ABCD Matrix and q-parameter evolution

ABCD Matrix Convention

There are two conventions for the definition of (x, x').
The first one is

$$x' = \frac{dx}{dz}$$

used by Kogelnik & Lee, and Gerhard Kloos.
The other one is

$$x' = n\frac{dx}{dz}$$

used by Siegman.
K&L assumes that the input and output planes are always in the vacuum (n=1).
Siegman deals with more general cases where the input and output planes may be in different medium.

q-parameter

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda}{\pi w^2(z)}$$

λ is always the wavelength of the light in the transmitting medium.

$$\lambda = \frac{\lambda_0}{n}$$

At the interface of two media, the beam size should not change.
This means that the value of q must change even if it is a flat interface plane with normal incidence.
The reduced q-parameter qr = q/n is thus convenient, because it does not change between two media.

ABCD transformation rule

For q-parameter:

$$\frac{q_2}{n_2} = \frac{A(q_1/n_1) + B}{C(q_1/n_1) + D}$$

For reduced q-parameter:

$$q_{r2} = \frac{A\,q_{r1} + B}{C\,q_{r1} + D}$$

Propagation in medium with index n

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & L/n \\ 0 & 1 \end{pmatrix}$$

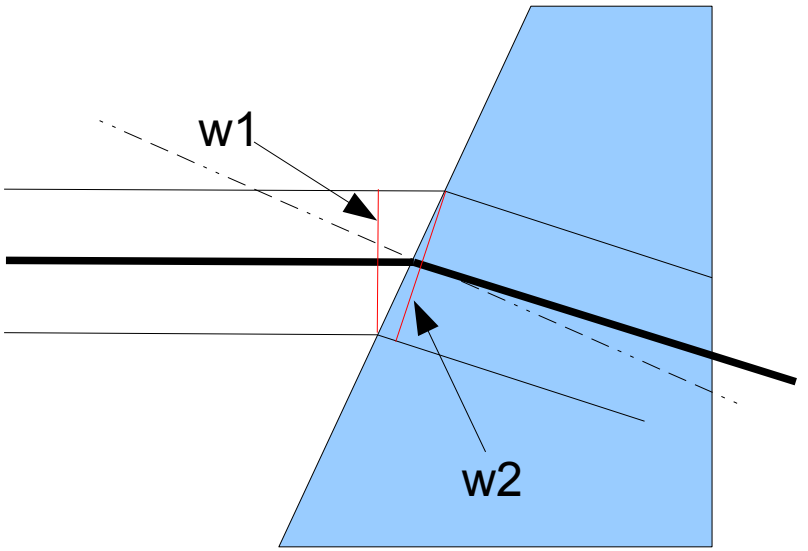
$$q_2 = q_1 + L \qquad\qquad q_{r2} = q_{r1} + L/n$$

Slanted incidence to a plane interface

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \frac{\cos\theta_2}{\cos\theta_1} & 0 \\ 0 & \frac{\cos\theta_1}{\cos\theta_2} \end{pmatrix}$$

The beam size changes between the interface. This can be derived from a purely geometrical consideration.

Note that after a transmission of an oblique optics, the beam becomes elliptic.



q-parameter in gtrace

In gtrace, q-parameters of a beam are stored in five attributes.

q: average q-parameter of the beam. If the beam is elliptic, this is a q-parameter of a circular beam best matching the elliptic one.

qx: q-parameter of the beam in x-direction.

qrx: reduced q-parameter in x-direction. qrx = qx/n

qy: q-parameter of the beam in y-direction.

qry: reduced q-parameter in y-direction. qry = qy/n

If any of the last 4 parameters are changed, all the other related parameters are automatically updated.
ABCD transformation is calculated with the reduced q-parameters. The result is reflected to the normal q-parameters immediately.