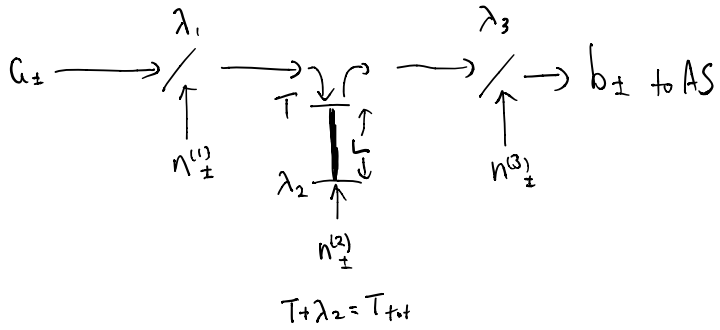

$a_\pm := a_{\omega_0 \pm \Omega}$, and so on.

$a_1 = \frac{1}{\sqrt{2}}(a_+ + a_-^\dagger)$, $a_2 = \frac{1}{i\sqrt{2}}(a_+ - a_-^\dagger)$, and so on

$Cov_{a_i a_j} = sq_{ij}$

$Cov_{n_i n_j} = \delta_{ij}$

Reflectivity of the filter cavity for U/L sidebands, $a_\pm$

$$r_\pm = -\sqrt{1-T} + \frac{T\sqrt{1-\lambda_2}\, e^{2iL(\omega_0 \pm \Omega)/c}}{1 - \sqrt{1-T_{tot}}\, e^{2iL(\omega_0 \pm \Omega)/c}}$$

$$b_\pm = \sqrt{1-\lambda_1}\sqrt{1-\lambda_3}\, r_\pm a_\pm + \sqrt{\lambda_1}\sqrt{1-\lambda_3}\, r_\pm n^{(1)}_\pm + \sqrt{1-\lambda_3}\sqrt{1-|r_\pm|^2}\, n^{(2)}_\pm + \sqrt{\lambda_3}\, n^{(3)}_\pm$$

$$b_1 = \sqrt{1-\lambda_1}\sqrt{1-\lambda_3}\,\frac{1}{\sqrt{2}}\left(r_+ a_+ + r_-^* a_-^\dagger\right) + \sqrt{\lambda_1}\sqrt{1-\lambda_3}\,\frac{1}{\sqrt{2}}\left(r_+ n_+^{(1)} + r_-^* n_-^{\dagger(1)}\right) + \sqrt{1-\lambda_3}\,\frac{1}{\sqrt{2}}\left(\sqrt{1-|r_+|^2}\, n_+^{(2)} + \sqrt{1-|r_-|^2}\, n_-^{\dagger(2)}\right) + \sqrt{\lambda_3}\, n_1^{(3)}$$

$\frac{1}{2}(r_+ + r_-^*)a_1 + \frac{i}{2}(r_+ - r_-^*)a_2$

$$b_2 = \sqrt{1-\lambda_1}\sqrt{1-\lambda_3}\,\frac{1}{i\sqrt{2}}\left(r_+ a_+ - r_-^* a_-^\dagger\right) + \sqrt{\lambda_1}\sqrt{1-\lambda_3}\,\frac{1}{i\sqrt{2}}\left(r_+ n_+^{(1)} - r_-^* n_-^{\dagger(1)}\right) + \sqrt{1-\lambda_3}\,\frac{1}{i\sqrt{2}}\left(\sqrt{1-|r_+|^2}\, n_+^{(2)} - \sqrt{1-|r_-|^2}\, n_-^{\dagger(2)}\right) + \sqrt{\lambda_3}\, n_2^{(3)}$$

$-i\frac{1}{2}(r_+ - r_-^*)a_1 + \frac{1}{2}(r_+ + r_-^*)a_2$

$a_+ = \frac{1}{\sqrt{2}}(a_1 + i a_2)$, $a_-^\dagger = \frac{1}{\sqrt{2}}(a_1 - i a_2)$

$Cov_{ij} := Cov_{b_i b_j}$

$$Cov_{11} = (1-\lambda_1)(1-\lambda_3)\left[\frac{1}{4}|r_+ + r_-^*|^2 sq_{11} + \frac{1}{4}|r_+ - r_-^*|^2 sq_{22} + \frac{i}{2}Re\left[(r_+^* + r_-)\cdot i(r_+ - r_-^*)\right] sq_{12}\right] + \lambda_1(1-\lambda_3)\cdot\frac{1}{2}\left(|r_+|^2 + |r_-|^2\right)$$

$$+ (1-\lambda_3)\cdot\frac{1}{2}\left(2 - |r_+|^2 - |r_-|^2\right) + \lambda_3$$

$$= (1-\lambda_1)(1-\lambda_3)\left[\frac{1}{4}|r_+ + r_-^*|^2 \cdot sq_{11} + \frac{1}{4}|r_+ - r_-^*|^2 \cdot sq_{22} - Im(r_+ r_-)\cdot sq_{12}\right] + \lambda_1(1-\lambda_3)\cdot\frac{1}{2}\left(|r_+|^2 + |r_-|^2\right) + (1-\lambda_3)\cdot\frac{1}{2}\left(2 - |r_+|^2 - |r_-|^2\right) + \lambda_3$$

$$Cov_{12} = (1-\lambda_1)(1-\lambda_3)\left[\frac{1}{4i}(r_+ - r_-^*)(r_+^* + r_-)sq_{11} + \frac{1}{4i}(r_+ + r_-^*)(r_+^* - r_-)sq_{22} + \frac{1}{4}\left(|r_+ + r_-|^2 - |r_+ - r_-^*|^2\right)sq_{12}\right] + \lambda_1(1-\lambda_3)\frac{1}{2i}\left(|r_+|^2 - |r_-|^2\right) - (1-\lambda_3)\frac{1}{2i}\left(|r_+|^2 - |r_-|^2\right)$$

$$Cov_{21} = (Cov_{12})^*$$

$$Cov_{22} = (1-\lambda_1)(1-\lambda_3)\left[\frac{1}{4}|r_+ - r_-^*|^2 sq_{11} + \frac{1}{4}|r_+ + r_-^*|^2 sq_{22} + \frac{1}{2}Re\left[(r_+^* r_-)\cdot i(r_+ + r_-^*)\right]sq_{12}\right] + \lambda_1(1-\lambda_3)\cdot\frac{1}{2}\left(|r_+|^2 + |r_-|^2\right) + (1-\lambda_3)\cdot\frac{1}{2}\left(2 - |r_+|^2 - |r_-|^2\right) + \lambda_3$$

$$= (1-\lambda_1)(1-\lambda_3)\left[\frac{1}{4}|r_+ - r_-^*|^2 sq_{11} + \frac{1}{4}|r_+ + r_-^*|^2 sq_{22} + Im\left[r_+ r_-\right]\cdot sq_{12}\right] + \lambda_1(1-\lambda_3)\cdot\frac{1}{2}\left(|r_+|^2 + |r_-|^2\right) + (1-\lambda_3)\cdot\frac{1}{2}\left(2 - |r_+|^2 - |r_-|^2\right) + \lambda_3$$

$$\begin{bmatrix} a_1^{out} \\ a_2^{out} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + (\text{ITF loss terms}) + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}\sqrt{2K}\,\tau\,\frac{h}{hsql}$$

ITF output

, Measured quadrature $\rightarrow a_\xi^{out} = \sin\xi \cdot a_1^{out} + \cos\xi \cdot a_2^{out}$

$$\langle (a_\xi^{out})^\dagger a_\xi^{out} \rangle = \left\langle \begin{bmatrix} b_1^\dagger & b_2^\dagger \end{bmatrix}\begin{bmatrix} C_{11}^* & C_{21}^* \\ C_{12}^* & C_{22}^* \end{bmatrix}\begin{bmatrix} \sin\xi \\ \cos\xi \end{bmatrix}\begin{bmatrix} \sin\xi & \cos\xi \end{bmatrix}\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}\right\rangle + (\text{loss term})$$

$$= \left\langle \begin{bmatrix} b_1^\dagger & b_2^\dagger \end{bmatrix}\cdot\left(\begin{bmatrix} C_{11}^*\sin\xi + C_{21}^*\cos\xi \\ C_{12}^*\sin\xi + C_{22}^*\cos\xi \end{bmatrix}\begin{bmatrix} C_{11}\sin\xi + C_{21}\cos\xi & C_{12}\sin\xi + C_{22}\cos\xi \end{bmatrix}\right)\cdot\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}\right\rangle$$

$$= |C_{11}\sin\xi + C_{21}\cos\xi|^2 Cov_{11} + |C_{12}\sin\xi + C_{22}\cos\xi|^2 Cov_{22} + 2Re\left[(C_{11}^*\sin\xi + C_{21}\cos\xi)(C_{12}\sin\xi + C_{22}\cos\xi)\cdot Cov_{12}\right] + (\text{loss term})$$