Notes on capacitor precision for the satellite amplifier board

JGW-T1808388-v1

created: 2018, June 15th

Kiwamu Izumi

1 Overview of this document

This document summarizes the reasoning behind choosing the precision required for the capacitor that is going to be staffed in the first amplification stage of the satellite amplification board [1] in the context of the recently proposed modification [2]. As stated in this document, **a 10% precision capacitor** seems reasonable for our typical applications.

The required precision is derived based on the size of cross-couplings when sensing multiple mechanical degrees of freedom using a set of the photo sensing systems, amplified by the satellite amplifier board. We require such cross-couplings to be smaller than a few % in the magnitude at 10 Hz and frequencies below.

2 Cross-coupling in a two-sensor system

2.1 Setup

For simplicity, we consider a specific situation where we have two photo-sensing-based sensors to sense two degrees of freedom for a mechanical object.

The two sensors are supposed to sense two mechanical degrees of freedom, say the yaw and longitudinal degrees of freedom. Assuming a lever length of ~ 1 m as a typical yaw amplification factor, we obtain the sensing matrix as

$$\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} H_1 & H_1 \\ H_2 & -H_2 \end{bmatrix} \begin{bmatrix} L \\ Y \end{bmatrix} \tag{1}$$

where L and Y are displacements of the mechanical object for the longitudinal and yaw degrees of freedom, respectively, S_1 and S_2 are the resulting amplified signals for sensors 1 and 2, respectively, and where H_1 and H_2 are the frequency responses of the satellite amplifier boards 1 and 2, respectively. Note that the first amplification stage is equipped with an RC low-pass filter and therefore we assume H_1 and H_2 to be a single pole system.

In most of applications, one decomposes the S_1 and S_2 signals into the mechanical degrees of freedom by applying a matrix in the digital system, so that he or she can estimate the motion of each degree of freedom independently as

$$\begin{bmatrix} S_L \\ S_Y \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}. \tag{2}$$

Plugging this into the first equation, we get

$$\begin{bmatrix} S_L \\ S_Y \end{bmatrix} = \frac{1}{2} \begin{bmatrix} H_1 + H_2 & H_1 - H_2 \\ H_1 - H_2 & H_1 + H_2 \end{bmatrix} \begin{bmatrix} L \\ Y \end{bmatrix}. \tag{3}$$

While the diagonal terms are the dominant elements in the matrix, the off-diagonal terms can end up being non-zero transfer functions only when $H_1 \neq H_2$. The off-diagonal elements must be sufficiently small in order not to introduce unnecessary cross-couplings between the longitudinal and yaw degrees of freedom.

2.2 Evaluating the size of the cross-coupling

Now, we consider a situation in that the capacitor values for satellite amplifiers 1 and 2 are different by $\pm\Delta$, so that their cut-off frequencies become

$$f_1 = \frac{1}{2\pi RC\,(1+\Delta)} \quad \text{and} \quad f_2 = \frac{1}{2\pi RC\,(1-\Delta)}, \tag{4}$$

where R and C are the nominal resistance and capacitor values. We set $R = 38.3\,\mathrm{k\Omega}$ and $C = 100\,\mathrm{nF}$ [2]. If $\Delta = 0$, this results in $f_1 = f_2 = 41\,\mathrm{Hz}$.

Figure 1 shows the computed transfer functions for the diagonal and off-diagonal terms. The off-diagonal terms are computed with two different precision values, namely $\Delta = 5\%$ and 10%. It is clear that for both precision values, the off-diagonal terms are smaller than a few % below 10 Hz.

3 Conclusion

Let's use a 10% precision capacitor.

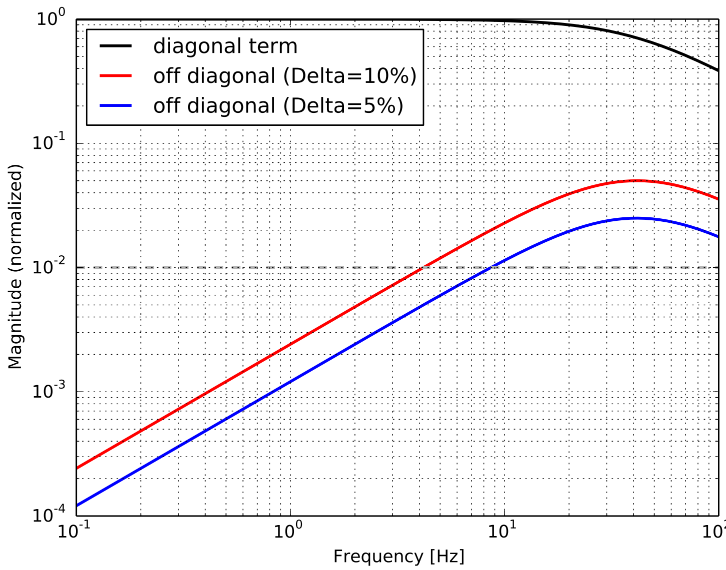

Figure 1: The normalized transfer functions for the diagonal and off-diagonal terms.

References

[1] M. Kamiizumi, "4Ch Satellite Amp Board," JGW-D1503499-v2 (2016) https://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=3499

[2] K. Izumi, "Proposal for modification of the satellite box," JGW-G1808352-v1 (2018) https://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=8352

[3] Y. Aso *et al.,* "Interferometer design of the KAGRA gravitational wave detector, " Phys. Rev. D **88,** 043007 (2013)