

Sensitivity Optimization of Cryogenic Gravitational Wave Detectors

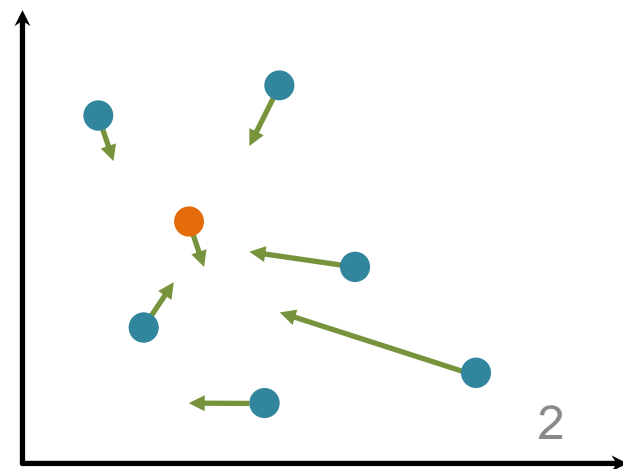
Yuta Michimura

Department of Physics, University of Tokyo

Kentaro Komori, Atsushi Nishizawa, Hiroki Takeda,
Koji Nagano, Yutaro Enomoto, Kazuhiro Hayama,
Kentaro Somiya, Masaki Ando

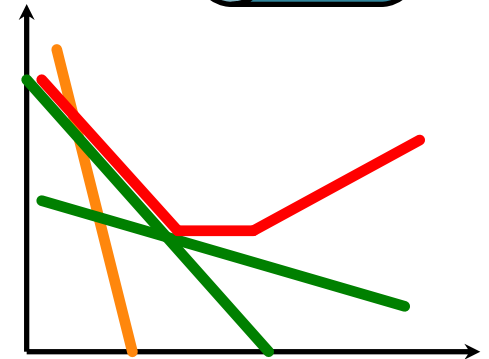
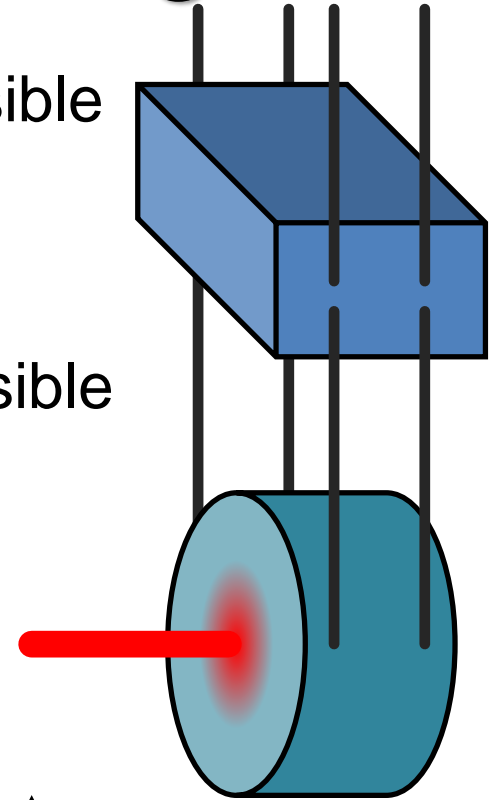
Overview

- **Cryogenic** GW detectors require careful design
 - tradeoff between laser power and temperature
 - simultaneous tuning of multiple parameters
- **Particle swarm optimization**
 - **simple** algorithm, few design variables
 - **fast** even with many parameters
- Applied for **KAGRA** sensitivity design
 - re-tuned 7 parameters
 - **inspiral range** optimization
 - **sky localization** optimization
- YM+ [arXiv:1804.09894](https://arxiv.org/abs/1804.09894)



Room Temperature Design

- **Seismic noise**: reduce as much as possible
multi-stage suspensions
underground
- **Thermal noise**: reduce as much as possible
larger mirror
thinner and longer wires
- **Quantum noise**: optimize the shape
input laser power
tune signal recycling parameters



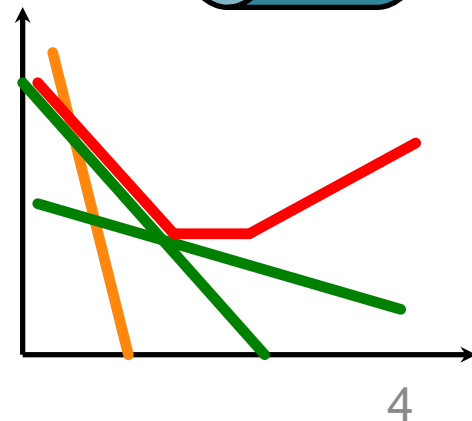
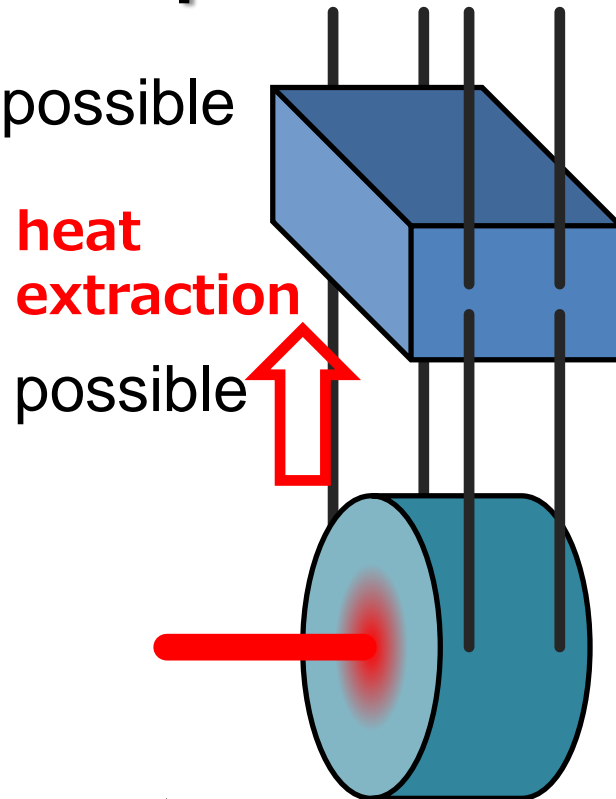
Cryogenic Design is Complicated

- **Seismic noise**: reduce as much as possible
multi-stage suspensions
underground

- **Thermal noise**: reduce as much as possible
larger mirror
thinner and longer wires

cryogenic cooling

- **Quantum noise**: optimize the shape
input laser power
tune signal recycling parameters



Cryogenic Design is Complicated

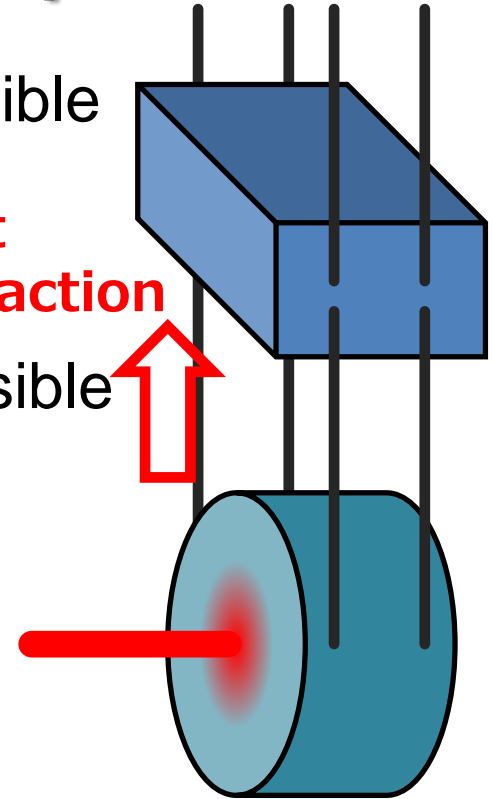
- **Seismic noise**: reduce as much as possible
multi-stage suspensions
underground

- **Thermal noise**: reduce as much as possible
larger mirror
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cryogenic cooling

- **Quantum noise**: optimize the shape
input laser power
tune signal recycling parameters

heat
extraction

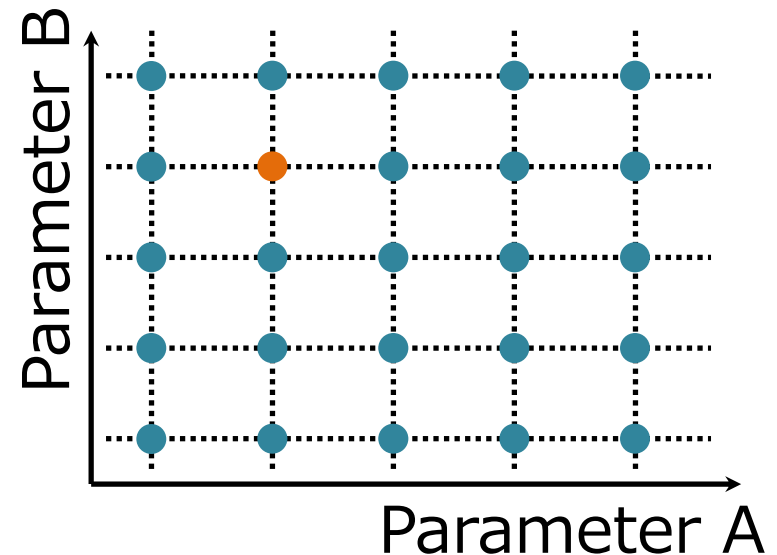


worse cooling power
mirror heating

Ancient Method is not Scalable

- Sensitivity design is an optimization problem
- Grid-based parameter search
 - deterministic
 - computational cost grows **exponentially** with number of parameters

- Future GW detectors (with crazy quantum ideas) require more parameters to be optimized
- **Almost impossible** with grid-based approach



Particle Swarm Optimization!

- Particles search the parameter space based on **own best** position and **entire swarm's best** known position

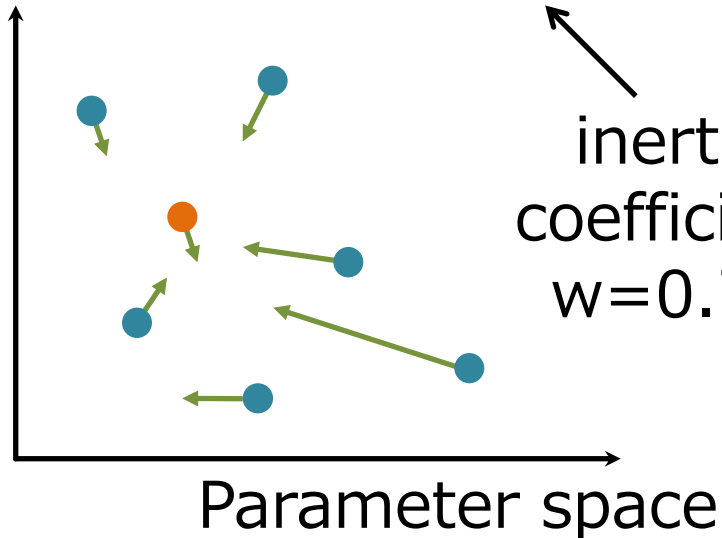
$$x_k(t+1) = x_k(t) + v_k(t)$$

personal best position so far global best position so far

$$v_k(t+1) = wv_k(t) + c_1r_1(\hat{x}_k - x_k(t)) + c_2r_2(\hat{x}_g - x_k(t))$$

inertia coefficient
 $w=0.72$

acceleration coefficient $c=1.19$
random number $r \in [0,1]$

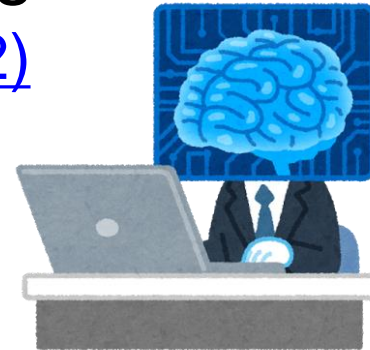


[Kennedy & Eberhart \(1995\)](#)





values for w and c are from [Standard PSO 2006](#)

PSO for GW Related Research

- CBC search
 - Weerathunga & Mohanty, [PRD 95, 124030 \(2017\)](#)
 - Wang & Mohanty, [PRD 81, 063002 \(2010\)](#)
 - Bouffanais & Porter, [PRD 93, 064020 \(2016\)](#)
- Continuous GW search using pulsar timing array
 - Wang, Mohanty & Jenet, [ApJ 795, 96 \(2014\)](#)
- Cosmological parameter estimation using CMB
 - Prasad & Souradeep, [PRD 85, 123008 \(2012\)](#)
- Gravitational lens modeling
 - Rogers & Fiege, [ApJ 727, 80 \(2011\)](#)
- Sensor correction filter design
 - Conor Mow-Lowry, [LIGO-G1700841](#) [LIGO-T1700541](#)



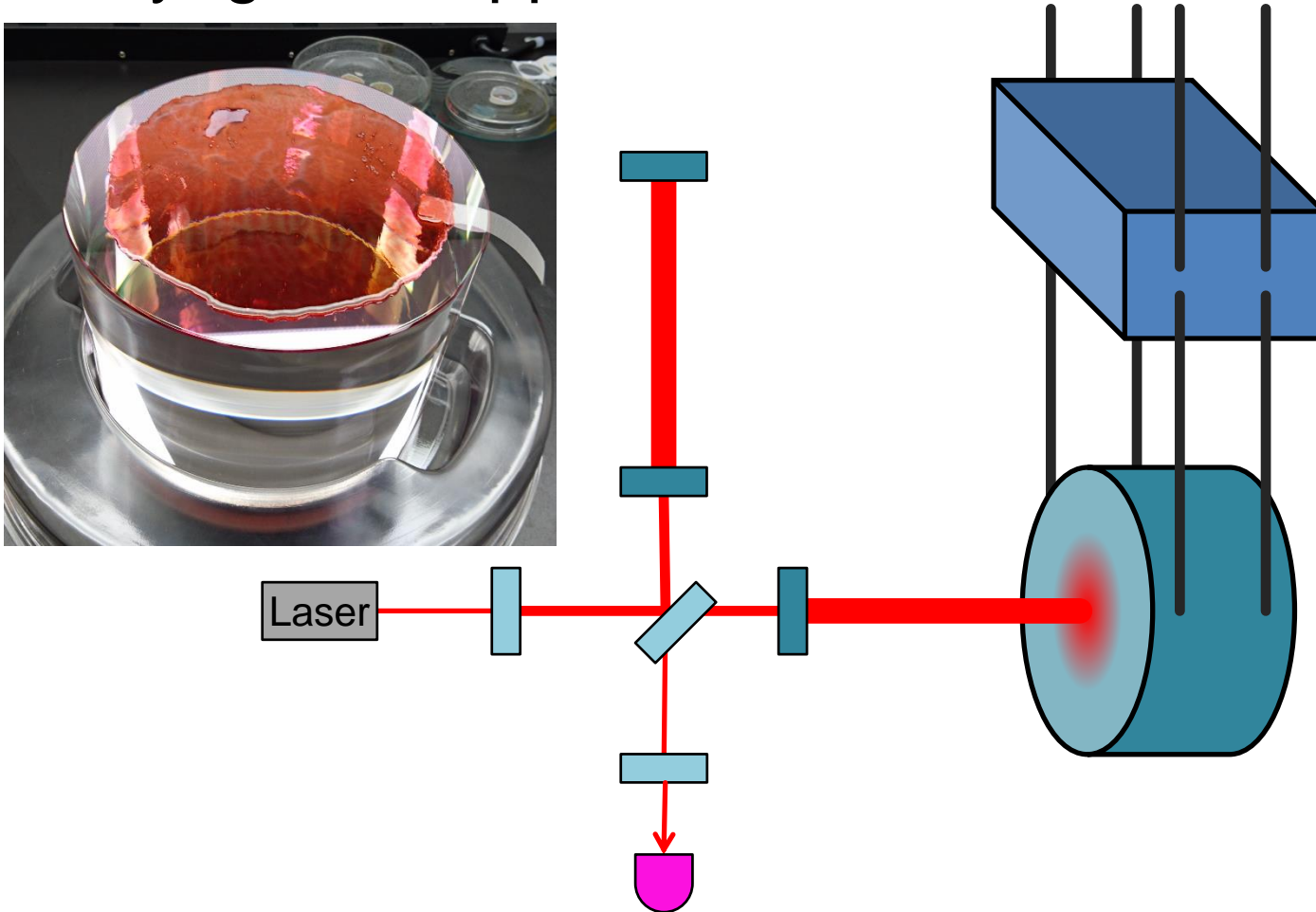
Pros and Cons of PSO

- **Fast** even for highly **multidimensional** parameter space 
uses entire swarm's information to search
- Requires **small number of design variables** and **little prior information** 
basically only swarm size and termination criterion
prior information is only search range
- **No guarantee** for convergence to global maximum 
stochastic method
- Do not give **error** of the parameters 
no direct information on stability of the solution

→ Sounds great for detector design

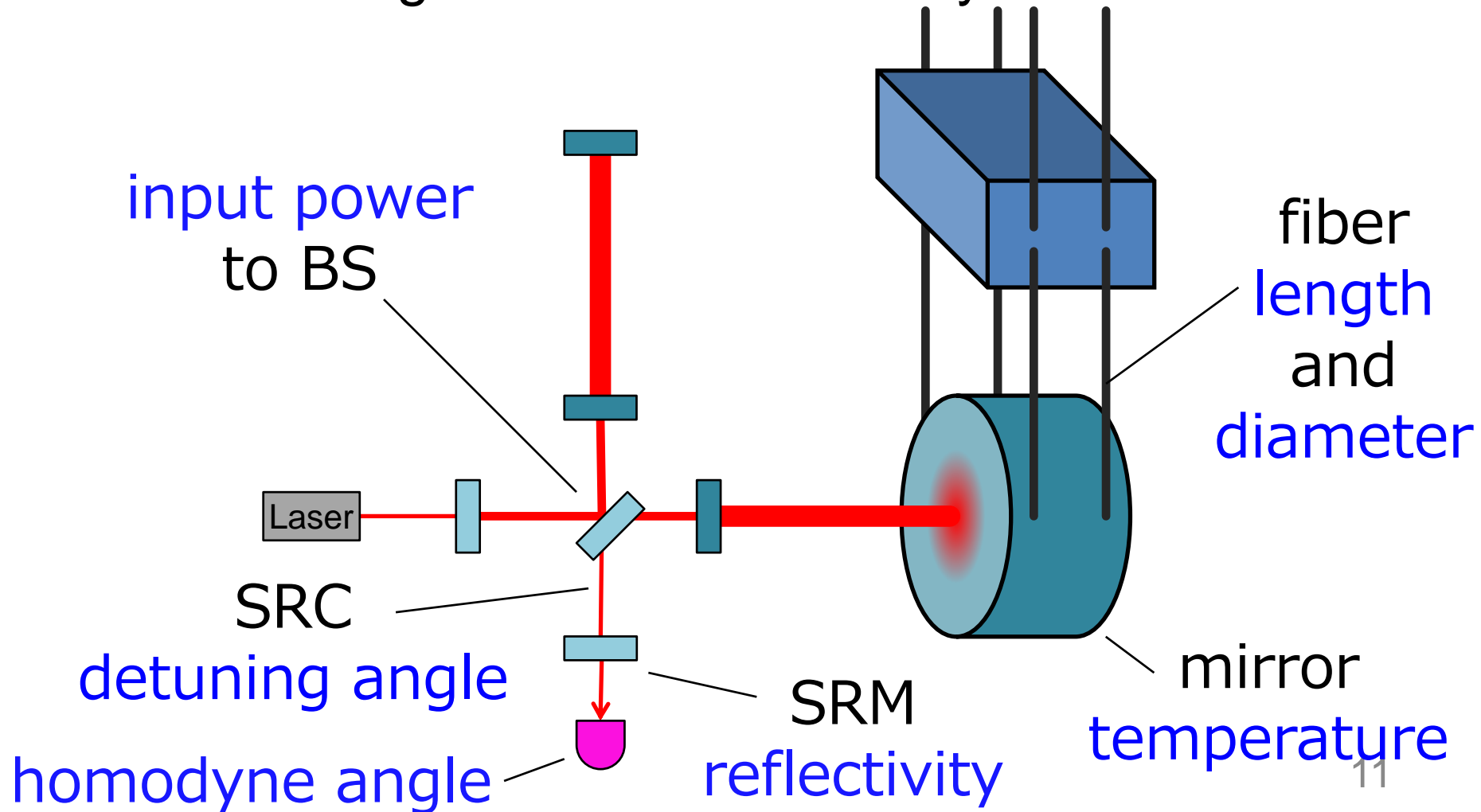
Apply PSO for KAGRA Design

- RSE interferometer
- Cryogenic sapphire test masses



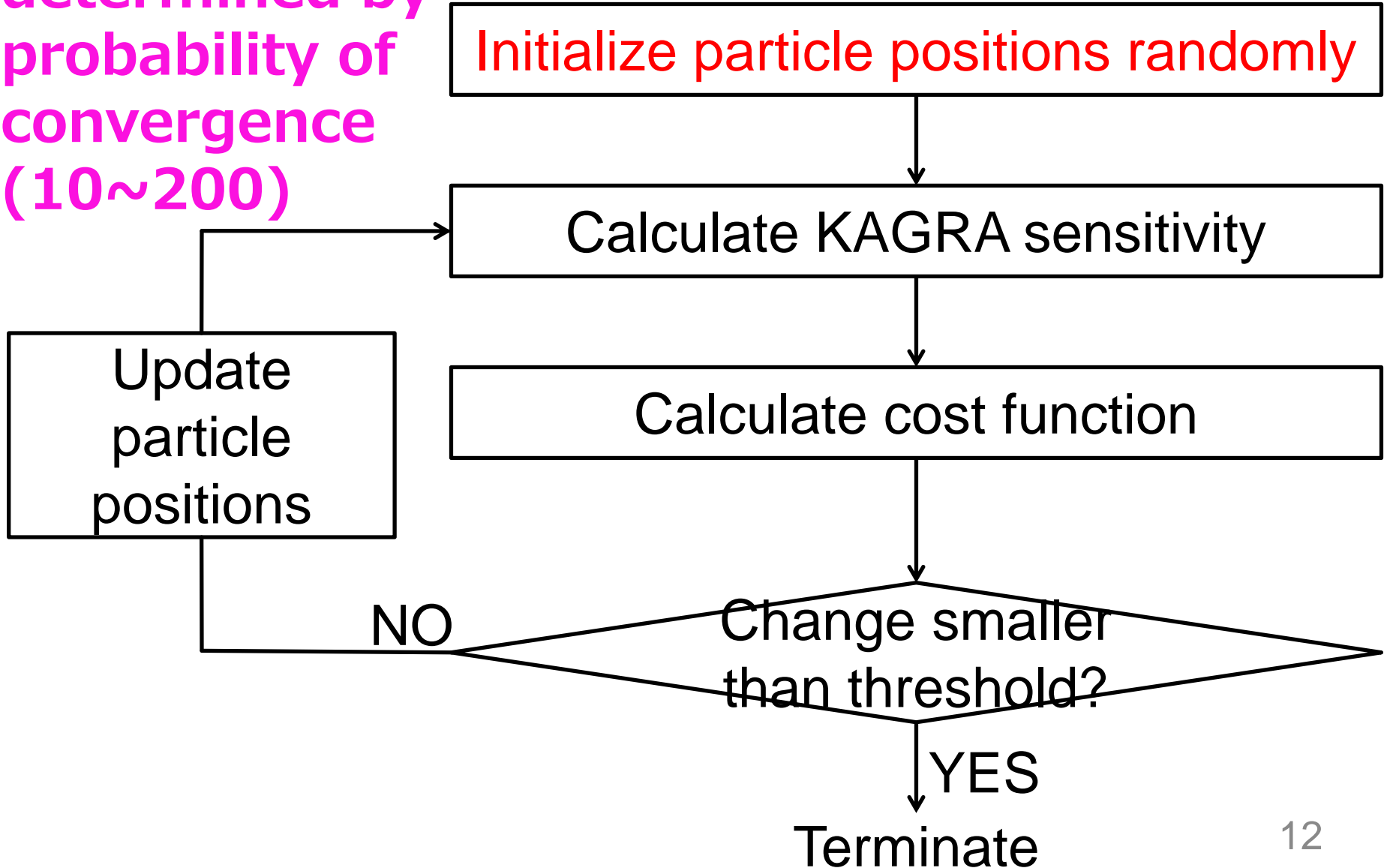
KAGRA Parameters to Optimize

- 7 parameters relatively easy to be retuned
- Search range based on feasibility



PSO Algorithm

Swarm size determined by probability of convergence (10~200)



PSO Algorithm

Initialize particle positions randomly

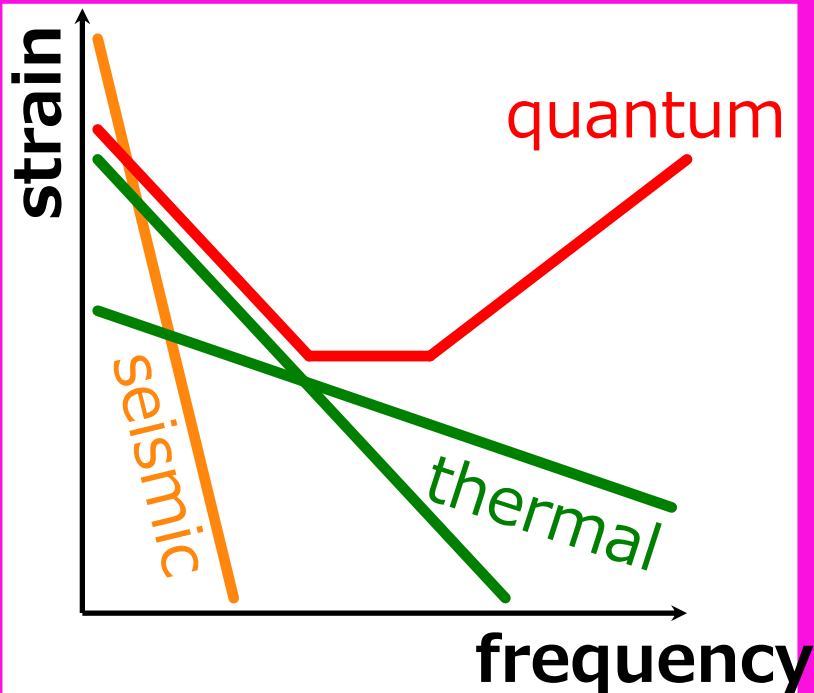
Calculate KAGRA sensitivity

Calculate cost function

Change smaller than threshold?

YES

Terminate



PSO Algorithm

Initialize particle positions randomly

**BNS inspiral range
as a cost function**

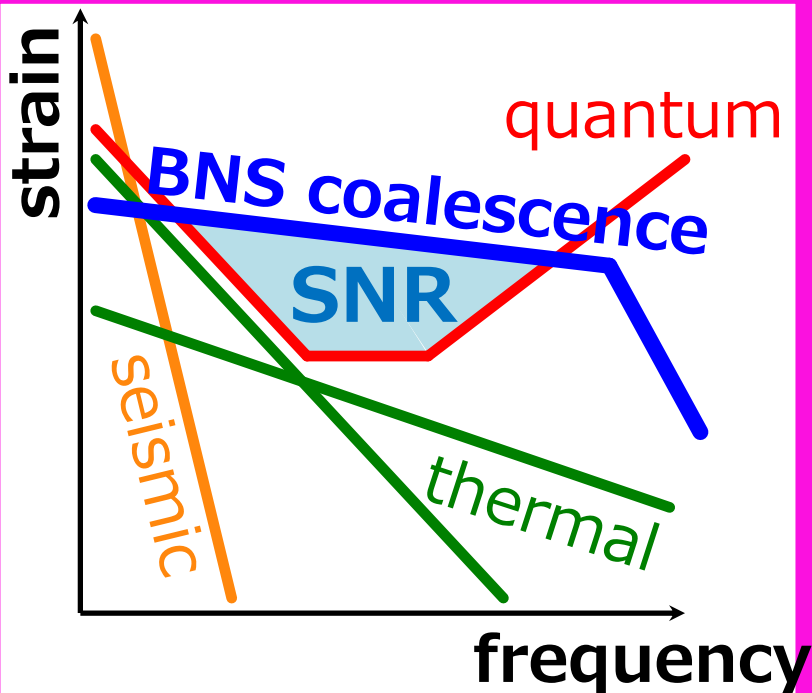
Calculate KAGRA sensitivity

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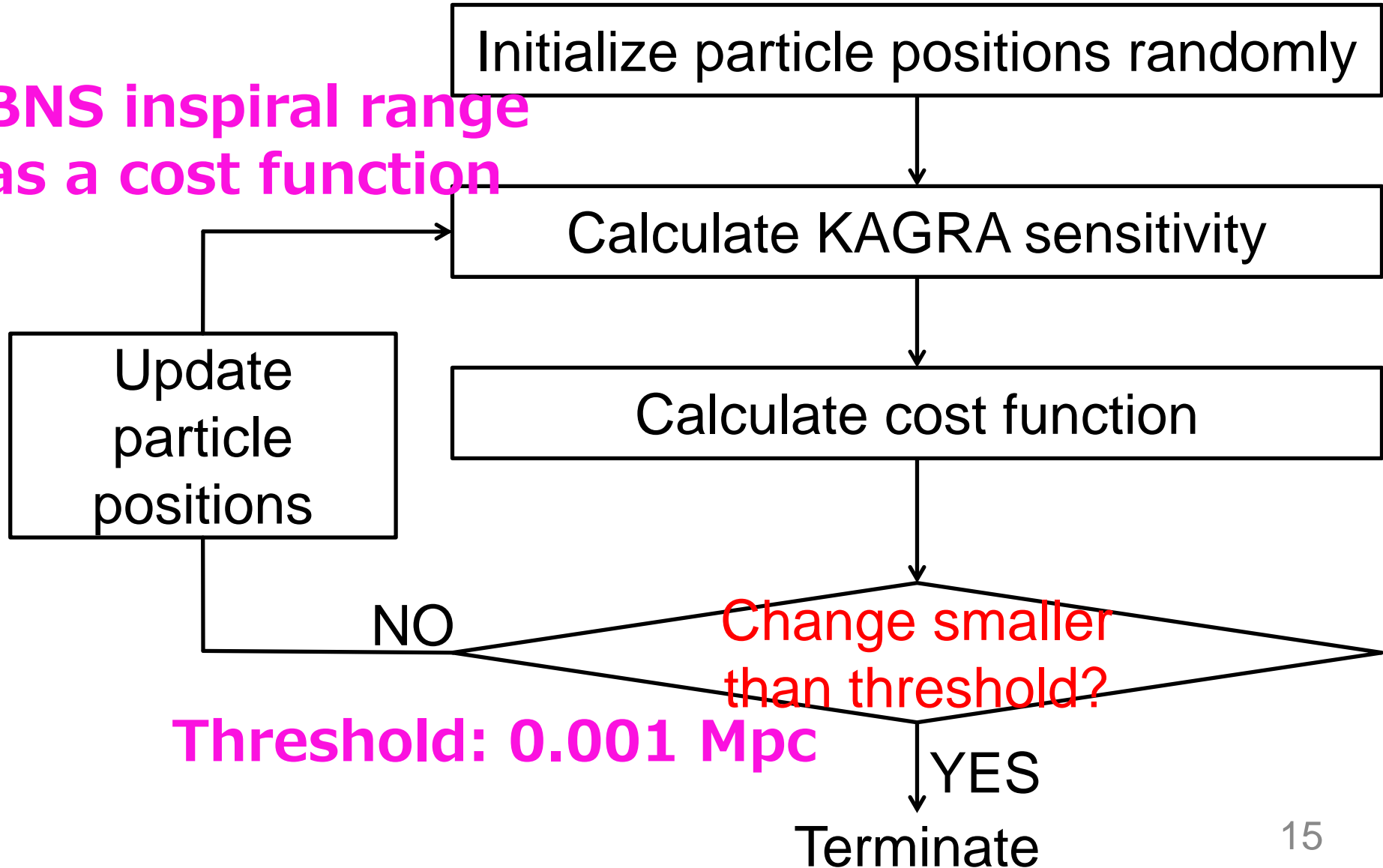
YES

Terminate

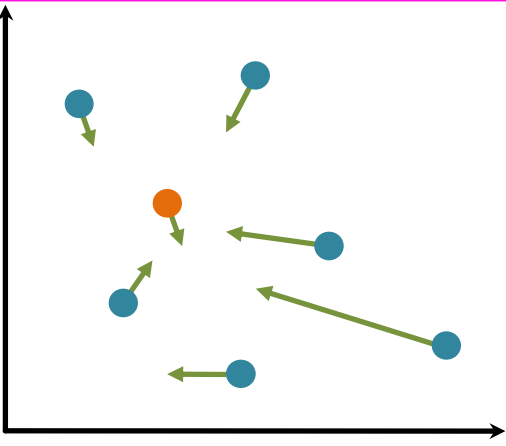


PSO Algorithm

**BNS inspiral range
as a cost function**



PSO Algorithm



Initialize particle positions randomly

Calculate KAGRA sensitivity

Calculate cost function

Update
particle
positions

NO

Change smaller
than threshold?

YES

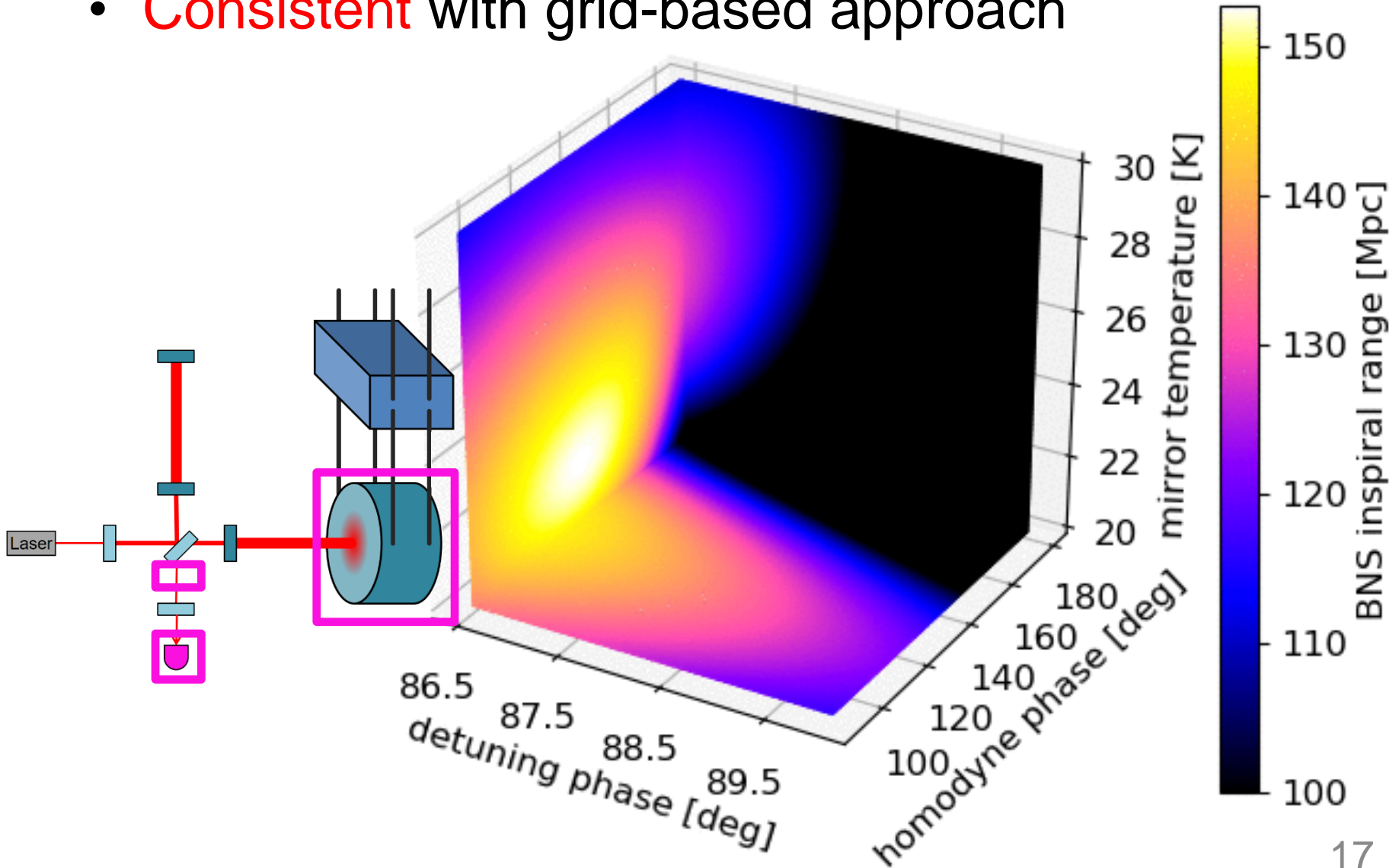
Terminate

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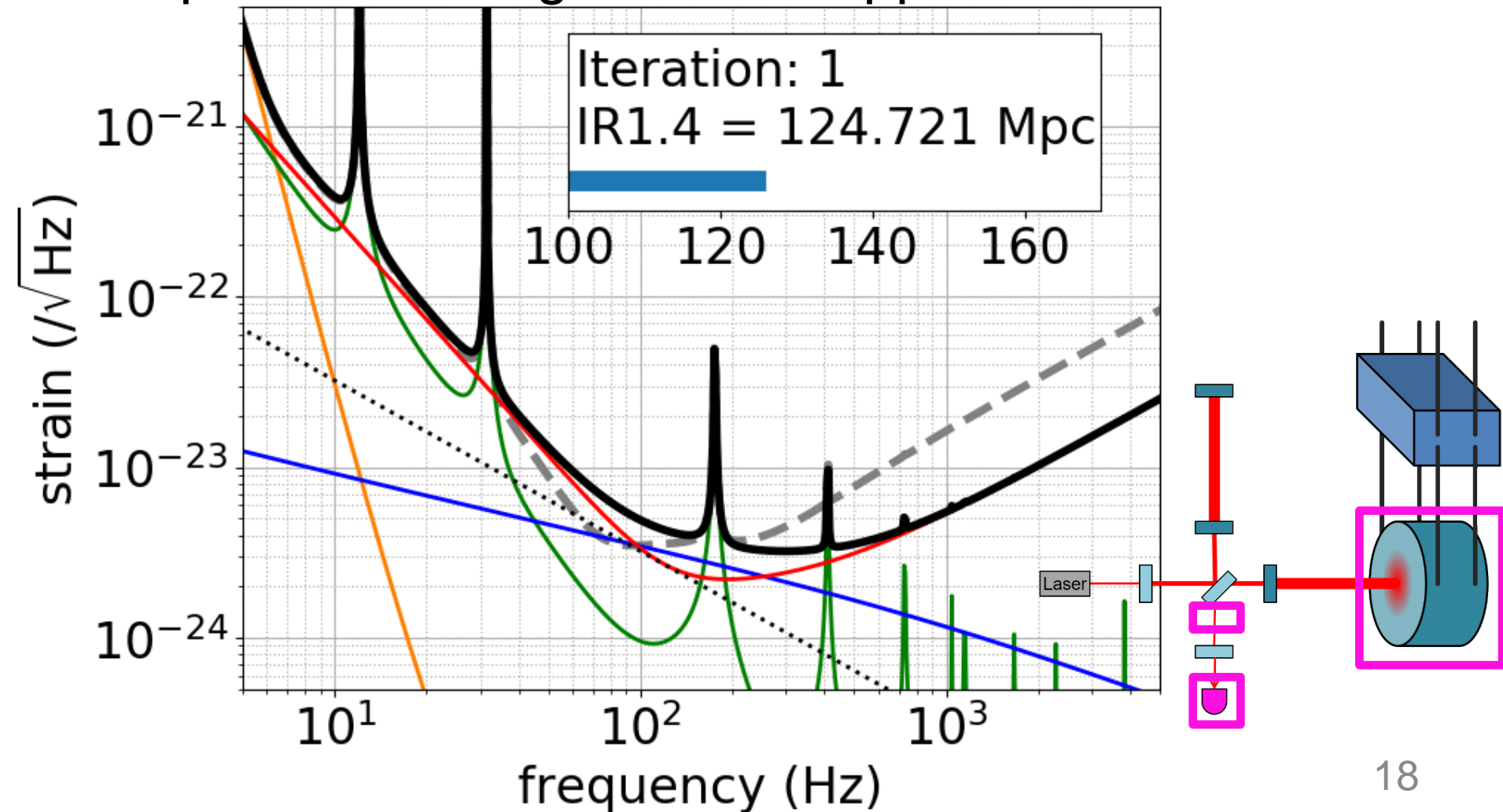
3 Parameter Optimization

- **Consistent** with grid-based approach



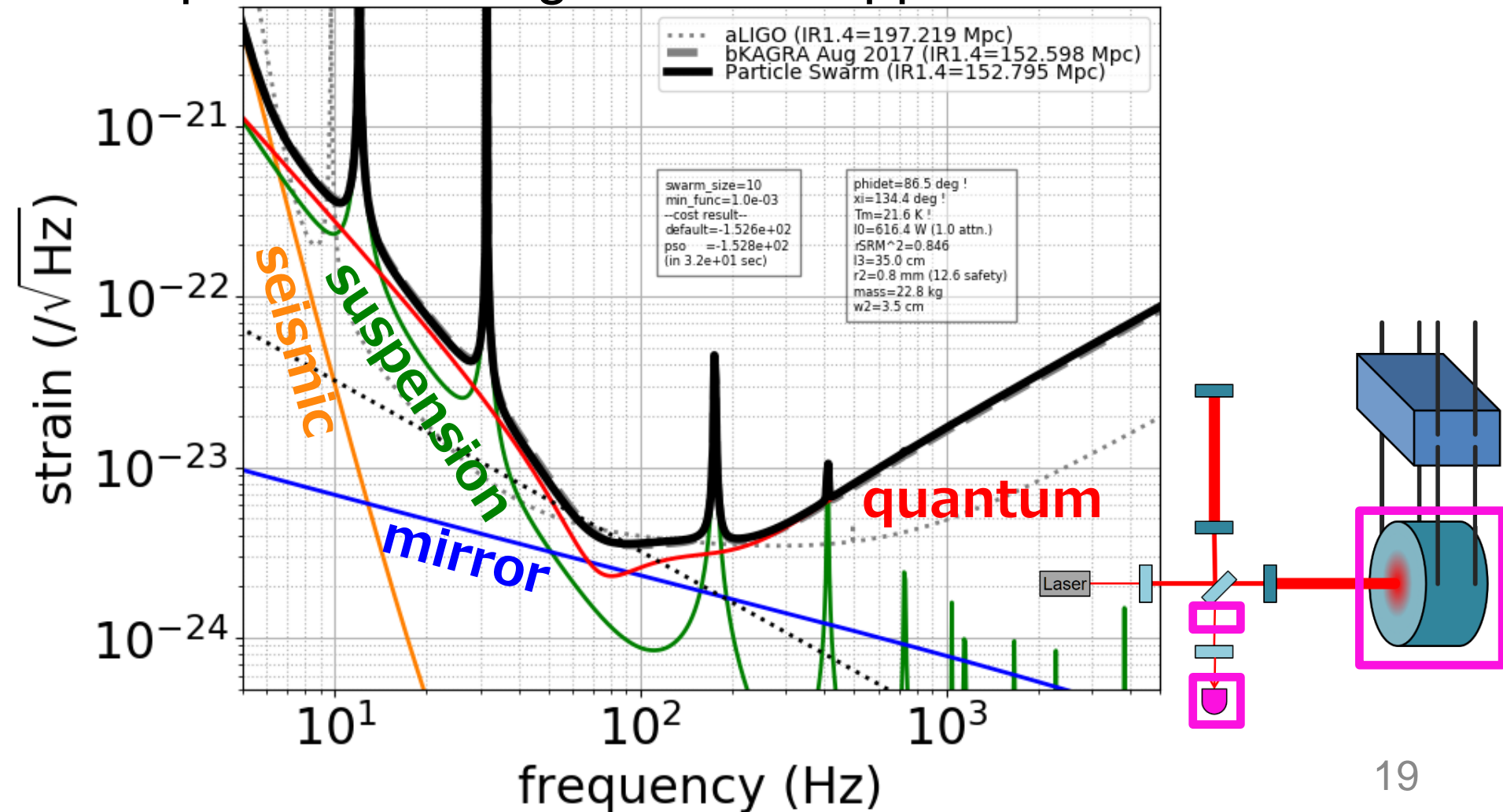
3 Parameter Optimization

- Consistent with current designed sensitivity optimized with grid-based approach



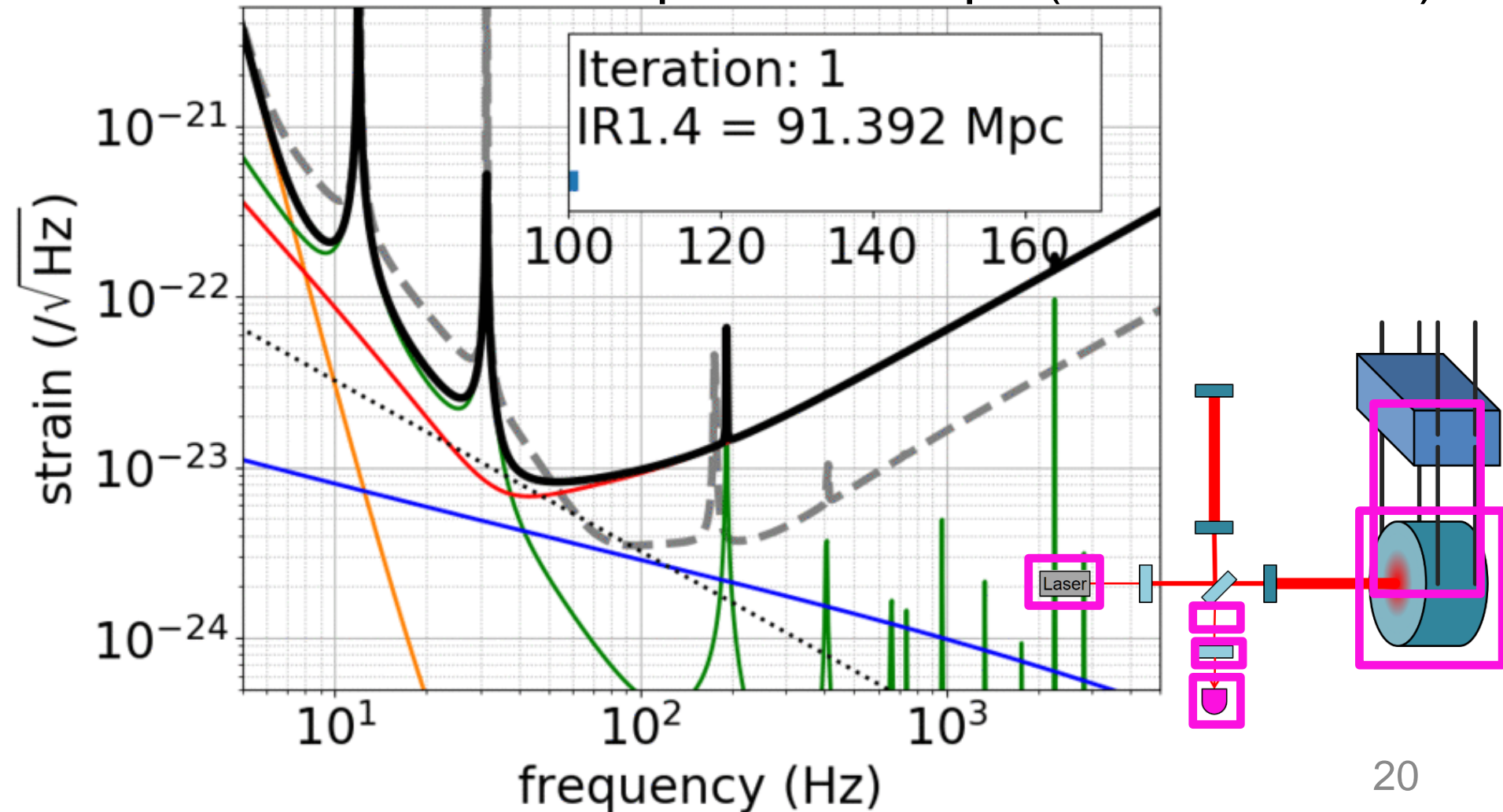
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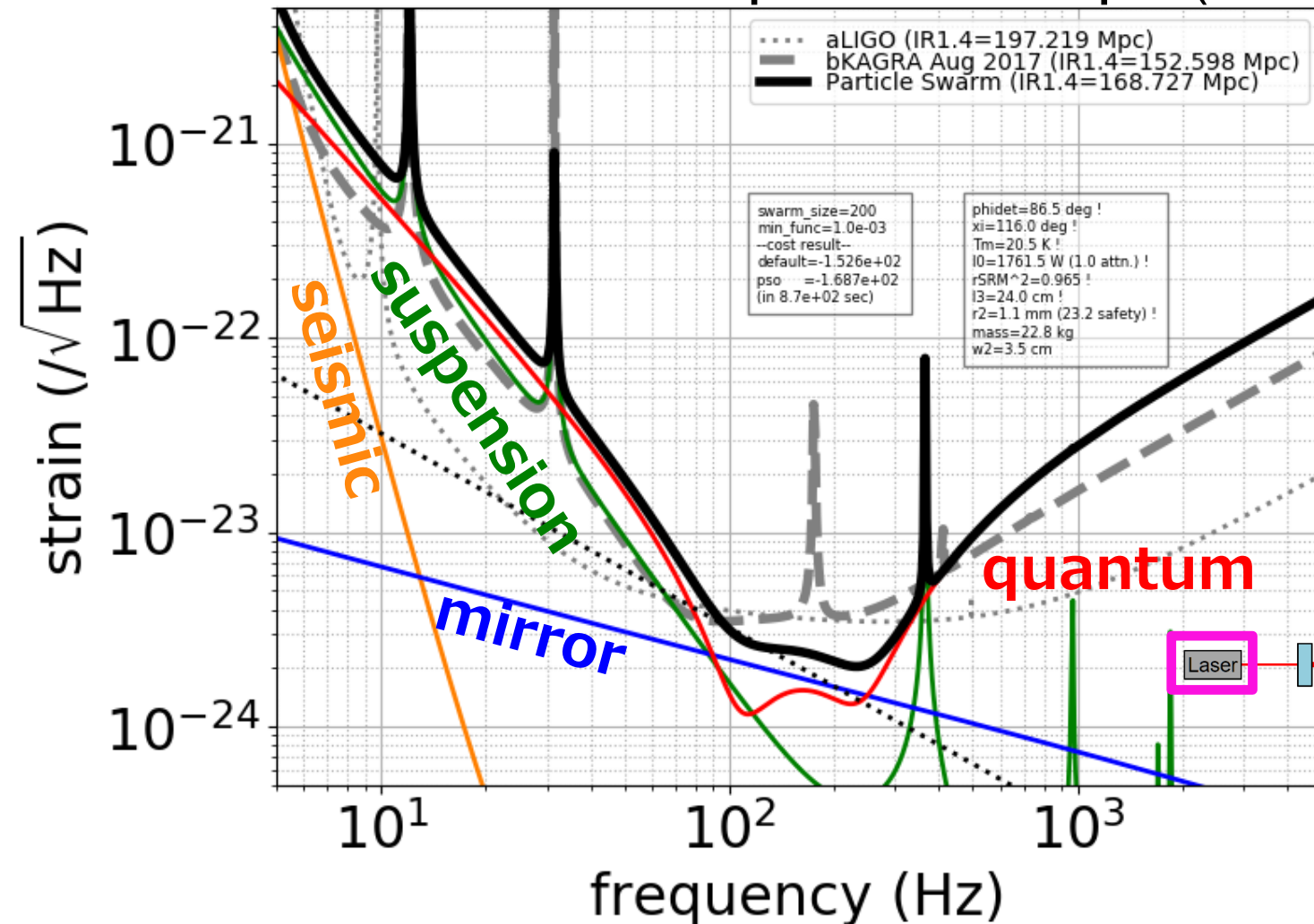
7 Parameter Optimization

- Changing suspension fibers and SRM increases BNS IR from 153 Mpc to 169 Mpc (10% increase)

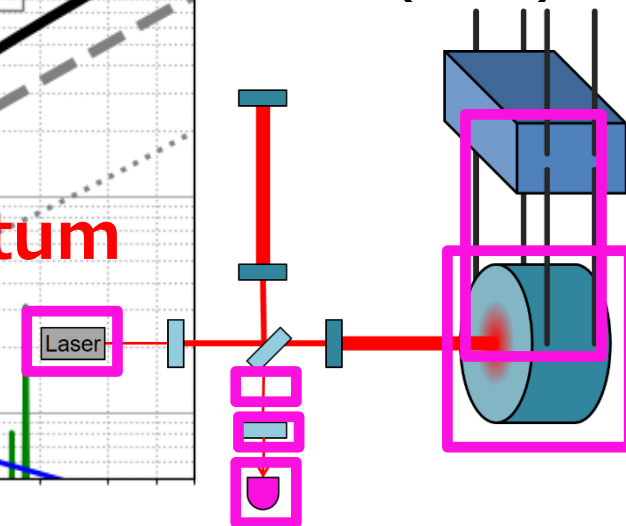


7 Parameter Optimization

- Changing suspension fibers and SRM increases BNS IR from 153 Mpc to 169 Mpc (**10% increase**)



shorter and thicker to put more power with 20 K (min)



Sensitivity Design with PSO is Fast

- Optimization done in **O(10) minutes** with my laptop
- Number of cost function evaluations

	Grid-based	PSO
3 params	$\sim 10^5$ □	$10 \times (52 \pm 13)$ □
5 params	$\sim 10^9$ □	$20 \times (73 \pm 16)$ □
7 params	$\sim 10^{14}$ □	$200 \times (60 \pm 18)$ □

* In case optimization is done at precision of 0.1 Mpc

- Computational cost **do not grow exponentially** with dimensionality of parameter space
- **Useful** for optimization with many parameters, computationally expensive cost function

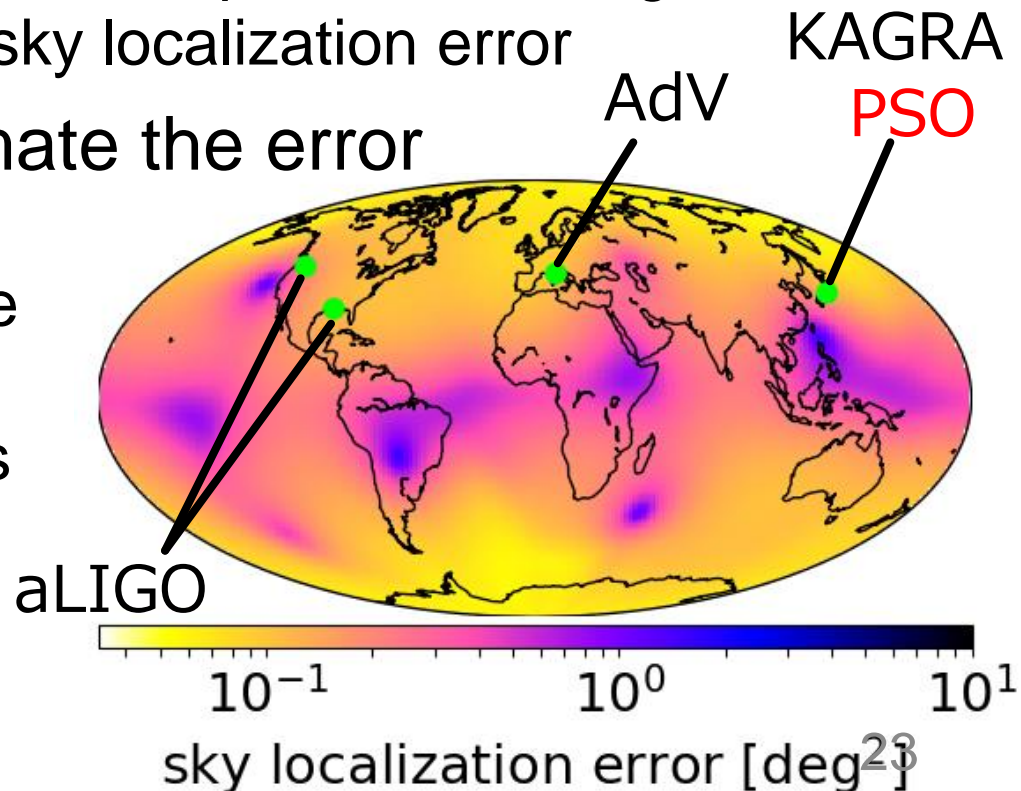
Sky Localization Optimization

- Cost function:
 - **sky localization** of **GW170817**-like binary
 - 1.25-1.5 Msun at 40 Mpc, inclination 28 deg
 - zero spins, no precession
 - **108 sets** of sky location and polarization angle to derive median of sky localization error

- **Fisher matrix** to estimate the error

- inspiral waveform to
 - 3.0 PN in amplitude
 - 3.5 PN in phase
- 11 binary parameters

- **HLVK** global network

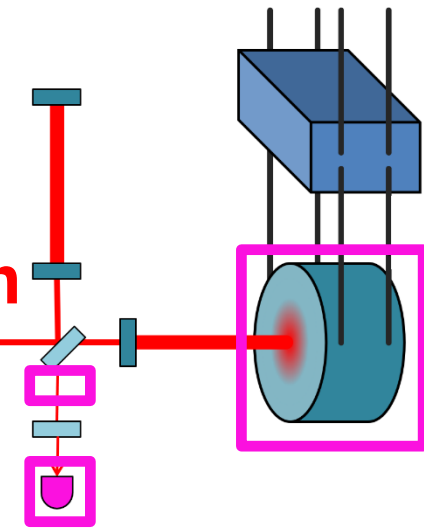
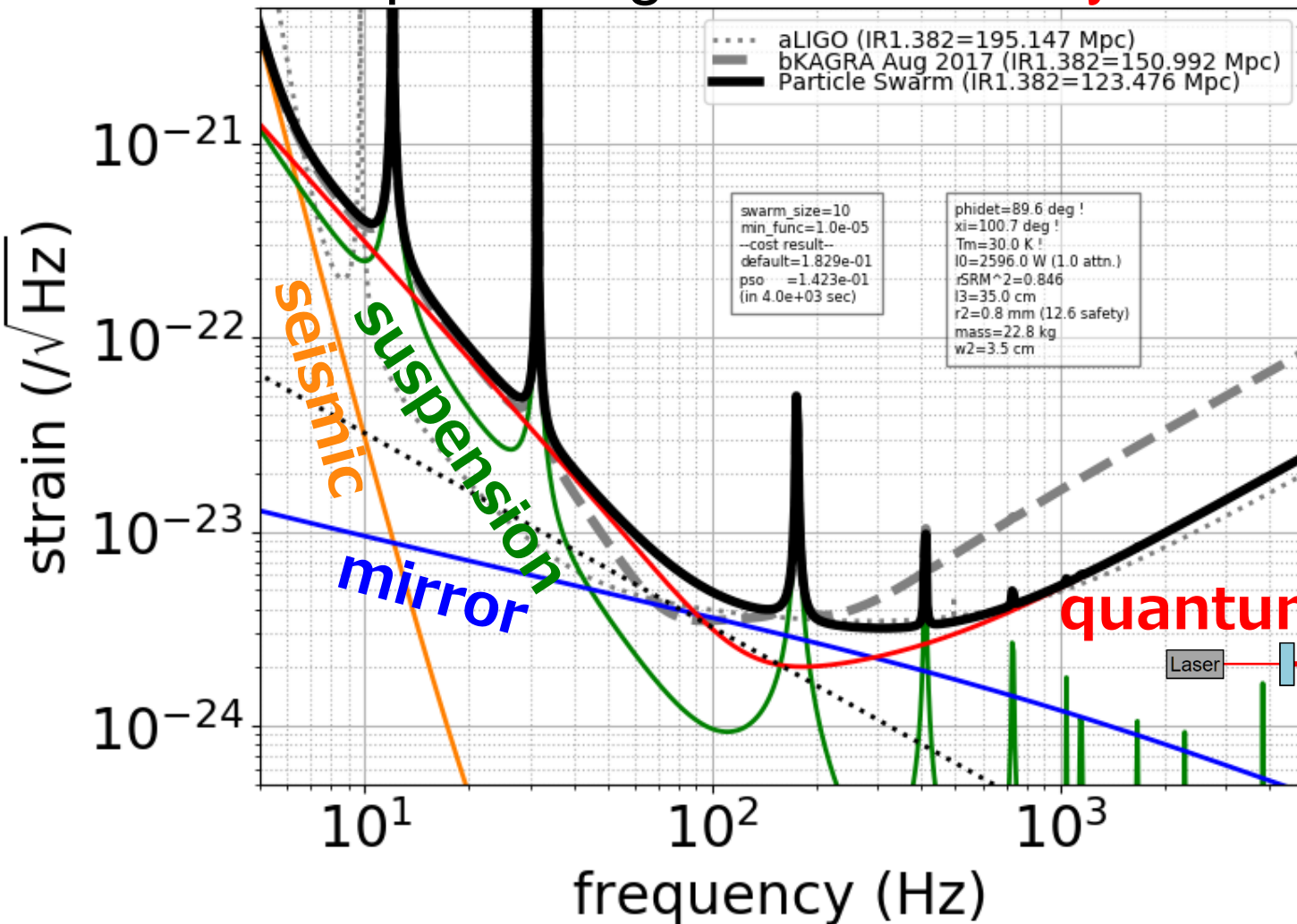


3 Parameter Optimization

- Sky localization **improved by a factor of 1.3**
but inspiral range is **reduced by 20%**

0.18 deg²
→ 0.14 deg²

2600 W at BS
30 K

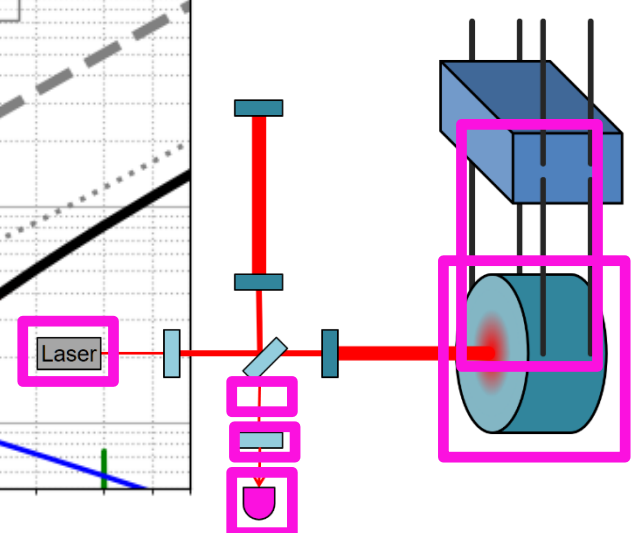
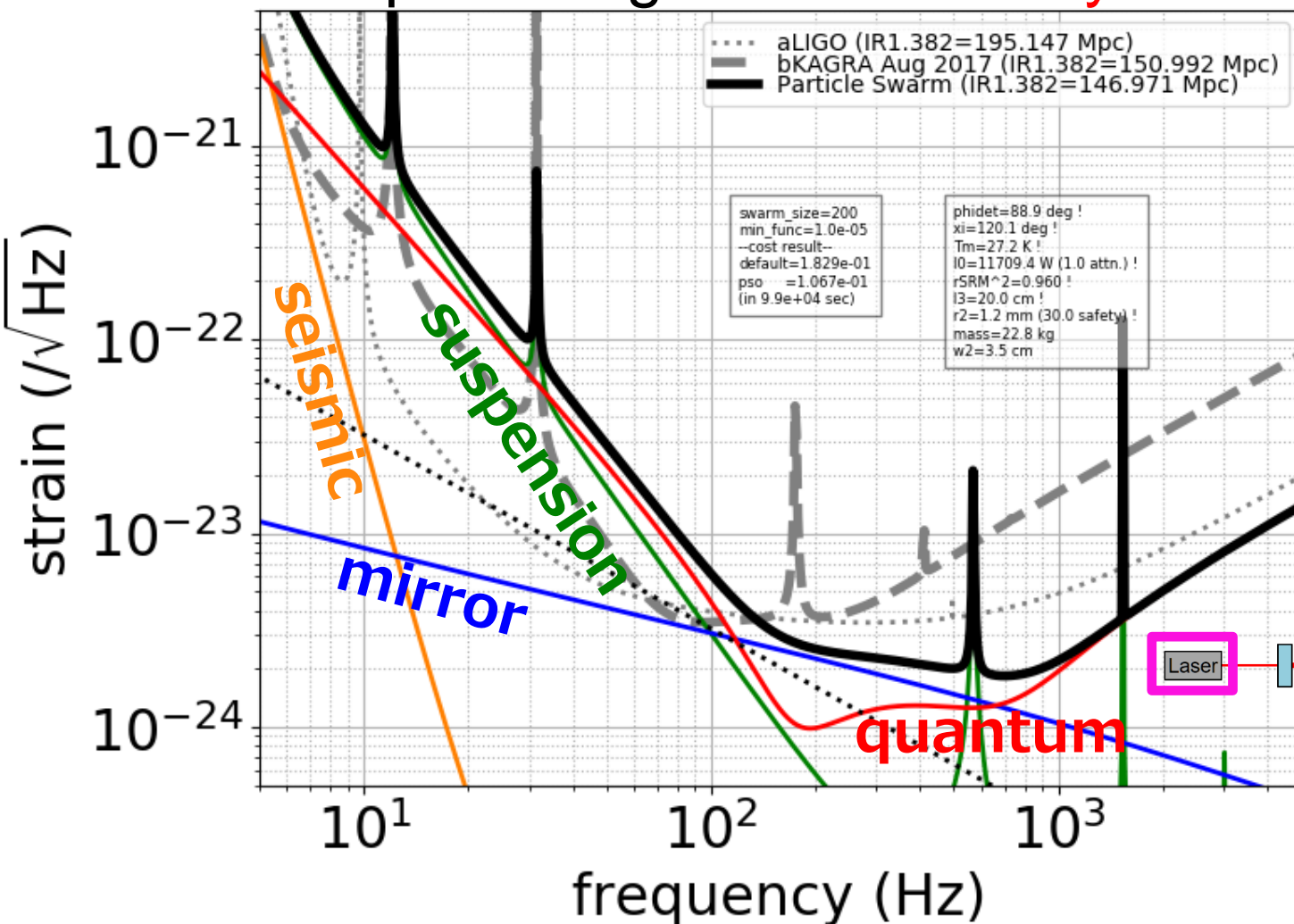


7 Parameter Optimization

- Sky localization improved by a factor of 1.6
but inspiral range is reduced by 2%

0.18 deg²
→ 0.11 deg²

shortest and
thickest fibers
1.1 kW at BS
27 K



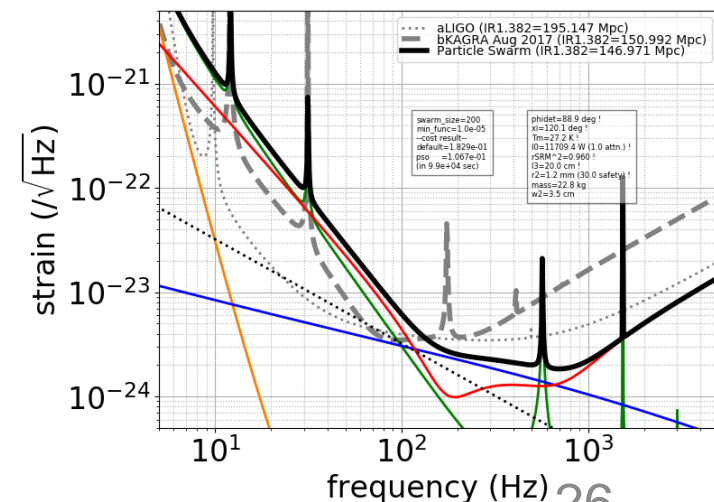
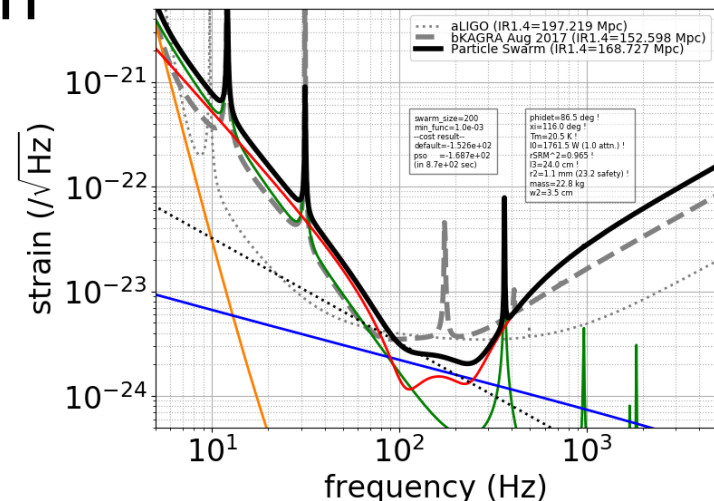
Summary of Strategy Difference

- BNS inspiral range optimization

- **high detuning**,
- high SRM reflectivity
- shorter and thicker fiber
- **lower temperature** to keep mirror thermal low

- BNS sky localization optimization

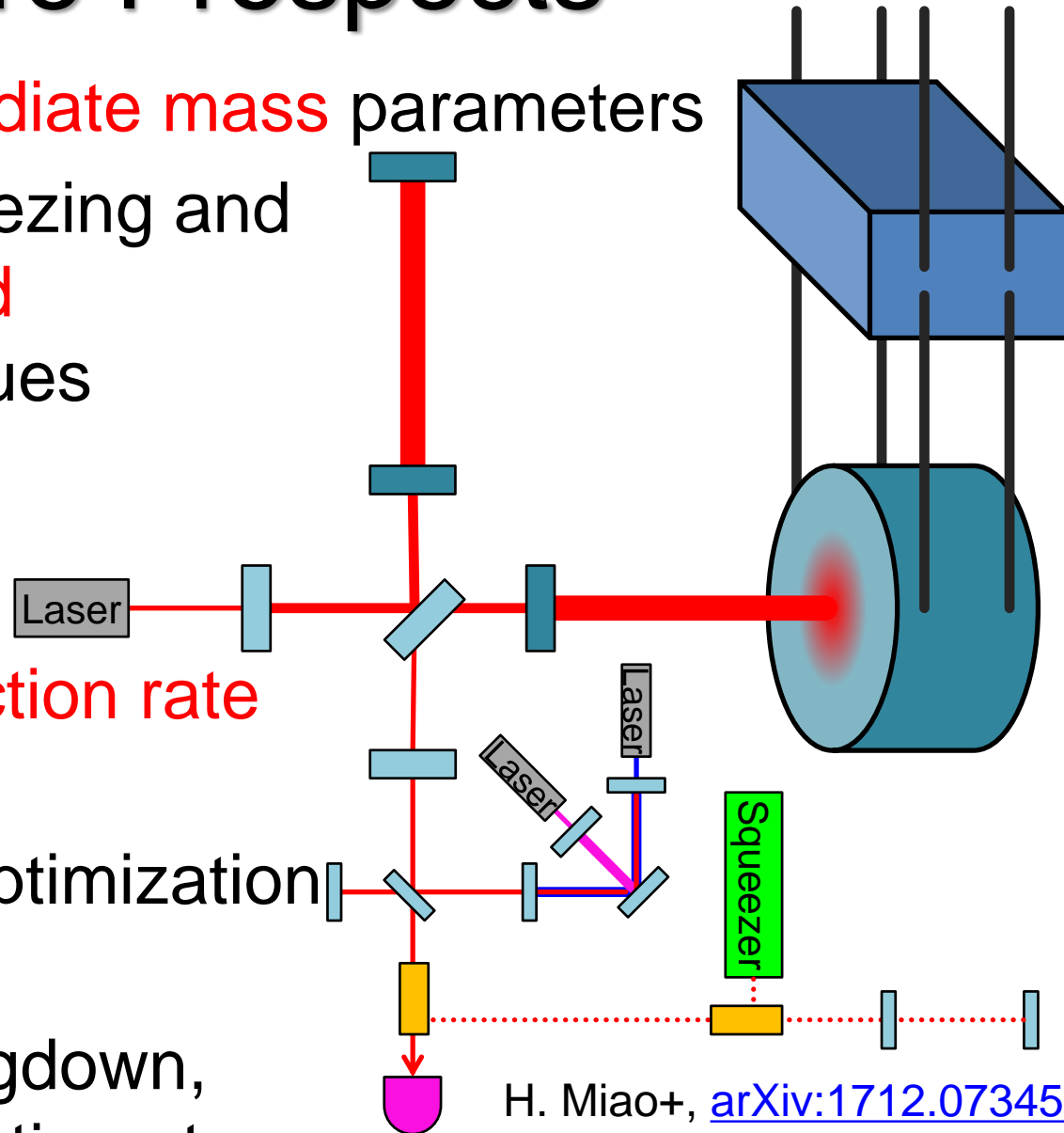
- **no detuning**,
- conventional readout
- shorter and thicker fiber
- **higher temperature** to put more laser power



Future Prospects

- Optimize **intermediate mass** parameters
- Incorporate squeezing and more **complicated** quantum techniques

- Incorporate **detection rate** into cost function
- **Global network** optimization
- Optimization for NS EoS, BH ringdown, non-GR polarization etc...



Summary

- Demonstrated sensitivity design with **PSO**
- Application to KAGRA shows
 - BNS **inspiral range** can be **increased by 10 %**
 - BNS **sky localization** can be **improved by 1.6**by retuning 7 parameters of existing components
- Showed that PSO is **useful** for future GW detectors
 - low computational cost
 - little prior information necessary
 - allows more sophisticated cost function
- YM+ [arXiv:1804.09894](https://arxiv.org/abs/1804.09894)

Supplementary Slides

Pyswarm

- Python package **Pyswarm** was used for this work

<https://pythonhosted.org/pyswarm/>

<https://github.com/tisimst/pyswarm/>

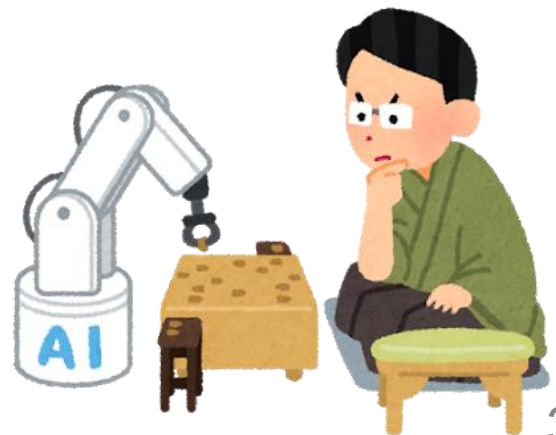
- PSO as easy as

```
xopt, fopt = pso(func, lb, ub)
```

optimal parameter set optimal cost function cost function lower and upper bounds

Other Optimization Methods

- Simulated annealing
tuning cooling schedule is troublesome
- Genetic algorithm
too many design variables
- Markov chain Monte Carlo
tend to be dependent on prior distribution
gives error from posterior distribution
takes time
- Machine learning
if the problem well-modeled,
you don't need ML



Swarm Size Determination

- **Probability of convergence**: ratio of PSO trials resulted within 0.1 Mpc or 10^{-3} deg²
- Increased swarm size until probability of convergence is larger than **90%**

number of params	3	5	7
number of particles	10	20	200
number of iterations	52 ± 13	73 ± 16	60 ± 18
probability of convergence	98 %	96 %	91 %

* From 100 PSO trials

IFO Parameter Search Range

	Lower bound	Upper bound	KAGRA Default	Precision
Detuning angle [deg]	86.5 (or 60) *	90	86.5	0.1
Homodyne angle [deg]	90	180	135.1	3
Mirror temperature [K]	20	30	22	0.09
Power attenuation	0.01	1	1	0.02
SRM reflectivity	0.5	1	0.92 (85%)	6e-4
Wire length [cm]	20	100	35	0.02
Wire safety factor	3	30	12.57 (0.8 mm)	0.07

* Considering SRC nonlinearity, maximum detuning is 3.5 deg
(see Y. Aso+ [CQG 29, 124008](#))

- *Reflecting wall* boundary:
 $x=x_{\max}$, $v=-v$ if $x>x_{\max}$
 $x=x_{\min}$, $v=-v$ if $x<x_{\min}$

step size which changes
BNS inspiral range by 0.1 Mpc

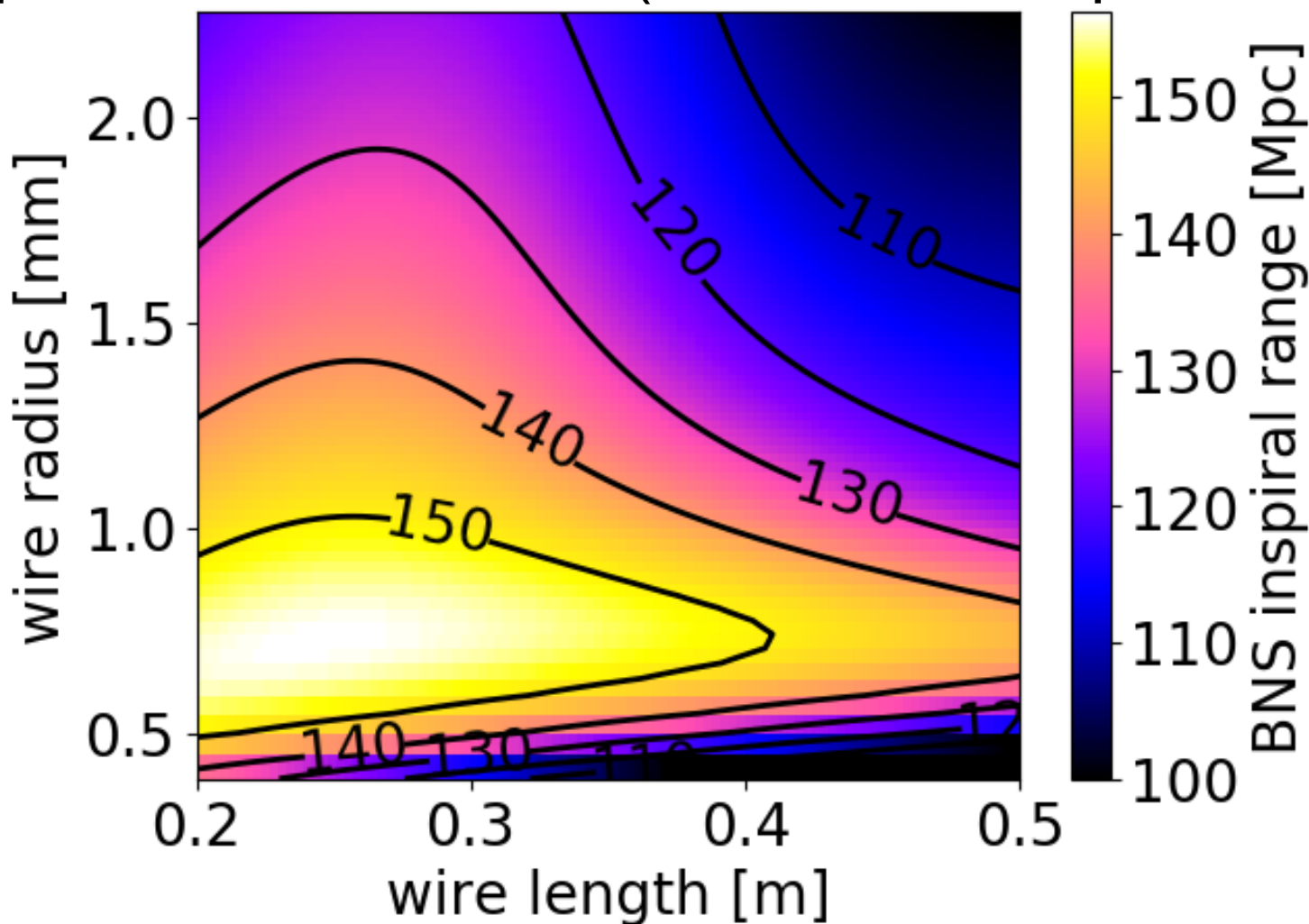


Money

- Detuning angle and homodyne angle can be retuned without additional cost
- Mirror temperature and input power can be retuned without additional cost if power at BS is less than ~ 1 kW (~ 100 W entering PRM)
- Change in SRM reflectivity require ~ 0.1 Million USD
- Change in wire parameters require ~ 0.01 Million USD/fiber
- Change in wire length require test mass suspension design change at ~ 0.1 Million USD/mirror
- Change in the test mass require ~ 0.5 Million USD/mirror

Fiber Length and Diameter

- 25cm/ ϕ 1.4mm is optimum for BNS IR if other parameters are fixed (default: 35cm/ ϕ 1.6mm)



Fisher Matrix Analysis

- Fisher matrix

$$\Gamma_{ij} = 4\Re \int_{f_{\min}}^{f_{\max}} \sum_k \frac{\partial h_k^*(f)}{\partial \lambda^i} \frac{\partial h_k(f)}{\partial \lambda^j} \frac{df}{S_{n,k}(f)}$$

- Covariance

$$\sqrt{\langle (\delta \lambda^i \delta \lambda^j) \rangle} = \sqrt{(\Gamma^{-1})^{ij}}$$

- 11 binary parameters considered

mc: chirp mass

eta: symmetric mass ratio

tc, **phic**: time and phase for coalescence

dL: luminosity distance

chis, **chia**: symmetric/asymmetric spin $\chi_{s/a} = (\chi_1 \pm \chi_2)/2$

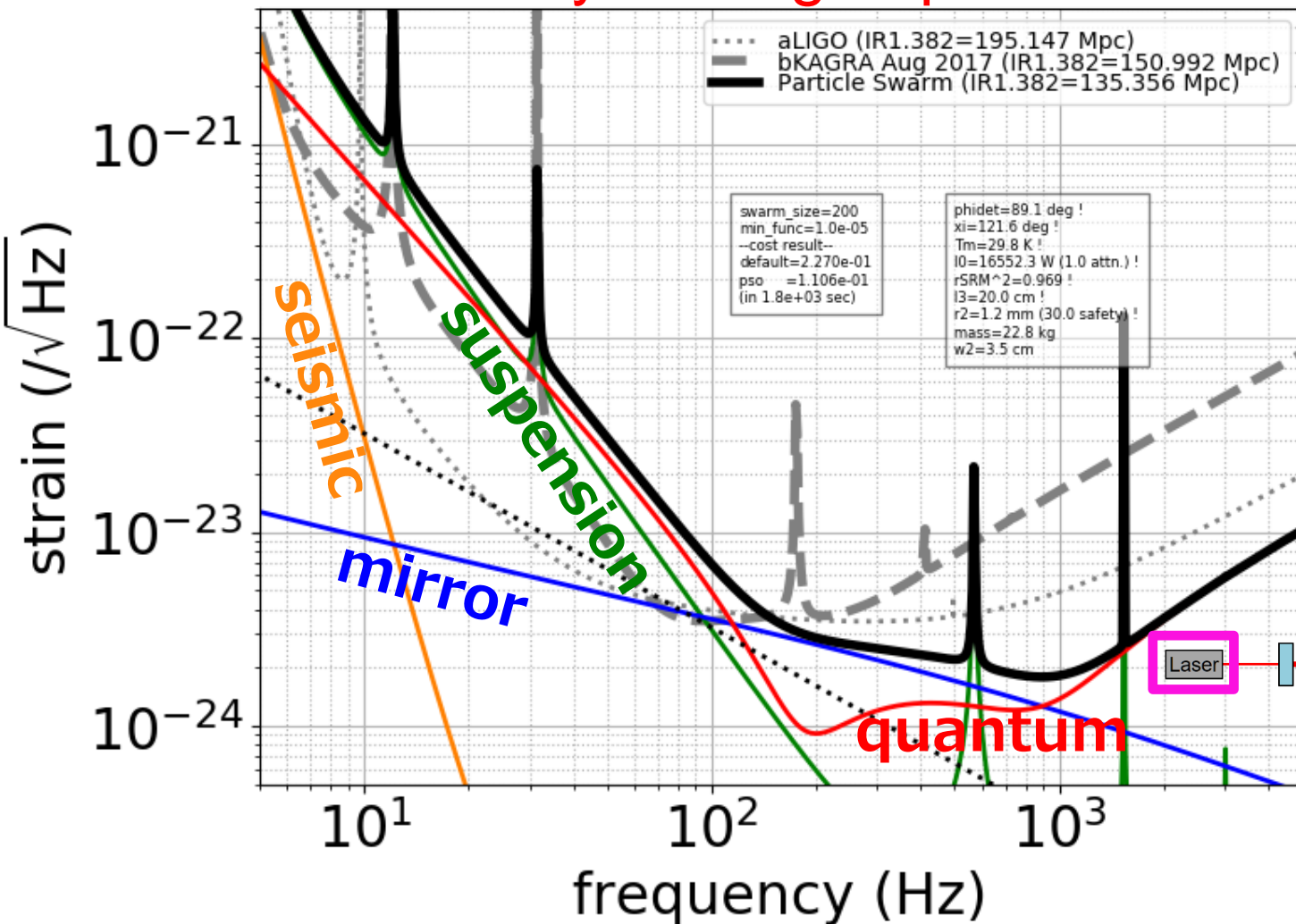
thetas, **phis**: colatitude / longitude of source

cthetai: inclination angle

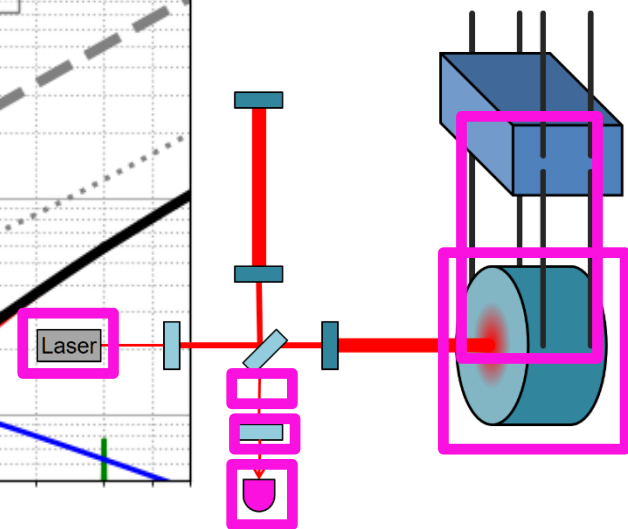
psip: polarization angle

Optimization for Fixed Sky Location

- Result for fixed sky location and polarization angle is **similar to sky average optimization**



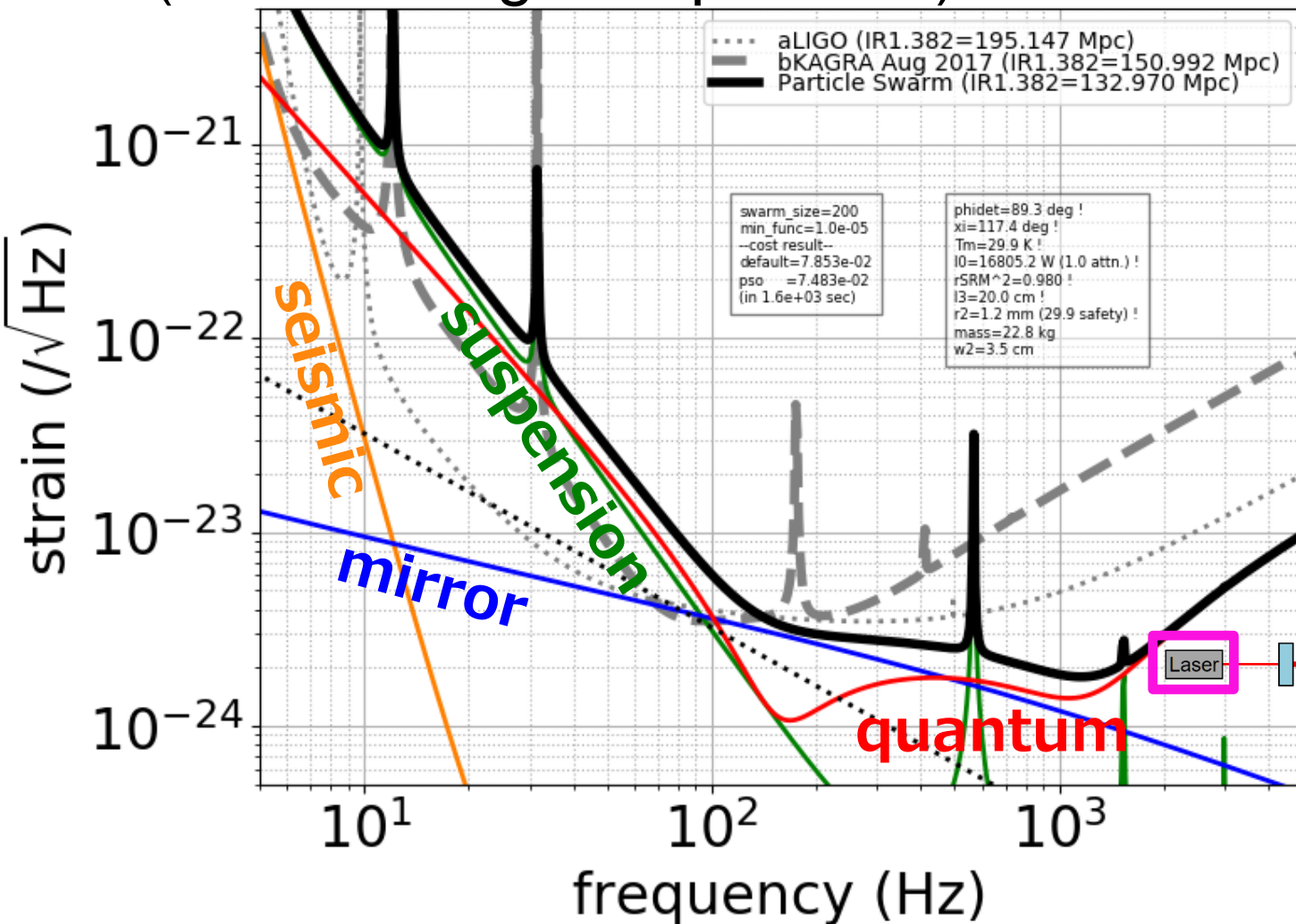
shortest and
thickest fibers
1.7 kW at BS
30 K (max)



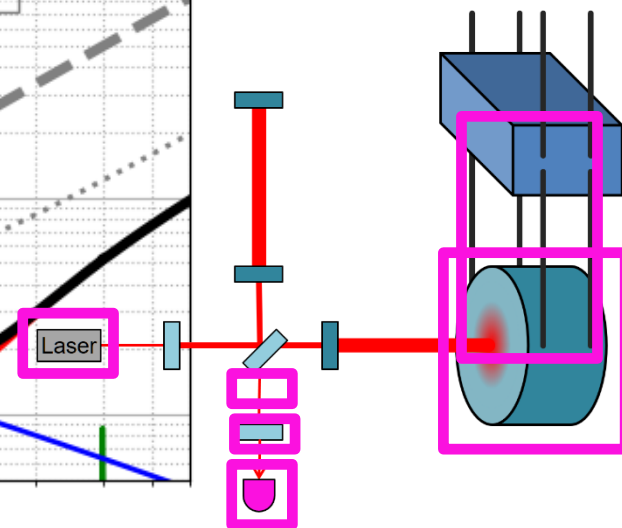
Symmetric Spin Optimization

- Similar to sky localization optimization (focus on high frequencies)

$$\chi_s = \frac{\chi_1 + \chi_2}{2}$$



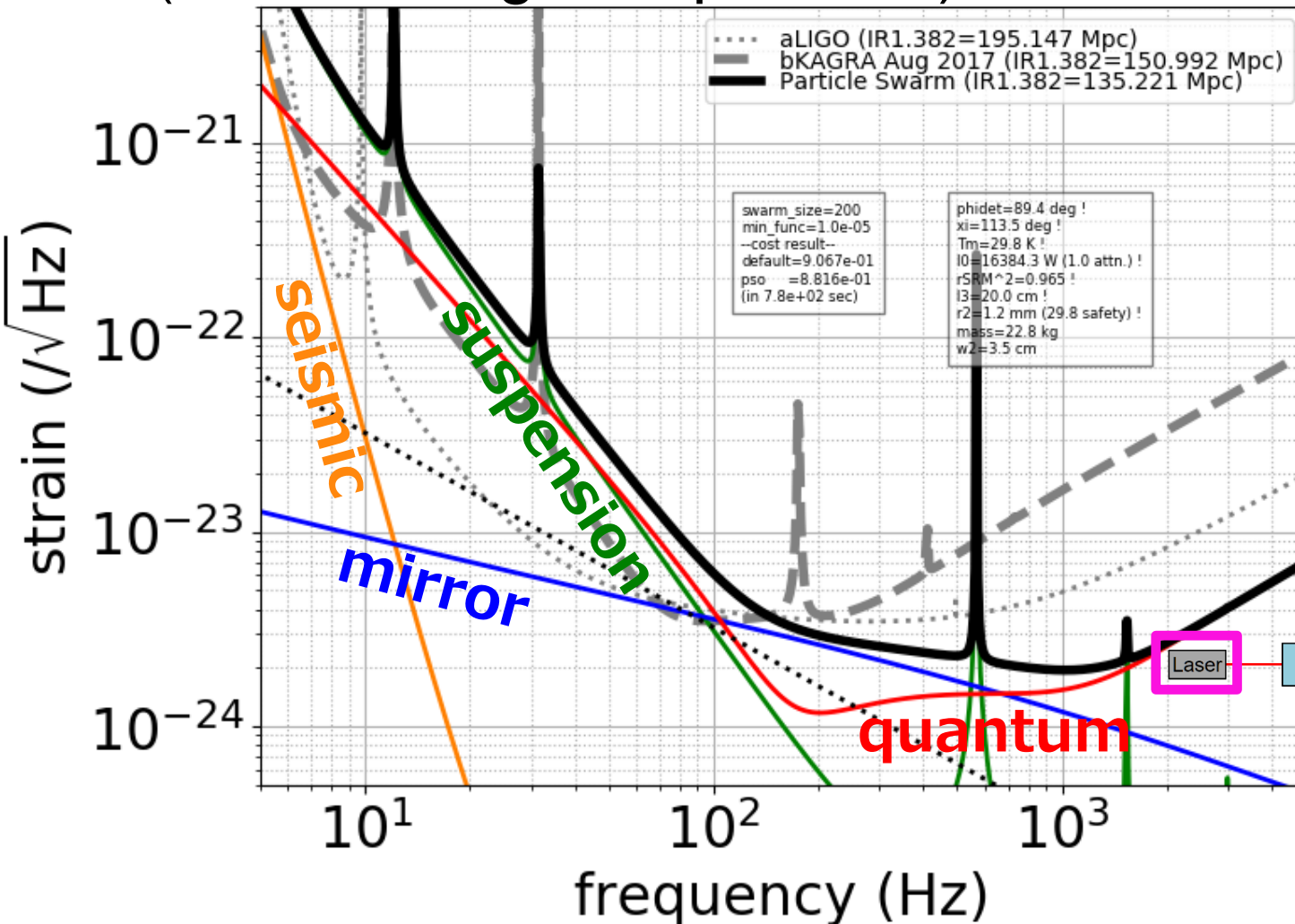
shortest and
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30 K (max)



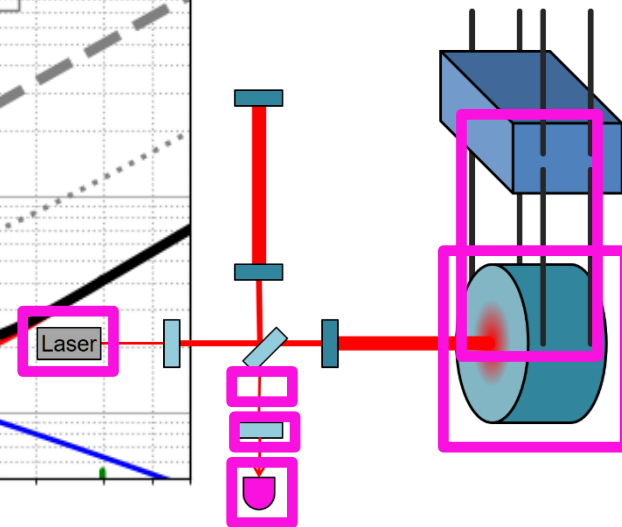
Asymmetric Spin Optimization

- **Similar to sky localization** optimization (focus on high frequencies)

$$\chi_a = \frac{\chi_1 - \chi_2}{2}$$

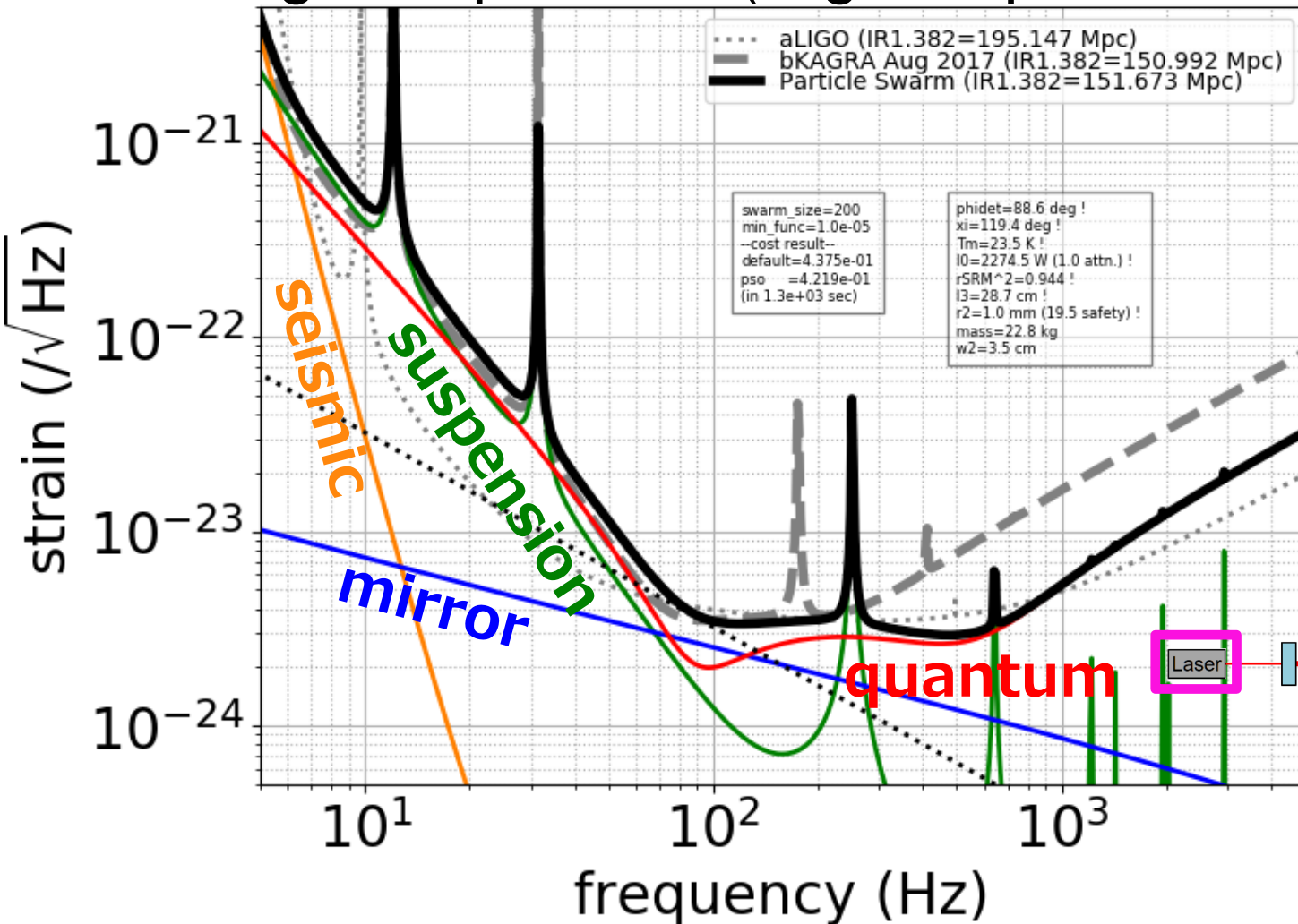


shortest and
thickest fibers
1.7 kW at BS
30 K (max)

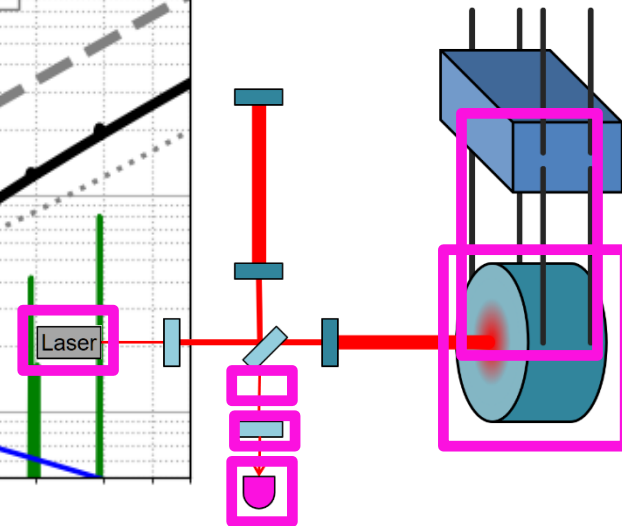


Distance Optimization

- Similar to inspiral range optimization, but slight shift to high frequencies (slight improvement by 2%)

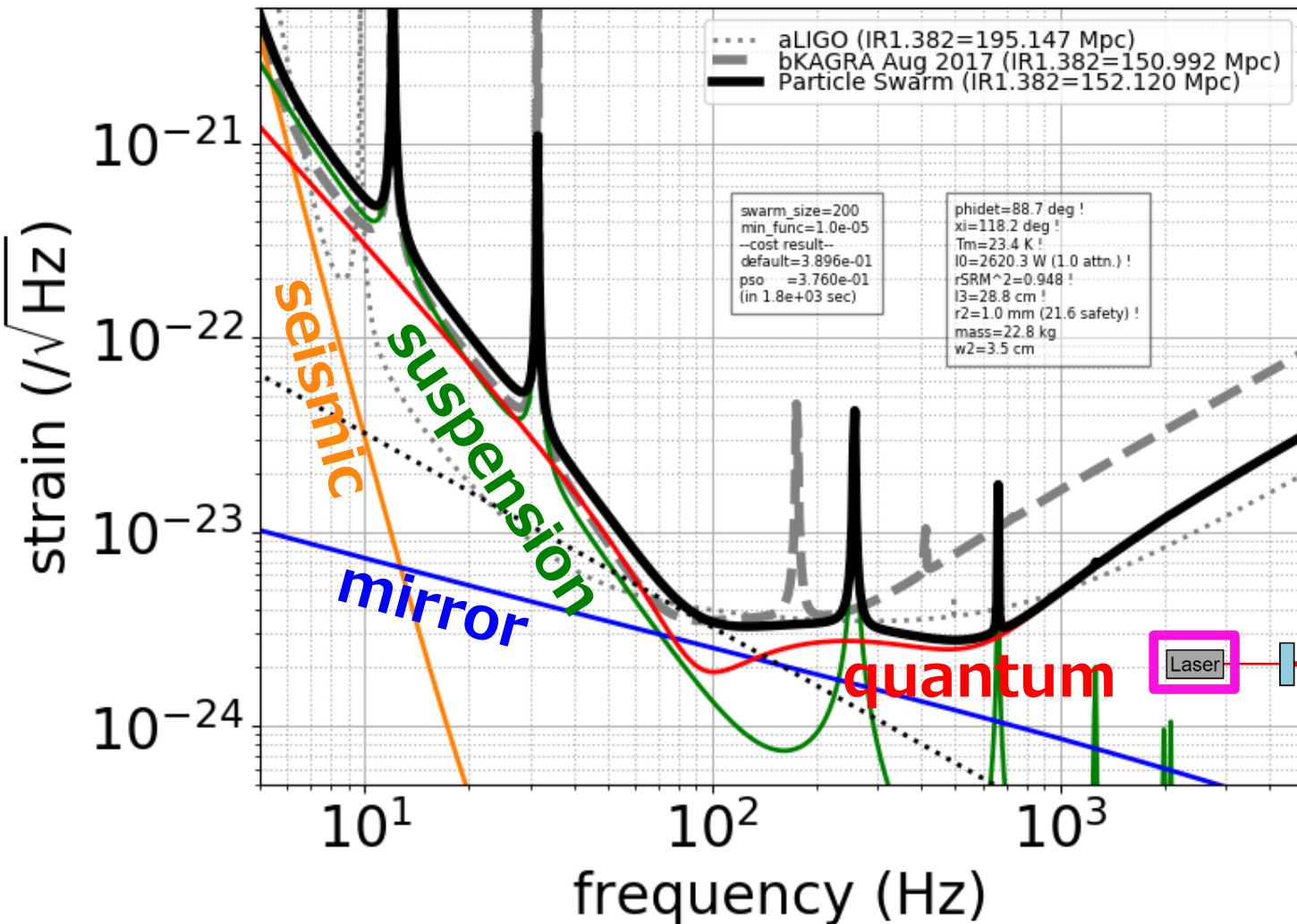


shorter and thicker fibers
2.3 kW at BS
24 K

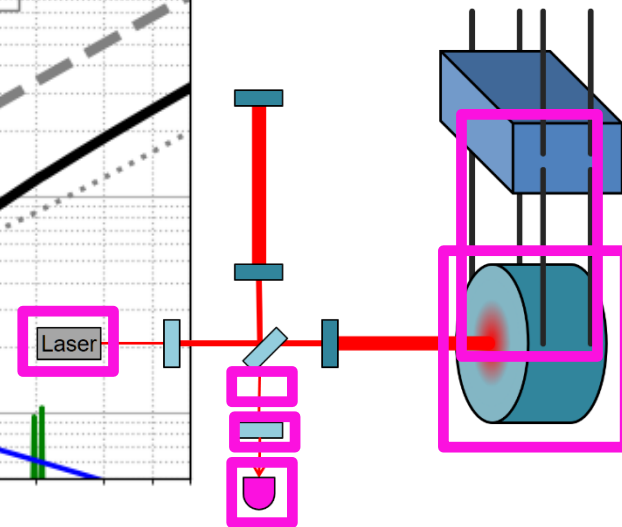


Inclination Angle Optimization

- Similar to distance optimization (PE degeneracy)

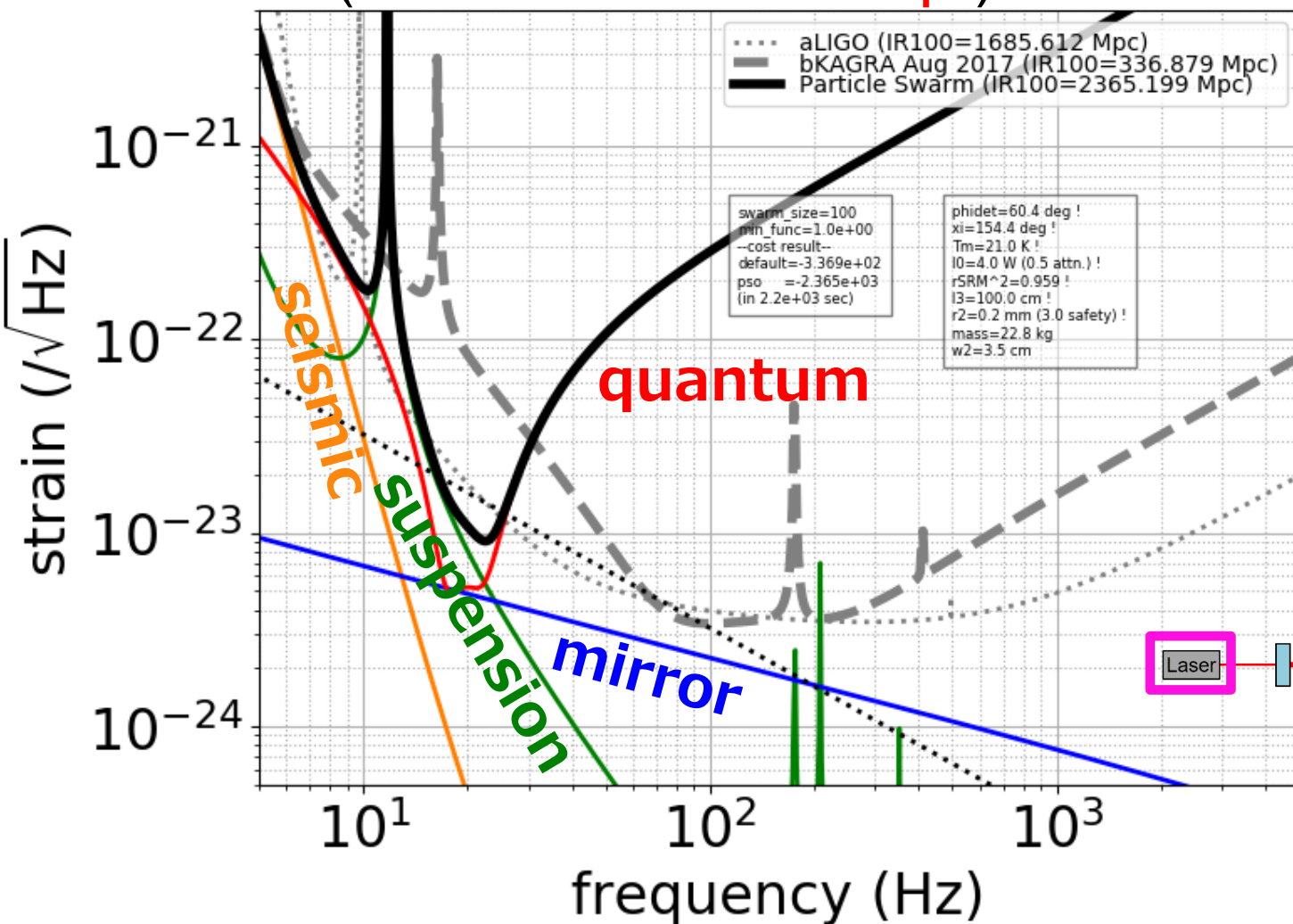


shorter and thicker fibers
2.3 kW at BS
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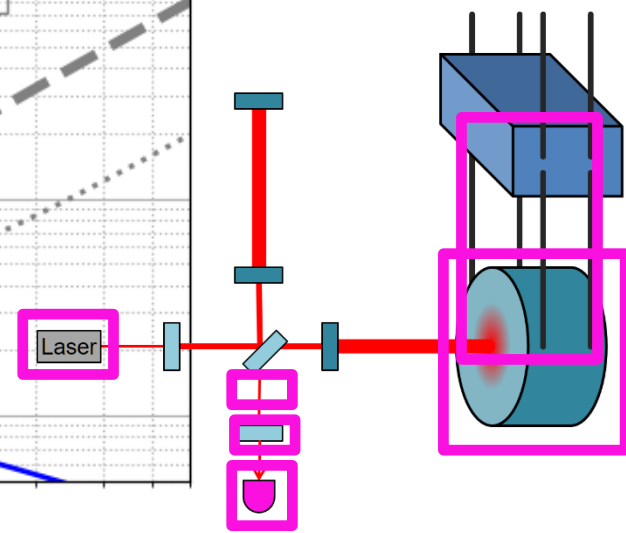


BBH100 IR Optimization

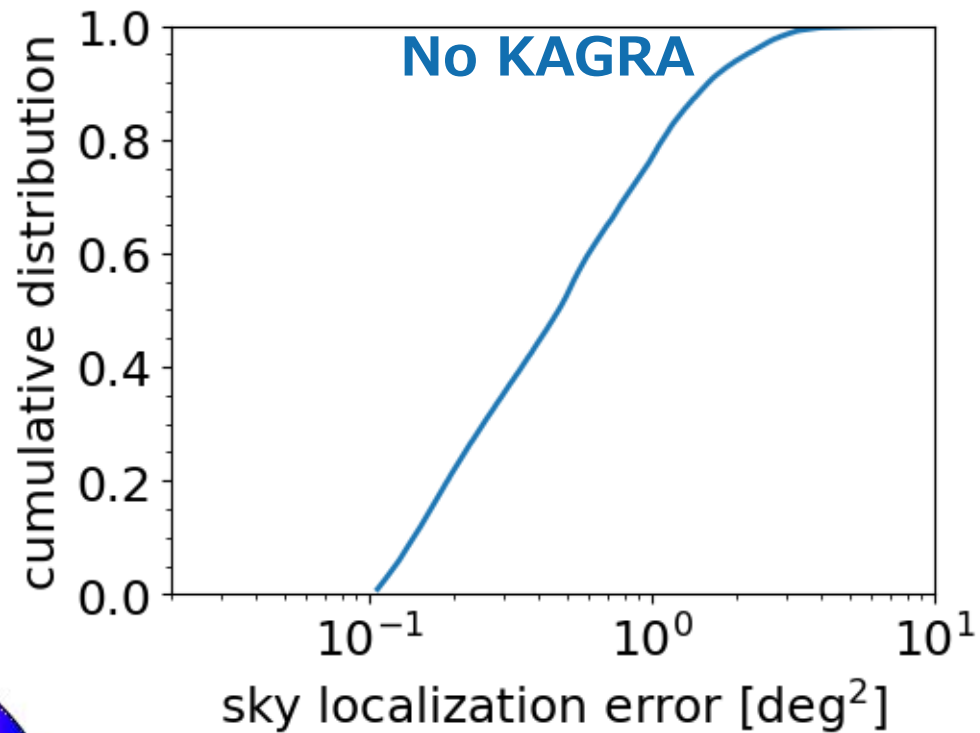
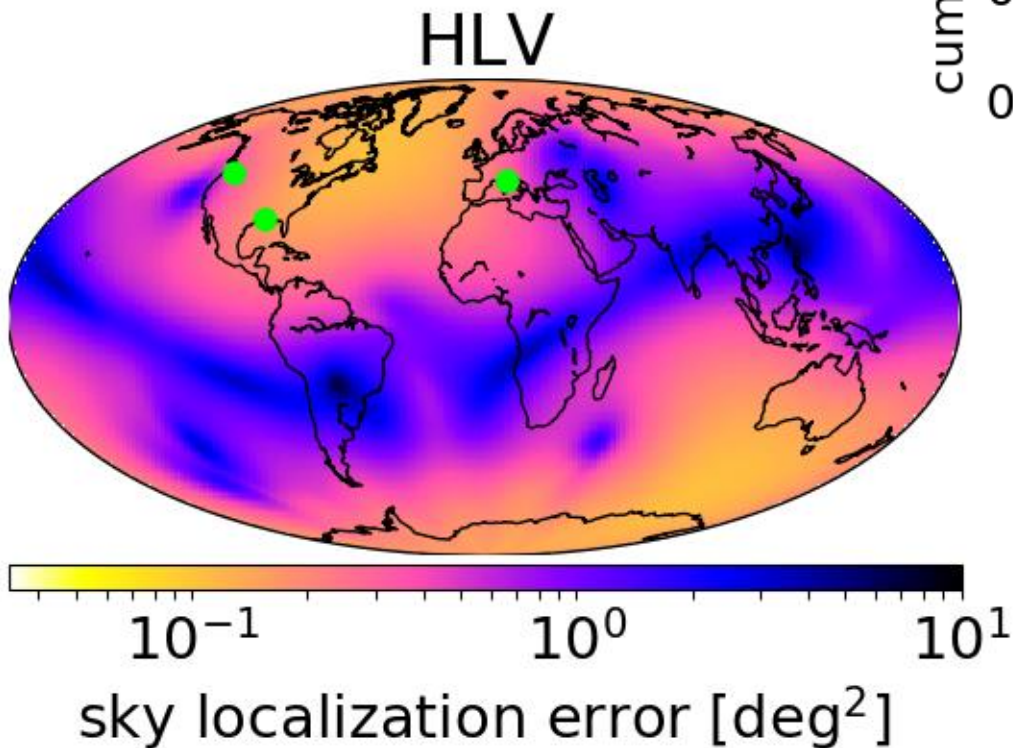
- Low power, low temperature with thin and longer fibers (KAGRA+ **LF concept**)



x4 heavier IM
with thinner IM
suspension
Less ambient heat
Allowed higher
detuning

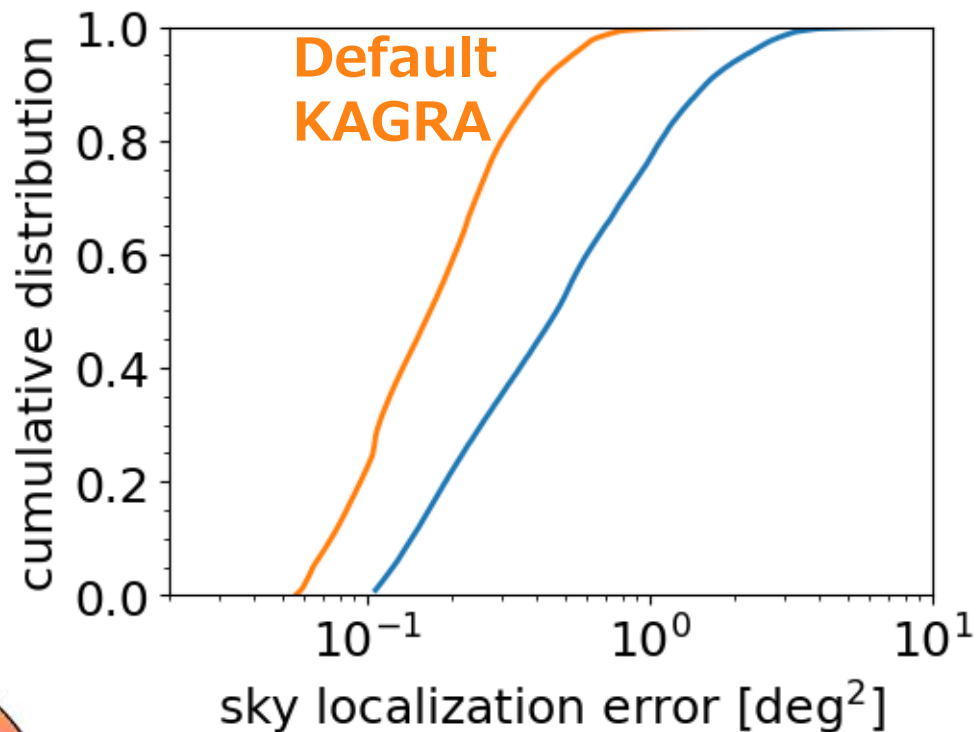
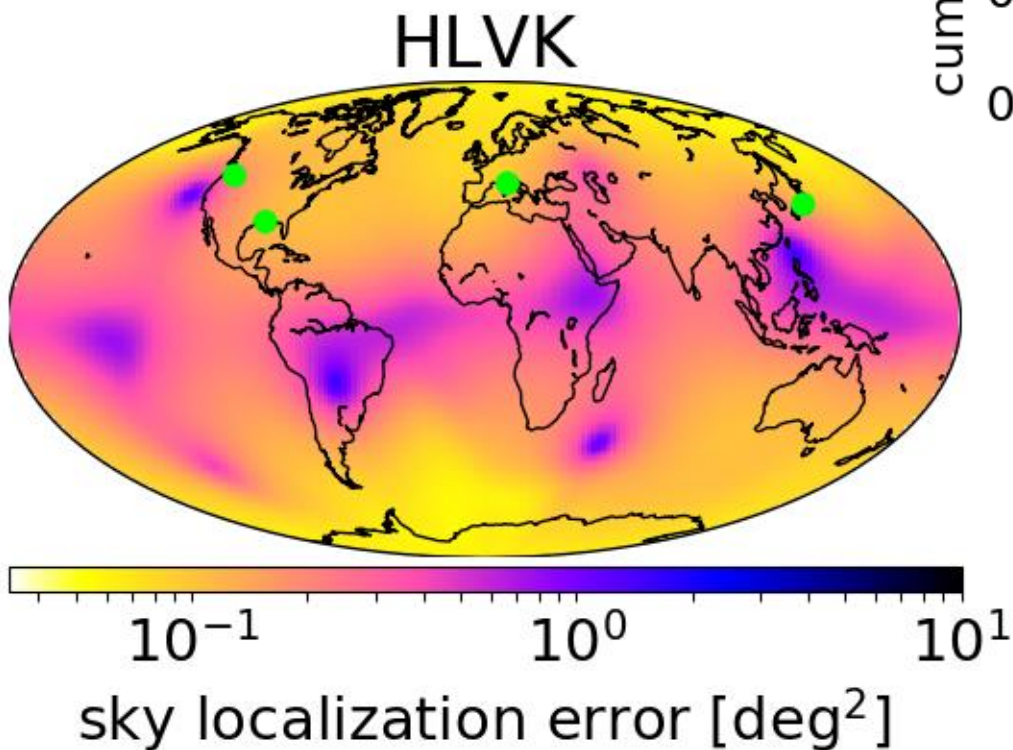


Sky Localization with HLV



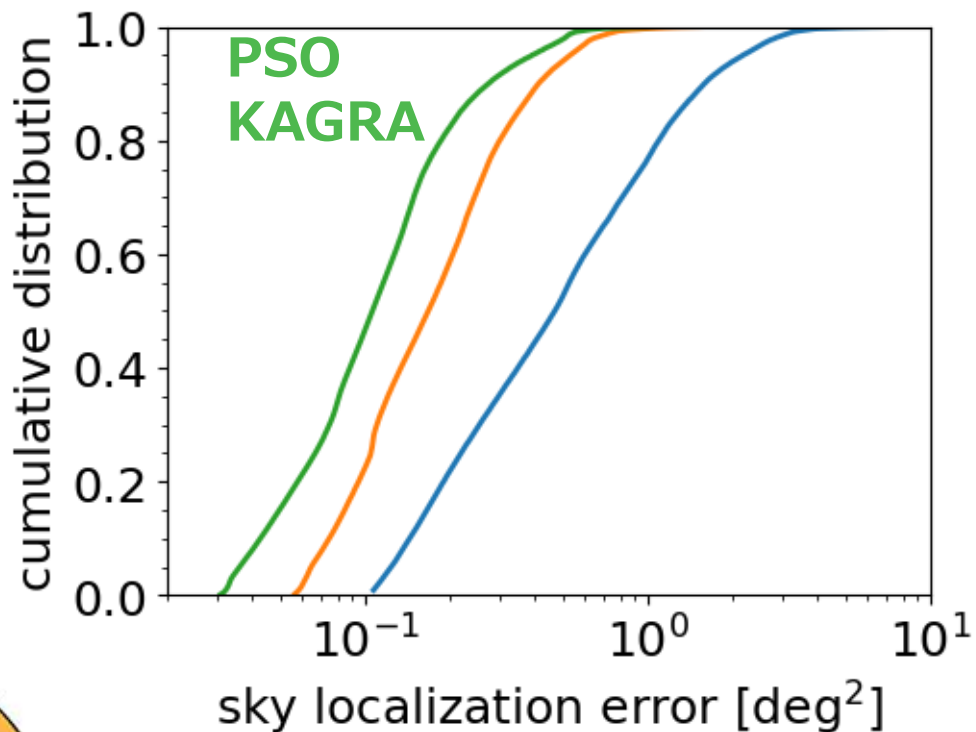
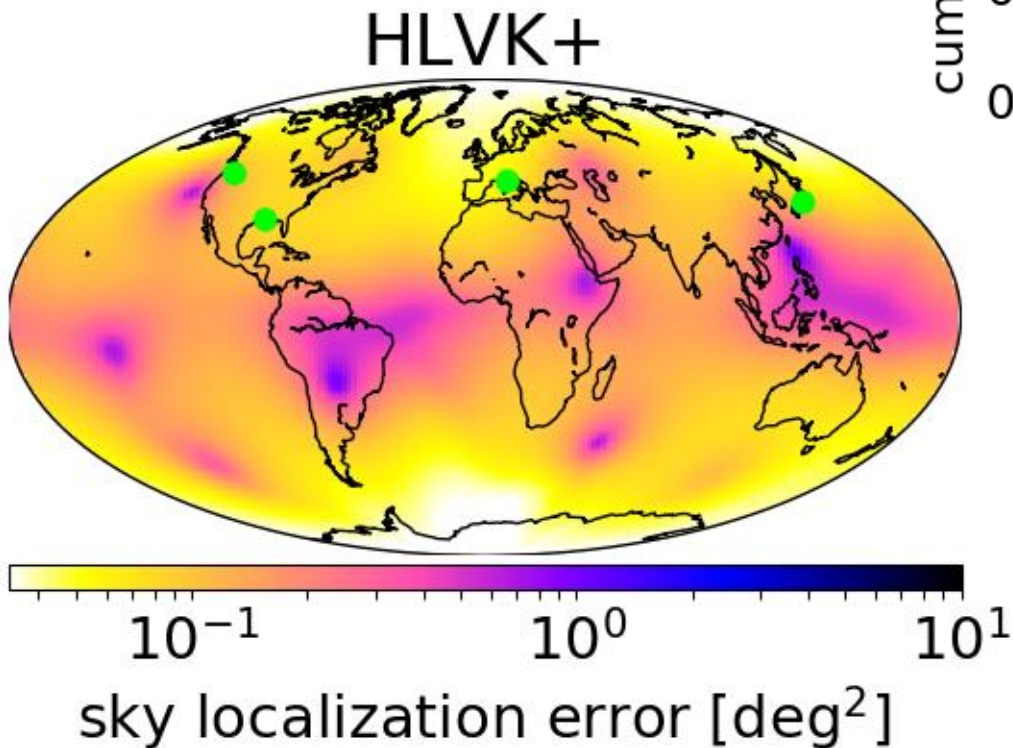
	median
HLV	0.472 deg ²
HLVK	
HLVK+	

Sky Localization with HLVK



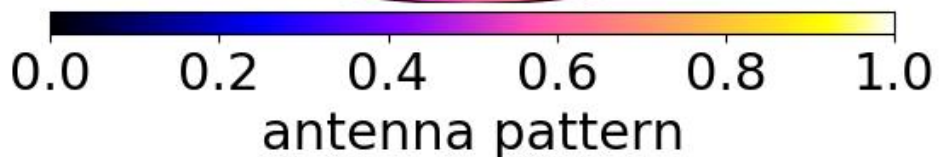
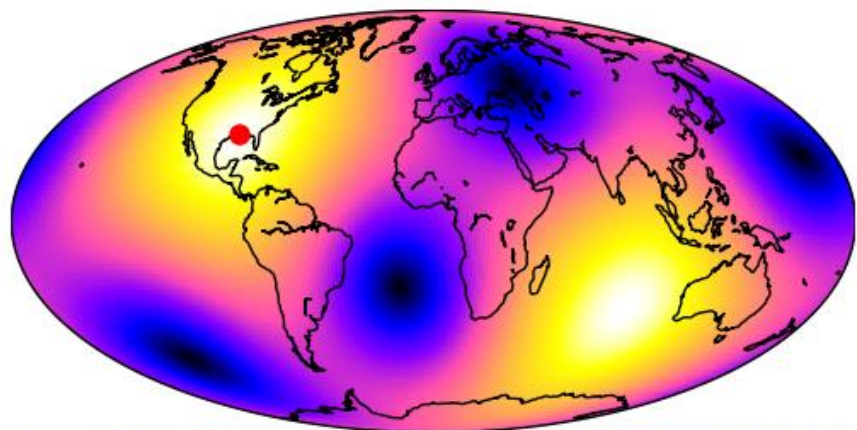
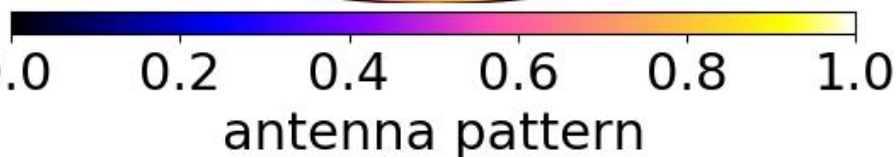
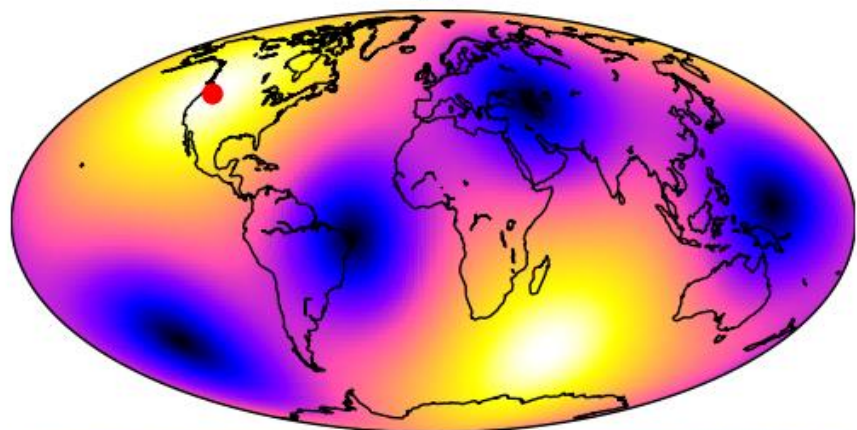
	median
HLV	0.472 deg ²
HLVK	0.168 deg ²
HLVK+	

Sky Localization with HLVK+

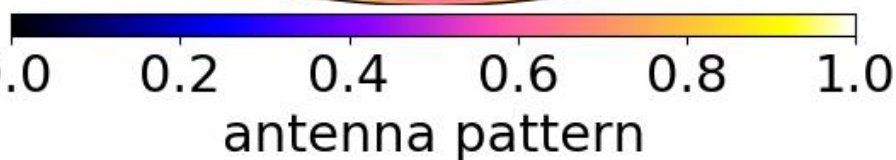
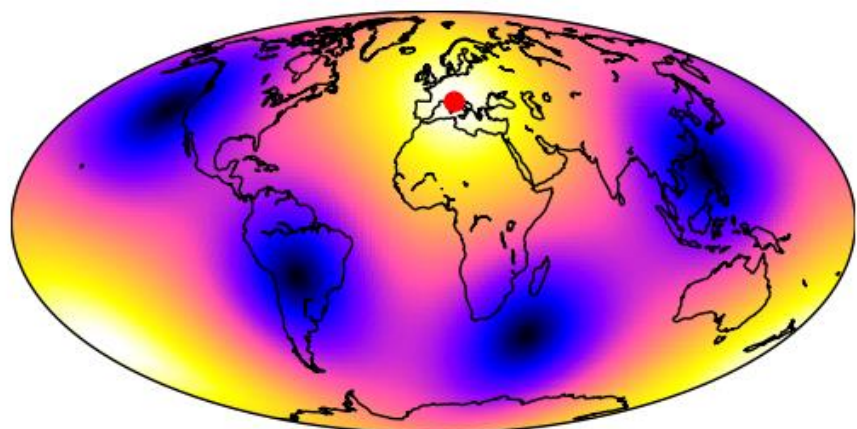


	median
HLV	0.472 deg^2
HLVK	0.168 deg^2
HLVK+	0.107 deg^2

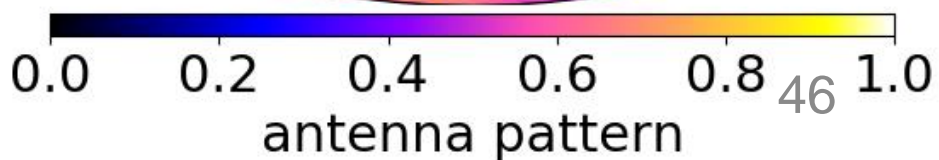
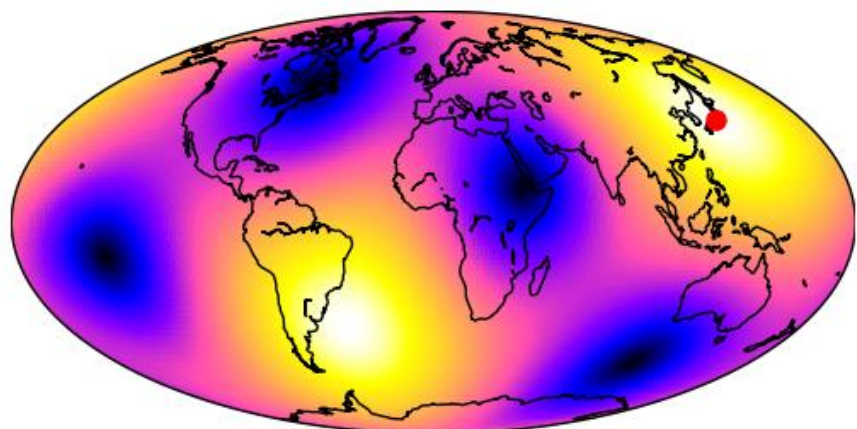
H Antenna Pattern L



V



K



2G/2G+ Parameter Comparison

	KAGRA	AdVirgo	aLIGO	A+	Voyager
Arm length [km]	3	3	4	4	4
Mirror mass [kg]	23	42	40	80	200
Mirror material	Sapphire	Silica	Silica	Silica	Silicon
Mirror temp [K]	22	295	295	295	123
Sus fiber	35cm Sap.	70cm SiO ₂	60cm SiO ₂	60cm SiO ₂	60cm Si
Fiber type	Fiber	Fiber	Fiber	Fiber	Ribbon
Input power [W]	67	125	125	125	140
Arm power [kW]	340	700	710	1150	3000
Wavelength [nm]	1064	1064	1064	1064	2000
Beam size [cm]	3.5 / 3.5	4.9 / 5.8	5.5 / 6.2	5.5 / 6.2	5.8 / 6.2
SQZ factor	0	0	0	6	8
F. C. length [m]	none	none	none	16	300

KAGRA Detailed Parameters

K. Komori *et al.*, [JGW-T1707038](#)

- **Optical parameters**
 - Mirror transmission: 0.4 % for ITM, 10 % for PRM, 15.36 % for SRM
 - Power at BS: 674 W
 - Detune phase: 3.5 deg (DRSE case)
 - Homodyne phase: 135.1 deg (DRSE case)
- **Sapphire mirror parameters**
 - TM size: 220 mm dia., 150 mm thick
 - TM mass: 22.8 kg
 - TM temperature: 22 K
 - Beam radius at ITM: 3.5 cm
 - Beam radius at ETM: 3.5 cm
 - Q of mirror substrate: $1e8$
 - Coating: tantala/silica
 - Coating loss angle: $3e-4$ for silica, $5e-4$ for tantala
 - Number of layers: 22 for ITM, 40 for ETM
 - Coating absorption: 0.5 ppm
 - Substrate absorption: 50 ppm/cm
- **Suspension parameters**
 - TM-IM fiber: 35 cm long, 1.6 mm dia.
 - IM temperature: 16 K
 - Heat extraction: 5800 W/m/K at 20 K
 - Loss angle: $5e-6/2e-7/7e-7$ for CuBe fiber/sapphire fiber/sapphire blade
- **Inspirial range calculation**
 - SNR=8, $f_{min}=10$ Hz, sky average constant 0.442478
- Seismic noise curve includes vertical coupling, vibration from heatlinks and Newtonian noise from surface and bulk

KAGRA Cryopayload

Provided by T. Ushiba and T. Miyamoto

Platform
(SUS, 65 kg)

Marionette
(SUS, 22.5 kg)

Intermediate Mass
(SUS, 20.1 kg,
16 K)

Test Mass
(Sapphire, 23 kg,
22 K)

3 CuBe blade springs

MN suspended by 1 Maraging steel fiber
(35 cm long, 2-7mm dia.)

MRM suspended by 3 CuBe fibers

Heat link attached to MN

IM suspended by 4 CuBe fibers
(24 cm long, 0.6 mm dia)

IRM suspended by 4 CuBe fibers

4 sapphire blades

TM suspended by 4 sapphire fibers
(35 cm long, 1.6 mm dia.)

RM suspended by 4 CuBe fibers

