Mode matching without PRM

JGW-T1910582-v1

created: 2019, August, 22nd

Kiwamu Izumi

1 Overview

This document summarizes an estimation of the degradation in the mode matching when PRM is taken out of the main interferometer beam path. The estimation employs a number of assumptions for simplicity such that the estimation can be done in a timely manner.

As described in the succeeding sections, our results indicate that the degradation would be very small and in fact the mode matching in terms of the optical power can be higher than 99%. Therefore we conclude that the removal of PRM does not pose a critical issue as far as the mode matching is concerned.

2 Setups and assumptions

2.1 Setups

The complex q of the beam incident on IMMT1 propagating from the input mode cleaner is assumed to have

$$q_{\mathrm{immt1}} = 6.118 + 16.8465\,\mathrm{i}. \tag{1}$$

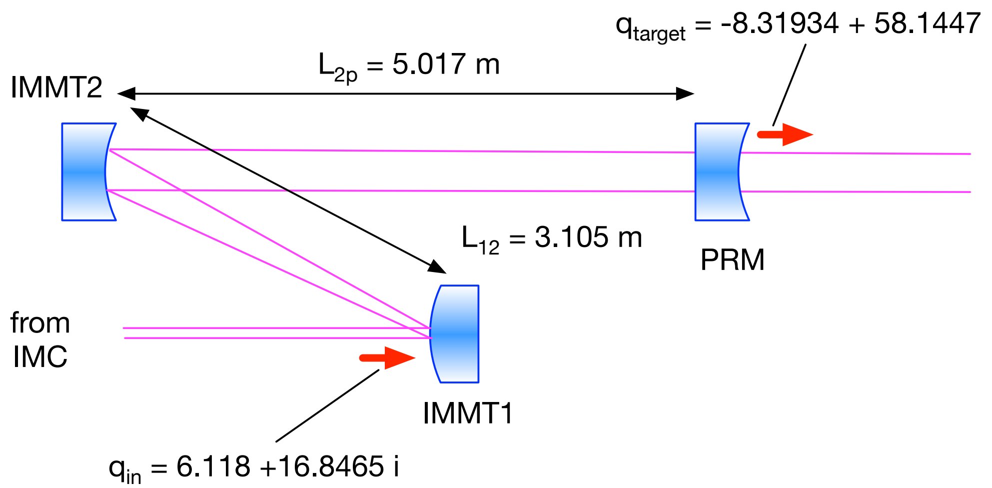

Figure 1: The setup under consideration.

This value is taken from the wiki page for the IOO parameters [2]. Similarly, the target q value for the beam at the HR surface of PRM propagating towards PRM [3] is set to be

$$q_{\text{target}} = -8.31934 + 58.1447\,\mathrm{i}. \tag{2}$$

The nominal distances, radii of curvature and other relevant optical parameters are summarized in figure 1 and table 1.

2.2 Assumptions

- No astigmatisms are included. This is equivalent to virtually set every incident angle to zero.

- The current realization (as of writing) in the actual interferometer is

Description	Symbol	Value	references
RoC of IMMT1	R_1	-8.91038 m	[4] [2]
RoC of IMMT2	R_2	14.005 m	[4] [2]
RoC of PRM	R_p	458.1285 m	[5] [1]
Thickness of PRM	h_p	0.1 m	[5]
Distance between IMMTs 1 and 2	L_{12}	3.105 m	[2]
Distance between IMMT2 and PRM	L_{2p}	5.017 m	[2] †
Refractive index at 1064 nm	n	1.4496	[7]

Table 1: A list of the parameters and actual values used for the estimation. †: I assumed this to be the distance to the AR surface of PRM in the calculation.

equal to the ones expected from the nominal parameters listed in figure 1 and table 1.

- The measurement values for the radii of curvature for IMMTs are used as opposed to the use of those from the specifications.
- All the calculations presented in this document are derived by applying the ray transfer matrix analysis (i.e., ABCD matrix).

2.3 How good is it at the moment?

Based on the assumptions described above, one can estimate how good the mode-matching is at this moment. Propagating $q_{\rm in}$ to that at the HR surface of PRM, one can obtain a q value of

$$q_{\rm PRM,HR} = -4.7310 + 56.8174\,{\rm i}. \quad \text{(with PRM included)}. \tag{3}$$

This corresponds to a mode-overlap of 99.55%. This may be optimistic according to the actual beam size measurement performed back in 2017 [6]. However, we think this is a good representation of the current mode-matching.

The mode overlap is computed using the following expression [8],

$$(\text{mode overlap}) = \frac{4 z_{R1} z_{R2}}{(z_{R1} + z_{R2})^2 + (d_1 - d_2)^2}, \tag{4}$$

where z_R's are Rayleigh ranges and d's are the distances to the waist location.

3 Calculation and results

3.1 Calculation

We now repeat the same calculation as performed in the previous section, but this time without PRM. Instead of letting the beam go through the mirror substrate of PRM, it now propagates through a free space for a distance of 0.1 m which is equal to the thickness of PRM. This gave a q value at the point where the HR surface of PRM was as

$$q_{\text{PRM,HR}} = -7.7924 + 56.1168\,\mathrm{i}. \quad (\text{without PRM}). \tag{5}$$

The mode overlap with respect to the target q value (2) can be then computed to be 99.85%. Comparing it against the one with PRM included (see section 2.3), one can notice that the mode-overlap slightly improves as PRM is taken out. But this seems to be just by chance.

Additionally, we move the position of IMMT2 in the calculation to see

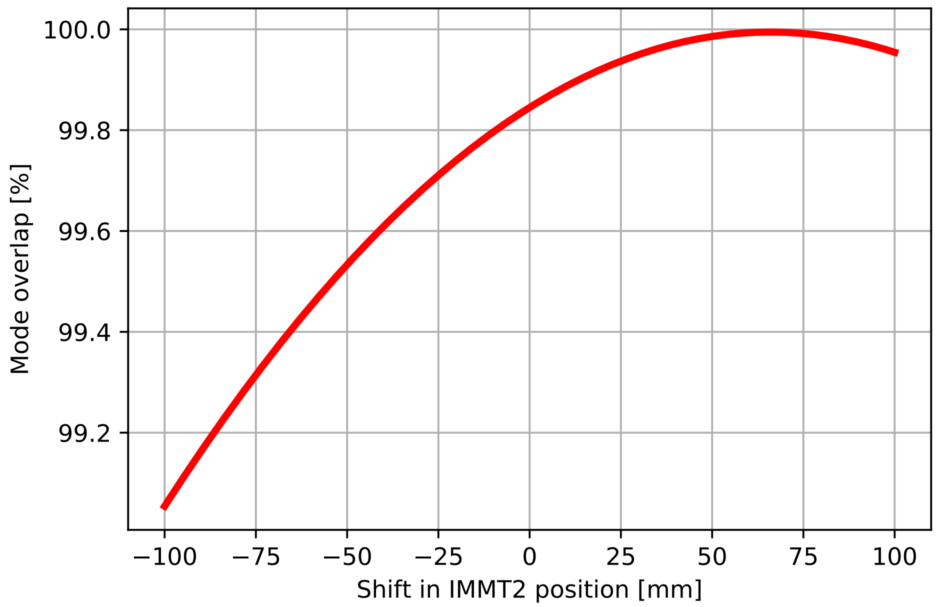

Figure 2: The mode overlap as a function of shift in the IMMT2 position. Positive values in the horizontal axis correspond to expansion of the distance between IMMTs.

how sensitive the mode overlap is against variation in its position. This was done by varying the distance between IMMTs or L_{12}. The distance from IMMT2 to PRM is also changed at the same time accordingly in order to simulate realistic relocation of IMMT2. The result is shown in figure 2. As shown in the figure, the mode matching seems robust against uncertainties in the position of IMMT2 as long as the error is on the order of 100 mm.

3.2 Interpretation

The results described above indicates that PRM does not act as a strong lens for the interferometer beam. We now verify this statement by performing an order-estimation.

Starting from the ideal interferometer beam incident on the AR side of PRM. Such a mode-matched beam must satisfy the following relation,

$$1/q_{\mathrm{target}} = \frac{n-1}{|R_{\mathrm{p}}|} + \frac{1}{q_{\mathrm{in}}}, \tag{6}$$

where we intentionally took the absolute value of R_{p} in order to avoid ambiguities due to the polarity of the curvature, and where we approximated PRM to be a thin lens. Extracting the real part of the equation above, one can obtain

$$R_{\mathrm{in}} = -\frac{|R_{\mathrm{p}}|}{n}, \tag{7}$$

where R_{in} is the curvature of the beam incident on PRM. Therefore, the incident q must be

$$q_{\mathrm{in}} = \left(-\frac{|R_{\mathrm{p}}|}{n} - i\frac{\lambda}{\pi w^2}\right)^{-1}, \tag{8}$$

where w is the beam radius of the beam at PRM. Since $(n/R_{\mathrm{p}})^2 \ll (\lambda/(\pi w^2))^2$ in our case, q_{in} can be approximated to be

$$q_{\mathrm{in}} \approx -\left(\frac{\pi w^2}{\lambda}\right)^2 \frac{n}{|R_{\mathrm{p}}|} + \mathrm{i}\frac{\pi w^2}{\lambda}. \tag{9}$$

Similarly, one can obtain q for the target mode as

$$q_{\mathrm{target}} \approx -\left(\frac{\pi w^2}{\lambda}\right)^2 \frac{1}{|R_{\mathrm{p}}|} + \mathrm{i}\frac{\pi w^2}{\lambda} \tag{10}$$

These expressions indicate that the Rayleigh range (i.e., the imaginary part of q) remains almost the same when the beam propagates through PRM. The only difference is the waist location i.e., the real part in q.

We are now ready to evaluate the mode overlap (4) of $q_{\rm in}$ with respect to the one after PRM i.e., $q_{\rm target}$. We first use the relation $z_{R1} = z_{R2}$ so that

$$\text{(mode overlap)} = \left\{ 1 + \left(\frac{\Delta d}{2 z'_R} \right)^2 \right\}^{-1}, \tag{11}$$

where Δd is the difference in the waist locations between the two modes defined by

$$\Delta d = \left(\frac{\pi w^2}{\lambda} \right)^2 \frac{1-n}{|R_{\rm p}|} \approx \frac{(z'_R)^2 (1-n)}{|R_{\rm p}|}, \tag{12}$$

and z'_R is the Rayleigh range. We approximated that $\pi w^2/\lambda \approx z'_R$ which is not always true, but this is valid in our case because the beam at around PRM is well within its Rayleigh range.

Finally, expanding equation (11) to the first order of Δd and plugging equation (12), one can arrive at

$$\text{(mode overlap)} \approx 1 - \left(\frac{z'_R (1-n)}{2 R_{\rm p}} \right)^2 = 1 - 8.1 \times 10^{-4}, \tag{13}$$

where $z'_R = 58.144$ m and $R_{\rm p} = 458.125$ m are used. This means that the mode overlap degrades by only a very small amount when PRM is taken out.

Therefore, we conclude that PRM is not a strong lens and in fact the mode overlap would not significantly be affected by the removal of PRM.

References

[1] Y. Aso et al., "Interferometer design of the KAGRA gravitational wave detector," Phys. Rev. D, 88, 043007 (2013)

[2] IOO optical parameters, JGW wiki (as of August 22nd, 2019) http://gwwiki.icrr.u-tokyo.ac.jp/JGWwiki/KAGRA/Subgroups/IOO/OptParam

[3] This value is picked up from the IOO optical parameters page in JGW wiki (as of August 22nd, 2019). However, the number appears to be the one for the beam propagating towards PRM as opposed to that towards PR2. So I flipped the sign of the real part.

[4] E. Hirose, "ROC measurement of the KAGRA IMMT1 nad IMMT2," JGW-T1605981-v1 (2016) https://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=5981

[5] "Actual Optical Parameters of the Main Interferometer," JGW wiki (as of 2019/Aug/23) http://gwwiki.icrr.u-tokyo.ac.jp/JGWwiki/LCGT/subgroup/ifo/MIF/OptParam

[6] H. Takeda et al.,"Beam profile for a beam from IFI to IMMT2 and reflected beam at IMMT2," klog 3245 (2017) http://klog.icrr.u-tokyo.ac.jp/osl/?r=3245

[7] Refractiveindex.info
https://refractiveindex.info/?shelf=glass&book=fused_silica&page=Malitson

[8] E. Siegman, “Lasers” (1986)