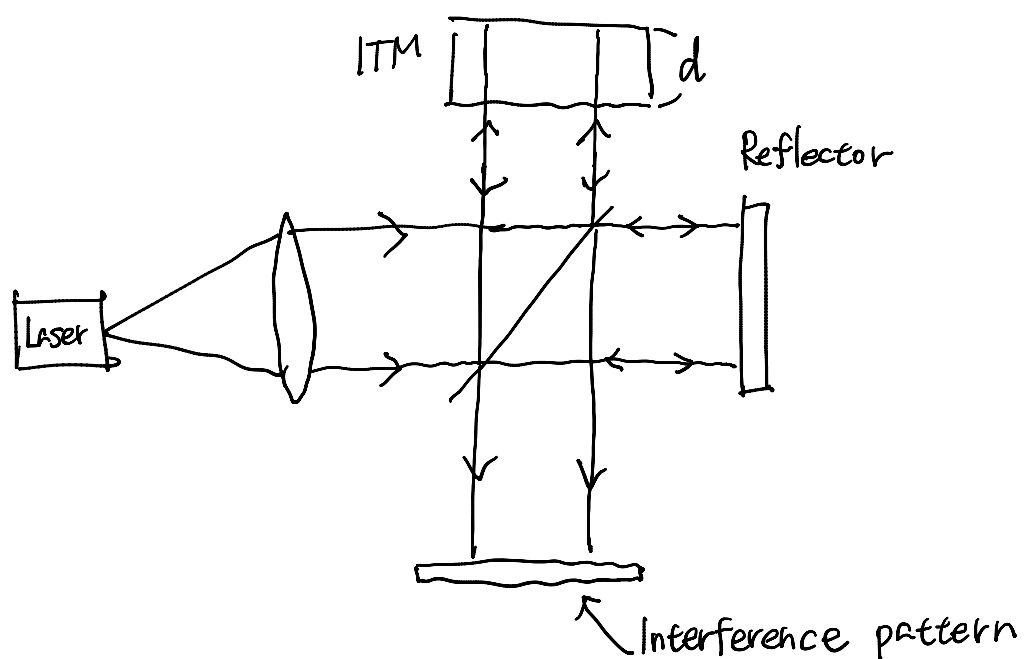


Birefringence map

2019年6月27日 木曜日 10:11

We assume the following measurement set up.

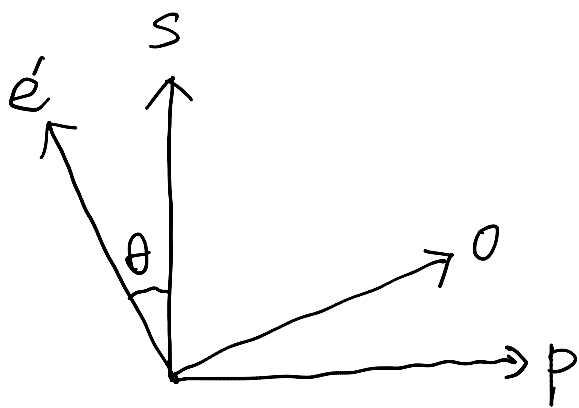


For each point on the ITM, we want to know the followings:

$$\alpha_e \equiv 2\pi \cdot \frac{2d \cdot n_{e'}}{\lambda} : \text{Round trip phase of the ITM transmission for } e' \text{ axis.}$$

$$\alpha_o \equiv 2\pi \cdot \frac{2d \cdot n_o}{\lambda} : \text{Same as above for } o\text{-axis}$$

θ : Angle between e' -axis and s-polarization axis



Assume that pure s-pol beam enters the ITM with amplitude of 1.

Projection of this field onto the e' - and o -axes after making a round trip transmission are:

$$E_e = \cos\theta e^{i\alpha_e}, \quad E_o = \sin\theta e^{i\alpha_o}$$

Projecting these fields back to, s- and p- axes:

$$E_s = \cos\theta E_e + \sin\theta E_o = \cos^2\theta e^{i\alpha_e} + \sin^2\theta e^{i\alpha_o} \\ - \sin\theta \cos\theta (e^{i\alpha_e} - e^{i\alpha_o})$$

$$\alpha_+ \equiv \frac{\alpha_e + \alpha_o}{2}$$

$$\alpha_- \equiv \frac{\alpha_e - \alpha_o}{2}$$

$$\alpha_e = \alpha_+ + \alpha_- \\ \alpha_o = \alpha_+ - \alpha_-$$

$$= e^{i\alpha t} (\cos^2 \theta e^{i\alpha_-} + \sin^2 \theta e^{-i\alpha_-})$$

$$E_p = -\sin \theta E_e + \cos \theta E_o = -\sin \theta \cos \theta e^{i\alpha_e} + \sin \theta \cos \theta e^{i\alpha_o}$$

$$= -\sin \theta \cos \theta e^{i\alpha t} (e^{i\alpha_-} - e^{-i\alpha_-}) = -2 \sin \theta \cos \theta \sin \alpha_-$$

At the output port, E_s and E_p are superposed with the reference field E_r

$$E_r = E_r \cdot e^{i\frac{\pi}{2}} \quad (\text{The phase is chosen to } \frac{\pi}{2} \text{ so that the interference is at the mid-fringe})$$

The output power is:

$$P_{out} = |E_s + E_r|^2 + |E_p|^2 \quad (E_p \text{ and } E_r \text{ do not interfere})$$

$$|E_s + E_r|^2 = |E_r \cdot e^{i\frac{\pi}{2}} + e^{i\alpha t} (\cos^2 \theta e^{i\alpha_-} + \sin^2 \theta e^{-i\alpha_-})|^2$$

$$= E_r^2 + E_r \cos^2 \theta \left(e^{i(\frac{\pi}{2} - \alpha t - \alpha_-)} + e^{-i(\frac{\pi}{2} - \alpha t - \alpha_-)} \right)$$

$$+ E_r \sin^2 \theta \left(e^{i(\frac{\pi}{2} - \alpha t + \alpha_-)} + e^{-i(\frac{\pi}{2} - \alpha t + \alpha_-)} \right)$$

$$+ \cos^4 \theta + \sin^4 \theta + 2 \sin^2 \theta \cos^2 \theta \cos 2\alpha_-$$

$$= E_r^2 + E_r \cos^2 \theta \cdot \sin \alpha_- e + E_r \sin^2 \theta \sin \alpha_-$$

$$+ \cos^4 \theta + \sin^4 \theta + 2 \sin^2 \theta \cos^2 \theta - 4 \sin^2 \theta \cos^2 \theta \sin^2 \alpha_-$$

$$|E_p|^2 = 4 \sin^2 \theta \cos^2 \theta \sin^2 \alpha_- \quad \leftarrow \text{Cancel out}$$

$$P(\theta) = E_r^2 + 1 + E_r (\cos^2 \theta \sin \alpha_- e + \sin^2 \theta \sin \alpha_-)$$

Now, we compute the following quantity.

$$A(\varphi) = P(\theta + \varphi) - P(\theta + \varphi + \frac{\pi}{2})$$

When $\varphi = 0$

$$A(0) = P(\theta) - P(\theta + \frac{\pi}{2}) = E_r (\cos^2 \theta - \sin^2 \theta) (\sin \alpha_- e - \sin \alpha_-)$$

$$\doteq E_r (\cos^2 \theta - \sin^2 \theta) (\alpha_- - \alpha_0)$$

$$|\alpha_-| \ll 1$$

$$|\alpha_0| \ll 1$$

When $\varphi = \frac{\pi}{4}$

$$A(\frac{\pi}{4}) \doteq -2 E_r \sin \theta \cos \theta (\alpha_- - \alpha_0)$$

T.L. the ratio between them

Take the ratio between them

$$b \equiv \frac{A(0)}{A(\frac{\pi}{4})} = -\frac{1}{2} \left(\frac{1}{\tan \theta} - \tan \theta \right)$$

$$x \equiv \tan \theta$$

$$b = -\frac{1}{2x} + \frac{x}{2} \Rightarrow x^2 - 2bx - 1 = 0$$

$$x = \frac{2b \pm \sqrt{4b^2 + 4}}{2} = b \pm \sqrt{b^2 + 1}$$

$$\theta = \tan^{-1}(b \pm \sqrt{b^2 + 1})$$

With θ known, we can compute

$$\alpha_e - \alpha_o = \frac{A(0)}{E_r(\cos^2 \theta - \sin^2 \theta)}$$

E_r is unknown. However, from the calibration of the original measurement, this parameter can be eliminated.

The original measurement by Hirose-san assume no birefringence. It is equivalent to assume $\theta = 0$ everywhere.

$$\text{Then, } P = E_r + E_r \cdot \alpha_e.$$

Ignoring the DC component, the varying part of P is,

$$\Delta P = E_r \cdot \alpha_e$$

therefore,

$$\frac{\Delta P}{E_r} = \alpha_e = \frac{4\pi \cdot dn_o}{\lambda}$$

$$\Rightarrow \frac{\Delta P \cdot \lambda}{4\pi E_r} = dn_o \equiv \Delta l$$

This Δl is the actual output of the original measurement.

$$A(0) = P(\theta) - P(\theta + \frac{\pi}{2}) = \frac{4\pi E_r}{\lambda} (\Delta l(\theta) - \Delta l(\theta + \frac{\pi}{2}))$$

$$\Rightarrow \alpha_e - \alpha_o = \frac{A(0)}{E_r(\cos^2 \theta - \sin^2 \theta)} = \frac{4\pi(\Delta l(\theta) - \Delta l(\theta + \frac{\pi}{2}))}{\lambda(\cos^2 \theta - \sin^2 \theta)}$$