0.1 Optic displacement sensing

The longitudinal displacement L, pitch P and yaw Y of the payload are sensed by an optical system composed of a tilt sensing optical lever (OL) and a length sensing "OL".

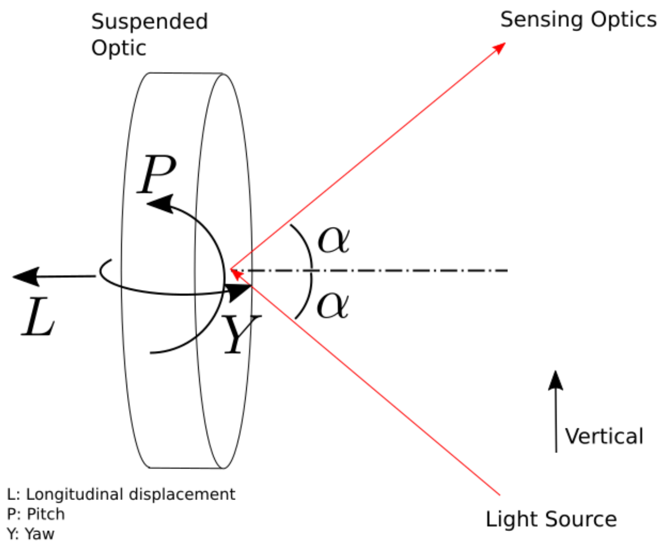

Figure 1: Sketch of the optical lever setup of the Type B SAS with the degrees of freedom labeled

The beam goes vertically from the bottom via a lower viewport of the vacuum chamber, then bounces off the payload and lastly exits the the vacuum chamber through an upper viewport and reaches a breadboard where the optics and sensing devices are located.

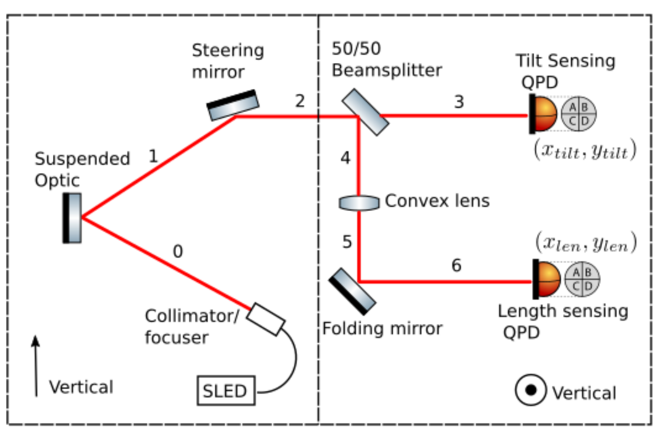

Figure 2: Sketch of optic layout of the sensing optics

For the tilt sensing OL, the OL beam travels some distance from the suspended optic to the sensor. So, From the ray transfer matrix for a beam traveling some distance r from point a to point b,

$$\begin{pmatrix} X_b \\ \theta_b \end{pmatrix} = \begin{pmatrix} 1 & r \\ 0 & 1 \end{pmatrix} \begin{pmatrix} X_a \\ \theta_a \end{pmatrix}. \tag{1}$$

As for the length sensing "OL", the beam travels some distance from the suspended optic to a convex lens and then travel some other distance to reach the length sensing QPD. So, for a beam traveling some distance l, through a convex lens with a focal length of f and then traveling some other distance d, the ray transfer matrix is,

$$\begin{aligned} \begin{pmatrix} X_b \\ \theta_b \end{pmatrix} &= \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} X_a \\ \theta_a \end{pmatrix} \\ &= \begin{pmatrix} 1-\frac{d}{f} & d+l\left(1-\frac{d}{f}\right) \\ \frac{-1}{f} & 1-\frac{1}{f} \end{pmatrix} \begin{pmatrix} X_a \\ \theta_a \end{pmatrix}. \end{aligned} \tag{2}$$

While the QPD is sensing X_b but not θ_b, it is obvious that if the length sensing quadrant photodiode is placed at $d = \frac{lf}{l-f}$, it will only be sensitive to the beam displacement X_a but not the beam angle θ_a, i.e.,

$$X_b = \left[1 - \frac{\left(\frac{lf}{l-f}\right)}{f}\right] X_a = \left(\frac{-f}{l-f}\right) X_a \tag{3}$$

From figure 2, the effective optical lever arm r is the length of path 1-2-3. So, the arm length for pitch is $r_p = r$ while that for yaw is $r_y = r\cos\alpha$ since the vertical portion of the beam path does not amplify the yaw angular displacement. Here, we define the length of the beam path 1-2-4 as r_l and the path 5-6 as d. We further assume that d is very close to, if not exactly $\frac{r_l f}{r_l - f}$. So, according to (2), the length sensing QPD is only sensitive to the longitudinal displacement of the optic.

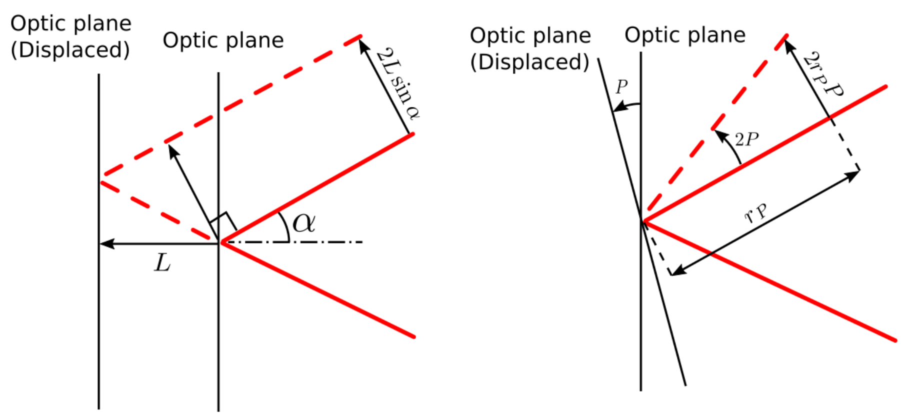

Figure 3: Left: optic shifted by L in longitudinal direction , Right: optic rotated by P in pitch direction.

Let (x_{tilt}, y_{tilt}) and (x_{len}, y_{len}) be displacement of the beamspot at the tilt and length sensing QPDs respectively (x is horizontal and y is vertical). According to (1) and (3), the relationship between the displacements of the beam at the QPDs and the optic's degrees of freedom (L, P, Y) is

$$x_{tilt} = 2r_Y\, Y, \tag{4}$$

$$y_{tilt} = 2r_P\, P + 2L\sin\alpha, \tag{5}$$

$$y_{len} = \frac{-2f\sin\alpha}{r_l - f} L. \tag{6}$$

Note that, a longitudinal displacement of L of the optic would cause the ray to move parallel to the optical axis by a amount of $2L\sin\alpha$ while a tilt (P, Y) in the optic would cause the ray to change slope by $(2P, 2Y)$. See figure 3 for a graphical description. The rotational angles P and Y are in the order of $10^{-6} rad$. Therefore, small-angle assumption is applied throughout all calculations.

0.1.1 Cross-couplings and diagonalization

Inevitably, there will be some cross-couplings between the one degree of freedom and another. The simplest case is longitudinal to pitch and yaw cross-coupling. As shown in (5), the sensor output y_{tilt} is a linear combination of both longitudinal and pitch. For the current setup, r_P is in the order of one meter and $\sin\alpha$ is in the order of 0.5. So, in fact, the pitch that is detected by purely a tilt sensing QPD is coupled with longitudinal displacement with a non-negligible coupling ratio around $2\,rad : 1\,m$. Therefore, in order to obtain pure pitch, it is necessary to obtain correction from the length sensing QPD.

Moreover, another similar correction to the tilt sensing signal would also be needed if the beam of the optical lever has a non-zero horizontal component that is perpendicular to the longitudinal axis. In that case, equation (4) would look more like (5) but with a different angle of incident β which should be very small compare to the usual vertical angle of incident α. So, the R.H.S of (4) should be replaced by $2r_Y\,Y + 2L\sin\beta$. In most cases, β is small enough to be negligible. So, this correction is only needed when there is some obvious longitudinal to yaw cross-coupling seen in the measurements.

Furthermore, the length sensing QPD is not guaranteed to only detect longitudinal displacement. Instead, it would be mostly sensitive a longitudinal shift of the suspended optic but with some cross-coupling from both pitch and yaw. The light ray is not guaranteed to be aligned to the absolute center of the optic. As a consequence, the displacement of the beam at the optic will also change if the optic purely tilts without any longitudinal shifts. And, this additional change in beam displacement at the optic plane will be picked up by the length sensing QPD and tilt sensing QPD. Therefore, L in the equations should be replaced by a linear combination of longitudinal displacement, pitch and yaw $L+\delta_P P+\delta_Y Y$ where δ_P and δ_Y represents roughly the vertical and horizontal (diagonal to longitudinal) displacements.

Nevertheless, since the beam of the optical lever has to be bend to horizontal by a steering mirror, a slight misalignment in the steering mirror could easily introduce a rotation to the beam spot's displacement at the tilt QPD plane w.r.t. the tilt QPD frame. Therefore, the equalities (4) and (5) will not hold. Instead, it would involve a rotational transformation from the QPD frame to the (L, P, Y) frame,

$$\begin{pmatrix} \cos\theta_t & \sin\theta_t \\ -\sin\theta_t & \cos\theta_t \end{pmatrix} \begin{pmatrix} x_{tilt} \\ y_{tilt} \end{pmatrix} = \begin{pmatrix} 2r_Y\,Y \\ 2r_P\,P + 2L\sin\alpha \end{pmatrix}, \tag{7}$$

where θ_t is some angle between the tilt QPD frame and the (L, P, Y) frame. Similarly, equation (6) should be rewritten as

$$\begin{pmatrix} \cos\theta_l & \sin\theta_l \\ -\sin\theta_l & \cos\theta_l \end{pmatrix} \begin{pmatrix} x_{len} \\ y_{len} \end{pmatrix} = \begin{pmatrix} V_{null} \\ \frac{-2f\sin\alpha}{r_l - f} L \end{pmatrix}, \tag{8}$$

where θ_l is some angle between the len QPD frame and the (L, P, Y) frame and V_{null} is some vector perpendicular to L and is supposedly a constant. Note that these rotation corrections are needed in general for compensating installation error.

By applying the aforementioned transformations $2r_Y\,Y \rightarrow (2r_Y\,Y + 2L\sin\beta)$ and $L \rightarrow (L + \delta_P P + \delta_Y Y)$, and assembling the matrices, we get the diagonalization matrix which converts the QPD readout $(x_{tilt}, y_{tilt}, x_{len}, y_{len})$ to the displacement of the optic (L, P, Y):

$$\begin{pmatrix} \cos\theta_t & \sin\theta_t & 0 & 0 \\ -\sin\theta_t & \cos\theta_t & 0 & 0 \\ 0 & 0 & -\sin\theta_l & \cos\theta_l \end{pmatrix} \begin{pmatrix} x_{tilt} \\ y_{tilt} \\ x_{len} \\ y_{len} \end{pmatrix} = \begin{pmatrix} 2\sin\beta & 2\delta_P\sin\beta & 2r_Y + 2\delta_Y\sin\beta \\ 2\delta_Y\sin\alpha & 2r_P + 2\delta_P\sin\alpha & 2\sin\alpha \\ \frac{-2f\sin\alpha}{r_l-f} & \frac{-2f\delta_P\sin\alpha}{r_l-f} & \frac{-2f\delta_Y\sin\alpha}{r_l-f} \end{pmatrix} \begin{pmatrix} L \\ P \\ Y \end{pmatrix}$$
$$\rightarrow \begin{pmatrix} L \\ P \\ Y \end{pmatrix} = \begin{pmatrix} 2\sin\beta & 2\delta_P\sin\beta & 2r_Y + 2\delta_Y\sin\beta \\ 2\sin\alpha & 2r_P + 2\delta_P\sin\alpha & 2\delta_Y\sin\alpha \\ \frac{-2f\sin\alpha}{r_l-f} & \frac{-2f\delta_P\sin\alpha}{r_l-f} & \frac{-2f\delta_Y\sin\alpha}{r_l-f} \end{pmatrix}^{-1} \begin{pmatrix} \cos\theta_t & \sin\theta_t & 0 & 0 \\ -\sin\theta_t & \cos\theta_t & 0 & 0 \\ 0 & 0 & -\sin\theta_l & \cos\theta_l \end{pmatrix} \begin{pmatrix} x_{tilt} \\ y_{tilt} \\ x_{len} \\ y_{len} \end{pmatrix}. \tag{9}$$

In spite of the cross-coupling mechanisms mentioned above, there are more ways one degree of freedom would coupled to another. For example, if the LEN QPD is misplaced, from equation (2), the QPD will be less sensitive to a parallel shift of the beam but will become more sensitive to the tilt of the optic. This means that there will be some tilt to longitudinal cross-coupling and equation (6) would need further modification to cancel the coupling. However, the effect of this type of cross-coupling is very similar to the aforementioned tilt to longitudinal cross-coupling due to an off-centered beam. We further assume that length sensing QPD is placed reasonably close to $\frac{r_l f}{r_l - f}$, i.e. at the image plane. Therefore, the coupling ratio should be small compared to other types of cross-coupling and is negligible.

Is it worth it to mention that ghost beams could introduce cross-couplings?