GAS (Geometric Anti Spring) filter and LVDT (Linear Variable Differential Transformer)

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Lecture 2

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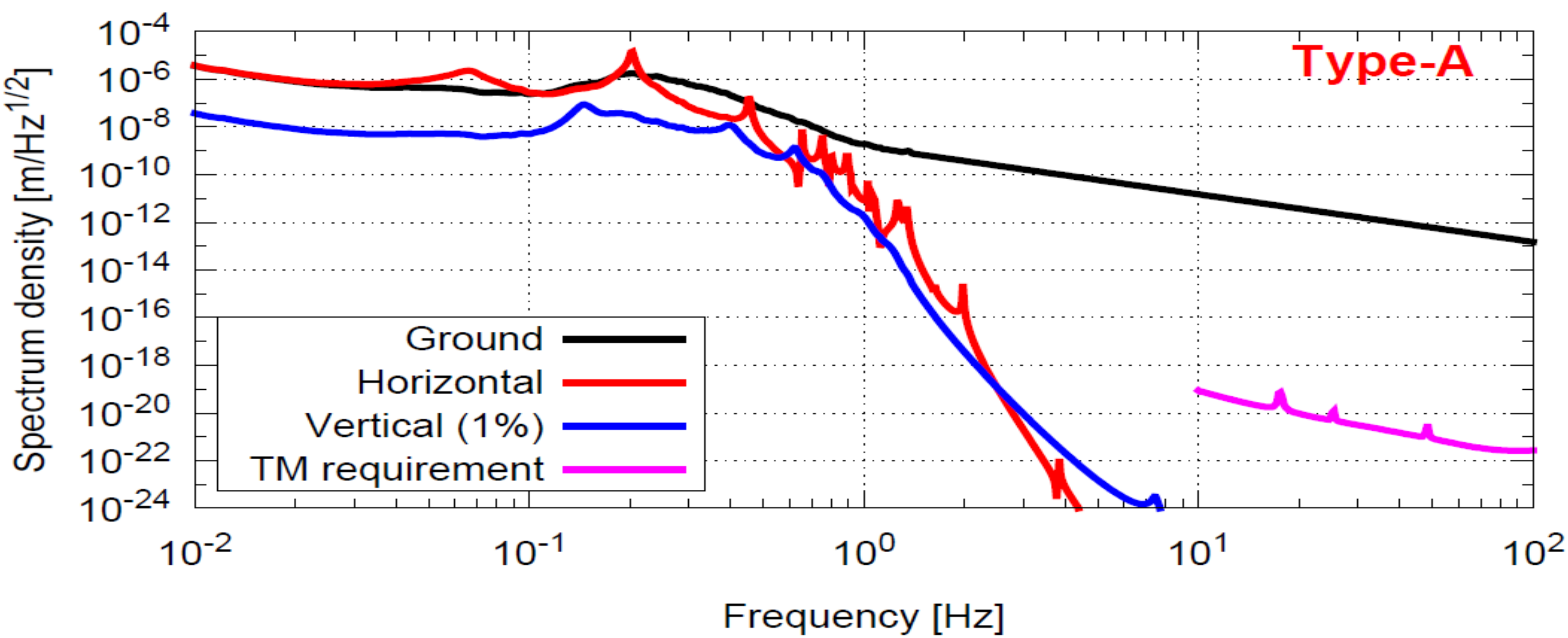
Vibration Isolation Systems

GW event induces a relative length change of about 10^-21 ~ 10^-22 (called strain).

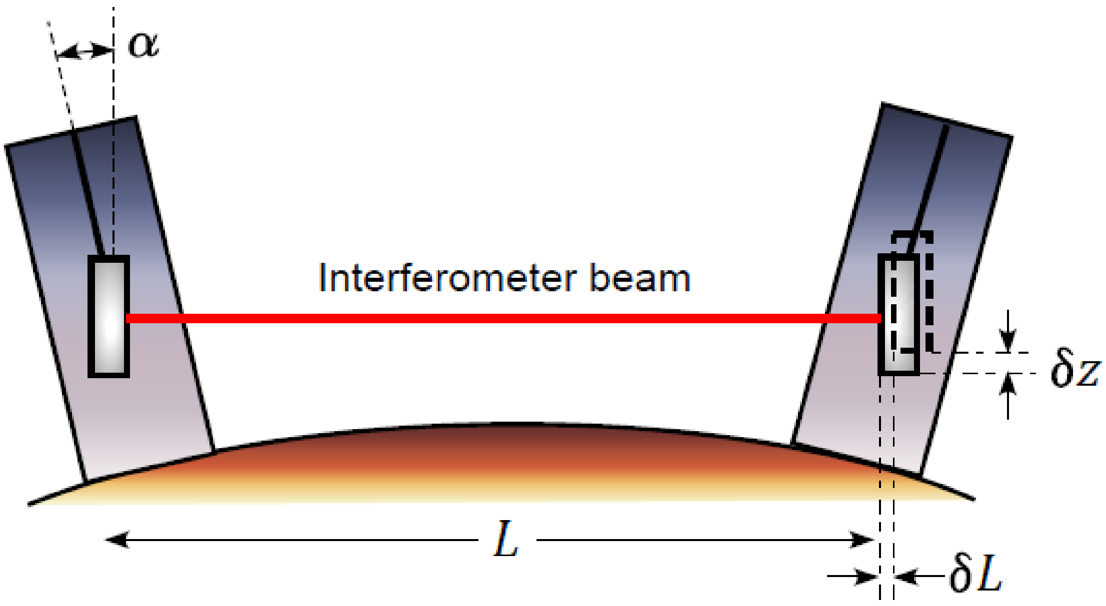
$$h = \frac{\Delta d}{d} = \frac{1}{2} h_+$$

Kagra is design to have a sensitivity of 3x10^-22 at 10 Hz. This means that a wave would induce a change of the length between test masses of 3000x3x10^-22/10(safety margin)~10^-19 m. So the systems must be designed to suppress the seismic noise below that value.



Vertical vibration isolation 1

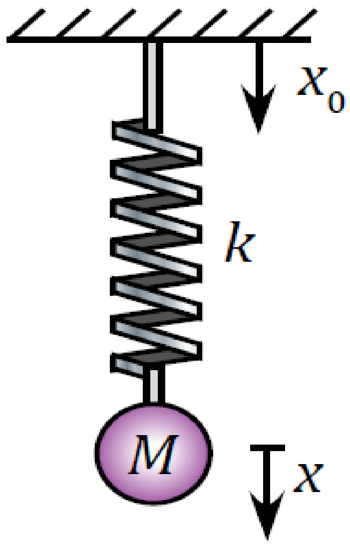
Horizontal ground vibration is the most critical because it transmits directly to the horizontal motions of the mirrors and contributes to the changes in the optical path lengths of the interferometer.

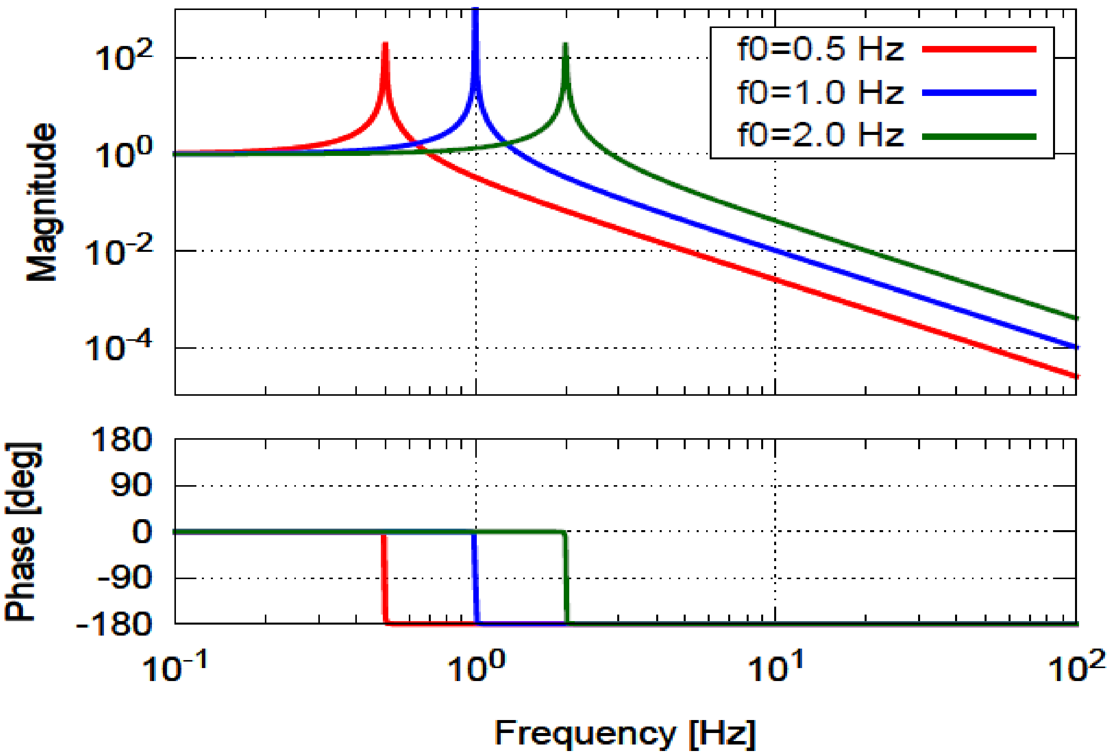


But 0.1-1% of the vertical motion is transferred to the horizontal direction by mechanical imperfections (crosscoupling) in each attenuation stage and by the non-parallelism of the verticality at locations kilometers apart in the interferometer.

Vertical vibration isolation 2

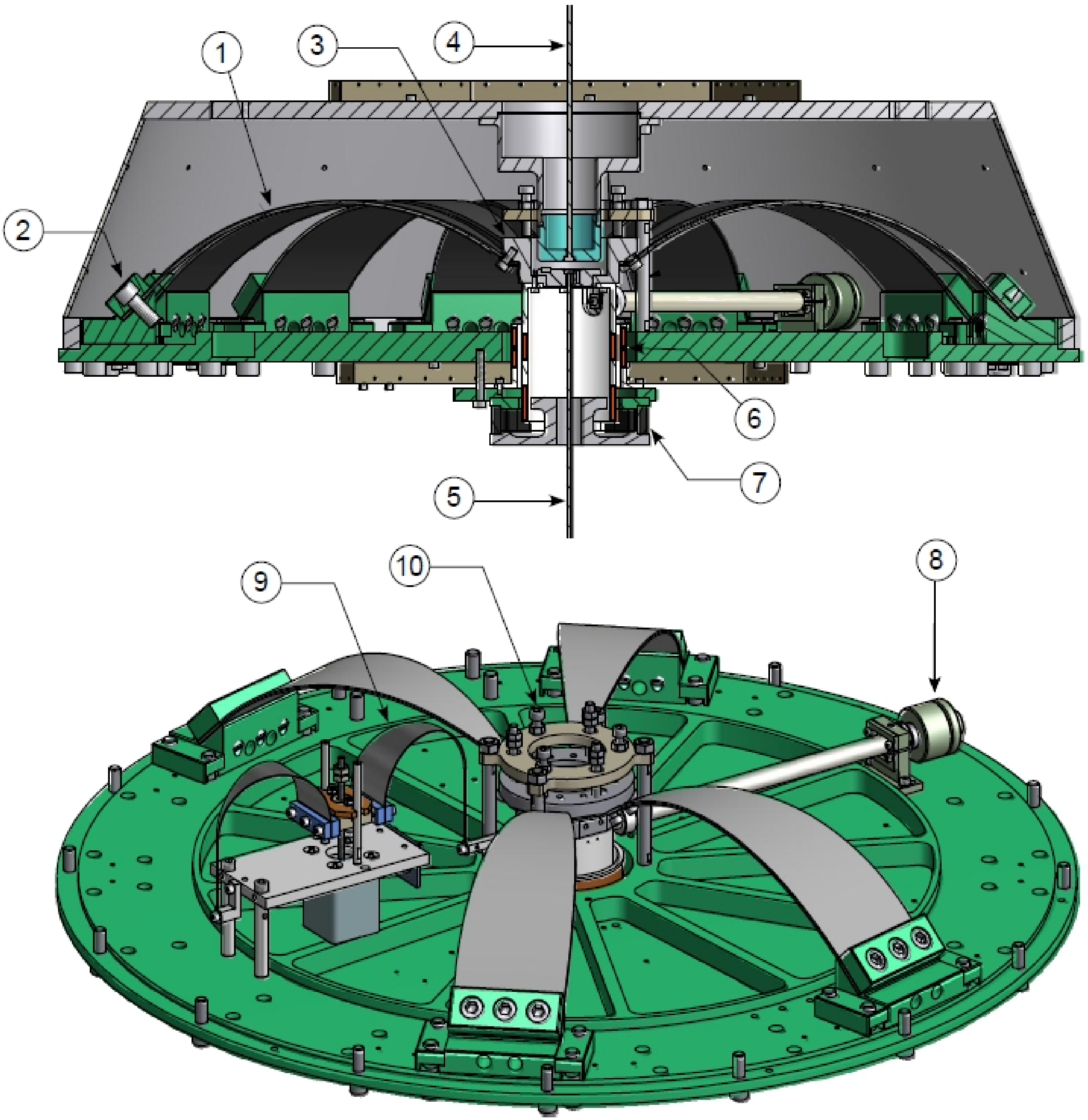
The isolation <u>could</u> be done using vertical springs.
But we don't use normal springs for this solution... Why?





Task 1: If we want to have a resonant frequency f_0=0.3 Hz by means of a linear spring. How much elongation should have this system?

GAS (Geometric anti-spring) filter



(1) Blades.

(2) Blade attachment to the base.

(3) Keystone.

(4) Upper rod supporting the weight to the GAS filter and the mass below it.

(5) Lower rod connected to the lower stage (It moves the Keystone).

(6) LVDT (it measures the displacement of the Keystone).

(7) Coil magnet actuator.

(8) Magic wand (to improve the saturation value of isolation).

(9) Fishing rod (to move the Keystone).

(10) Locking system screws.

GAS filter model 1

Case 1: The equation of motion of a GAS filter as harmonic oscillator:

$$M\ddot{x} = -k(x - x_0)$$

We can write the transfer function of a linear system as the ratio between the Fourier transform of the output and of the input.

$$H_X \equiv \frac{X}{X_0} = \frac{\omega_0^2}{\omega_0^2 - \omega^2}$$

GAS filter model 2

Case 2: Considering the spring mass and introducing viscous damping, we can write:

$$M\ddot{x} = -k(1+i\phi)(x-x_0) - m\ddot{x}_0 - \gamma\dot{x},$$

Where m is the mass of the spring, γ is the damping coefficient, ϕ is the "loss angle" (measure of spring anelasticity).

Using the Fourier tranformation we obtain:

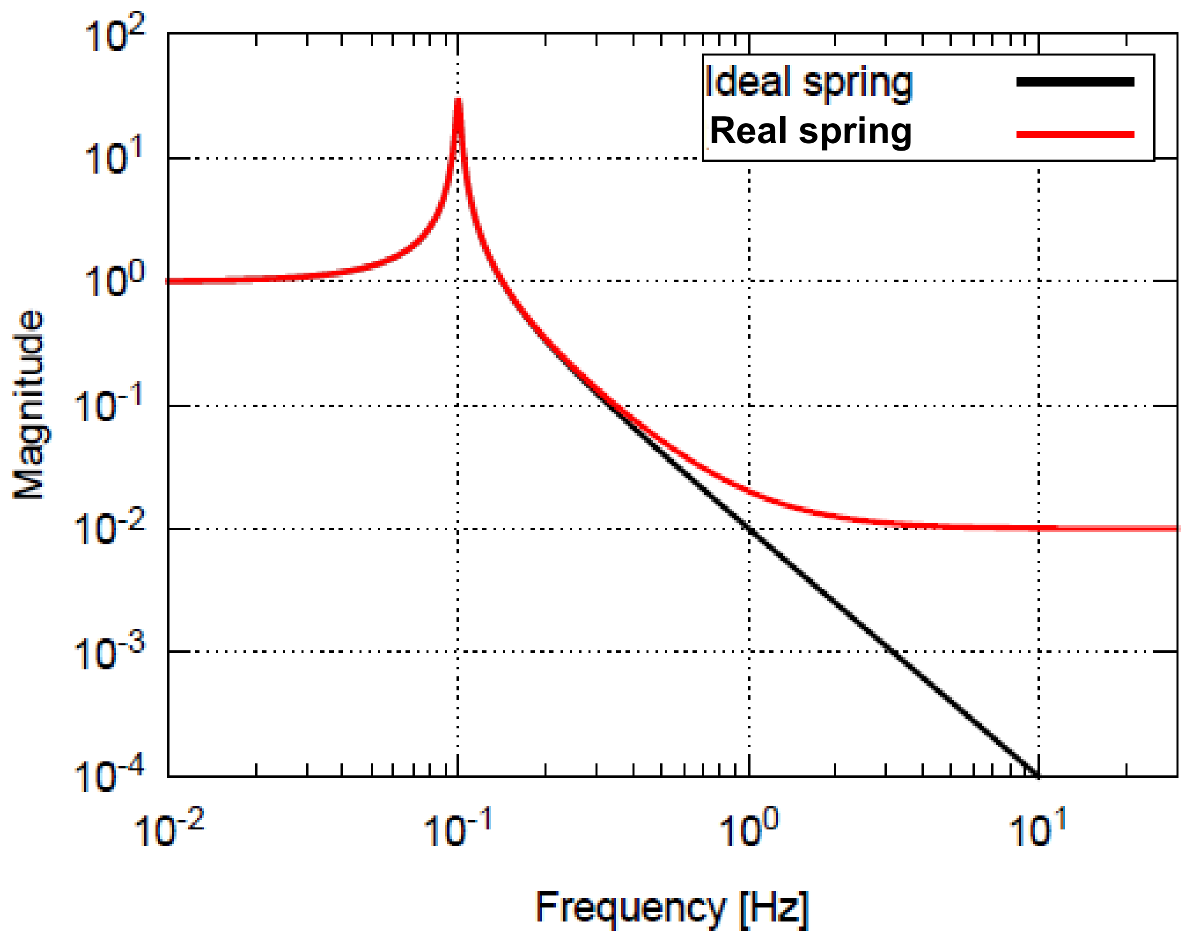
$$-M\omega^2 X - m\omega^2 X_0 = -k(1+i\phi)(X-X_0) - i\gamma\omega X$$

And the transfer function for the displacement:

$$H_X = \frac{\omega_0^2(1+i\phi) + \frac{m}{M}\omega^2}{\omega_0^2(1+i\phi) - \omega^2 + i\frac{\gamma}{M}\omega}$$

GAS filter Transfer Function (TF) 1 and 2

The TF of a real spring mass system saturates at the level m/M.



GAS filter model 3

Case 3: Considering the spring mass and introducing viscous damping, and the internal pendulum modes, we can write:

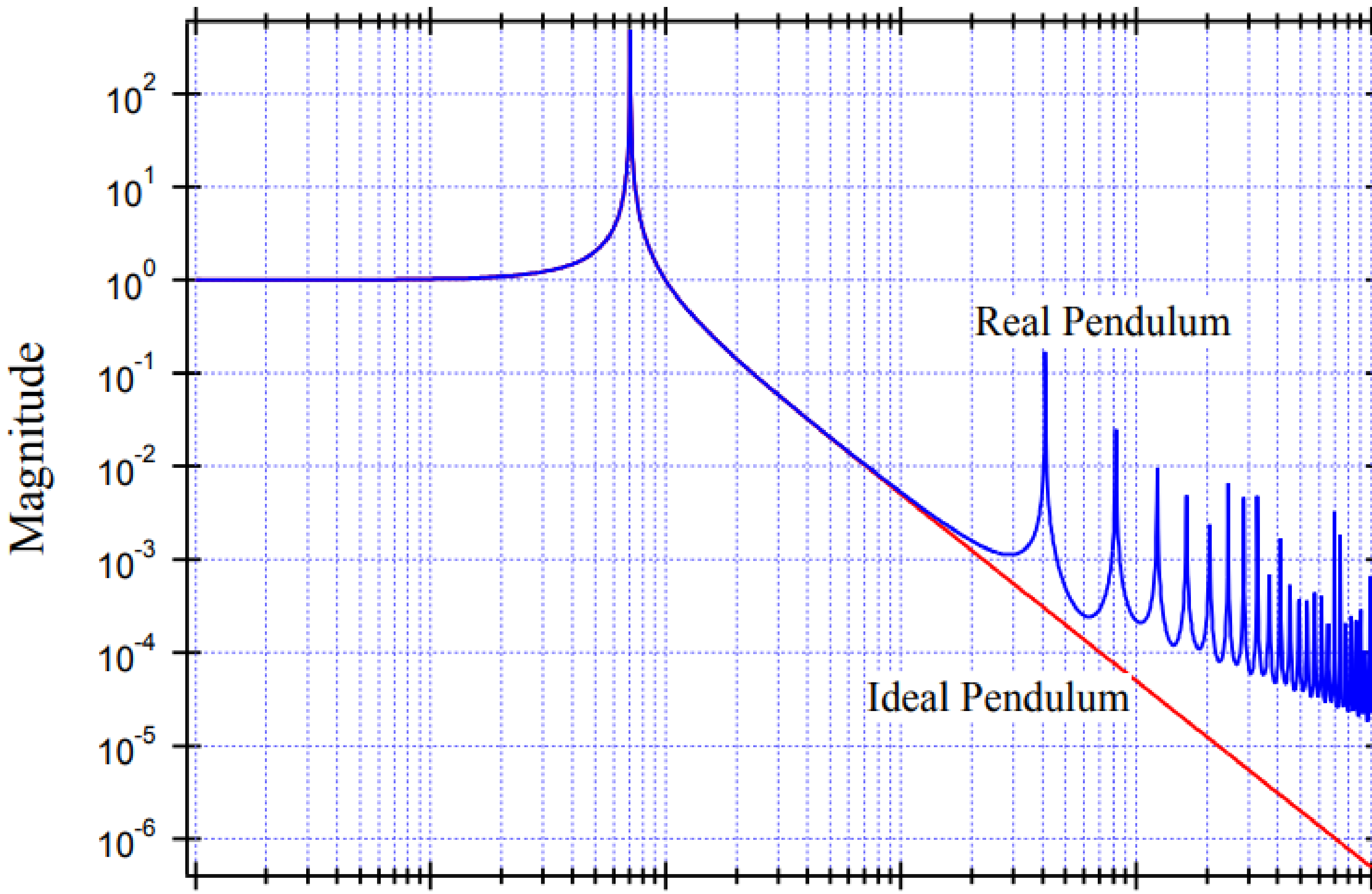
$$H_{x_0 \to x} = \sum_{n=0}^{\infty} \frac{M}{\mu_n \cos(k_n l)} \frac{\omega_n^2[1 + \mathrm{i}\phi_n(\omega)]}{\omega_n^2[1 + \mathrm{i}\phi_n(\omega)] - \omega^2}$$

$$\approx \frac{\omega_{\mathrm{p}}^2[1 + \mathrm{i}\phi_0(\omega)]}{\omega_{\mathrm{p}}^2[1 + \mathrm{i}\phi_0(\omega)] - \omega^2}$$

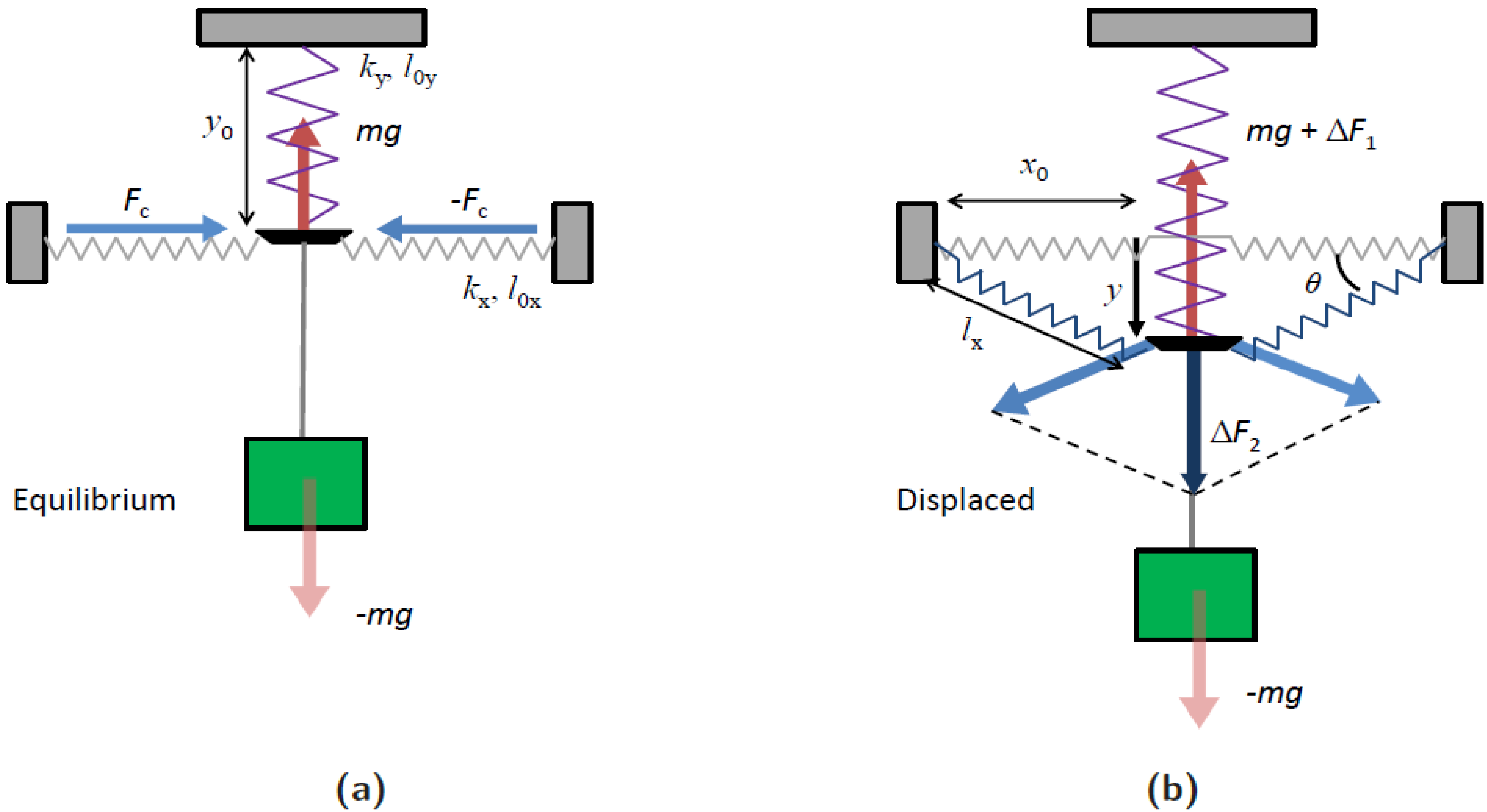
$$+ 2\sum_{n=1}^{\infty} \frac{(-1)^n \omega_{\mathrm{p}}^2[1 + \mathrm{i}\phi_n(\omega)]}{\omega_n^2[1 + \mathrm{i}\phi_n(\omega)] - \omega^2},$$

GAS filter Transfer Function (TF) 3

The amplitude of the TF disagrees from the TF of the ideal mass spring system above some tens of Hz. The slope 1/f^2 is degraded to 1/f due to the harmonics of internal modes.



GAS filter linearized model 1



This model is useful to describe the qualitative behavior of such a mechanical system in an intuitive way.

GAS filter linearized model 2

In the situation on the left of the last figure, the forces of the horizontal springs cancel and the force of the vertical spring and the gravitational force on the suspended mass cancel. In this case the resonance frequency of the system will be minimal.

We call this special situation the working point. In the working point the equation of motion of the system is:

$$m\ddot{y} = -k_{\mathrm{y}}\left(y_0 - l_{0\mathrm{y}}\right) - mg = 0,$$

For small excursions *y* from the working point the horizontal forces of the horizontal springs will still cancel.
The force of the vertical spring will increase as: $\Delta F_1 = -k_{\mathrm{y}} y$

And it will be partially canceled by the vertical component of the forces of the compressed springs.

$$\Delta F_2 \approx -k_{\mathrm{x}}(l_{\mathrm{x}} - l_{0\mathrm{x}})\sin\theta = -k_{\mathrm{x}}\left(1 - l_{0\mathrm{x}}/l_{\mathrm{x}}\right)y,$$

Where *l*x is the length of the compressed horizontal springs.

GAS filter linearized model 3

For amplitudes *y* much smaller than the length of the compressed horizontal spring, the equation of motion of the GAS filter can be written as:

$$(y^2 \ll x_0^2) \qquad m\ddot{y} = \Delta F_1 + \Delta F_2 = -k_{\mathrm{y}} y - k_{\mathrm{x}} \left(1 - \frac{l_{0\mathrm{x}}}{l_{\mathrm{x}}}\right) y.$$

For small amplitudes the system behaves as a harmonic oscillator with an effective stiffness:

$$k_{\mathrm{eff}} = k_{\mathrm{y}} + k_{\mathrm{x}} \left(1 - \frac{l_{0\mathrm{x}}}{l_{\mathrm{x}}}\right) \qquad l_{0\mathrm{x}}/l_{\mathrm{x}} > 1$$

The term in brackets is negative and the stiffness of the system is lowered.
This is the principle of the linear anti-spring effect!

We can also define the compression rate as: $K = \dfrac{l_{0\mathrm{x}} - l_{\mathrm{x}}}{l_{0\mathrm{x}}}.$

GAS filter linearized model 4

Taking into acount that: $l_{\mathrm{x}} = \sqrt{y^2 + x_0^2},$

Then the previous equations can be written as:

Stiffness:

$$k_{\mathrm{eff}} = k_{\mathrm{y}} + k_{\mathrm{x}} \left(1 - \frac{l_{0\mathrm{x}}}{\sqrt{y^2 + x_0^2}}\right) \qquad k_{\mathrm{eff}}(y)$$

Compresion rate:

$$K = 1 - \frac{\sqrt{y^2 + x_0^2}}{l_{0\mathrm{x}}}. \qquad K(x_0).$$

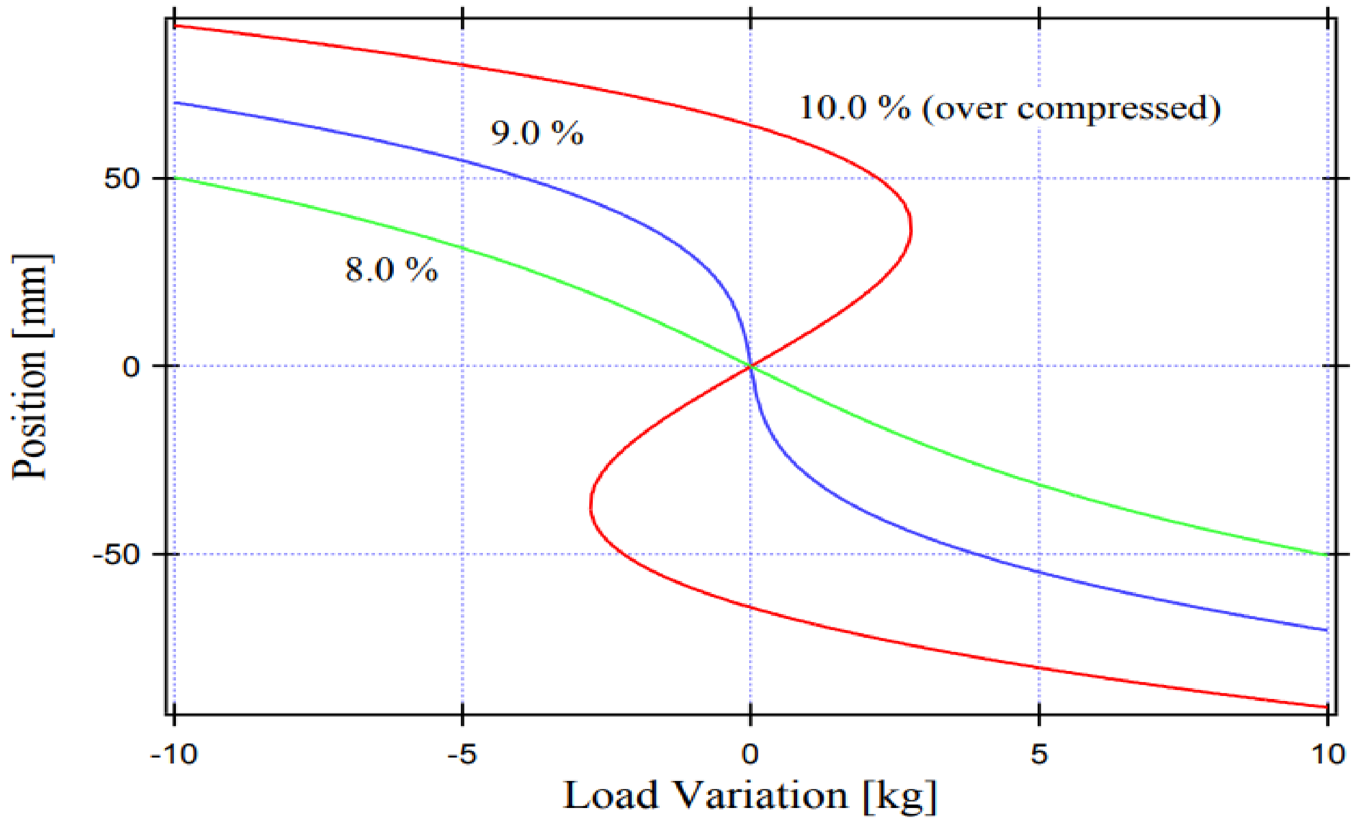
GAS Equilibrium position 1

At the working point the vertical force of the blades is exactly balanced by the load of the filter and the filter will have a minimal frequency.

When the load is changed by an small amount new equilibrium position will no longer coincide with the working point. It will change by:

$$\Delta y_{\mathrm{eq}} = \frac{\Delta F}{k_{\mathrm{eff}}} = \frac{g\Delta m}{k_{\mathrm{eff}}} = \frac{g}{k_{\mathrm{y}} + k_{\mathrm{x}}\left(1 - \frac{l_{0\mathrm{x}}}{\sqrt{y^2+x_0^2}}\right)}\Delta m.$$

GAS Equilibrium position 2



When the system is not overcompressed, it has single working point where the resonant frequency takes its minimum value.
But when the horizontal spring is compressed too much (critical value), the system has two stable and one unstable working points.

Increasing the load of the filter, the vertical position of the keystone would at some point jump to a low position and vice versa.

GAS Resonant Frequency 1

The resonant frequency of the filter at a particular vertical position is given by:

$$\omega(y) = \sqrt{\frac{k_{\text{eff}}(y)}{m(y)}},$$

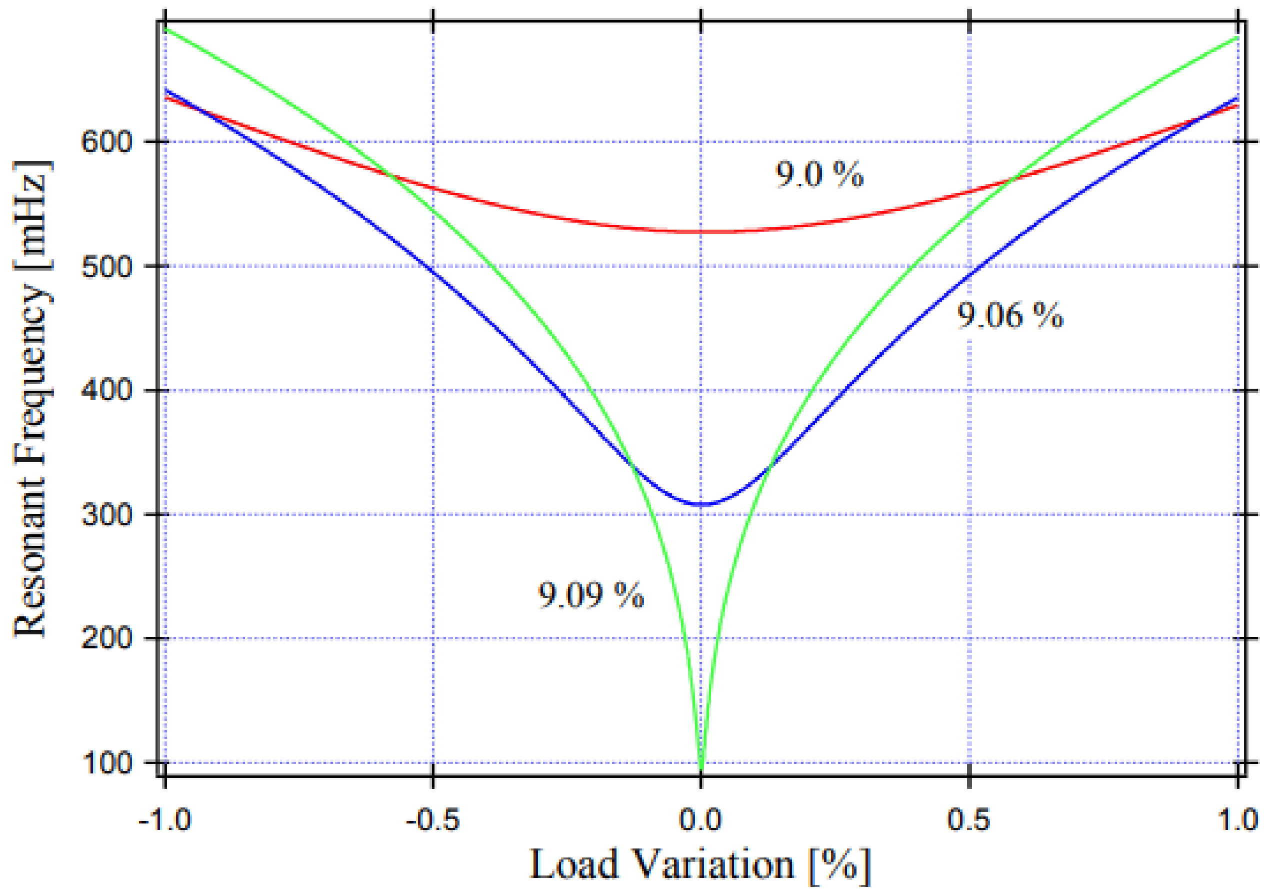
Remembering that Δy_{eq} **is just a small variation from the working point** y **we obtain:**

$$m(y) \; = \; m_0 + \Delta m = m_0 + y k_{\text{eff}}(y)/g$$

And then the resonant frequency becomes:

$$\omega(y) = \sqrt{\frac{k_{\text{eff}}(y)}{m_0 + \frac{y}{g} k_{\text{eff}}(y)}},$$

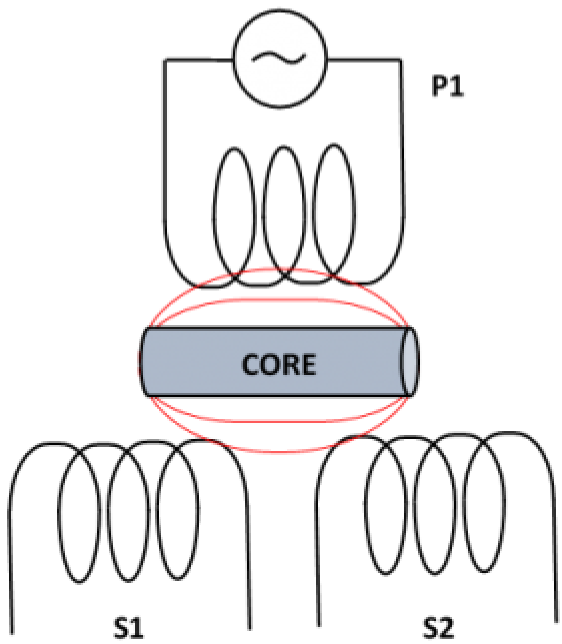
GAS Resonant Frequency 2



At a compression K larger than 9.0% the slope of the load-height curve (page 15) approaches infinity and the frequency of the filter goes to 0. Compressing the filter even more, the frequency becomes imaginary, at which point the system is no longer able to perform stable oscillations. In such a state, the filter has two equilibrium positions. It is bistable!

LVDT 1
(Linear Variable Differential Transformer)

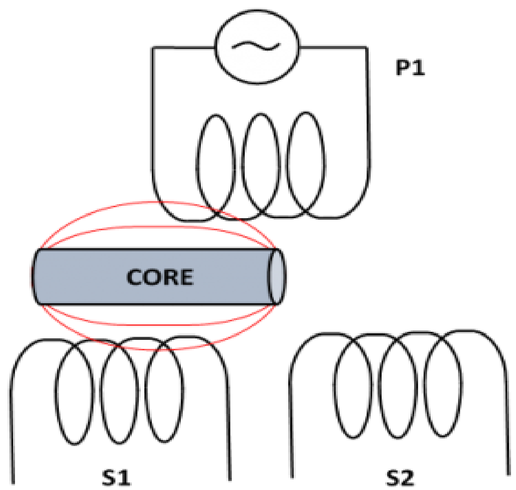
Now that we know the behavior of the GAS filter, we are interested to measure the displacement of the keystone at any moment. And for this we use the LVDTs.

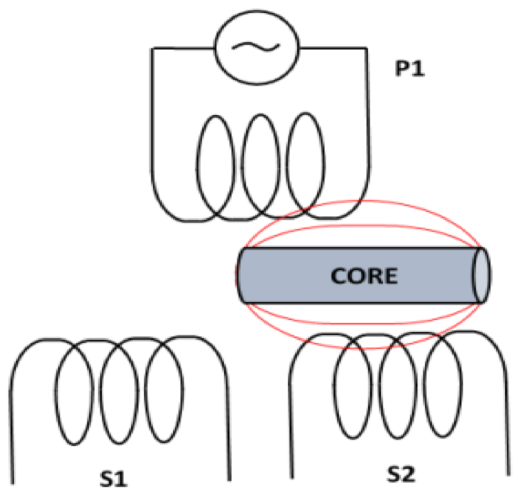


A Linear Variable Differential Transformer (LVDT) is a sensor that converts the linear position or motion of a measured object to a proportional electrical output.

LVDT 2

An LVDT has 3 components: the single primary winding, two secondary windings S1 and S2, and an <u>optional</u> ferromagnetic movable core (we do not use this core for the GAS filters). There is no physical contact between the housing and the core. The single primary coil is centered in the housing and energized with an AC signal.



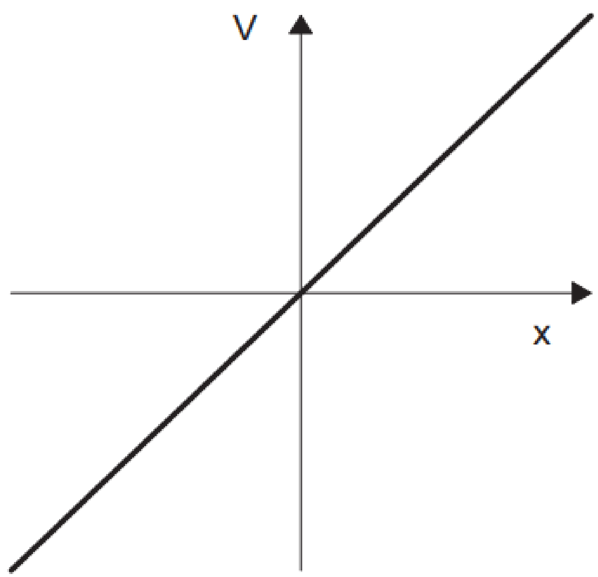


Magnetically coupled by the core, a voltage is induced in each of two symmetrical secondary windings connected in a series-opposing circuit. The effective voltage and LVDT output is the difference between each secondary.

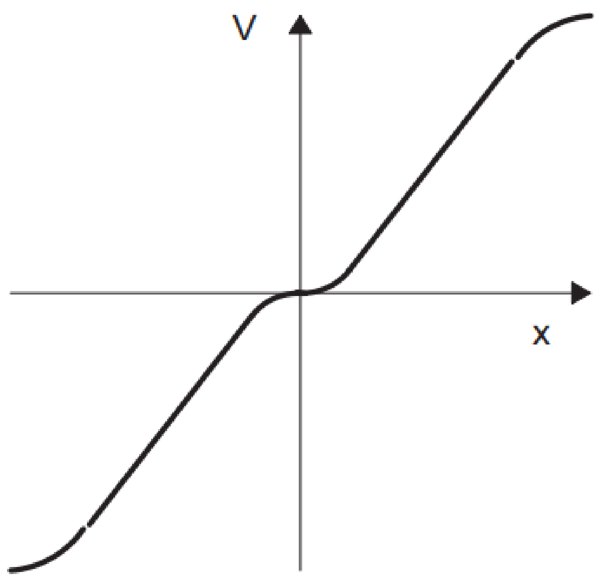
When the core moves away from the center (null) of the LVDT, the signal from the primary will be coupled to one secondary more than the other. As the core moves over S1, the voltage output of S1 increases. As the core moves over S2, the output of S2 increases.

LVDT 3

The value of (S1 - S2) and (S2 – S1) becomes a linear function of the core position as it moves toward S1 and S2, respectively.



a) Ideal output/displacement curve



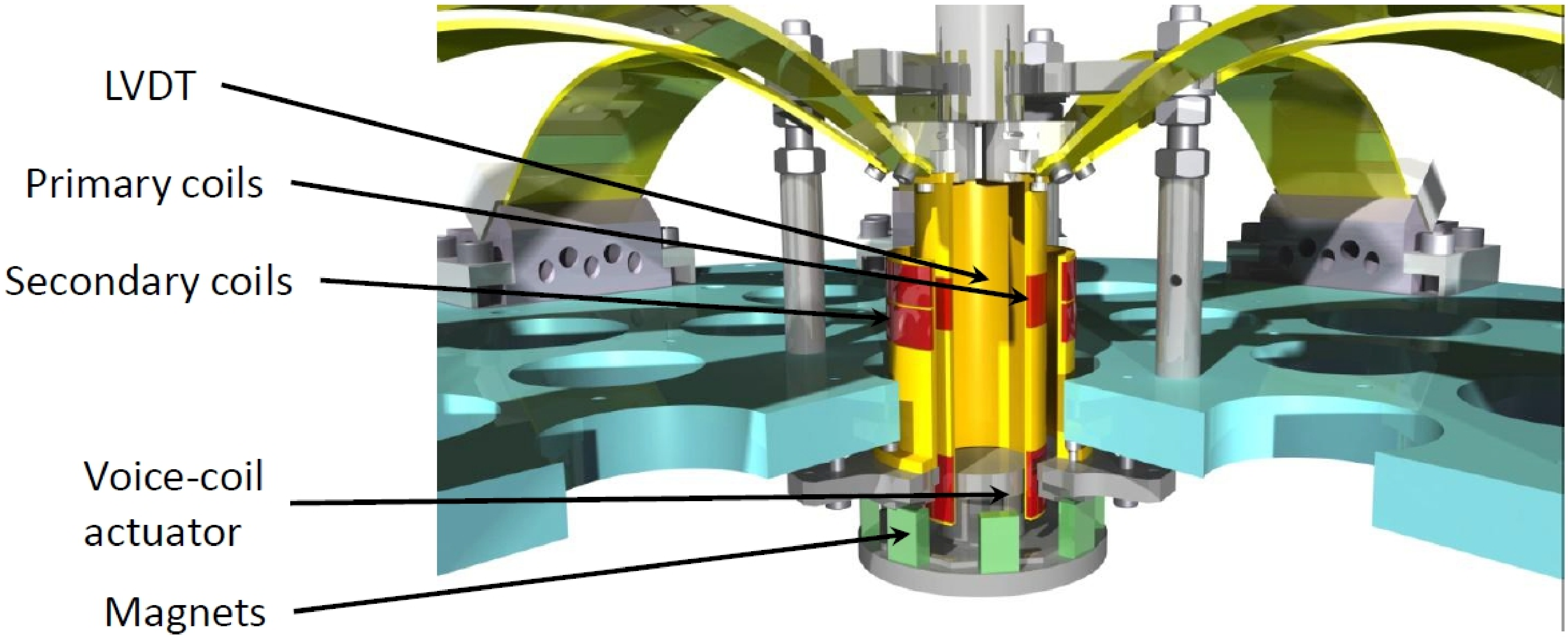
b) Exaggerated non−linear curve

Roll-off near the extremes of core displacement arises from transducer construction and phase effects, while non-linearity near the zero crossing is the effect of residual signal at the null point of the transducer. In a purely analog system, both effects are very difficult to correct.

For GAS filters LVDTs we use a digital signal processing (DSP) which can significantly increase the measurement accuracy.

GAS LVDT 1

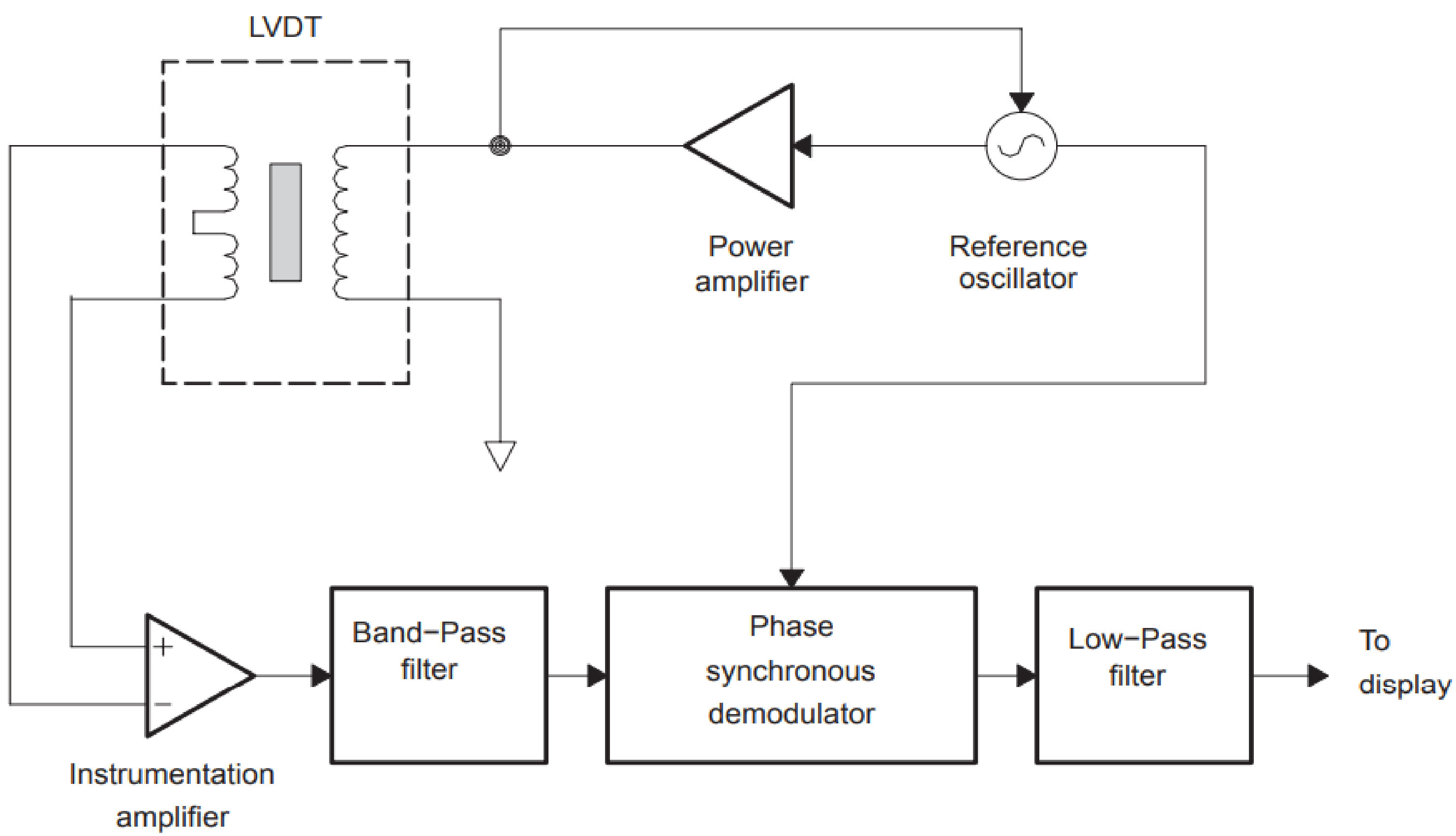
To measure the keystone relative position of the GAS filters. The LVDT primary coil is wounded around a PEEK (material) cylindrical piece fixed together with the keystone.



GAS LVDT 2

•Typically we use a sinusoidal excitation signal of 5 Vpp at 10 kHz for the primary coil which produces a magnetic field around the two receiver (secondary) coils and provide oscillating voltages.

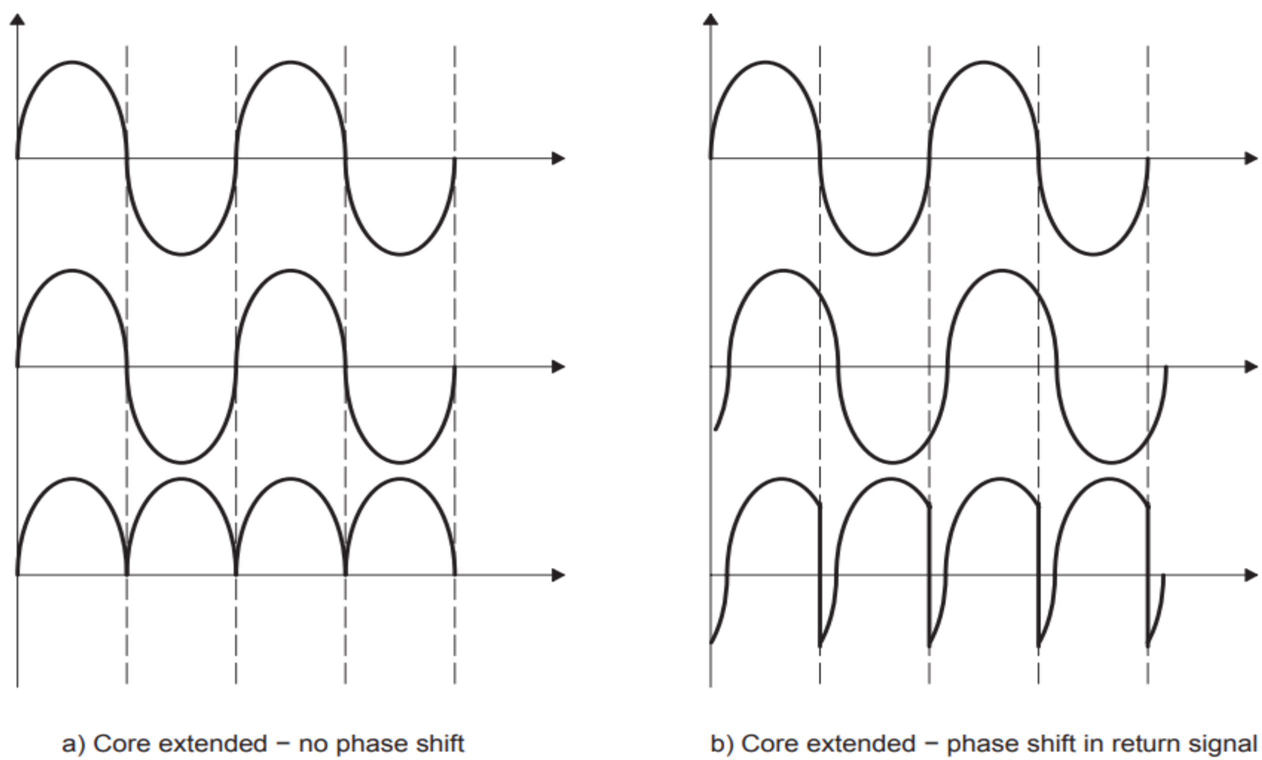
•When the emitter coil is displaced from the center, the mutual inductance is changed and that makes a difference in the induced voltages. The induced differential voltage is then introduced to a mixer which demodulates the oscillation signals and produce DC signals proportional to the oscillation amplitude.



GAS LVDT 3

The demodulation process is multiplying the return signal with the reference oscillator being fed to the primary. <u>If both signals are in-phase</u>, the result is of the form: $sin^2\theta$ which is a rectified sine wave which can be smoothed with a low-pass filter to give a stable voltage linearly proportional to core position. The technique works well in the absence of phase shift between reference and return signals (left). When there is a phase offset (right), the zero crossing points of the return signal no longer align with those of the reference signal, and negative "spikes" are seen in the rectified trace at the demodulator output which leads to non-linear measurement error (AD630 Balanced demodulator used as a precision phase comparator).

More here: https://www.st-andrews.ac.uk/~www_pa/Scots_Guide/RadCom/part13/page1.html



Effect of phase shift on demodulated signal.

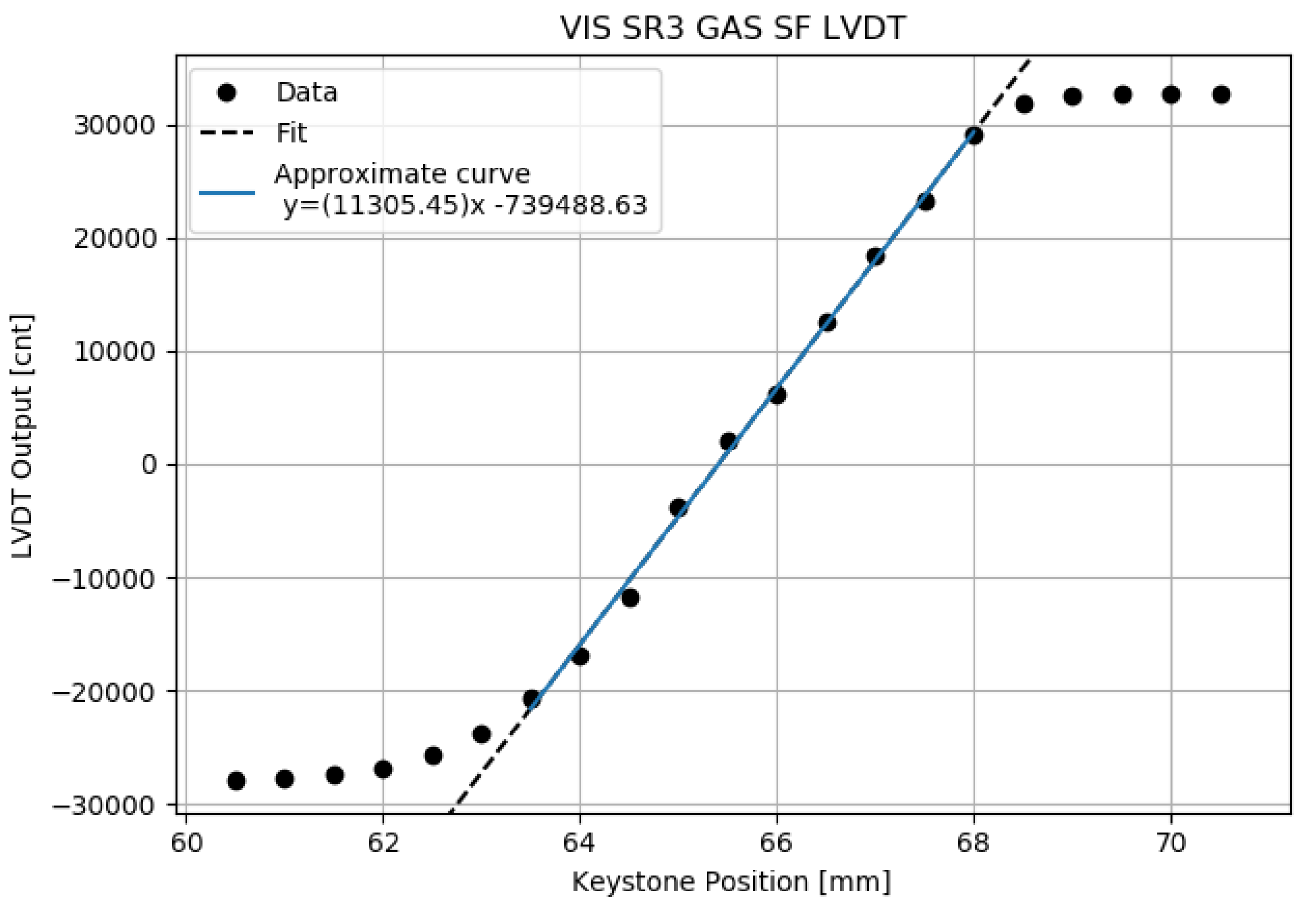
GAS LVDT Calibration 1

For the calibration process we need to set in phase the return signal (from secondary coils) with the local oscillator by adjusting the phase trimmers in the driver card. In order to have maximum output (see figure in page 17).

Adjustable from the front (use screwdriver)

GAS LVDT Calibration 2

Once the signals are in phase, we can move the keystone manually up and down for a range of ±5 mm around the nominal position and register the reading from the digital system.



The calibration factor is the inverse of the slope of this graph.

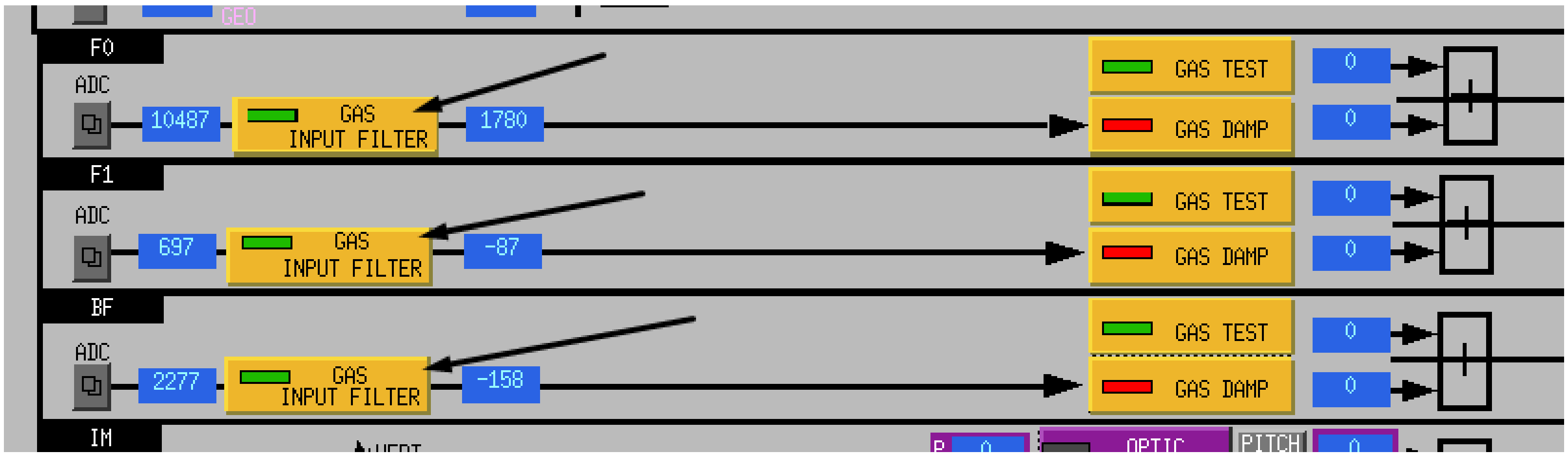
cal_f = 1/11305.45 [cnt/mm]

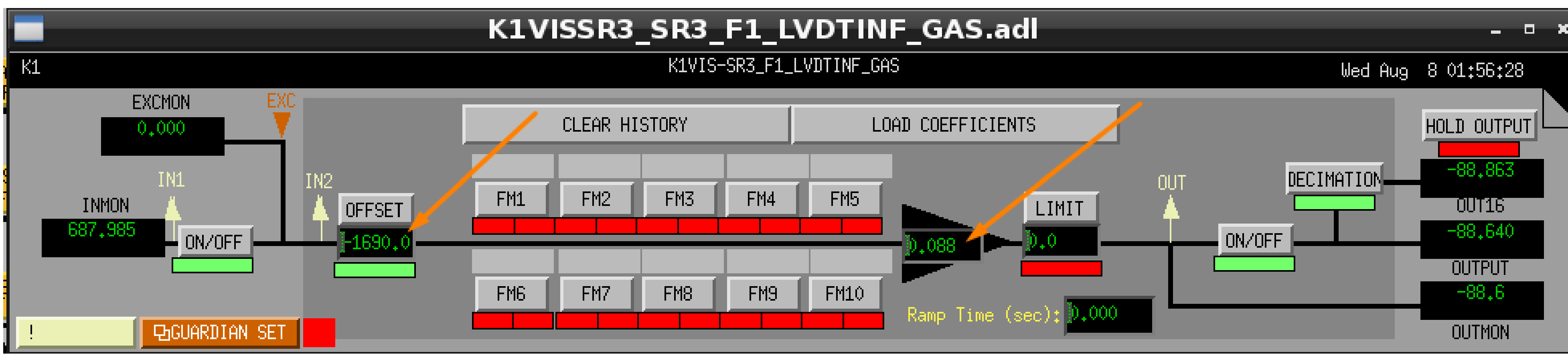
cal_f = 0.08845 [µm/cnt]

Offset: 1690 [cnt]
Readout at nominal height (65.5mm)

GAS LVDT Calibration 3

Then we need to insert the calibration factor and subtract the offset in the digital system.

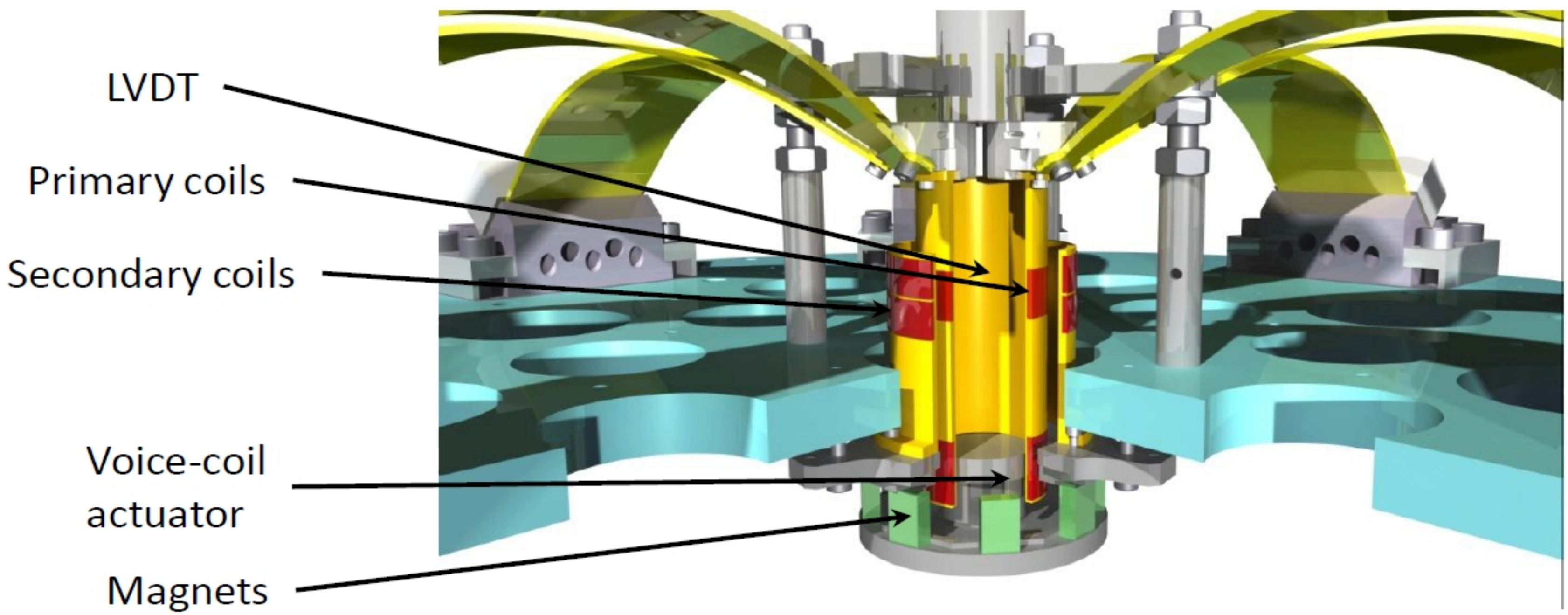




Voice coil actuator 1

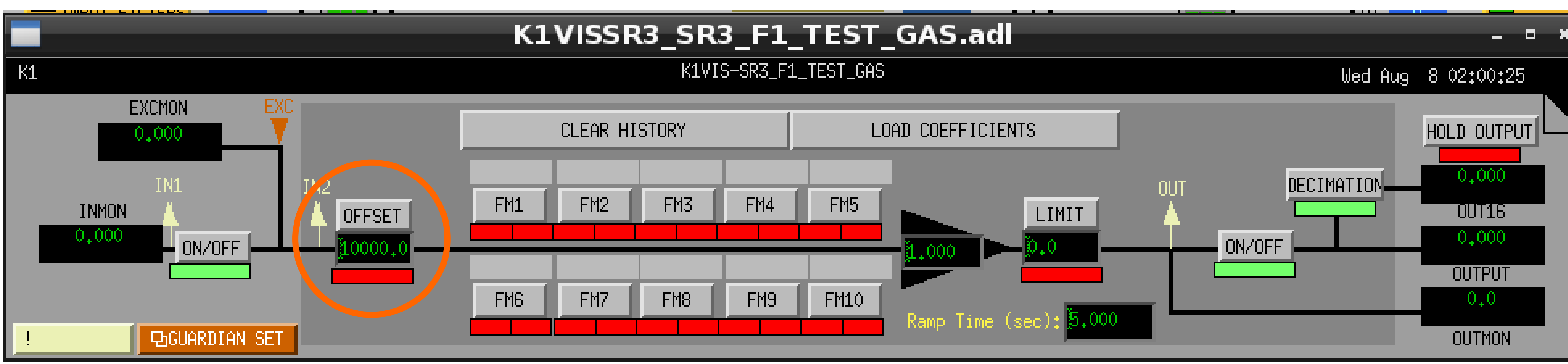
We can move the position of the keystone by applying current to the force coil (below the LVDT).
The vertical actuators are co-axially located with the LVDTs at the center of the GAS filters.

A yoke containing permanent magnets is hanging from the springbox. These actuators need to deliver a constant force, within 1%, in a region of 10 mm in the horizontal plane. To accomplish this special care has been taken in the design of the magnetic yoke. With these actuators, the top stage can be positioned within the resolution of the LVDTs.



Voice coil actuator 2

Excitation value



Static positioning

We use stepper motors for DC positioning and alignment of the suspension System, which are controlled from the digital system through EPICS. Stepper motors are driven by using a TMCM-6110 controller/driver board, which supports up to 6 bipolar stepper motors and has various communication interfaces for digital controls. The TMCM-6110 board is connected to the digital system LAN with RS485 (2-wire) serial interface, through an Ethernet-serial convertor NPort 5110A.

But we will hear more details about this at another opportunity.

Task

(1) If we want to have a resonant frequency $f_0=0.3$ Hz by means of a linear spring. How much elongation should have this system?

(2) Apply different excitation values on the digital system (between -20000 and 20000) for one of the GAS filters and measure the position of the keystone. Use StripTool.

(3) Apply different excitation values on the digital system (between -20000 and 20000) and measure the position of the keystone of one of the GAS filters (BF, SF and F0) make a graph and compare your result with other people.

Next Lecture

Plan for next weeks:

- GAS Filters details. ○
- OSEMs details.
- FFTs, windowing, aliasing, leakage, ...
- Use of diaggui.
- Transfer Functions (TF).
- Inverted pendulum.

Suggestions?

- How do the pico motor and stepper motor work?
- How does the geophone work?.
- Explaination about OSEMs.
- Terrance will explain and prepare documentation about oplevs.
- Kozu-kun will explain the suspension modelling of the payload.

KAGRA

Bibliography

NAOJ National Astronomical Observatory of Japan 国立天文台

• Low frequency vibration isolation system for Large Scale Gravitational Wave Detectors by T. Tsekiguchi.

• Seismic attenuation for Advanced Virgo. Vibration isolation for the external injection bench by M. Blom.

• Pendulum with finite mass system N. Mio.

• Low frequency seismic isolation for gravitational wave detectors by A. Takamori.

• LVDT signal conditioning. Texas Instruments.

You can get these and more pdf files from the KAGRA wiki page:

http://gwwiki.icrr.u-tokyo.ac.jp/JGWwiki/KAGRA/Subgroups/VIS/Introduction