

Distance measures

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Distance measures

Distance measures are important Figure of Merits to evaluate the detector sensitivity, the typical ones are:

- Horizon distance
- Range distance

In this study we extend Range distance from the single detector into the multi-detectors network.

We also apply Range not only for the detection (SNR) but also for the sky localization confidence area (CA).

Distance measure : Horizon

- Horizon distance, d^h is defined as the farthest luminosity distance the given source could ever be detected above threshold [[Finn, Chernoff, PRD 47, 2198](#)].
- If we consider the inspiral only, SNR, ρ is

$$\rho^2 = \frac{5}{96\pi^{4/3}} \widehat{\mathcal{A}}^2 \widehat{\mathcal{M}}^{5/3} f_{7/3}, \text{ where } f_{7/3} = \int_0^\infty df \left[f^{7/3} S_h(f) \right]^{-1}$$

- In the actual calculation, we should use IMR waveform (particularly for BBH) instead of TaylorF2, and redshifted with $M_{\text{obs}} = (1+z)M_{\text{src}}$

Distance measures : Range

- Range, R is defined as $4/3\pi R^3 = V_z$ where V_z is “Redshifted Volume” defined with comoving distance, D_c [[arxiv:1709.08079](https://arxiv.org/abs/1709.08079)]:

$$V_z = \frac{\int_{D_c < d^h} \frac{D_c^2}{1+z(D_c)} dD_c d\Omega \sin \iota d\iota d\psi}{\int \sin \iota d\iota d\psi}$$

- In the small redshift ($z < 0.1$), where the Universe is well described by Euclidean geometry, R can be approximated as [[Finn, Chernoff](#)]: $R \sim 1/2.264 d^h$

Latest BNS sensitivities ($1/2.264 d^h$)

- O3 (actual)
140 Mpc (L1), 112 Mpc (H1), 51 Mpc (V1)
- O4 (estimated)
low : 180 Mpc (LIGO), 95 Mpc (Virgo)
high : 205 Mpc (LIGO), 127 Mpc (Virgo)
- O5 (estimated)
low : 397 Mpc (A+), 161 Mpc (AdV+)
high : 397 Mpc (A+), 305 Mpc (AdV+)

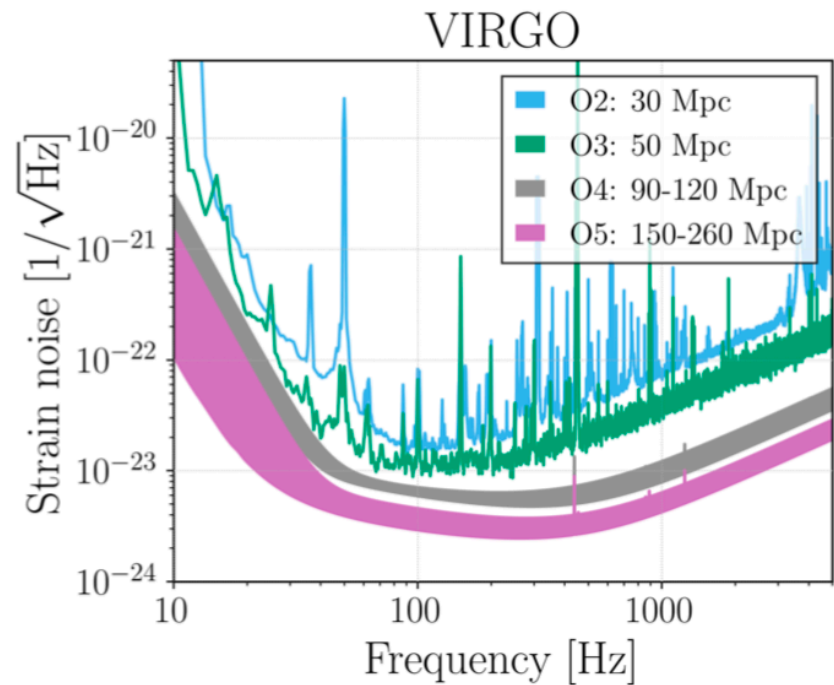
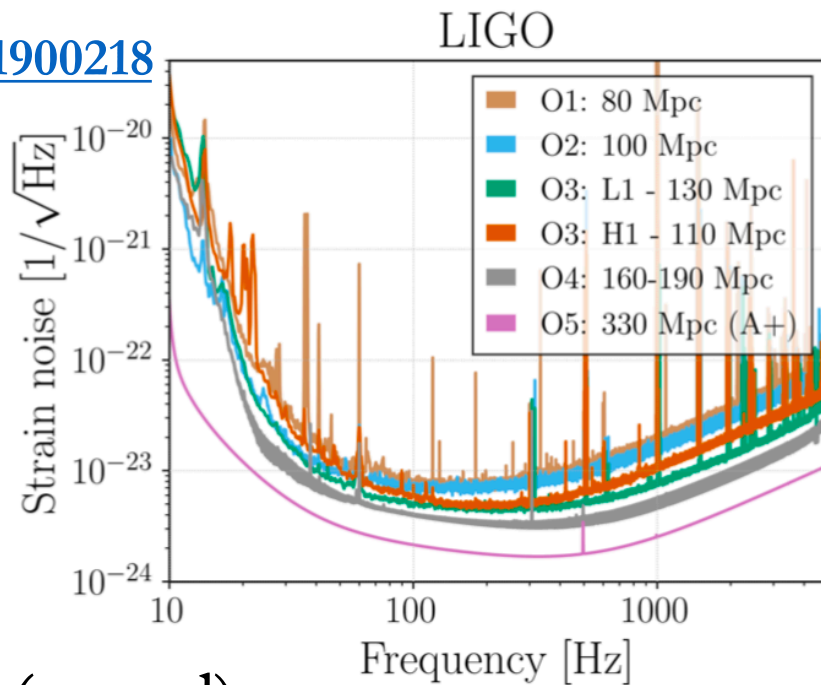
Overestimated with Euclidean geometry approx.

Latest BNS sensitivities (Range)

- O3 (actual)
130 Mpc (L1), 106 Mpc (H1), 50 Mpc (V1)
- O4 (estimated)
low : 165 Mpc (LIGO), 90 Mpc (Virgo)
high : 185 Mpc (LIGO), 119 Mpc (Virgo)
- O5 (estimated)
low : 332 Mpc (A+), 149 Mpc (AdV+)
high : 332 Mpc (A+), 265 Mpc (AdV+)

Close to the official numbers at [LIGO-P1900218](#)

LIGO-P1900218



- O3 (actual)
130 Mpc (L1), 106 Mpc (H1), 50 Mpc (V1)
- O4 (estimated)
low : 165 Mpc (LIGO), 90 Mpc (Virgo)
high : 185 Mpc (LIGO), 119 Mpc (Virgo)
- O5 (estimated)
low : 332 Mpc (A+), 149 Mpc (AdV+)
high : 332 Mpc (A+), 265 Mpc (AdV+)

Network Range

- Now we extend the idea of Range ($4/3\pi R^3 = V_z$) to multi-detector network with the same formula:

$$V_z = \frac{\int_{D_c < d^h} \frac{D_c^2}{1+z(D_c)} dD_c d\Omega \sin \iota d\iota d\psi}{\int \sin \iota d\iota d\psi}$$

- $d^h(\theta, \varphi, \psi, \iota)$ is the comoving distance which satisfies
 - Network SNR = ρ_n (considering antenna pattern and each duty cycle, ϵ)
 - At least N detectors have single SNR > ρ_swith typical values: N=2, $\rho_s=4$, $\rho_n=12$, $\epsilon=0.7$

Network Range

- If SNR of each detector is completely independent, the network range can be obtained as

$$\sqrt{\sum R_i^2}, \text{ where } R_i \text{ is the range for each detector.}$$

- This is not true because SNR is highly correlated with each detector due to the orbital inclination, ι and partially correlated with the antenna patterns
- KAGRA's contribution can be more appropriately evaluated as the network instead of single detector

BNS network ranges (100% duty cycle)

- O3 (actual)
114 Mpc (H1:106, L1:130, V1:50)
- O4 (estimated)
low : 162 Mpc (LIGO: 165, Virgo: 90)
high : 185 Mpc (LIGO: 185, Virgo: 119)
- O5 (estimated)
low : 320 Mpc (A+: 332, AdV+: 149)
high : 346 Mpc (A+: 332, AdV+: 265)

with $N=2$, $\rho_s=4$, $\rho_n=12$, $\varepsilon=1.0$

BNS network ranges (70% duty cycle)

- O3 (actual)
93 Mpc (H1:106, L1:130, V1:50)
- O4 (estimated)
low : 133 Mpc (LIGO: 165, Virgo: 90)
high : 154 Mpc (LIGO: 185, Virgo: 119)
- O5 (estimated)
low : 262 Mpc (A+: 332, AdV+: 149)
high : 288 Mpc (A+: 332, AdV+: 265)

with $N=2$, $\rho_s=4$, $\rho_n=12$, $\varepsilon=0.7$

Sky localization Range

- Now we extend the idea of Range ($4/3\pi R^3 = V_z$) from the detection (SNR) to the sky localization with the same formula but extended for d^h

$$V_z = \frac{\int_{D_c < d^h} \frac{D_c^2}{1+z(D_c)} dD_c d\Omega \sin \iota d\iota d\psi}{\int \sin \iota d\iota d\psi}$$

- $d^h(\theta, \varphi, \psi, \iota)$ is defined as the distance which satisfies
 - Sky localization (90% C.L.) = $\alpha \text{ deg}^2$
 - with at least N detectors (each duty cycle, ε)
with typical values: $N=3, \alpha=10, \varepsilon=0.7$

Sky localization area

- If the sky localization with $N \geq 3$ detectors and the confidence area is small ($< \sim 10 \text{ deg}^2$), it can be defined by the triangulation with the timing resolution of each detector
- The detector timing resolution can be estimated as $1/\rho/2\pi$, where ρ is

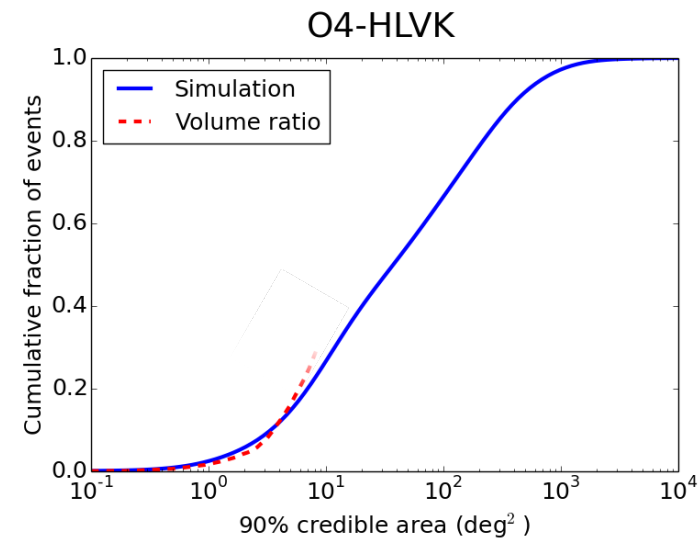
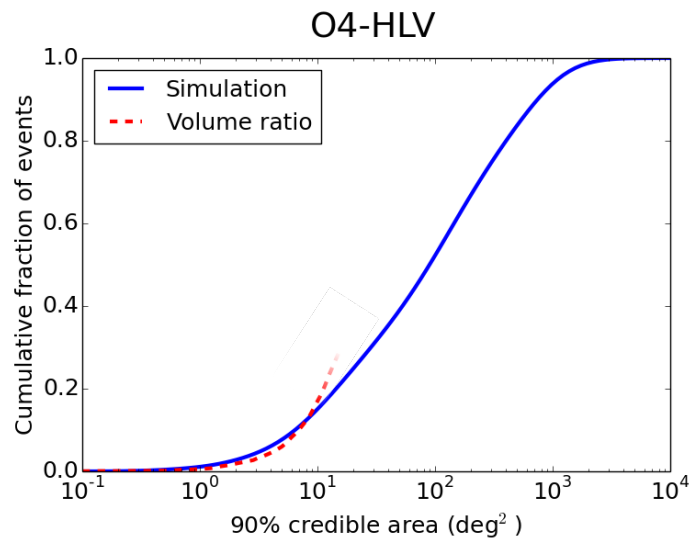
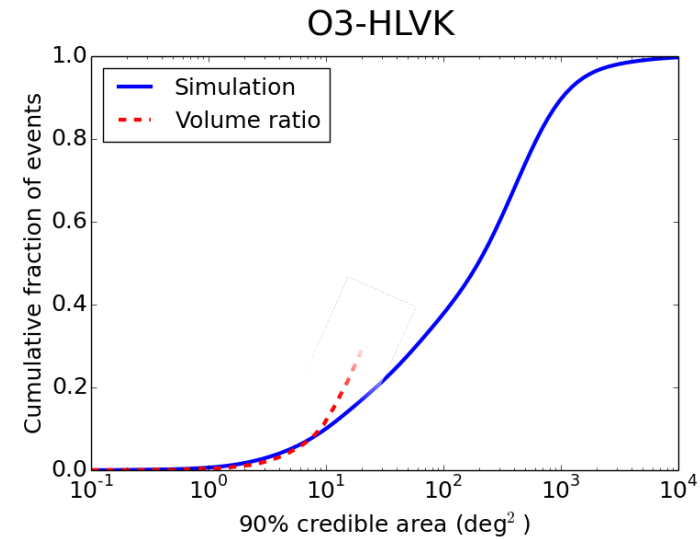
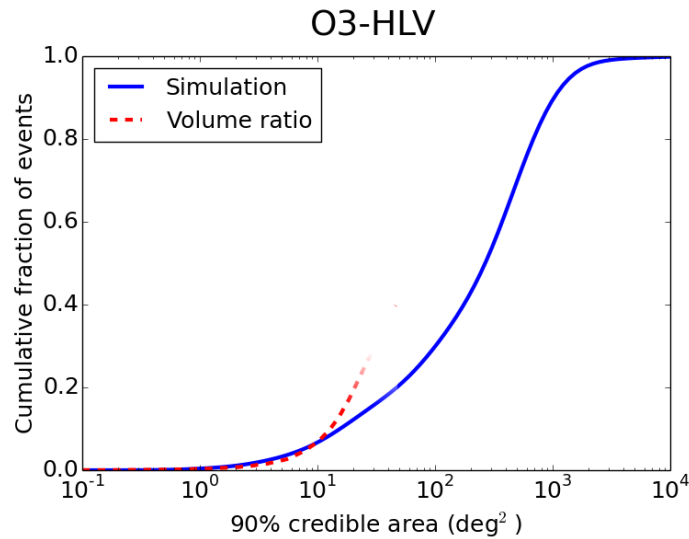
$$\rho^2 = \frac{5}{96\pi^{4/3}} \widehat{\mathcal{A}}^2 \widehat{\mathcal{M}}^{5/3} f_{16/3}, \quad f_{16/3} = \int_0^\infty df \left[f^{16/3} S_h(f) \right]^{-1}$$

Sky localization fraction

- The volume ratio, $(R_{CA}/R_{SNR})^3$ gives the event number fraction which satisfies the good sky localization (i.e. EM association)
- The volume ratio is compared with simulation results used in the Observing Scenario Paper (OSP) with Bayestar by L. Singer based on the sensitivities:
 - O3 : LIGO 120Mpc, Virgo 60 Mpc, KAGRA 25 Mpc
 - O4 : LIGO 175Mpc, Virgo 125 Mpc, KAGRA 80 Mpc

Comparison of sky area fraction

Agreement at 90% credible area $< 10 \text{ deg}^2$



Python codes

- All the ranges discussed above can be obtained by python codes available at:
<https://git.ligo.org/sadakazu.haino/kagragwinc>
- Standalone codes for cosmological calculation and waveform amplitude are implemented so it can work without LAL or other external libraries
- For the single detector ranges, agreements ($<1\%$) are checked with the original one (dependent on LAL)
<https://git.ligo.org/gwinc/inspiral-range>