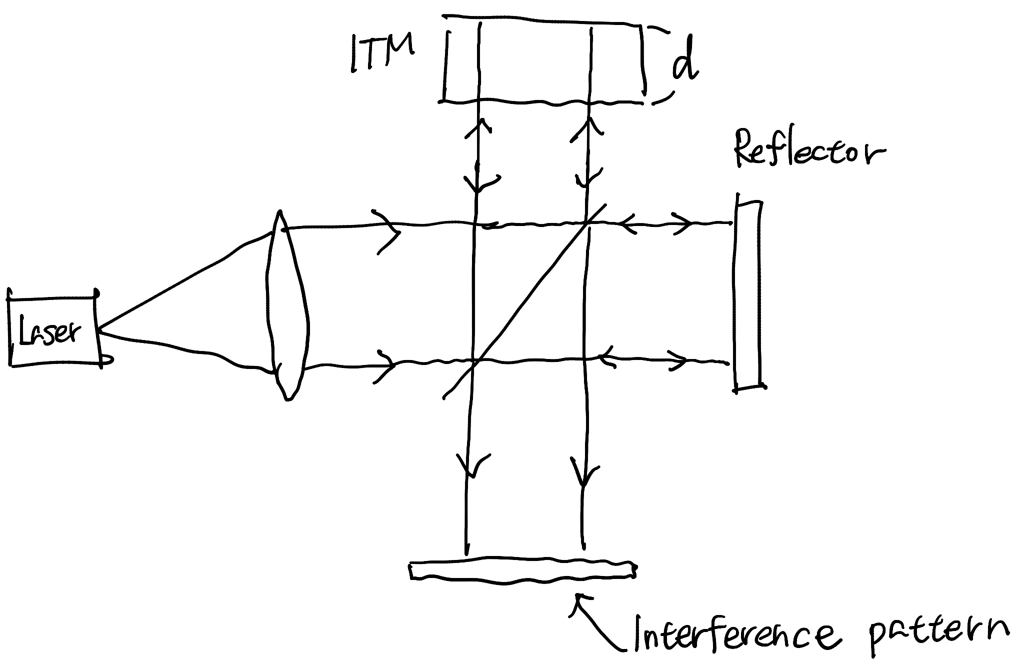
Birefringence map

2019年6月27日 木曜日　10:11

We assume the following measurement set up.

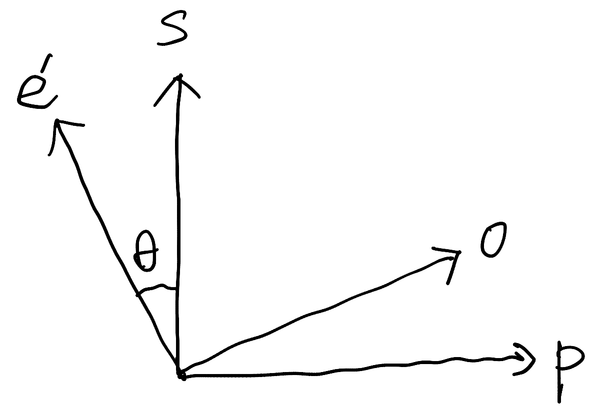


For each point on the ITM, we want to know the followings:

$$\alpha_e \equiv 2\pi \cdot \frac{2d \cdot n_e'}{\lambda}$$: Round trip phase of the ITM transmission for e' axis.

$$\alpha_o \equiv 2\pi \cdot \frac{2d \cdot n_o}{\lambda}$$: Same as above for o-axis

θ : Angle between e'-axis and s-polarization axis



Assume that pure s-pol beam enters the ITM with amplitude of 1.

Projection of this field on to the e'- and o-axes after making a round trip transmission are:

$$E_e = \cos\theta \, e^{i\alpha_e}, \quad E_o = \sin\theta \, e^{i\alpha_o}$$

$$\alpha_+ \equiv \frac{\alpha_e + \alpha_o}{2}$$

$$\alpha_- \equiv \frac{\alpha_e - \alpha_o}{2}$$

Projecting these fields back to, s- and p-axes;

$$\alpha_e = \alpha_+ + \alpha_-$$
$$\alpha_o = \alpha_+ - \alpha_-$$

$$E_s = \cos\theta E_e + \sin\theta E_o = \cos^2\theta \, e^{i\alpha_e} + \sin^2\theta \, e^{i\alpha_o}$$

$$= e^{i\alpha_+}\left(\cos^2\theta \, e^{i\alpha_-} + \sin^2\theta \, e^{-i\alpha_-}\right)$$

$$= e^{i\alpha_+}\left(\cos^2\theta\, e^{i\alpha_-} + \sin^2\theta\, e^{-i\alpha_-}\right)$$

$$E_p = -\sin\theta E_e + \cos\theta E_o = -\sin\theta\cos\theta\, e^{i\alpha_e} + \sin\theta\cos\theta\, e^{i\alpha_o}$$

$$= -\sin\theta\cos\theta\, e^{i\alpha_+}\left(e^{i\alpha_-} - e^{-i\alpha_-}\right) = -2\sin\theta\cos\theta\sin\alpha_-$$

At the output port, E_s and E_p are superposed with the reference field E_r

$$E_r = E_r \cdot e^{i\frac{\pi}{2}}$$ (The phase is chosen to $\frac{\pi}{2}$ so that the interference is at the mid-flinge)

The output power is:

$$P_{out} = |E_s + E_r|^2 + |E_p|^2$$ (E_p and E_r do not interfere)

$$|E_s+E_r|^2 = \left|E_r \cdot e^{i\frac{\pi}{2}} + e^{i\alpha_+}\left(\cos^2\theta\, e^{i\alpha_-} + \sin^2\theta\, e^{-i\alpha_-}\right)\right|^2$$

$$= E_r^2 + E_r\cos^2\theta\left(e^{i\left(\frac{\pi}{2}-\alpha_+-\alpha_-\right)} + e^{-i\left(\frac{\pi}{2}-\alpha_+-\alpha_-\right)}\right)$$

$$+ E_r\sin^2\theta\left(e^{i\left(\frac{\pi}{2}-\alpha_++\alpha_-\right)} + e^{-i\left(\frac{\pi}{2}-\alpha_++\alpha_-\right)}\right)$$

$$+ \cos^4\theta + \sin^4\theta + 2\sin^2\theta\cos^2\theta\cos 2\alpha_-$$

$$= E_r^2 + E_r\cos^2\theta \cdot \sin\alpha_e + E_r\sin^2\theta\sin\alpha_o$$

$$+ \cos^4\theta + \sin^4\theta + 2\sin^2\theta\cos^2\theta - 4\sin^2\theta\cos^2\theta\sin^2\alpha_-$$

$$|E_p|^2 = 4\sin^2\theta\cos^2\theta\sin^2\alpha_-$$ Cancel out

$$P(\theta) = E_r^2 + 1 + E_r\left(\cos^2\theta\sin\alpha_e + \sin^2\theta\sin\alpha_o\right)$$

Now, we compute the following quantity.

$$a(\varphi) = P(\theta+\varphi) - P\left(\theta+\varphi+\frac{\pi}{2}\right)$$

When $\varphi = 0$

$$a(0) = P(\theta) - P\left(\theta+\frac{\pi}{2}\right) = E_r\left(\cos^2\theta - \sin^2\theta\right)\left(\sin\alpha_e - \sin\alpha_o\right)$$

$$\fallingdotseq E_r\left(\cos^2\theta - \sin^2\theta\right)\left(\alpha_e - \alpha_o\right)$$

$|\alpha_e| \ll 1$
$|\alpha_o| \ll 1$

When $\varphi = \frac{\pi}{4}$

$$a\left(\frac{\pi}{4}\right) \fallingdotseq -2E_r\sin\theta\cos\theta\left(\alpha_e - \alpha_o\right)$$

Take the ratio between them

Take the ratio between them

$$b \equiv \frac{a(0)}{a(\frac{\pi}{4})} = -\frac{1}{2}\left(\frac{1}{\tan\theta} - \tan\theta\right)$$

$$x \equiv \tan\theta$$

$$b = -\frac{1}{2x} + \frac{x}{2} \Rightarrow x^2 - 2bx - 1 = 0$$

$$x = \frac{2b \pm \sqrt{4b^2+4}}{2} = b \pm \sqrt{b^2+1}$$

$$\theta = \tan^{-1}\left(b \pm \sqrt{b^2+1}\right)$$

With θ known, we can compute

$$\alpha_e - \alpha_o = \frac{a(0)}{E_r(\cos^2\theta - \sin^2\theta)}$$

E_r is unknown. However, from the calibration of the original measurement, this parameter can be eliminated.

The original measurement by Hirose-san assume no birefringence. It is equivalent to assume $\theta = 0$ everywhere.

Then, $P = E_r^2 + 1 + E_r \cdot \alpha_e$.

Ignoring the DC component, the varying part of P is,

$$\Delta P = E_r \cdot \alpha_e$$

therefore,

$$\frac{\Delta P}{E_r} = \alpha_e = \frac{4\pi \cdot dn_o}{\lambda}$$

$$\Rightarrow \frac{\Delta P \cdot \lambda}{4\pi E_r} = dn_o \equiv \Delta l$$

This Δl is the actual output of the original measurement.

$$a(0) = P(\theta) - P(\theta + \tfrac{\pi}{2}) = \frac{4\pi E_r}{\lambda}\left(\Delta l(\theta) - \Delta l(\theta + \tfrac{\pi}{2})\right)$$

$$\Rightarrow \alpha_e - \alpha_o = \frac{a(0)}{E_r(\cos^2\theta - \sin^2\theta)} = \frac{4\pi\left(\Delta l(\theta) - \Delta l(\theta + \frac{\pi}{2})\right)}{\lambda(\cos^2\theta - \sin^2\theta)}$$