JGW-G1910490

Distance measures

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Distance measures

Distance measures are important Figure of Merits
to evaluate the detector sensitivity, the typical ones are:

- Horizon distance
- Range distance

In this study we extend Range distance from
the single detector into the multi-detectors network.

We also apply Range not only for the detection (SNR)
but also for the sky localization confidence area (CA).

Distance measure : Horizon

- Horizon distance, d^{h} is defined as the farthest luminosity distance the given source could ever be detected above threshold [Finn, Chernoff, PRD 47, 2198].
- If we consider the inspiral only, SNR, ρ is

$$\rho^2 = \frac{5}{96\pi^{4/3}} \widehat{\mathcal{A}}^2 \widehat{\mathcal{M}}^{5/3} f_{7/3}, \text{ where } f_{7/3} = \int_0^\infty df \left[f^{7/3} S_h(f) \right]^{-1}$$

- In the actual calculation, we should use IMR waveform (particularly for BBH) instead of TaylorF2, and redshifted with $M_{\mathrm{obs}} = (1+z)M_{\mathrm{src}}$

Distance measures : Range

- Range, R is defined as $4/3\pi\ R^3 = V_z$
  where V_z is "Redshifted Volume" defined with comoving distance, D_c [arxiv:1709.08079]:

$$V_z = \frac{\int_{D_c < d^h} \frac{D_c^2}{1+z(D_c)} dD_c \, d\Omega \, \sin\iota \, d\iota \, d\psi}{\int \sin\iota \, d\iota \, d\psi}$$

- In the small redshift ($z < 0.1$), where the Universe is well described by Euclidean geometry, R can be approximated as[Finn, Chernoff]: $R \sim 1/2.264\ d^h$

Latest BNS sensitivities (1/2.264 d^{h})

- O3 (actual)
  140 Mpc (L1), 112 Mpc (H1), 51 Mpc (V1)
- O4 (estimated)
  low : 180 Mpc (LIGO), 95 Mpc (Virgo)
  high : 205 Mpc (LIGO), 127 Mpc (Virgo)
- O5 (estimated)
  low : 397 Mpc (A+), 161 Mpc (AdV+)
  high : 397 Mpc (A+), 305 Mpc (AdV+)

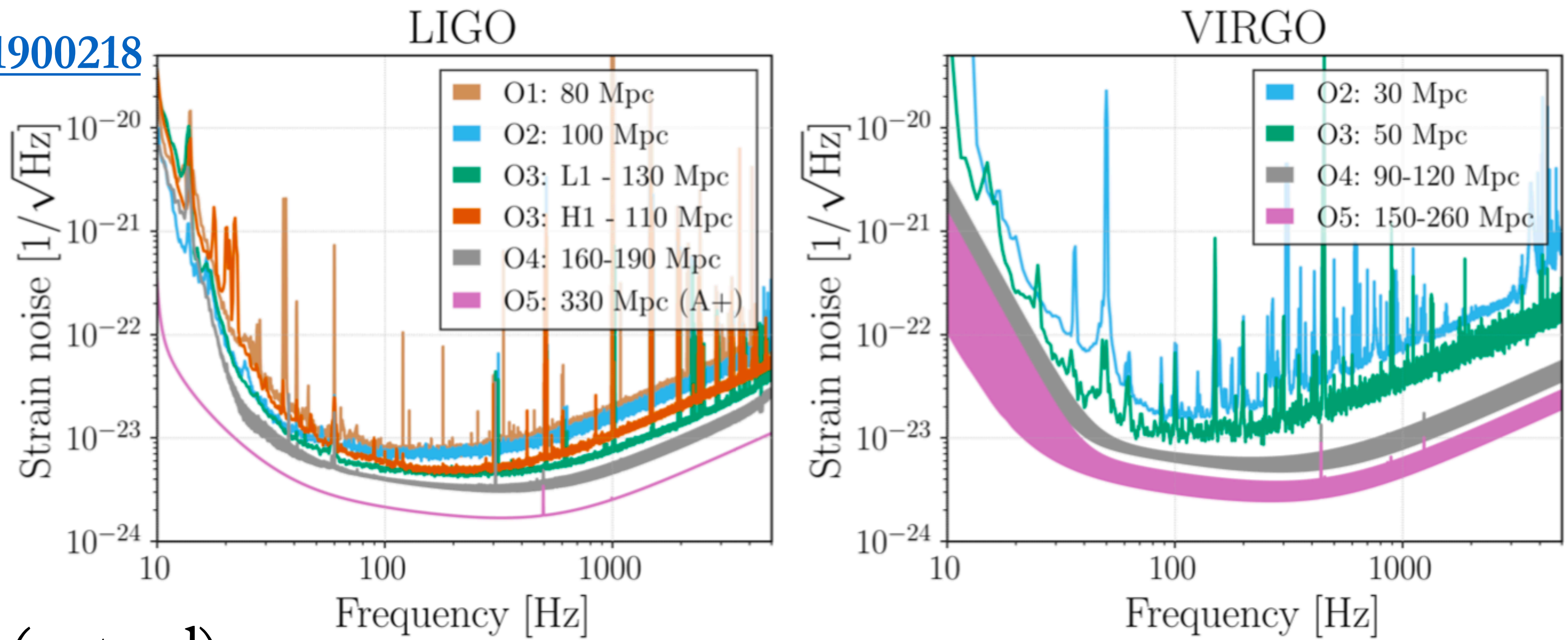
Overestimated with Euclidean geometry approx.

Latest BNS sensitivities (Range)

- O3 (actual)
  130 Mpc (L1), 106 Mpc (H1), 50 Mpc (V1)
- O4 (estimated)
  low : 165 Mpc (LIGO), 90 Mpc (Virgo)
  high : 185 Mpc (LIGO), 119 Mpc (Virgo)
- O5 (estimated)
  low : 332 Mpc (A+), 149 Mpc (AdV+)
  high : 332 Mpc (A+), 265 Mpc (AdV+)

Close to the official numbers at LIGO-P1900218

[LIGO-P1900218]



- O3 (actual)
  130 Mpc (L1), 106 Mpc (H1), 50 Mpc (V1)
- O4 (estimated)
  low : 165 Mpc (LIGO), 90 Mpc (Virgo)
  high : 185 Mpc (LIGO), 119 Mpc (Virgo)
- O5 (estimated)
  low : 332 Mpc (A+), 149 Mpc (AdV+)
  high : 332 Mpc (A+), 265 Mpc (AdV+)

Network Range

- Now we extend the idea of Range ($4/3\pi\ R^3 = V_z$) to multi-detector network with the same formula:

$$V_z = \frac{\int_{D_c<d^{\rm h}} \frac{D_c^2}{1+z(D_c)} dD_c\, d\Omega\, \sin\iota\, d\iota\, d\psi}{\int \sin\iota\, d\iota\, d\psi}$$

- $d^{\rm h}(\theta, \varphi, \psi, \iota)$ is the comoving distance which satisfies
  - Network SNR = $\rho_{\rm n}$ (considering antenna pattern and each duty cycle, ε)
  - At least N detectors have single SNR > $\rho_{\rm s}$

  with typical values: N=2, $\rho_{\rm s}$ =4, $\rho_{\rm n}$ =12, ε=0.7

Network Range

- If SNR of each detector is completely independent, the network range can be obtained as $\sqrt{\sum R_i^2}$, where R_i is the range for each detector.
- This is not true because SNR is highly correlated with each detector due to the orbital inclination, ι and partially correlated with the antenna patterns

- KAGRA's contribution can be more appropriately evaluated as the network instead of single detector

BNS network ranges (100% duty cycle)

- O3 (actual)
  114 Mpc (H1:106, L1:130, V1:50)
- O4 (estimated)
  low : 162 Mpc (LIGO: 165, Virgo: 90)
  high : 185 Mpc (LIGO: 185, Virgo: 119)
- O5 (estimated)
  low : 320 Mpc (A+: 332, AdV+: 149)
  high : 346 Mpc (A+: 332, AdV+: 265)

with N=2, ρ_s =4, ρ_n =12, ε=1.0

BNS network ranges (70% duty cycle)

- O3 (actual)
  93 Mpc (H1:106, L1:130, V1:50)
- O4 (estimated)
  low : 133 Mpc (LIGO: 165, Virgo: 90)
  high : 154 Mpc (LIGO: 185, Virgo: 119)
- O5 (estimated)
  low : 262 Mpc (A+: 332, AdV+: 149)
  high : 288 Mpc (A+: 332, AdV+: 265)

with N=2, ρ_s =4, ρ_n =12, ε=0.7

Sky localization Range

- Now we extend the idea of Range ($4/3\pi\, R^3 = V_z$) from the detection (SNR) to the sky localization with the same formula but extended for d^{h}

$$V_z = \frac{\int_{D_c < d^{\mathrm{h}}} \frac{D_c^2}{1+z(D_c)} dD_c\, d\Omega\, \sin\iota\, d\iota\, d\psi}{\int \sin\iota\, d\iota\, d\psi}$$

- d^{h} (θ, φ, ψ, ι) is defined as the distance which satisfies
  - Sky localization (90% C.L.) = α deg^2
  - with at least N detectors (each duty cycle, ε)
    with typical values: N=3, α=10, ε=0.7

Sky localization area

- If the sky localization with N>=3 detectors and the confidence area is small (<~10 deg^2), it can be defined by the triangulation with the timing resolution of each detector
- The detector timing resolution can be estimated as $1/\rho/2\pi$, where ρ is

$$\rho^2 = \frac{5}{96\pi^{4/3}} \widehat{\mathcal{A}}^2 \widehat{\mathcal{M}}^{5/3} f_{16/3}, \quad f_{16/3} = \int_0^\infty df \left[f^{16/3} S_h(f) \right]^{-1}$$

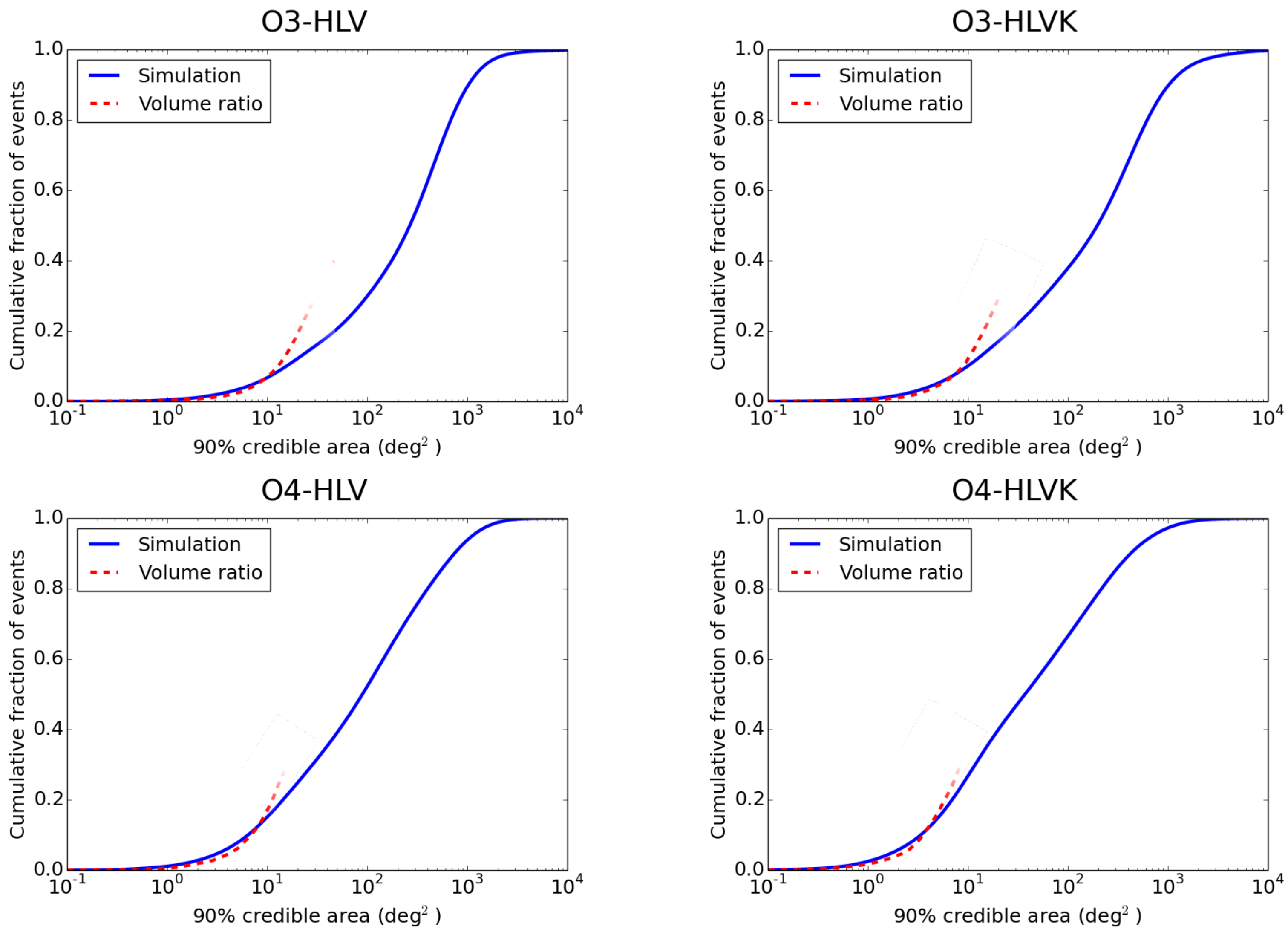
Sky localization fraction

- The volume ratio, $(R_{\mathrm{CA}}/R_{\mathrm{SNR}})^3$ gives the event number fraction which satisfies the good sky localization (i.e. EM association)

- The volume ratio is compared with simulation results used in the Observing Scenario Paper (OSP) with Bayester by L. Singer based on the sensitivities:
  - O3 : LIGO 120Mpc, Virgo 60 Mpc, KAGRA 25 Mpc
  - O4 : LIGO 175Mpc, Virgo 125 Mpc, KAGRA 80 Mpc

Comparison of sky area fraction

Agreement at 90% credible area < 10 deg2



Python codes

- All the ranges discussed above can be obtained by python codes available at: https://git.ligo.org/sadakazu.haino/kagragwinc
- Standalone codes for cosmological calculation and waveform amplitude are implemented so it can work without LAL or other external libraries
- For the single detector ranges, agreements (<1%) are checked with the original one (dependent on LAL) https://git.ligo.org/gwinc/inspiral-range

Distance measure : Horizon

- Horizon distance, d^{h} is defined as the farthest luminosity distance the given source could ever be detected above threshold [Finn, Chernoff, PRD 47, 2198].
- If we consider the inspiral (TaylorF2) only,

$$d^{\mathrm{h}} = 2\sqrt{\frac{5}{96}\frac{c\,(G\mathcal{M}c^3)^{5/6}}{\pi^{-2/3}}} \times \sqrt{I_7}, \quad \text{where} \quad I_7 = \int \frac{f^{-7/3}}{S_h(f)}\,df.$$

- In the actual calculation, we should use IMR waveform (particularly for BBH) instead of TaylorF2, and redshifted with $M_{\mathrm{obs}} = (1+z)M_{\mathrm{src}}$