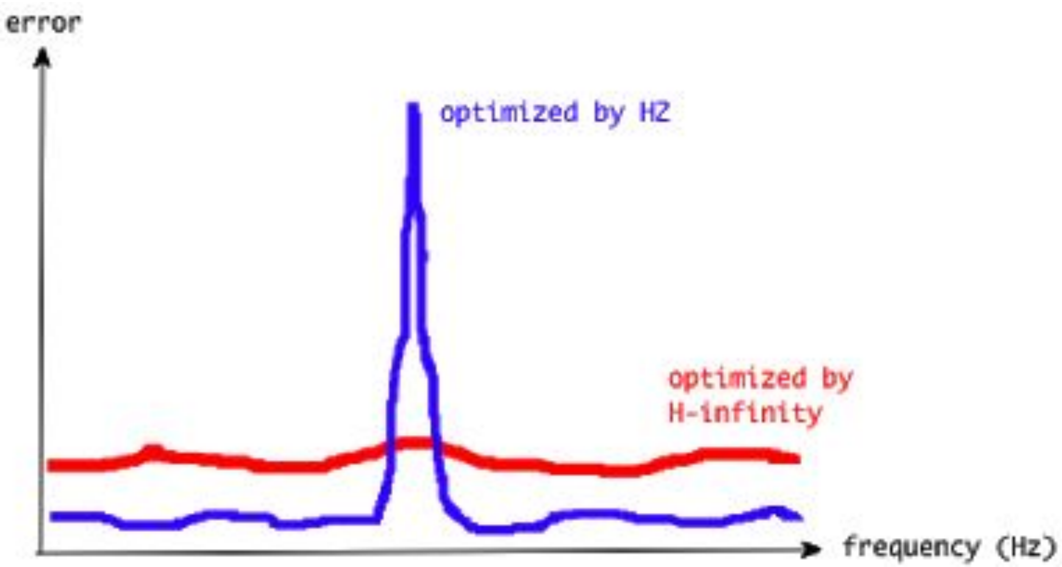
Classical Bode Plots (multiple SISO)



Singular values (MIMO)

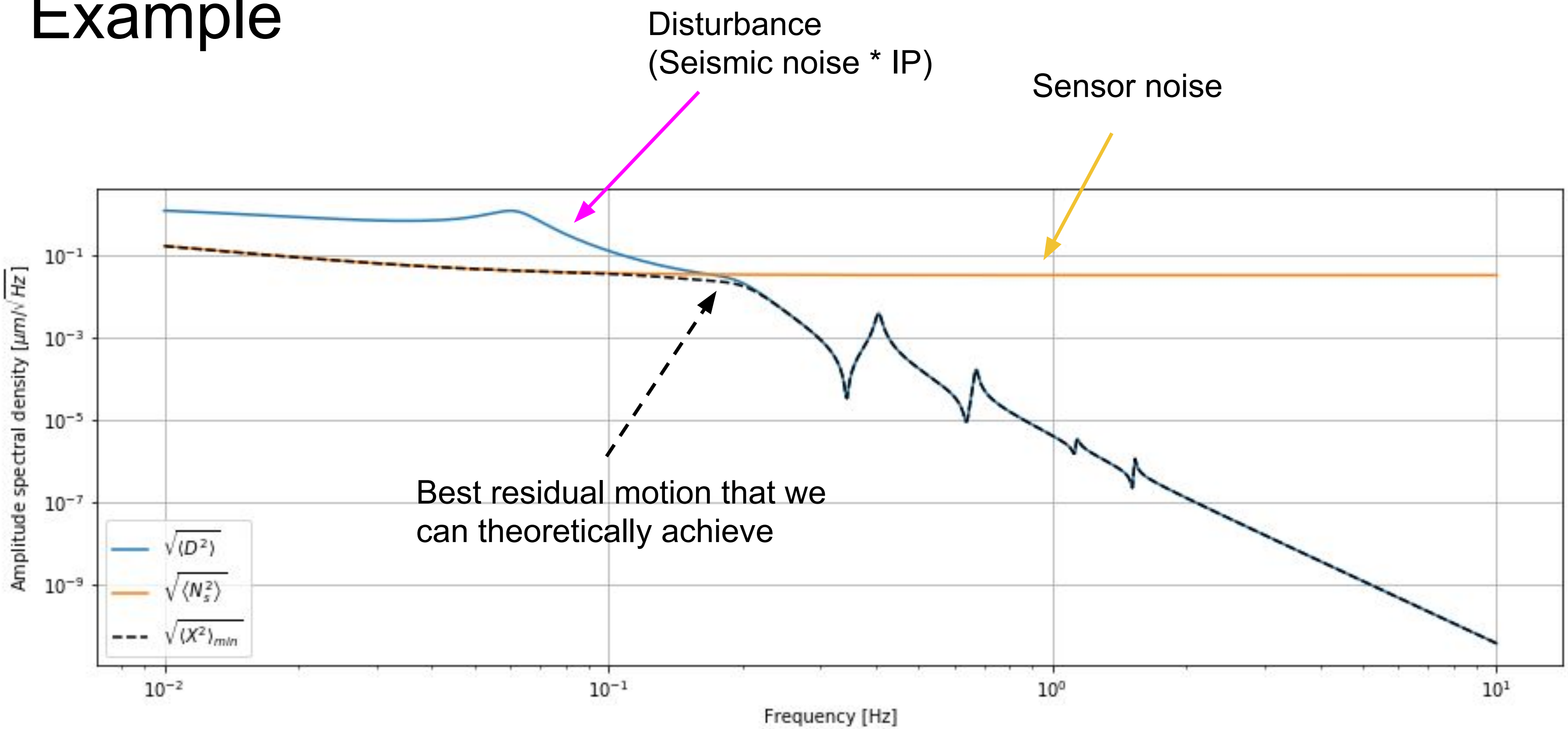


Difference between H_2 and H_∞ Optimal controllers



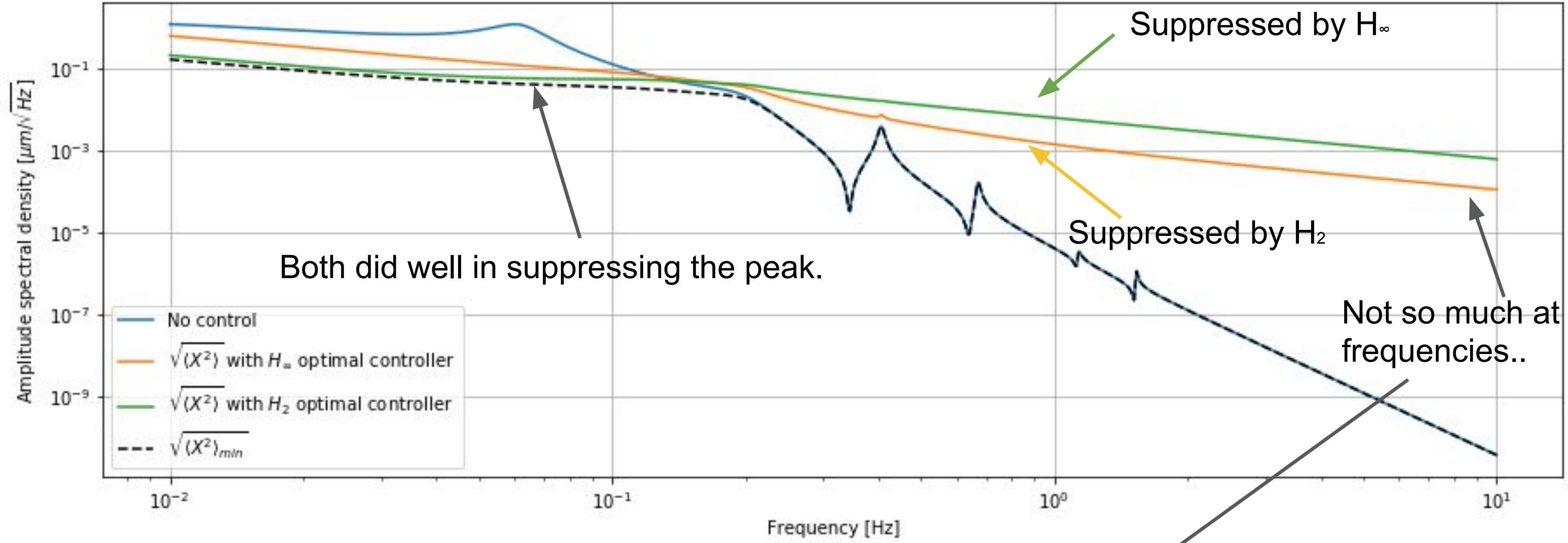
Credits: Masaaki Nagahara
Source: http://sparseland.blogspot.com/2012/05/h-2-versus-h.html

Example



Goal: as close to theoretical optimal as possible

Suppressed by H_2 and H_∞ Optimal Controller



Suppressed by H_∞

Suppressed by H_2

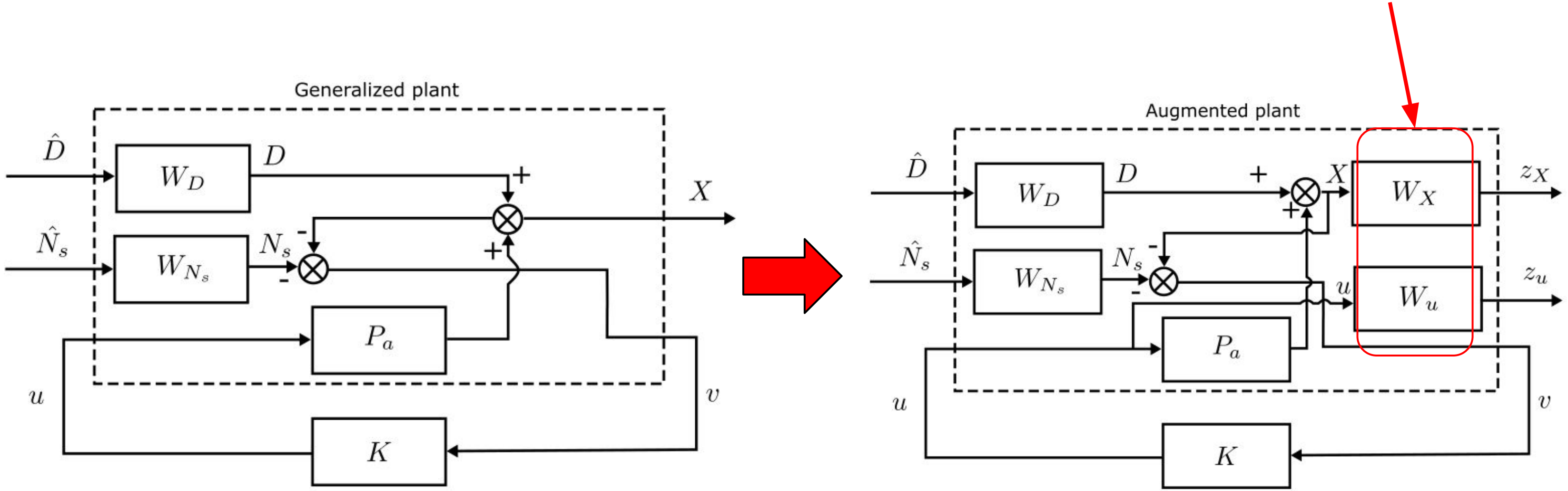
Both did well in suppressing the peak.

Not so much at high frequencies..

Because the noise is not significant compared to disturbance during the norm minimization.
How to fix?

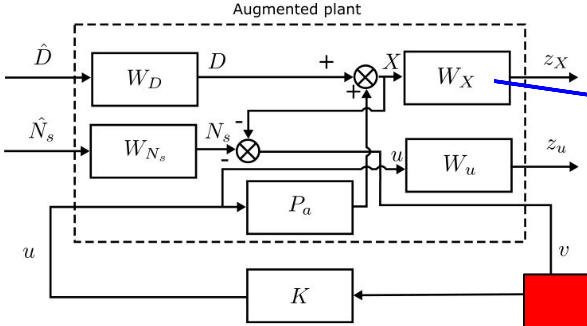
Performance weighting functions

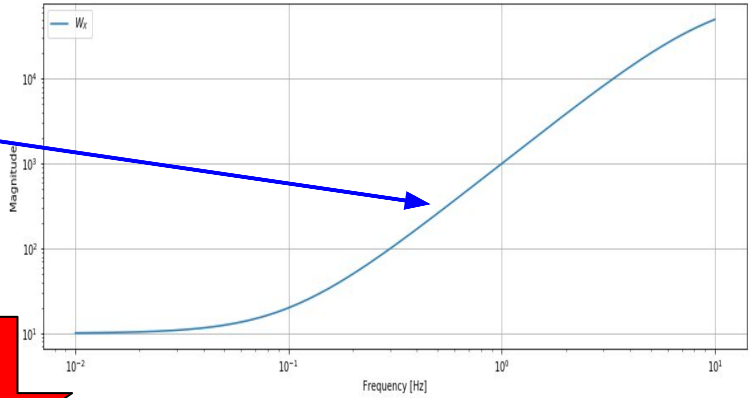
Performance weighting functions

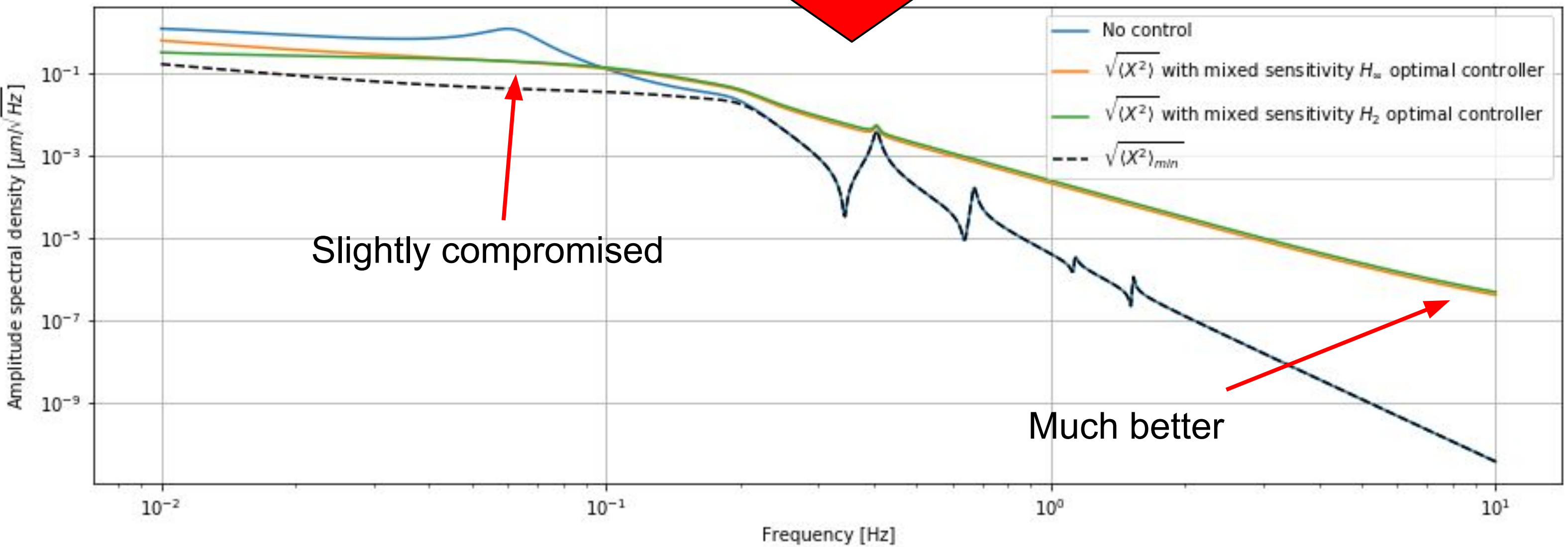


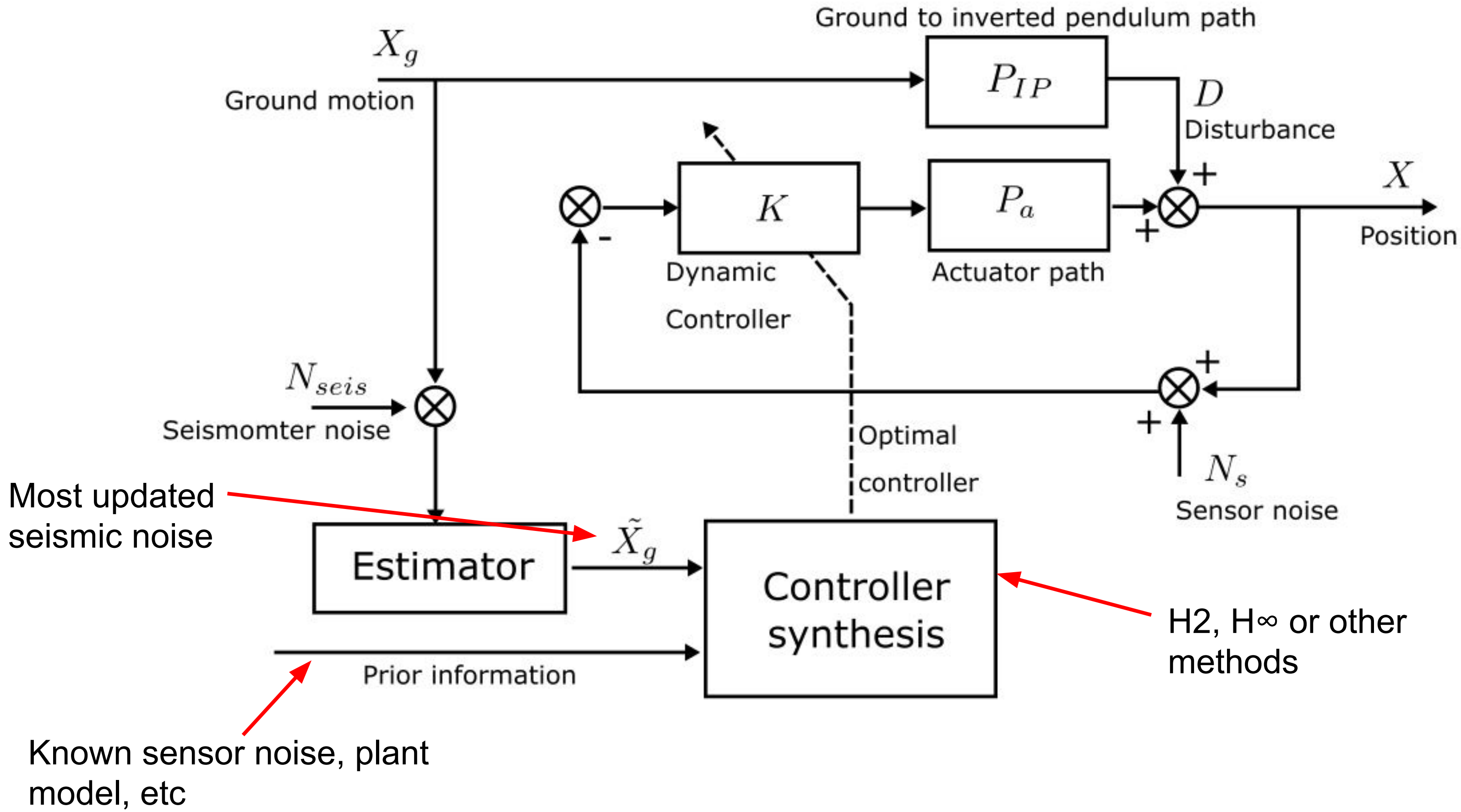
W_X: set target residual motion
W_u: Penalize unwanted actuation signal (high frequency)

Example


Augmented plant
$\hat{D}$
W_D
D
+
+
X
W_X
z_X
$\hat{N}_s$
W_{N_s}
N_s
-
-
u
W_u
z_u
P_a
u
v
K


W_X
Magnitude
Frequency [Hz]


No control
$\sqrt{\langle X^2 \rangle}$ with mixed sensitivity H_∞ optimal controller
$\sqrt{\langle X^2 \rangle}$ with mixed sensitivity H_2 optimal controller
$\sqrt{\langle X^2 \rangle_{min}}$
Amplitude spectral density [$\mu m/\sqrt{Hz}$]
Frequency [Hz]
Slightly compromised
Much better

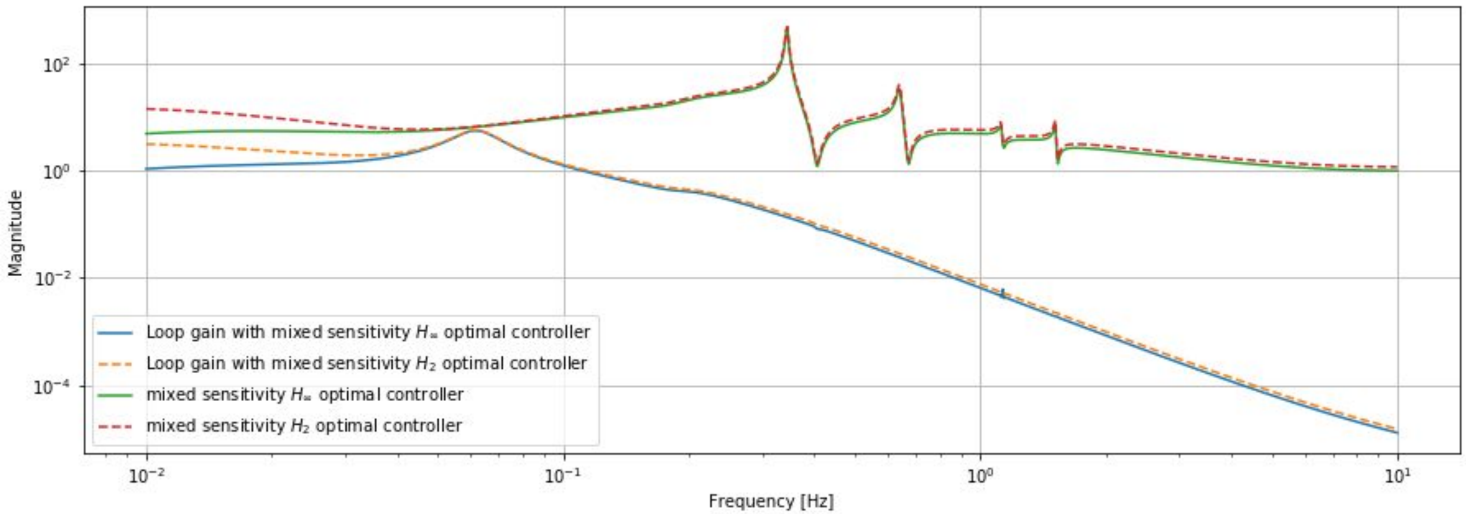

Ground to inverted pendulum path
X_g
Ground motion
P_{IP}
D
Disturbance
K
Dynamic
Controller
P_a
Actuator path
X
Position
N_{seis}
Seismomter noise
Optimal
controller
N_s
Sensor noise
Most updated
seismic noise
Estimator
$\tilde{X}_g$
Controller
synthesis
H2, H∞ or other
methods
Prior information
Known sensor noise, plant
model, etc

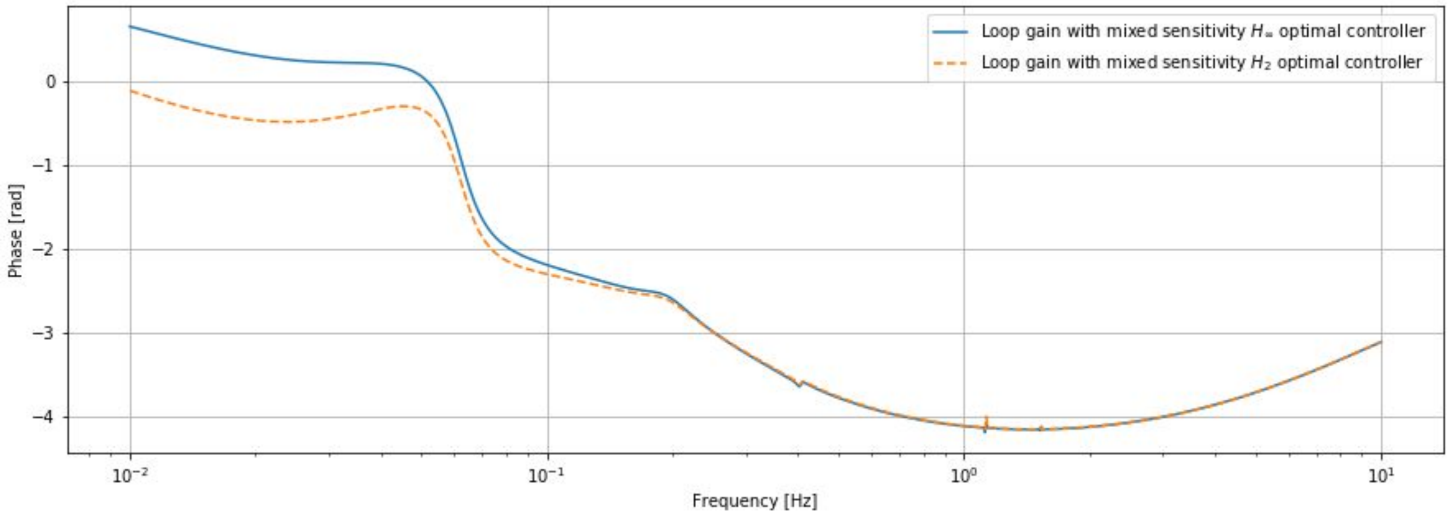
Other topics

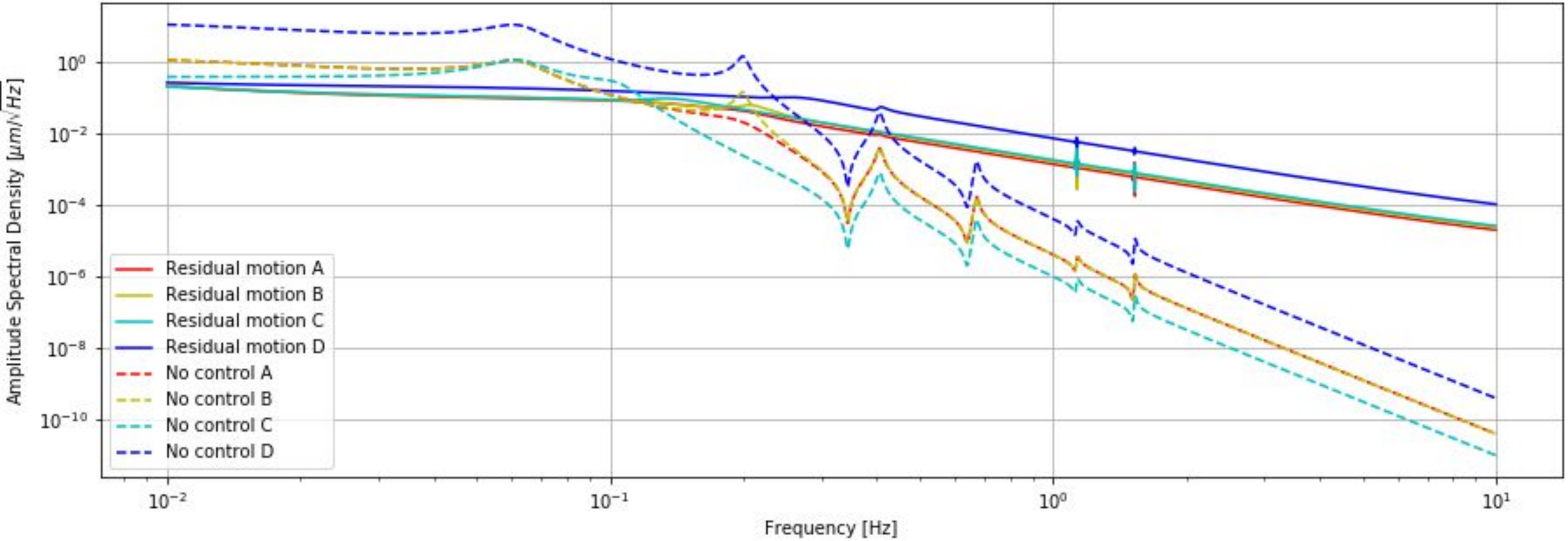
- Limitations and how do we bypass them.
- How to find good weighting functions?
- Switch modes, e.g. Calm down mode, Observation mode
- Generalization so it works for all degrees of freedom of a suspension?
- Other adaptive approaches for sensor correction, feedforward and coupling cancellation filters.

Please refer to the document for more detailed information.

Questions, comments.


Magnitude
Frequency [Hz]
Loop gain with mixed sensitivity H_∞ optimal controller
Loop gain with mixed sensitivity H_2 optimal controller
mixed sensitivity H_∞ optimal controller
mixed sensitivity H_2 optimal controller


Phase [rad]
Frequency [Hz]
Loop gain with mixed sensitivity H_∞ optimal controller
Loop gain with mixed sensitivity H_2 optimal controller



Amplitude Spectral Density [$\mu m/\sqrt{Hz}$]
Frequency [Hz]
Residual motion A
Residual motion B
Residual motion C
Residual motion D
No control A
No control B
No control C
No control D