

# GW Parameter Estimations and Simulations

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# Outline

- G.P.E. – GPU accelerated Parameter Estimation
- Comparison of results – GW events analysis
- Comparison of results – Simulation
- Immediate future – O3 simulation
- Physics targets – O5 simulation

# Acknowledgements

- Narikawa-san, Tagoshi-san, Morisaki-san,  
Michimura-san  
Discussions and suggestions
- Ting-Wai Chiu (NTNU/ASIoP)  
GPU developments and his GPU farm supports
- Chun-Yu Lin (NCHC)  
National Center for High-performance Computing
- Academia Sinica Grid Center (ASGC)  
GPU farm and computing supports

# Outline

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- Comparison of results – Simulation
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# Motivation

- Parameter Estimation (PE) is a time-consuming process; we need to find a new way to accelerate it in view of many-GW-detection era towards 3G
- Freq.-domain CBC PE on 15 parameters ( $3(M_c, q, \varphi)$  + 6(spin) + 6(ext.)) with
  - Nested sampling with MCMC sub-chains and
  - *IMRPhenomPv2* waveform model has been accelerated with GPU
- The codes are all newly written in C++ and CUDA and produce the same output for *cbcBayesPostProc*

# P.E. with Bayes' theorem

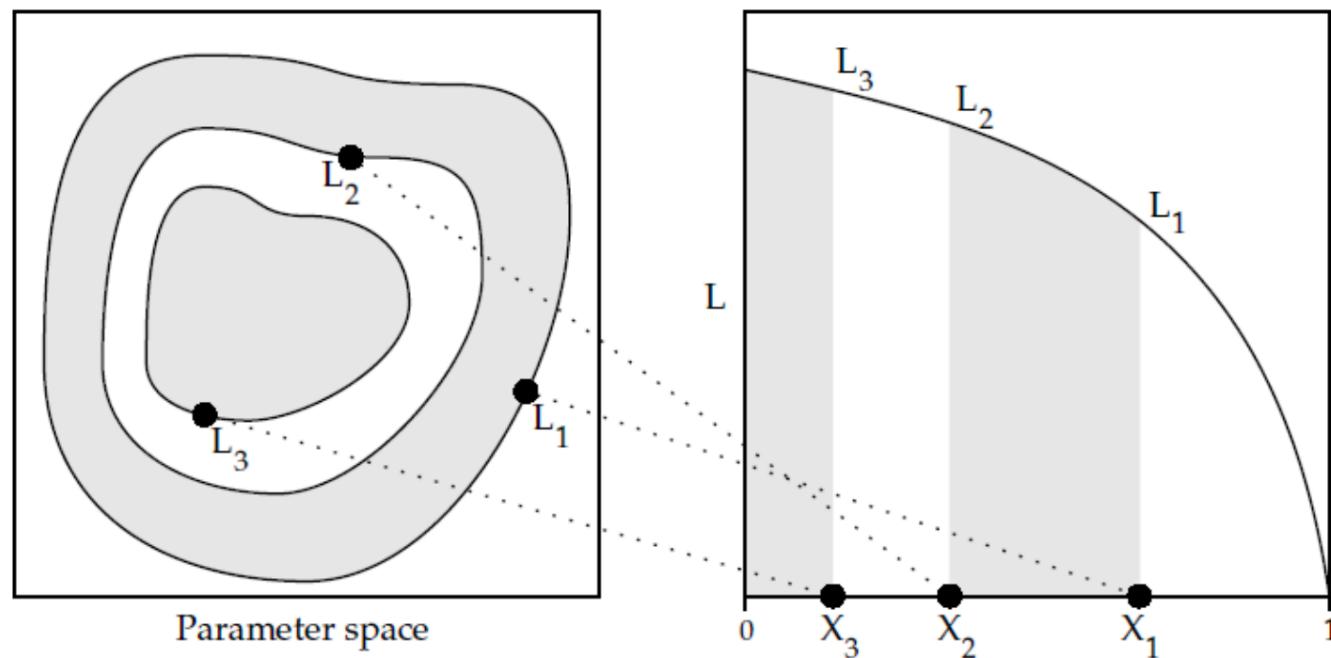
[arXiv:1409.7215](https://arxiv.org/abs/1409.7215)

Parameter estimation can then be performed using Bayes' theorem, where a prior probability distribution  $p(\theta|H)$  is updated upon receiving the new data  $d$  from the experiment to give a posterior distribution  $p(\theta|d, H)$ ,

$$p(\theta|d, H) = \frac{p(\theta|H)p(d|\theta, H)}{p(d|H)}. \quad (1)$$

# Nested sampling

- Introduced by J. Skilling to compute Bayesian evidence( $Z$ ) with MCMC sub-chains by transforming the Multi-Dimensional integral into 1-dimension over the prior volume      Ref(e.g.): *arXiv:1409.7215*



# Nested sampling in LAL

- Implemented in :  
*LALInferenceNest, LALInferenceNestedSampler,  
LALInferenceProposal, LALInferencePrior, ...*
- 5 methods used for Jump proposals (LAL default) :
  - *CovarianceEigenvectorJump*
  - *DifferentialEvolution*
  - *EnsembleStretch*
  - *EnsembleWalk*
  - *DistanceLikelihood*
- MCMC sub-chain length is determined at every  
( $N_{\text{live}}/10$ ) iterations from autocorrelation length

For more details please refer: [arXiv:1409.7215](https://arxiv.org/abs/1409.7215) or the source code itself

# GPU Approach

- Waveform and likelihood calculations are the dominant sources of time consumption for P.E.
    - $4,096(\text{srate}) \times 8(\text{seglen}) / 2 \times 2(\text{Nifo}) = 32,768 \text{ /call}$
    - ~1,000 times called per iteration
    - ~16,000 iterations per run
      - =>  $5 \times 10^{11} \text{ calculations / run}$
  - These particular parts are implemented in CUDA
    - Single Floating point is used ( $10^{-5} \text{--} 10^{-4}$  precision)
- \* In *LALSimulation*, waveform calculation can run in OpenMP but the performance didn't improve so significantly on Core™ i7

# Core of the GPU code

Each GPU core : each frequency bin

```
if (xmin <= x && x < Nfr) {  
    if (x < xmax) cimr(ph, hpr, hpi, hcr, hci, fr);  
    int k = blockIdx.y;  
    int l = k*Nfr+x;  
    core(fr, dr[l], di[l], ps[l], hpr, hpi, hcr, hci,  
         fp[k], fc[k], dt[k], Slen, cs[i], ss[i], rr[i], ri[i]);  
}  
__syncthreads();
```

cimr: IMRphenomP wave form calculation

core: Likelihood calculation

# Core of the GPU code

Each GPU core : each frequency bin

```
STATIC inline void core(real fr, real dtr, real dti, real psd,
                       real hpr, real hpi, real hcr, real hci,
                       real fp, real fc, real dt, int slen,
                       real &cs, real &ss, real &rr, real &ri)
{
    tsft(hpr, hpi, fr*_tc); tsft(hcr, hci, fr*_tc);
    Polarization and Detector antenna pattern
    real cpr = _czp*hpr+_szp*hcr, cpi = _czp*hpi+_szp*hci;
    real ccr = _czp*hcr-_szp*hpr, cci = _czp*hci-_szp*hpi;
    real tpr = _fp*cpr+_fc*ccr, tpi = _fp*cpi+_fc*cci;
    tsft(tpr, tpi, fr*dt);
    tpr, tpi: Waveform template
    real dT = 1./Rsmp, dtn = 2*dT/Rsmp/slen;
    real dfr = dtr-tpr, dfi = dti-tpi, ssq = psd*dT*dt;
    dfr, dfi: Data-template
    cs += -dtn*(dfr*dfi+dfi*dfi)/ssq; ss += dtn*(tpr*tpr+tpi*tpi)/ssq;
    rr += dtm*(tpr*dtr+tpi*dti)/ssq; ri += dtm*(tpr*dti-tpi*dtr)/ssq;
}
```

cs: Chis-square (log(likelihood)) element

# Performance test

- GW150914 data from LIGO Open Science Center
- LAL: *lalinference\_nest* with *IMRPhenomPv2*, *seglen=8*, *Nlive=500* are used as a reference
  - running on single CPU (4 parallel jobs / machine)
- GPE (GPU-accelerated P.E.) :
  - running on single CPU/machine and tested with 3 different GPU boards (NVIDIA™ GeForce™)
- 23 independent runs performed and time consumptions and output results are compared

# Performance comparison

Code	Hardware	Spec.	Wall Time Mean ± RMS	Acceleration w.r.t. LAL	Improvement
LAL	Core™ i7	4 cores (x2 HT) 3.6 GHz	24:27:24 ± 47:42		
GPE	GeForce™ GTX 1060	1152 cores 1.76 GHz 192 bit Bus	17:21 ± 0:28	× 84.5	
GPE	GeForce™ GTX 1070	1920 cores 1.68 GHz 256 bit Bus	13:58 ± 0:17	× 105.0	24% to 1060
GPE	GeForce™ GTX 1080	2560 cores 1.85 GHz 256 bit Bus	12:25 ± 0:15	× 118.1	40% to 1060 13% to 1070

Log Bayes factors  $\ln(B_{s/n})$  : 254.7±0.3 and 254.5±0.3 in LAL and GPE, respectively

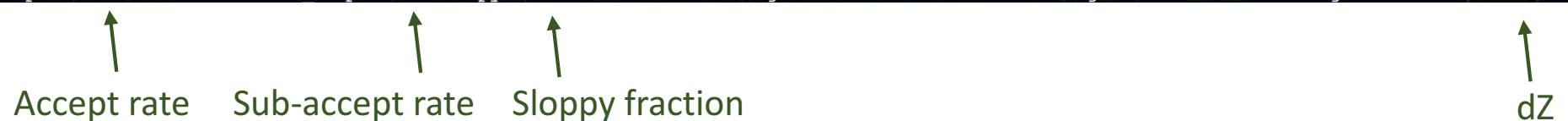
# Nested sampling parameters

## LALInference\_nest

```

16000: accpt: 0.253 Nmcmc: 5000 sub_accpt: 0.331 slpy: 74.5% H: 26.18 nats logL:-31862.067 ->-31861.838 logZ: -31890.951 deltaloglmax: 285.02 dZ: 0.14
16001: accpt: 0.352 Nmcmc: 5000 sub_accpt: 0.290 slpy: 74.6% H: 26.18 nats logL:-31862.067 ->-31861.416 logZ: -31890.951 deltaloglmax: 285.02 dZ: 0.14
16002: accpt: 0.388 Nmcmc: 5000 sub_accpt: 0.259 slpy: 74.7% H: 26.18 nats logL:-31862.066 ->-31862.049 logZ: -31890.951 deltaloglmax: 285.02 dZ: 0.14
16003: accpt: 0.259 Nmcmc: 5000 sub_accpt: 0.337 slpy: 74.6% H: 26.18 nats logL:-31862.066 ->-31862.017 logZ: -31890.951 deltaloglmax: 285.02 dZ: 0.14
16004: accpt: 0.342 Nmcmc: 5000 sub_accpt: 0.291 slpy: 74.7% H: 26.18 nats logL:-31862.066 ->-31861.974 logZ: -31890.951 deltaloglmax: 285.02 dZ: 0.14
16005: accpt: 0.244 Nmcmc: 5000 sub_accpt: 0.353 slpy: 74.6% H: 26.18 nats logL:-31862.065 ->-31861.798 logZ: -31890.951 deltaloglmax: 285.02 dZ: 0.14
16006: accpt: 0.266 Nmcmc: 5000 sub_accpt: 0.330 slpy: 74.5% H: 26.18 nats logL:-31862.065 ->-31861.219 logZ: -31890.951 deltaloglmax: 285.02 dZ: 0.14
16007: accpt: 0.364 Nmcmc: 5000 sub_accpt: 0.269 slpy: 74.6% H: 26.18 nats logL:-31862.065 ->-31861.519 logZ: -31890.951 deltaloglmax: 285.02 dZ: 0.14
16008: accpt: 0.406 Nmcmc: 5000 sub_accpt: 0.259 slpy: 74.7% H: 26.18 nats logL:-31862.065 ->-31861.810 logZ: -31890.951 deltaloglmax: 285.02 dZ: 0.14
16009: accpt: 0.233 Nmcmc: 5000 sub_accpt: 0.350 slpy: 74.6% H: 26.18 nats logL:-31862.064 ->-31861.960 logZ: -31890.951 deltaloglmax: 285.02 dZ: 0.14
16010: accpt: 0.528 Nmcmc: 5000 sub_accpt: 0.190 slpy: 74.7% H: 26.18 nats logL:-31862.064 ->-31861.260 logZ: -31890.950 deltaloglmax: 285.02 dZ: 0.14

```



## GPE

```

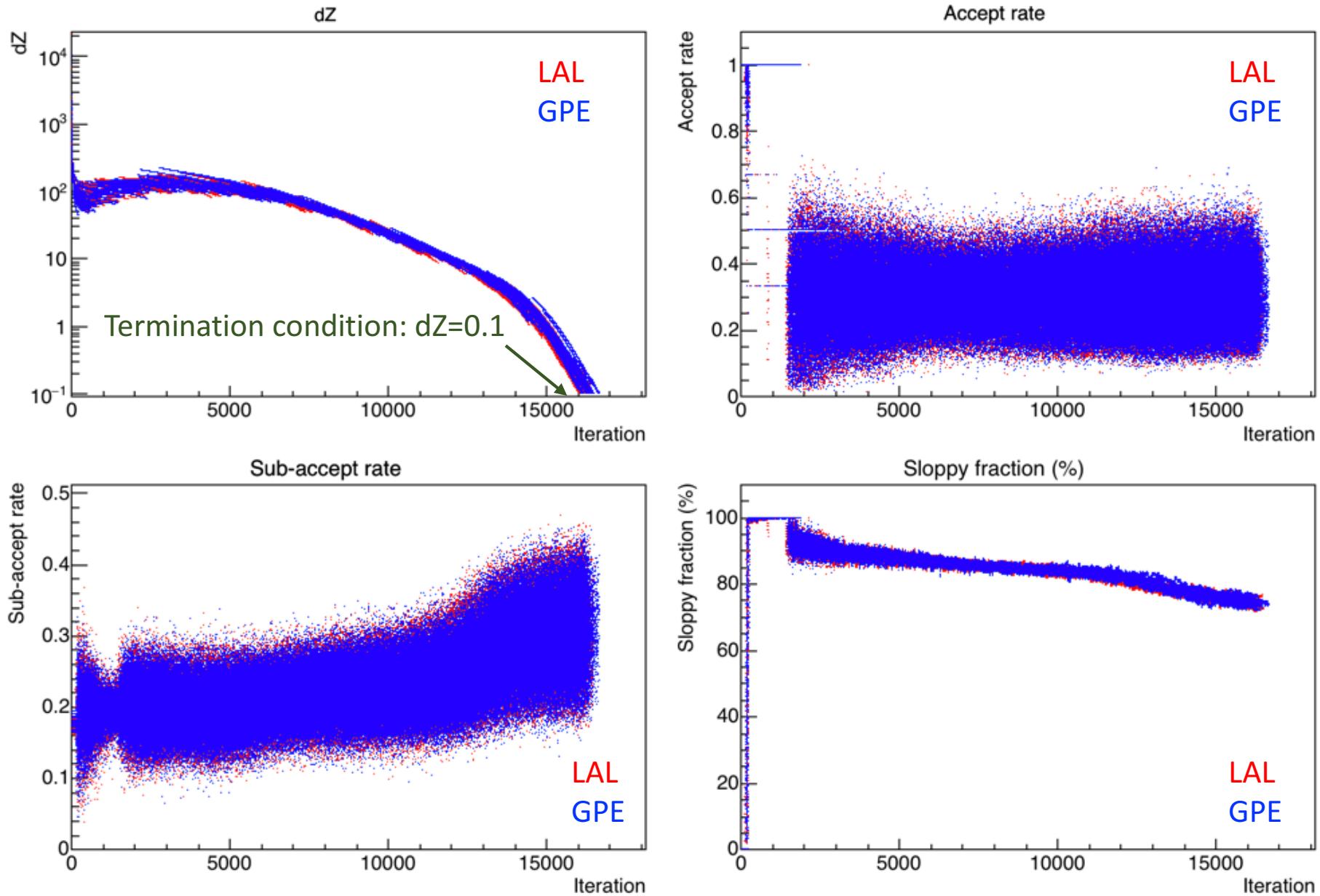
16034: accpt: 0.205691 Nmcmc: 5000 sa: 0.373176 slf: 75.3115 H: 26.5989 lm= -31862.262 -31861.559 lZ: -31891.545 dlm: 284.651 dZ: 0.170992
16035: accpt: 0.317409 Nmcmc: 5000 sa: 0.291025 slf: 75.4115 H: 26.5991 lm= -31862.262 -31861.883 lZ: -31891.545 dlm: 284.651 dZ: 0.170659
16036: accpt: 0.373984 Nmcmc: 5000 sa: 0.253248 slf: 75.5115 H: 26.5993 lm= -31862.262 -31861.738 lZ: -31891.545 dlm: 284.651 dZ: 0.170325
16037: accpt: 0.477551 Nmcmc: 5000 sa: 0.213272 slf: 75.6115 H: 26.5995 lm= -31862.262 -31861.582 lZ: -31891.544 dlm: 284.651 dZ: 0.169994
16038: accpt: 0.246721 Nmcmc: 5000 sa: 0.336998 slf: 75.5115 H: 26.5997 lm= -31862.262 -31861.648 lZ: -31891.544 dlm: 284.651 dZ: 0.169663
16039: accpt: 0.342857 Nmcmc: 5000 sa: 0.268239 slf: 75.6115 H: 26.5999 lm= -31862.258 -31861.516 lZ: -31891.544 dlm: 284.651 dZ: 0.169332
16040: accpt: 0.336066 Nmcmc: 5000 sa: 0.277234 slf: 75.7115 H: 26.6001 lm= -31862.258 -31862.051 lZ: -31891.544 dlm: 284.651 dZ: 0.169002
16041: accpt: 0.255144 Nmcmc: 5000 sa: 0.325005 slf: 75.6115 H: 26.6003 lm= -31862.258 -31861.871 lZ: -31891.544 dlm: 284.651 dZ: 0.168675
16042: accpt: 0.309836 Nmcmc: 5000 sa: 0.293424 slf: 75.7115 H: 26.6005 lm= -31862.258 -31862.215 lZ: -31891.544 dlm: 284.651 dZ: 0.168347
16043: accpt: 0.283128 Nmcmc: 5000 sa: 0.307815 slf: 75.6115 H: 26.6007 lm= -31862.254 -31862.109 lZ: -31891.544 dlm: 284.651 dZ: 0.168018

```



For more details please refer: [arXiv:1409.7215](https://arxiv.org/abs/1409.7215) or the source code itself

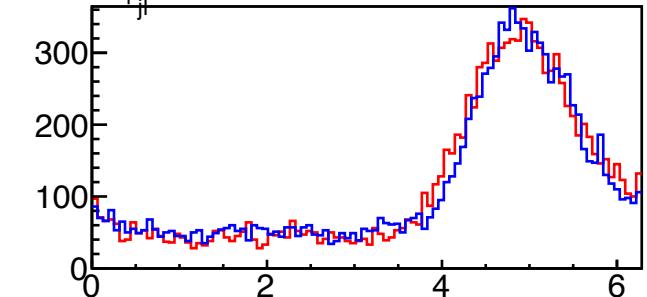
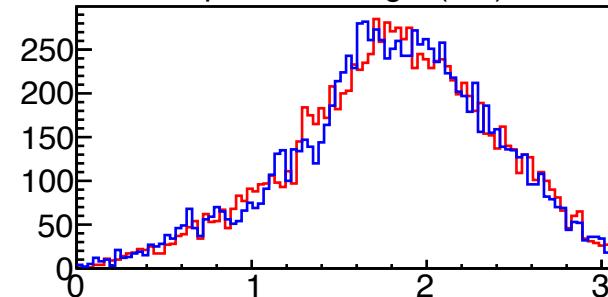
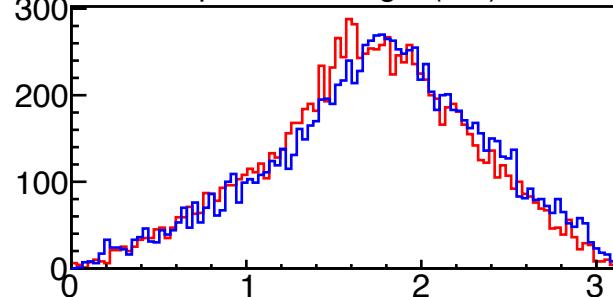
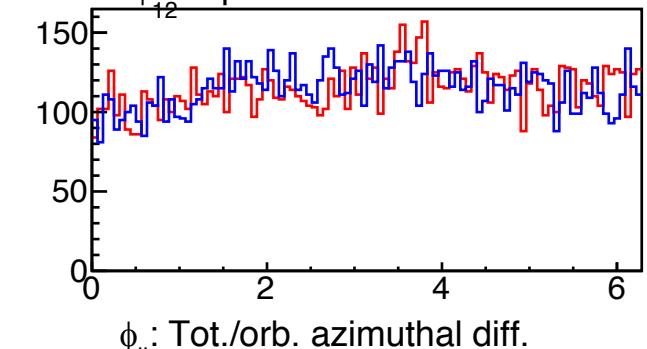
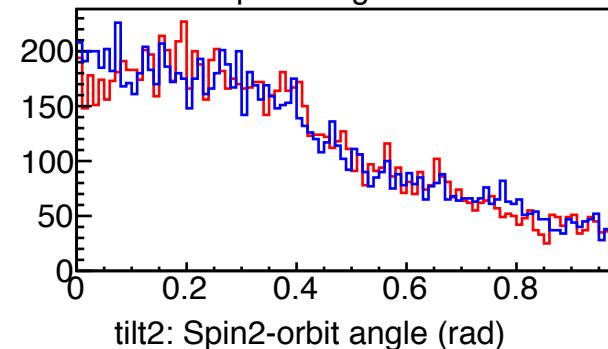
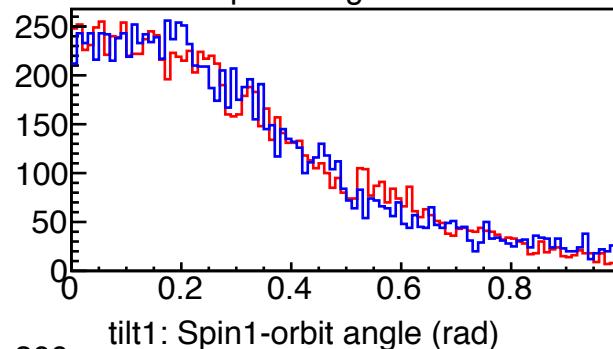
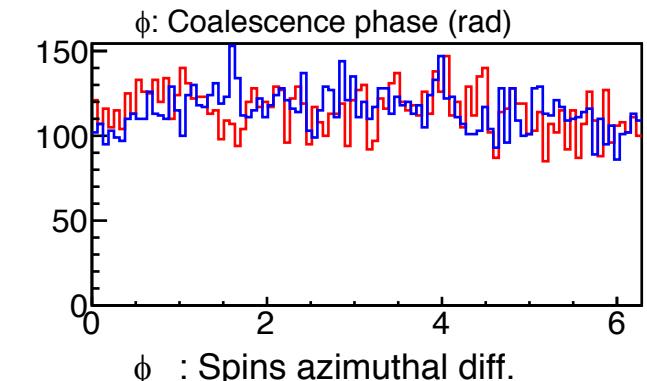
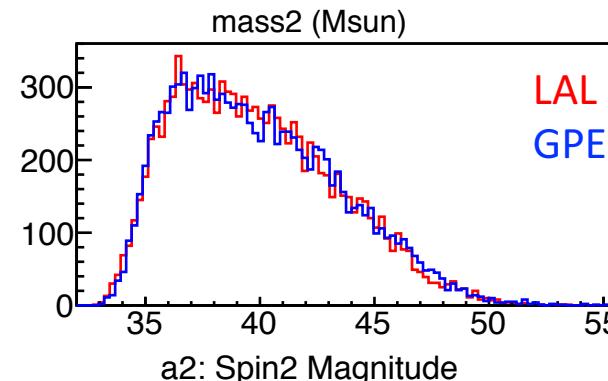
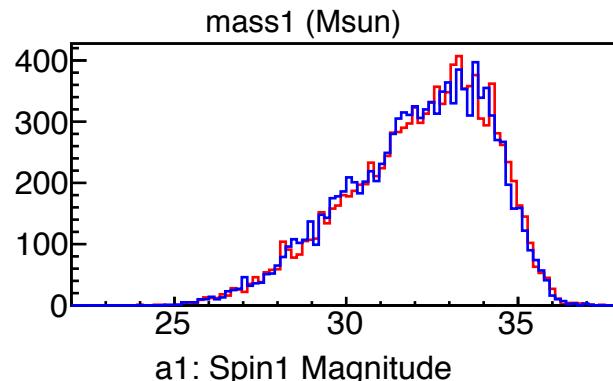
# Comparisons (Nested sampling parameters)



# Comparisons (CBC intrinsic parameters)

Parameters in the final 500 live points (from 23 independent runs)

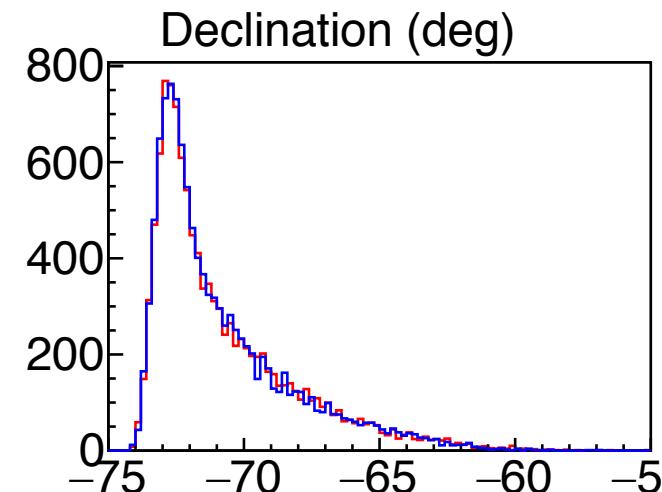
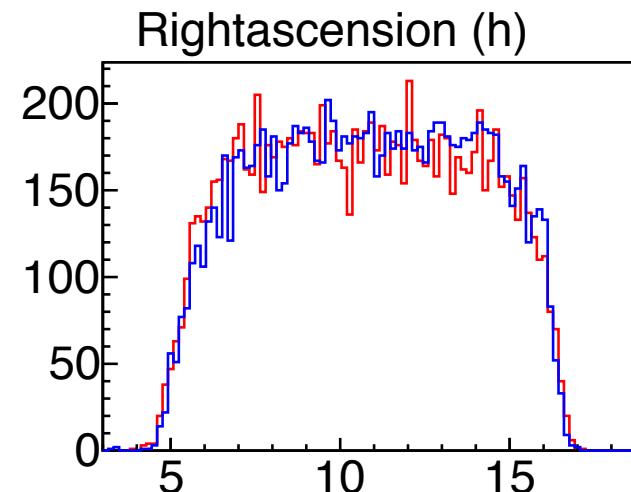
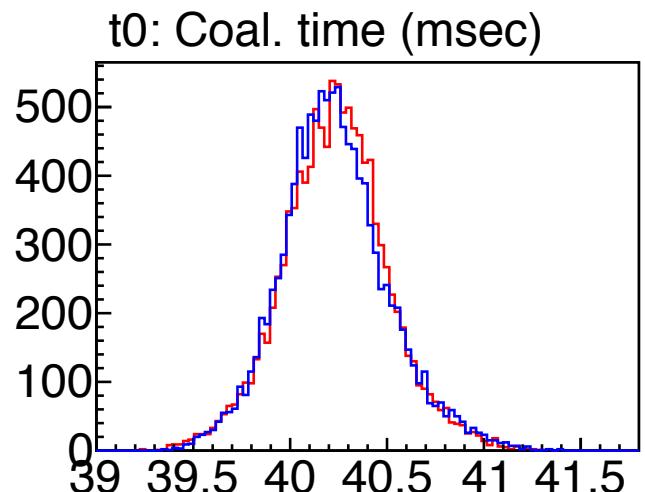
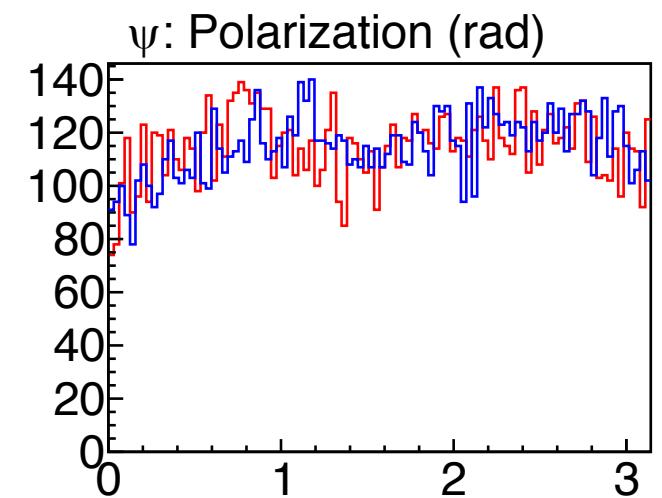
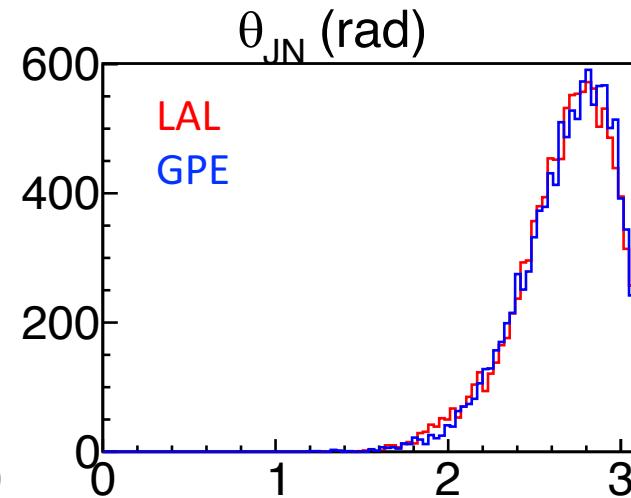
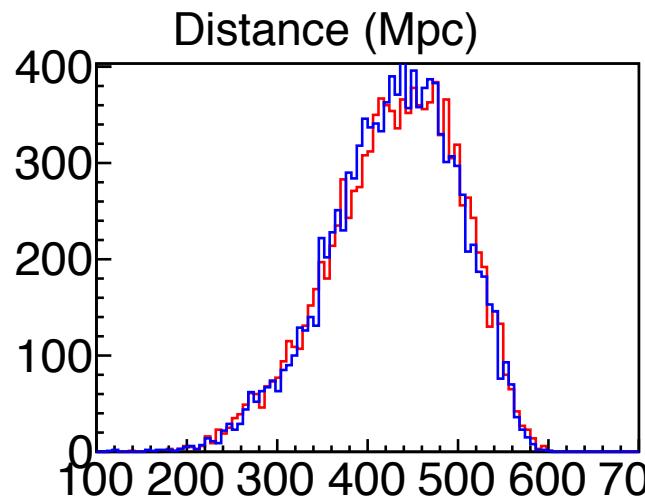
Note: they are not the posterior distributions



# Comparisons (CBC extrinsic parameters)

Parameters in the final 500 live points (from 23 independent runs)

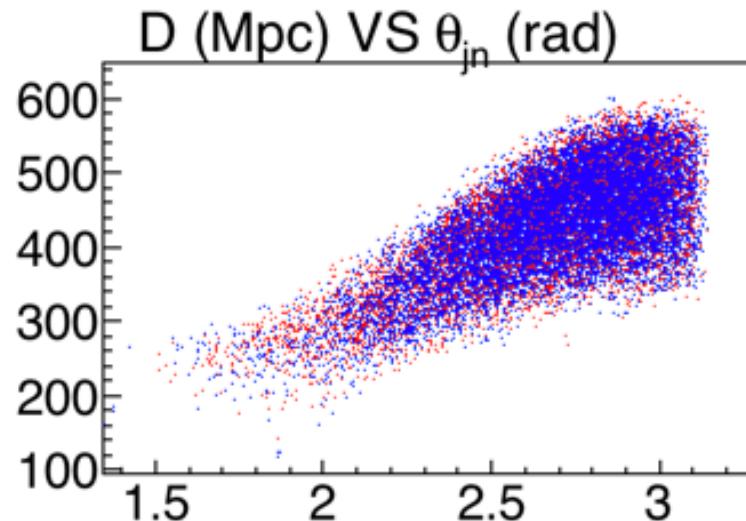
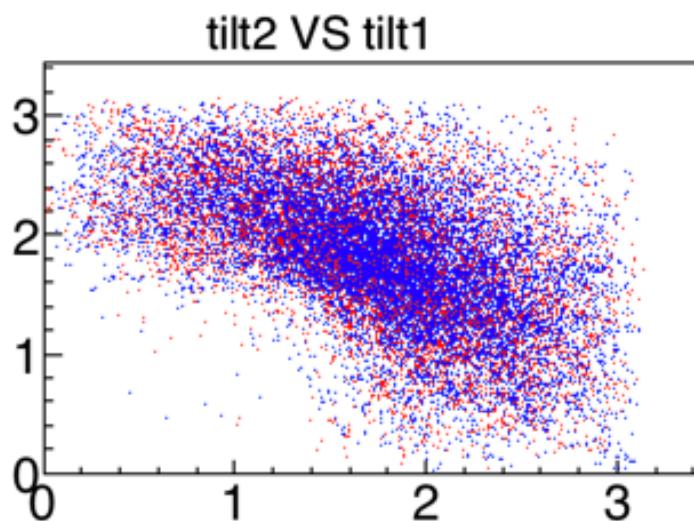
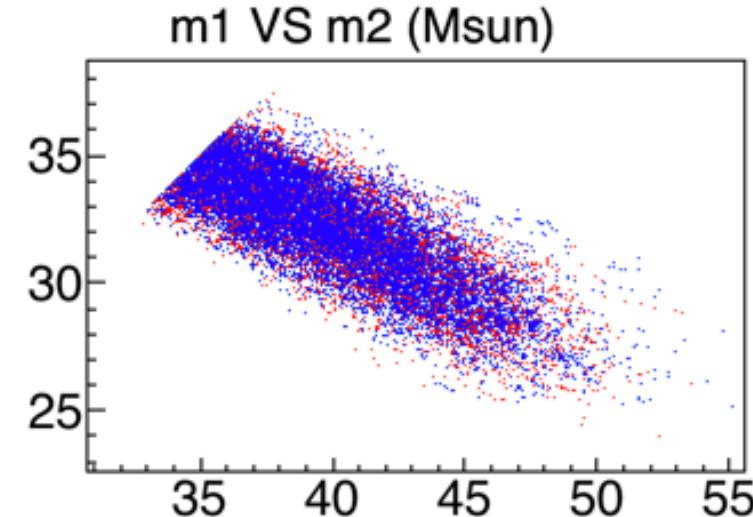
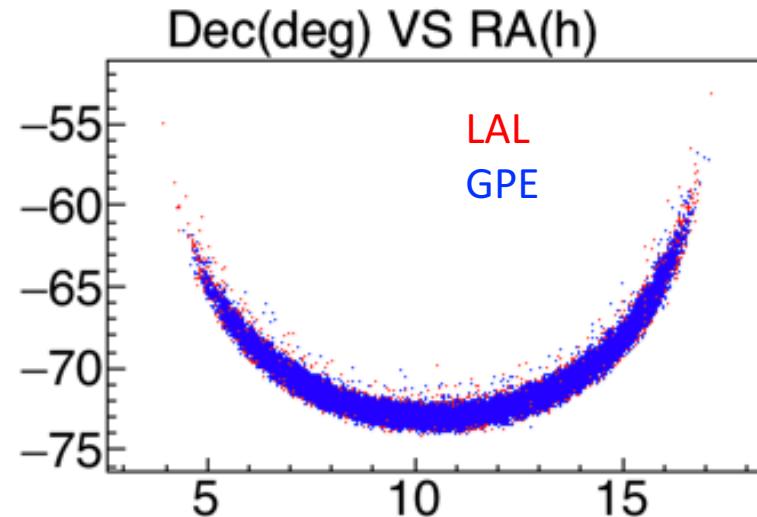
Note: they are not the posterior distributions



# Comparisons (CBC parameter correlations)

Parameters in the final 500 live points (from 23 independent runs)

Note: they are not the posterior distributions



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- Comparison of results – Simulation
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# GW events analysis

- GW150914
  - Detailed studies done by LVC  
many official results have been published
- GW151226
  - Weaker but longer signal
- GW170814
  - Localization by LV
- GW170817
  - The first and (so far) the only BNS events

$N_{\text{live}} = 2048$ , Time comsumption : 1~4 hours

# GW150914 references

Referenced compared in this study

- PRL 116, 061102 : First detection
- PRL 116, 241102 : CBC parameter estimation
- PRD 93, 122004 : Minimal assumptions
- PRX 6, 041014 : Improved analysis
- PRX 6, 041015 : Review of O1 BH-BH events

# PRX 6, 041014 – Improved analysis

- Full parameter (15) estimation with precessing-spin

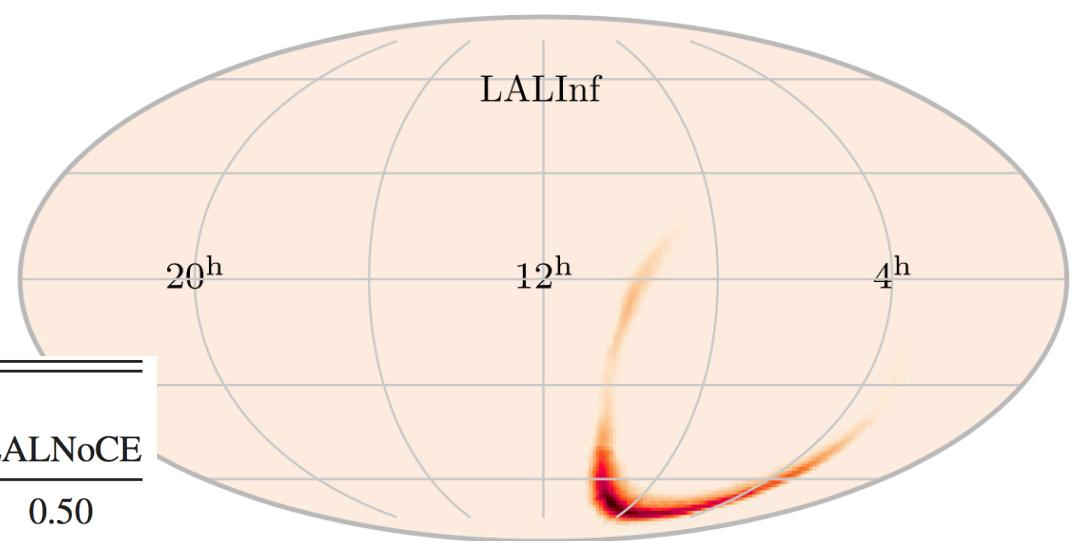
	Precessing EOBNR	Precessing IMRPhenom	Overall
Detector-frame total mass $M/M_\odot$	$71.6^{+4.3}_{-4.1}$	$70.9^{+4.0}_{-3.9}$	$71.3^{+4.3}_{-4.1}$
Detector-frame chirp mass $\mathcal{M}/M_\odot$	$30.9^{+2.0}_{-1.9}$	$30.6^{+1.8}_{-1.8}$	$30.8^{+1.9}_{-1.8}$
Detector-frame primary mass $m_1/M_\odot$	$38.9^{+5.1}_{-3.7}$	$38.5^{+5.6}_{-3.6}$	$38.7^{+5.3}_{-3.7}$
Detector-frame secondary mass $m_2/M_\odot$	$32.7^{+3.6}_{-4.8}$	$32.2^{+3.6}_{-4.8}$	$32.5^{+3.7}_{-4.8}$
Detector-frame final mass $M_f/M_\odot$	$68.3^{+3.8}_{-3.7}$	$67.6^{+3.6}_{-3.5}$	$68.0^{+3.8}_{-3.6}$
Source-frame total mass $M^{\text{source}}/M_\odot$	$65.6^{+4.1}_{-3.8}$	$65.0^{+4.0}_{-3.6}$	$65.3^{+4.1}_{-3.7}$
Source-frame chirp mass $\mathcal{M}^{\text{source}}/M_\odot$	$28.3^{+1.8}_{-1.7}$	$28.1^{+1.7}_{-1.6}$	$28.2^{+1.8}_{-1.7}$
Source-frame primary mass $m_1^{\text{source}}/M_\odot$	$35.6^{+4.8}_{-3.4}$	$35.3^{+5.2}_{-3.4}$	$35.4^{+5.0}_{-3.4}$
Source-frame secondary mass $m_2^{\text{source}}/M_\odot$	$30.0^{+3.3}_{-4.4}$	$29.6^{+3.3}_{-4.3}$	$29.8^{+3.3}_{-4.3}$
Source-frame final mass $M_f^{\text{source}}/M_\odot$	$62.5^{+3.7}_{-3.4}$	$62.0^{+3.7}_{-3.3}$	$62.2^{+3.7}_{-3.4}$
Mass ratio $q$	$0.84^{+0.14}_{-0.20}$	$0.84^{+0.14}_{-0.20}$	$0.84^{+0.14}_{-0.20}$
Effective inspiral spin parameter $\chi_{\text{eff}}$	$-0.02^{+0.14}_{-0.16}$	$-0.05^{+0.13}_{-0.15}$	$-0.04^{+0.14}_{-0.16}$
Effective precession spin parameter $\chi_p$	$0.28^{+0.38}_{-0.21}$	$0.35^{+0.45}_{-0.27}$	$0.31^{+0.44}_{-0.23}$
Dimensionless primary spin magnitude $a_1$	$0.22^{+0.43}_{-0.20}$	$0.32^{+0.53}_{-0.29}$	$0.26^{+0.52}_{-0.24}$
Dimensionless secondary spin magnitude $a_2$	$0.29^{+0.52}_{-0.27}$	$0.34^{+0.54}_{-0.31}$	$0.32^{+0.54}_{-0.29}$
Final spin $a_f$	$0.68^{+0.05}_{-0.05}$	$0.68^{+0.06}_{-0.06}$	$0.68^{+0.05}_{-0.06}$
Luminosity distance $D_L/\text{Mpc}$	$440^{+160}_{-180}$	$440^{+150}_{-180}$	$440^{+160}_{-180}$
Source redshift $z$	$0.094^{+0.032}_{-0.037}$	$0.093^{+0.029}_{-0.036}$	$0.093^{+0.030}_{-0.036}$
Upper bound on primary spin magnitude $a_1$	0.54	0.74	0.65
Upper bound on secondary spin magnitude $a_2$	0.70	0.78	0.75
Lower bound on mass ratio $q$	0.69	0.68	0.68

# Comparison of results – GW150914

	PRX 6, 041014 (IMRPhenom)	GPE with LOSC data
SNR	23.7	23.7
Chirp mass (det.)	$30.6 +1.8 -1.8$	$30.0 +2.2 -2.5$
$m_1$ (det.)	$38.5 +5.6 -3.6$	$38.9 +5.9 -4.4$
$m_2$ (det.)	$32.2 +3.6 -4.8$	$31.1 +4.6 -6.7$
Mass ratio	$0.84 +0.14 -0.20$	$0.80 +0.13 -0.16$
Chirp mass (source)	$28.1 +1.7 -1.6$	$27.6 +1.9 -2.2$
$m_1$ (source)	$35.3 +5.2 -3.4$	$35.7 +5.7 -3.9$
$m_2$ (source)	$29.6 +3.3 -4.3$	$28.5 +4.1 -6.1$
Luminosity distance (Mpc)	$440 +160 -180$	$431 +150 -174$
Source redshift	$0.094 +0.032 -0.037$	$0.091 +0.035 -0.029$
Effective inspiral spin $\chi_{\text{eff}}$	$-0.05 +0.13 -0.15$	$-0.03 +0.12 -0.14$
Effective precession spin $\chi_p$	$0.35 +0.45 -0.27$	$0.32 +0.45 -0.25$

# PRD93, 122004 – Minimal assumptions

- Using burst analysis (not CBC) but sky localization errors are compared between with and without calibration errors (10%,10 deg)

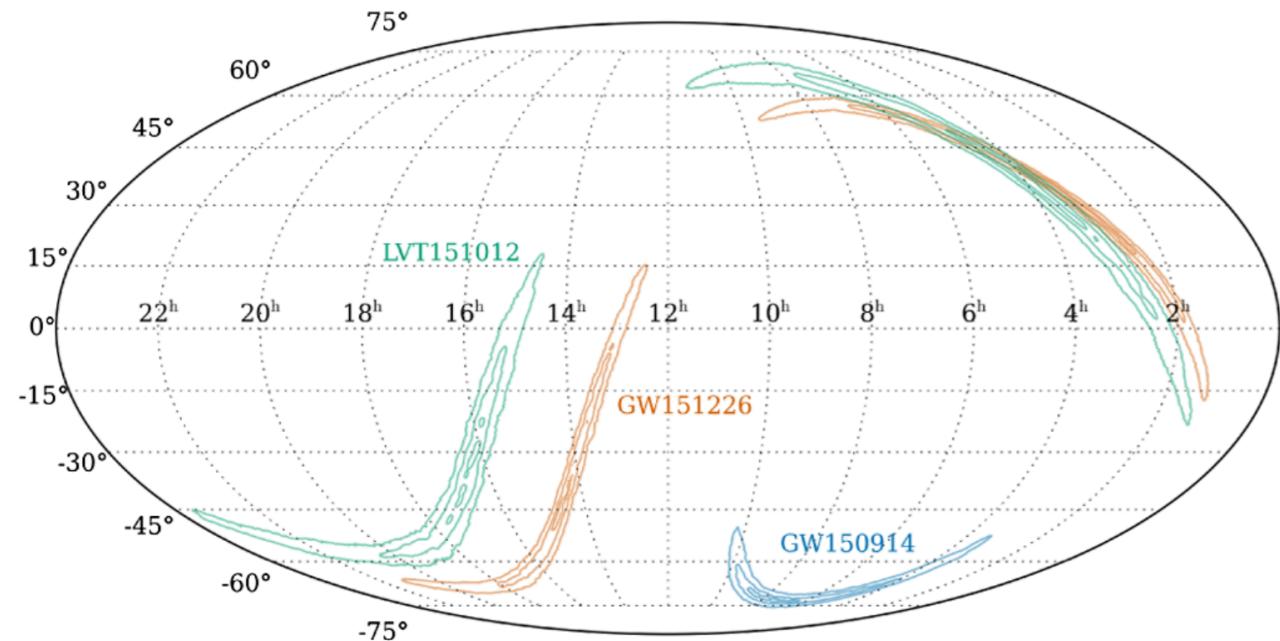


	Confidence regions		Fidelity			
	50%	90%	LIB	BW	LALInf	LALNoCE
cWB	98 deg <sup>2</sup>	308 deg <sup>2</sup>	0.55	0.55	0.51	0.50
LIB	208 deg <sup>2</sup>	746 deg <sup>2</sup>	...	0.45	0.68	0.28
BW	101 deg <sup>2</sup>	634 deg <sup>2</sup>	...	...	0.68	0.87
LALInf	150 deg <sup>2</sup>	610 deg <sup>2</sup>	...	...	...	0.81
LALNoCE	48 deg <sup>2</sup>	150 deg <sup>2</sup>	...	...	...	...

# PRX 6, 041015 – Review of O1 BBH events

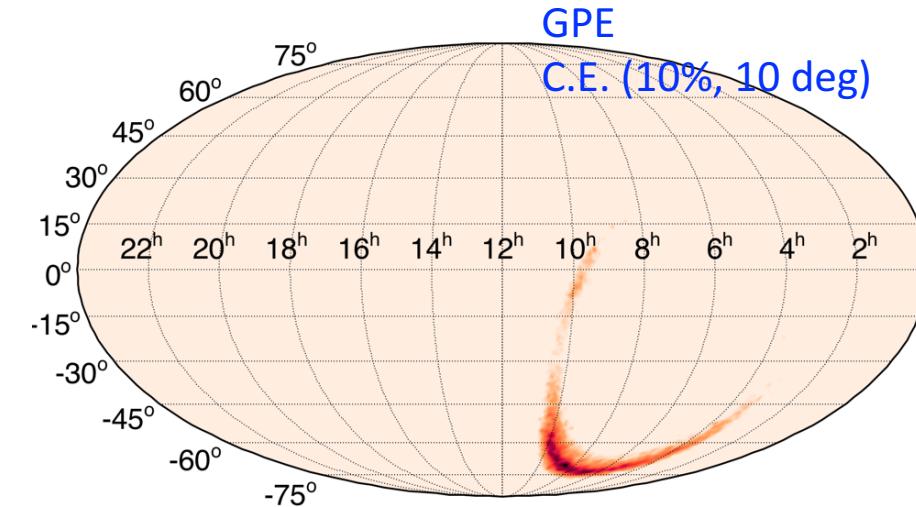
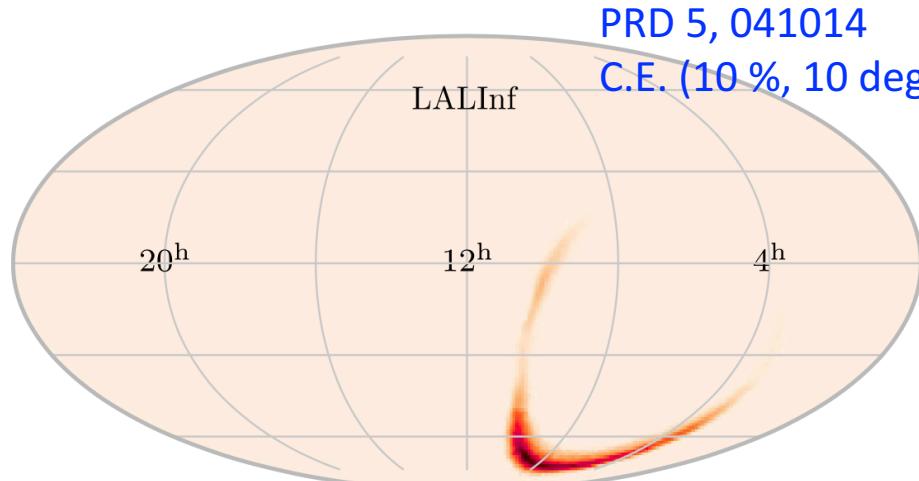
- Using the improved calibration errors

Event	Amplitude		Phase	
	Hanford	Livingston	Hanford	Livingston
GW150914	4.8%	8.2%	3.2 deg	4.2 deg
LVT151012	4.2%	8.3%	2.7 deg	4.3 deg
GW151226	4.2%	6.9%	2.7 deg	3.6 deg



# Comparison of results – GW150914

	PRD 6, 041014 (Burst)		GPE (LOSC data)	
	50 % C.L.	90 % C.L.	50 % C.L.	90 % C.L.
No C.E.	48 deg <sup>2</sup>	150 deg <sup>2</sup>	46 deg <sup>2</sup>	158 deg <sup>2</sup>
C.E. (5%, 5 deg)	—	—	78 deg <sup>2</sup>	296 deg <sup>2</sup>
C.E. (10%, 10 deg)	150 deg <sup>2</sup>	610 deg <sup>2</sup>	150 deg <sup>2</sup>	577 deg <sup>2</sup>
	PRX 6, 041015 (CBC)			
Improved C.E.	—	230 deg <sup>2</sup>	67 deg <sup>2</sup>	236 deg <sup>2</sup>



# GW151226 references

## References compared in this study

- PRL 116, 241103
- PRX 6, 041015

Primary black hole mass	$14.2^{+8.3}_{-3.7} M_{\odot}$
Secondary black hole mass	$7.5^{+2.3}_{-2.3} M_{\odot}$
Chirp mass	$8.9^{+0.3}_{-0.3} M_{\odot}$
Total black hole mass	$21.8^{+5.9}_{-1.7} M_{\odot}$
Final black hole mass	$20.8^{+6.1}_{-1.7} M_{\odot}$
Radiated gravitational-wave energy	$1.0^{+0.1}_{-0.2} M_{\odot} c^2$
Peak luminosity	$3.3^{+0.8}_{-1.6} \times 10^{56} \text{ erg/s}$
Final black hole spin	$0.74^{+0.06}_{-0.06}$
Luminosity distance	$440^{+180}_{-190} \text{ Mpc}$
Source redshift $z$	$0.09^{+0.03}_{-0.04}$

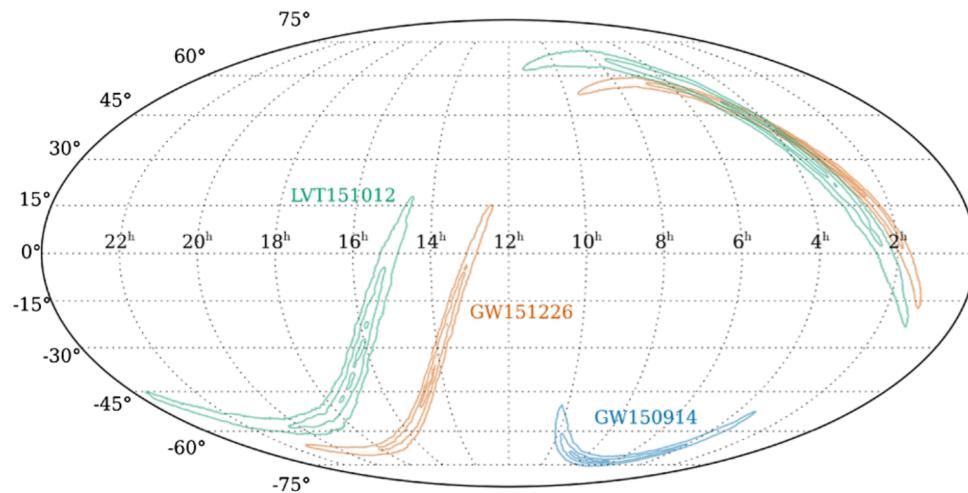
	EOBNR	GW151226 IMRPhenom	Overall
Detector frame			
Total mass $M/M_{\odot}$	$23.6^{+8.0}_{-1.3}$	$23.8^{+5.1}_{-1.5}$	$23.7^{+6.5 \pm 2.2}_{-1.4 \pm 0.1}$
Chirp mass $\mathcal{M}/M_{\odot}$	$9.71^{+0.08}_{-0.07}$	$9.72^{+0.06}_{-0.06}$	$9.72^{+0.07 \pm 0.01}_{-0.06 \pm 0.01}$
Primary mass $m_1/M_{\odot}$	$15.3^{+10.8}_{-3.8}$	$15.8^{+7.2}_{-4.0}$	$15.6^{+9.0 \pm 2.6}_{-4.0 \pm 0.2}$
Secondary mass $m_2/M_{\odot}$	$8.3^{+2.5}_{-2.9}$	$8.1^{+2.5}_{-2.1}$	$8.2^{+2.6 \pm 0.2}_{-2.5 \pm 0.5}$
Final mass $M_f/M_{\odot}$	$22.5^{+8.2}_{-1.4}$	$22.8^{+5.3}_{-1.6}$	$22.6^{+6.7 \pm 2.2}_{-1.5 \pm 0.1}$
Source frame			
Total mass $M^{\text{source}}/M_{\odot}$	$21.6^{+7.4}_{-1.6}$	$21.9^{+4.7}_{-1.7}$	$21.8^{+5.9 \pm 2.0}_{-1.7 \pm 0.1}$
Chirp mass $\mathcal{M}^{\text{source}}/M_{\odot}$	$8.87^{+0.35}_{-0.28}$	$8.90^{+0.31}_{-0.27}$	$8.88^{+0.33 \pm 0.01}_{-0.28 \pm 0.04}$
Primary mass $m_1^{\text{source}}/M_{\odot}$	$14.0^{+10.0}_{-3.5}$	$14.5^{+6.6}_{-3.7}$	$14.2^{+8.3 \pm 2.4}_{-3.7 \pm 0.2}$
Secondary mass $m_2^{\text{source}}/M_{\odot}$	$7.5^{+2.3}_{-2.6}$	$7.4^{+2.3}_{-2.0}$	$7.5^{+2.3 \pm 0.2}_{-2.3 \pm 0.4}$
Final mass $M_f^{\text{source}}/M_{\odot}$	$20.6^{+7.6}_{-1.6}$	$20.9^{+4.8}_{-1.8}$	$20.8^{+6.1 \pm 2.0}_{-1.7 \pm 0.1}$
Energy radiated $E_{\text{rad}}/(M_{\odot} c^2)$	$1.02^{+0.09}_{-0.24}$	$0.99^{+0.11}_{-0.17}$	$1.00^{+0.10 \pm 0.01}_{-0.20 \pm 0.03}$
Mass ratio $q$	$0.54^{+0.40}_{-0.33}$	$0.51^{+0.39}_{-0.25}$	$0.52^{+0.40 \pm 0.03}_{-0.29 \pm 0.04}$
Effective inspiral spin $\chi_{\text{eff}}$	$0.21^{+0.24}_{-0.11}$	$0.22^{+0.15}_{-0.08}$	$0.21^{+0.20 \pm 0.07}_{-0.10 \pm 0.03}$
Primary spin magnitude $a_1$	$0.42^{+0.35}_{-0.37}$	$0.55^{+0.35}_{-0.42}$	$0.49^{+0.37 \pm 0.11}_{-0.42 \pm 0.07}$
Secondary spin magnitude $a_2$	$0.51^{+0.44}_{-0.46}$	$0.52^{+0.42}_{-0.47}$	$0.52^{+0.43 \pm 0.01}_{-0.47 \pm 0.00}$
Final spin $a_f$	$0.73^{+0.05}_{-0.06}$	$0.75^{+0.07}_{-0.05}$	$0.74^{+0.06 \pm 0.03}_{-0.06 \pm 0.03}$
Luminosity distance $D_L/\text{Mpc}$	$450^{+180}_{-210}$	$440^{+170}_{-180}$	$440^{+180 \pm 20}_{-190 \pm 10}$
Source redshift $z$	$0.096^{+0.035}_{-0.042}$	$0.092^{+0.033}_{-0.037}$	$0.094^{+0.035 \pm 0.004}_{-0.039 \pm 0.001}$
Upper bound			
Primary spin magnitude $a_1$	0.68	0.83	$0.77 \pm 0.12$
Secondary spin magnitude $a_2$	0.90	0.89	$0.90 \pm 0.01$
Lower bound			
Mass ratio $q$	0.25	0.30	$0.28 \pm 0.04$
Log Bayes factor $\ln \mathcal{B}_{\text{s/n}}$	$59.5 \pm 0.1$	$60.2 \pm 0.2$	...
Information criterion DIC	$34296.4 \pm 0.2$	$34295.1 \pm 0.1$	...

# Comparison of results – GW151226

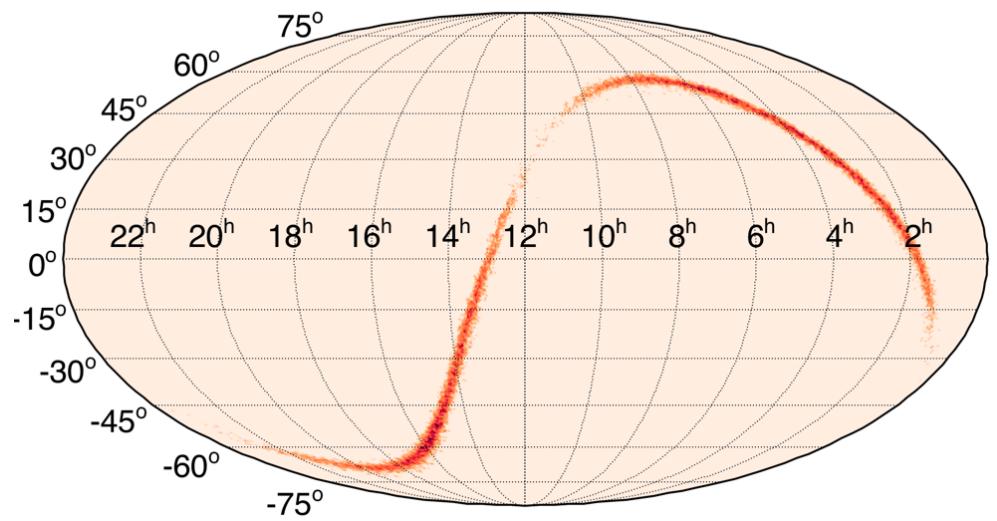
	PRX 6, 041015 (IMRPhenom)	GPE with LOSC data
SNR	13.0	12.1
Chirp mass (source)	8.90 +0.31 -0.27	8.92 +0.37 -0.33
m1 (source)	14.5 +6.6 -3.7	14.0 +6.2 -3.4
m2 (source)	7.4 +2.3 -2.0	7.7 +2.2 -2.0
Luminosity distance (Mpc)	440 +170 -180	441 +196 -195
Source redshift	0.092 +0.033 -0.037	0.095 +0.039 -0.037
Effective inspiral spin $\chi_{\text{eff}}$	0.22 +0.15 -0.08	0.23 +0.14 -0.11
Primary spin magnitude $a_1$	0.55 +0.35 -0.42	0.56 +0.35 -0.44
Primary spin magnitude $a_2$	0.52 +0.42 -0.47	0.56 +0.39 -0.50

SNR could not be reproduced exactly as in the publication  
 Maybe we need additional cleaning of LOSC public data

# Comparison of results – GW151226



GW151226: (PRX 6, 041015)  
90 %  $\Delta\Omega = 850 \text{ deg}^2$

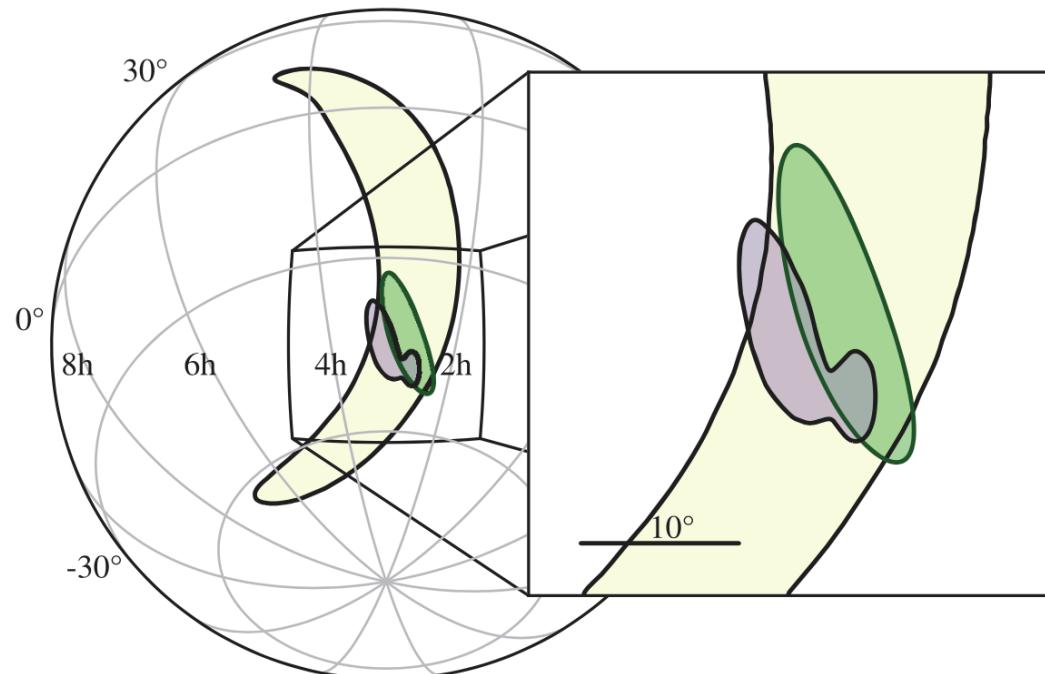


GPE  
90 %  $\Delta\Omega = 863 \text{ deg}^2$

# GW170814 reference

References compared in this study

- PRL 119, 141101

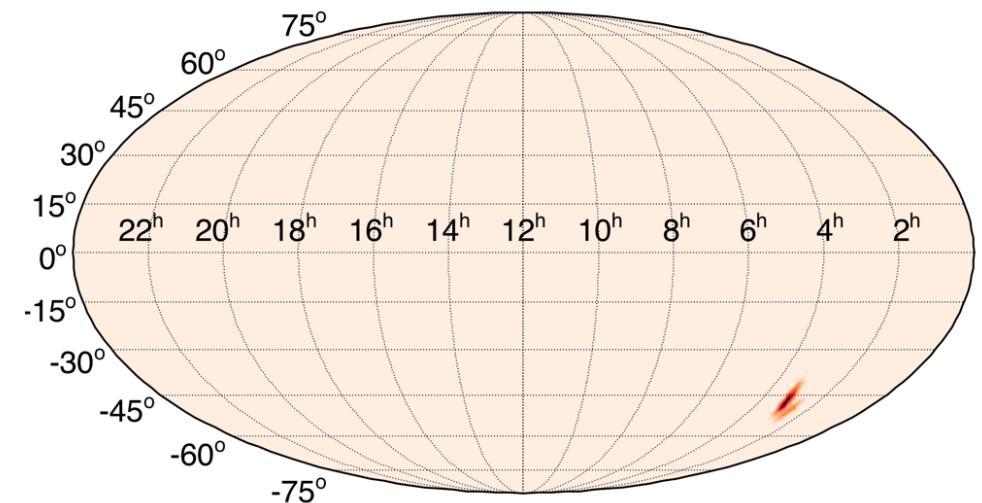
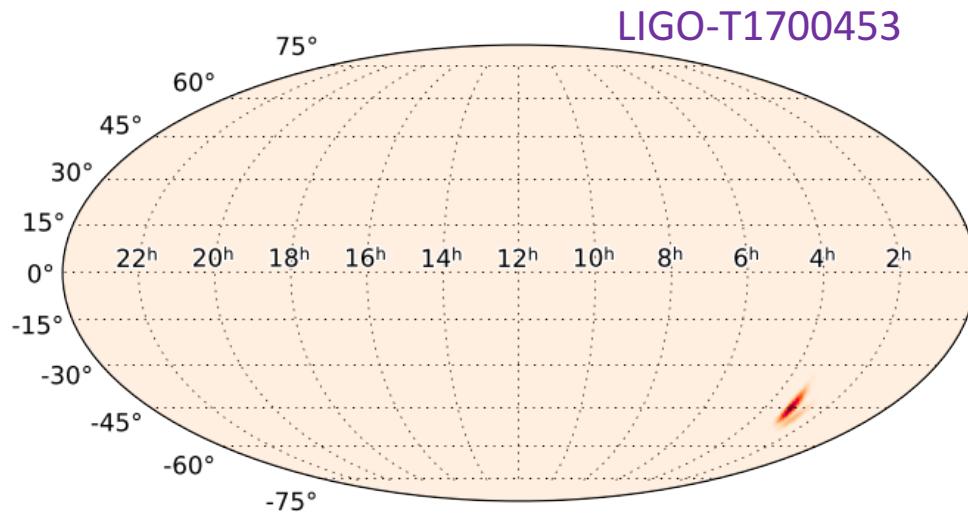


Primary black hole mass $m_1$	$30.5^{+5.7}_{-3.0} M_{\odot}$
Secondary black hole mass $m_2$	$25.3^{+2.8}_{-4.2} M_{\odot}$
Chirp mass $\mathcal{M}$	$24.1^{+1.4}_{-1.1} M_{\odot}$
Total mass $M$	$55.9^{+3.4}_{-2.7} M_{\odot}$
Final black hole mass $M_f$	$53.2^{+3.2}_{-2.5} M_{\odot}$
Radiated energy $E_{\text{rad}}$	$2.7^{+0.4}_{-0.3} M_{\odot} c^2$
Peak luminosity $\ell_{\text{peak}}$	$3.7^{+0.5}_{-0.5} \times 10^{56} \text{ erg s}^{-1}$
Effective inspiral spin parameter $\chi_{\text{eff}}$	$0.06^{+0.12}_{-0.12}$
Final black hole spin $a_f$	$0.70^{+0.07}_{-0.05}$
Luminosity distance $D_L$	$540^{+130}_{-210} \text{ Mpc}$
Source redshift $z$	$0.11^{+0.03}_{-0.04}$

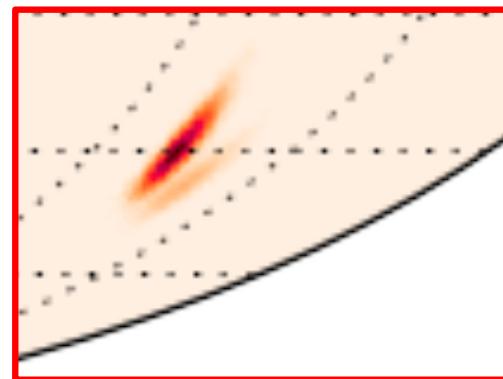
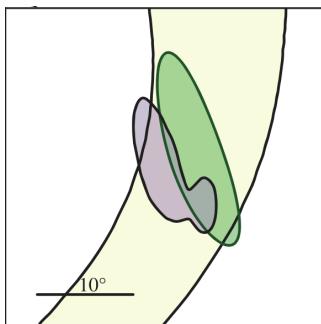
# Comparison of results – GW170814

	PRX 6, 041015 (IMRPhenom)	GPE with LOSC data (CLN)
SNR	16.1	16.6
SNR (H,L,V)	7.3, 13.7, 4.4	9.5, 13.1, 4.0
Chirp mass (source)	24.1 +1.4 -1.1	23.2 +2.0 -2.1
m1 (source)	30.5 +5.7 -3.0	30.7 +6.9 -4.1
m2 (source)	25.3 +2.8 -4.2	23.2 +4.1 -5.0
Luminosity distance (Mpc)	540 +130 -210	462 +157 -168
Source redshift	0.11 +0.03 -0.04	0.10 +0.03 -0.03
Effective inspiral spin $\chi_{\text{eff}}$	0.06 +0.12 -0.12	-0.06 +0.18 -0.21

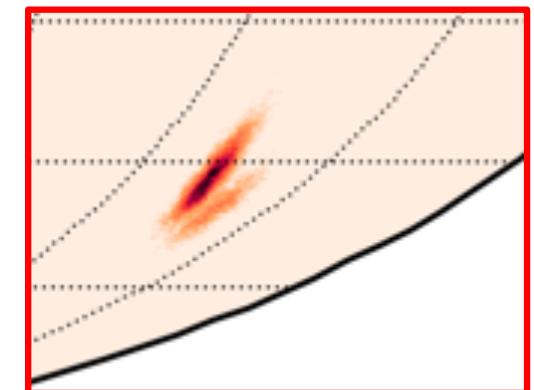
# Comparison of results – GW170814



PRL 119. 141101  
90 %  $\Delta\Omega = 60 \text{ deg}^2$



GPE  
90 %  $\Delta\Omega = 55 \text{ deg}^2$



# GW170817 reference

## References compared in this study

- PRL 119, 161101

	Low-spin priors ( $ \chi  \leq 0.05$ )	High-spin priors ( $ \chi  \leq 0.89$ )
Primary mass $m_1$	1.36–1.60 $M_{\odot}$	1.36–2.26 $M_{\odot}$
Secondary mass $m_2$	1.17–1.36 $M_{\odot}$	0.86–1.36 $M_{\odot}$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_{\odot}$	$1.188^{+0.004}_{-0.002} M_{\odot}$
Mass ratio $m_2/m_1$	0.7–1.0	0.4–1.0
Total mass $m_{\text{tot}}$	$2.74^{+0.04}_{-0.01} M_{\odot}$	$2.82^{+0.47}_{-0.09} M_{\odot}$
Radiated energy $E_{\text{rad}}$	$> 0.025 M_{\odot} c^2$	$> 0.025 M_{\odot} c^2$
Luminosity distance $D_L$	$40^{+8}_{-14}$ Mpc	$40^{+8}_{-14}$ Mpc
Viewing angle $\Theta$	$\leq 55^\circ$	$\leq 56^\circ$
Using NGC 4993 location	$\leq 28^\circ$	$\leq 28^\circ$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	$\leq 800$	$\leq 700$
Dimensionless tidal deformability $\Lambda(1.4M_{\odot})$	$\leq 800$	$\leq 1400$

# Comparison of results – GW170817

Low spin priors ( $|\chi| < 0.05$ )

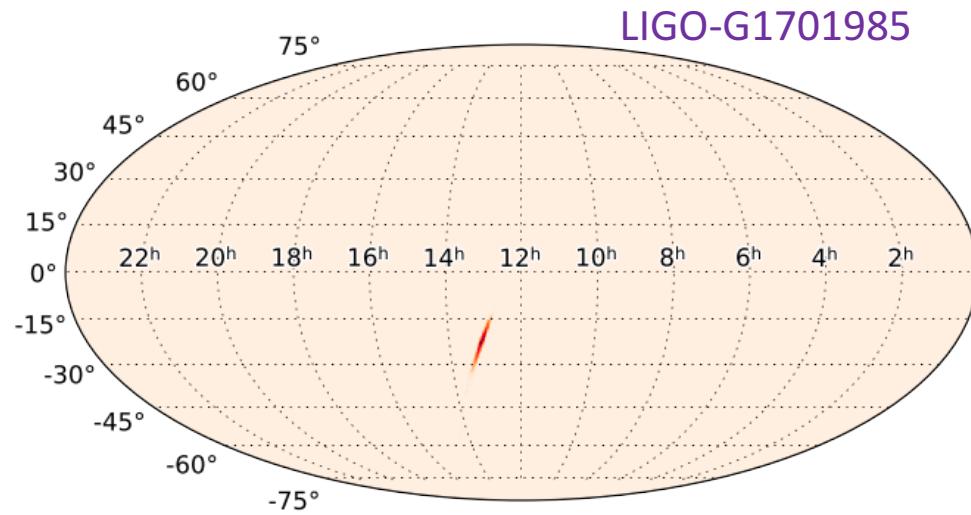
	PRX 6, 041015 (IMRPhenom)	GPE with LOSC data (CLN)
SNR	32.4	37.2
SNR (H,L,V)	18.8, 26.4, 2.0	21.9, 29.8, 3.8
Chirp mass (source)	$1.188 +0.004 -0.002$	$1.188 +0.005 -0.003$
$m_1$ (source)	$1.36 - 1.60$	$1.36 - 1.72$
$m_2$ (source)	$1.17 - 1.36$	$1.09 - 1.36$
Mass ratio	$0.7 - 1.0$	$0.63 - 0.98$
Distance	$40 +8 -14$	$36 +10 -20$

# Comparison of results – GW170817

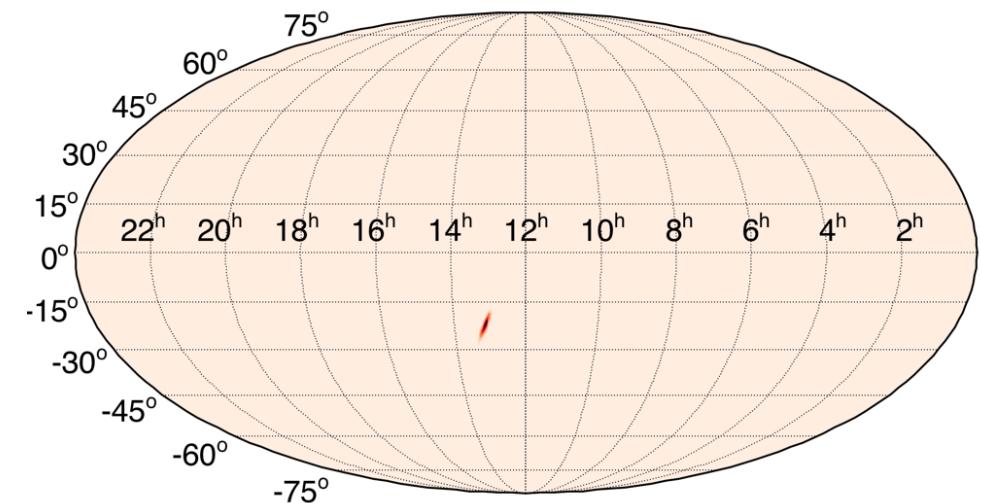
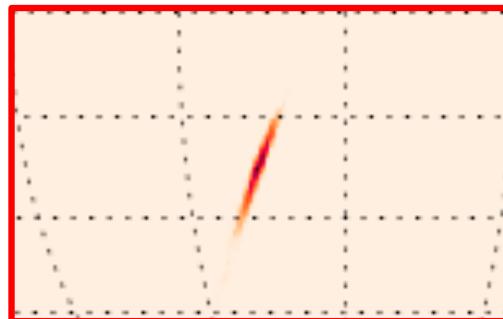
High spin priors ( $|x| < 0.89$ )

	PRX 6, 041015 (IMRPhenom)	GPE with LOSC data (CLN)
SNR	32.4	37.2
SNR (H,L,V)	18.8, 26.4, 2.0	21.9, 29.8, 3.7
Chirp mass (source)	$1.188 +0.004 -0.002$	$1.188 +0.005 -0.003$
m1 (source)	$1.36 - 2.26$	$1.36 - 2.12$
m2 (source)	$0.86 - 1.36$	$0.91 - 1.36$
Mass ratio	$0.4 - 1.0$	$0.43 - 0.97$
Distance	$40 +8 -14$	$36 +11 -20$

# Comparison of results – GW170817

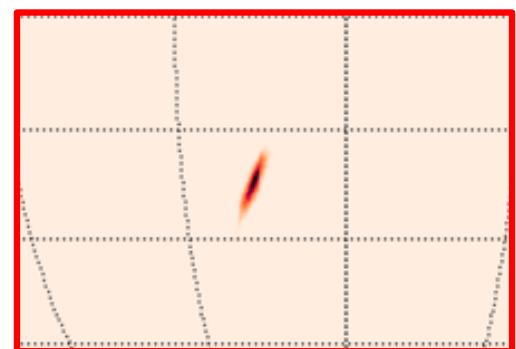


PRL 119. 141101  
90 %  $\Delta\Omega = 28 \text{ deg}^2$



GPE ( $|\chi| < 0.05$ )  
90 %  $\Delta\Omega = 13.5 \text{ deg}^2$

GPE ( $|\chi| < 0.89$ )  
90 %  $\Delta\Omega = 14.3 \text{ deg}^2$



# Outline

- G.P.E. – GPU accelerated Parameter Estimation
- Comparison of results – GW events analysis
- Comparison of results – Simulation
- Immediate future – O3 simulation
- Physics targets – O5 simulation

# Comparison (KAGRA 40 Mpc)

Based on Narikawa-san's list: TF2\_15BNS125\_40Mpc\_i30deg.xml

Event #	SNR(H)		SNR(L)		SNR(V)		SNR(K)		Network	
	TN	SH	TN	SH	TN	SH	TN	SH	TN	SH
10	42.2	42.2	43.2	43.2	9.2	9.2	7.8	7.8	61.6	61.6

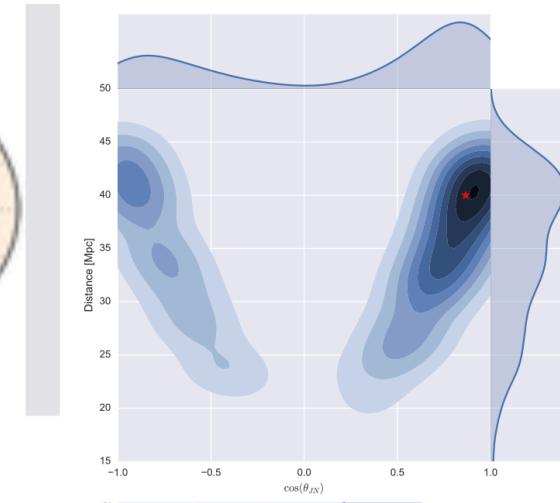
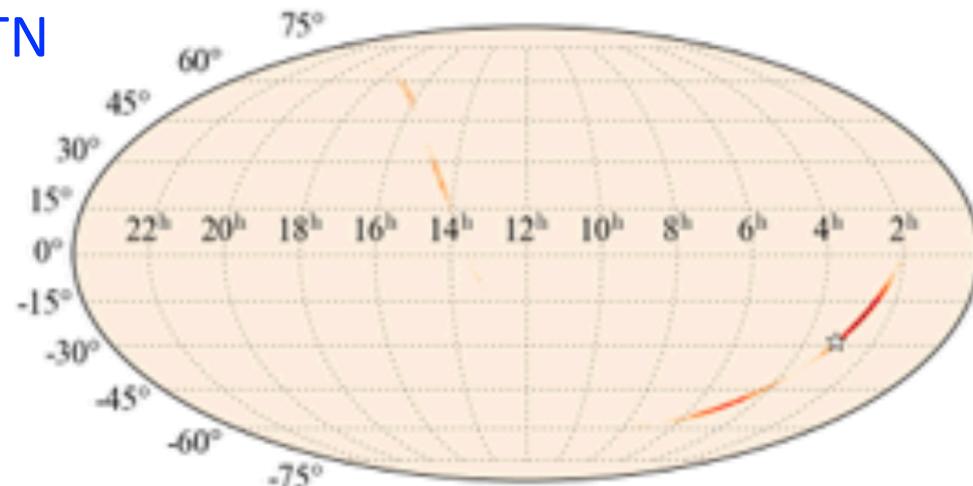
Ev. #	ΔΩ (HL)		ΔΩ (HLV)		ΔΩ (HLK)		ΔΩ (HVK)		ΔΩ (LVK)		ΔΩ (HLVK)	
	TN	SH	TN	SH	TN	SH	TN	SH	TN	SH	TN	SH
10	167	144	5.5	5.9	25.5	27.1	8.1	9.3	6.9	6.6	5.3	5.6

Agreements between TN and SH results

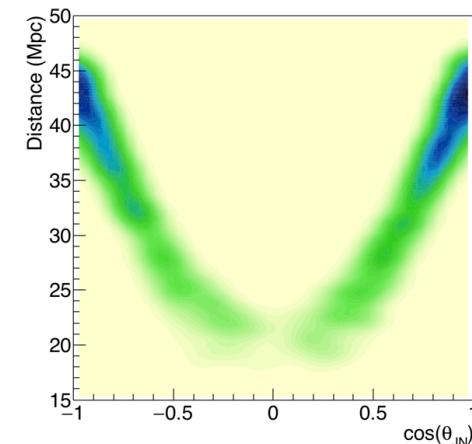
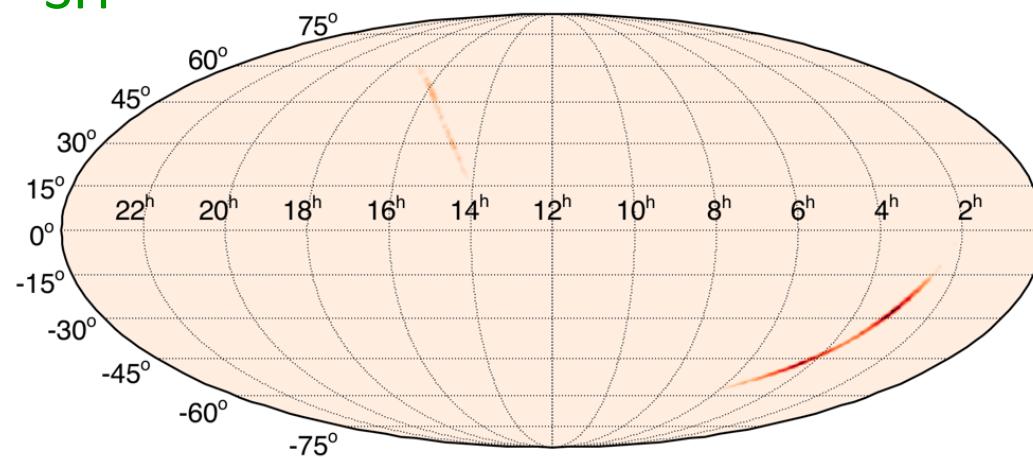
# Comparison

Event #10 <HL>  $\Delta\Omega = 167$  (TN), 144 (SH) deg<sup>2</sup>

TN



SH



# Injected signal SNR

Based on Narikawa-san's list: TF2\_15BNS125\_40Mpc\_i30deg.xml

Event #	SNR(H)		SNR(L)		SNR(V)		SNR(K)		Network	
	TN	SH	TN	SH	TN	SH	TN	SH	TN	SH
2	19.4	19.4	26.2	26.2	2.7	2.7	3.5	3.5	32.9	32.9
271	25.2	25.2	8.8	8.8	12.4	12.4	3.4	3.4	29.7	29.7
287	16.3	16.3	18.1	18.1	7.5	7.5	3.6	3.6	25.7	25.7
294	27.5	27.4	15.4	15.4	12.0	12.0	3.5	3.5	33.9	33.9
300	20.1	20.1	12.9	12.9	10.7	10.7	3.6	3.6	26.4	26.4
306	18.1	18.1	16.8	16.8	8.8	8.8	3.7	3.7	26.5	26.4
320	21.9	21.9	17.0	17.0	10.2	10.2	3.7	3.7	29.8	29.8

# Comparison

Wide Prior

comp- min	comp- max	Mc- min	Mc- max	q-min	flow	fhigh	srate	seglen	seed	Nlive
0.6	5.0	1.13	1.40244	0.125	23	2048	4096	128	1234	2048

test run

/home/handai/narikawa/MCMC/Toward\_03/BNS/event2  
TF2\_15BNS125\_40Mpc\_i30deg.xml

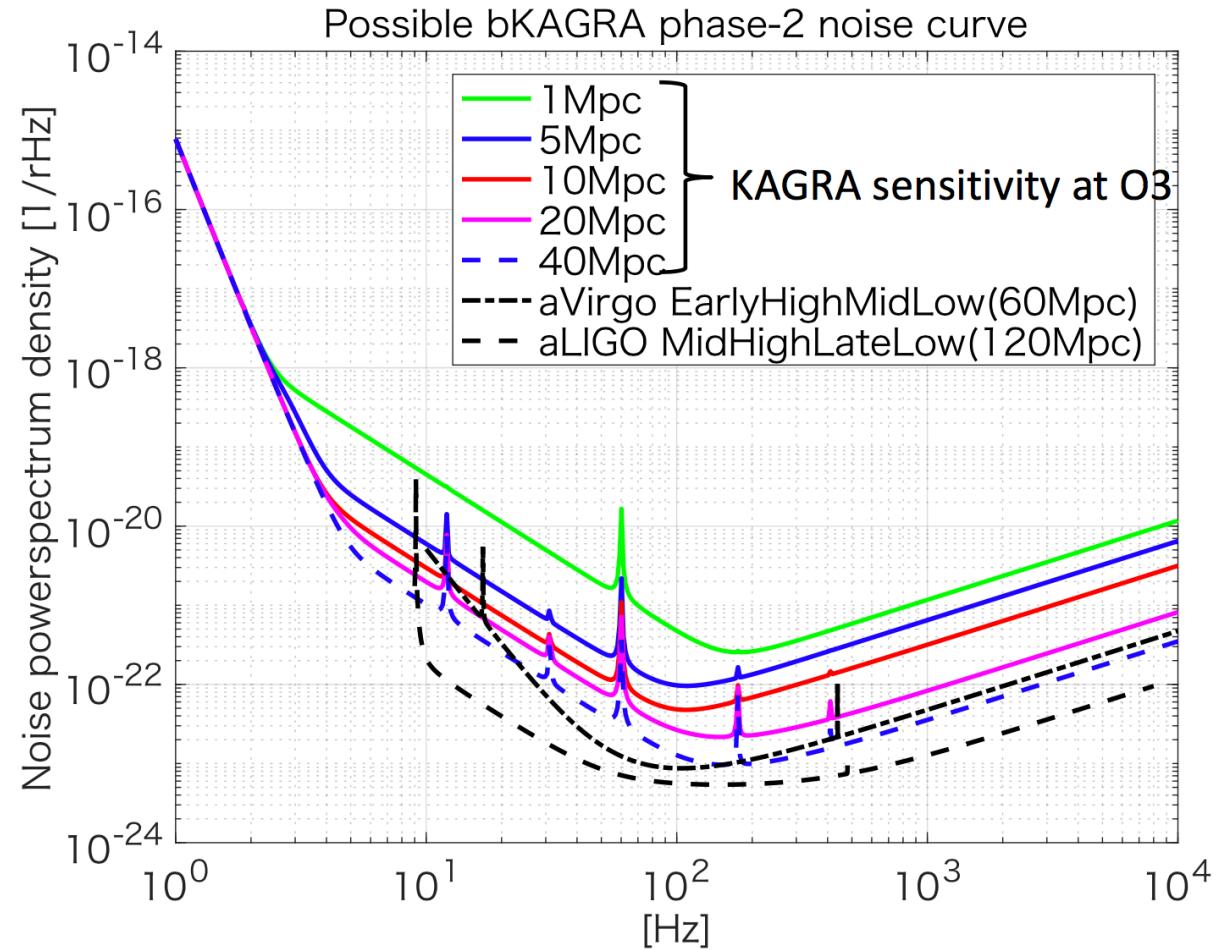
Event #	2	271	287	294	300	306	320
$\Delta\Omega$ TN	22.29	11.06	17.15	7.92	12.48	11.75	11.22
$\Delta\Omega$ SH	25.98	10.42	11.18	9.88	11.7	11.68	10.73

# Outline

- G.P.E. – GPU accelerated Parameter Estimation
- Comparison of results – GW events analysis
- Comparison of results – Simulation
- Immediate future – O3 simulation
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# O3 simulation

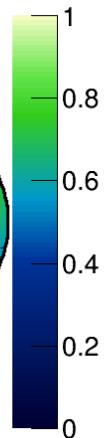
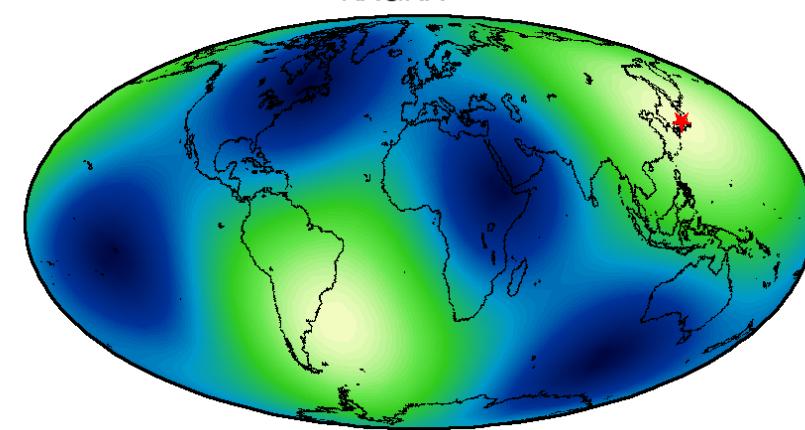
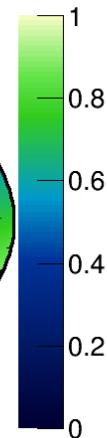
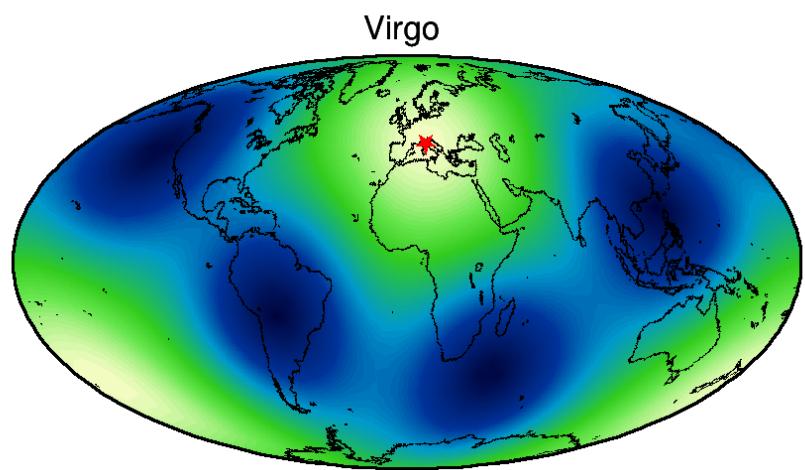
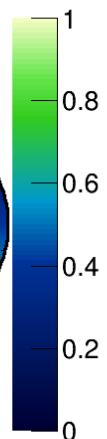
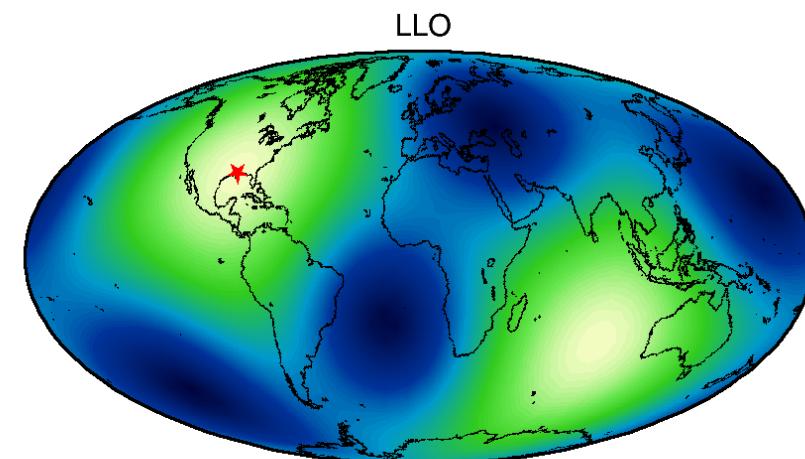
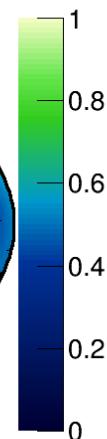
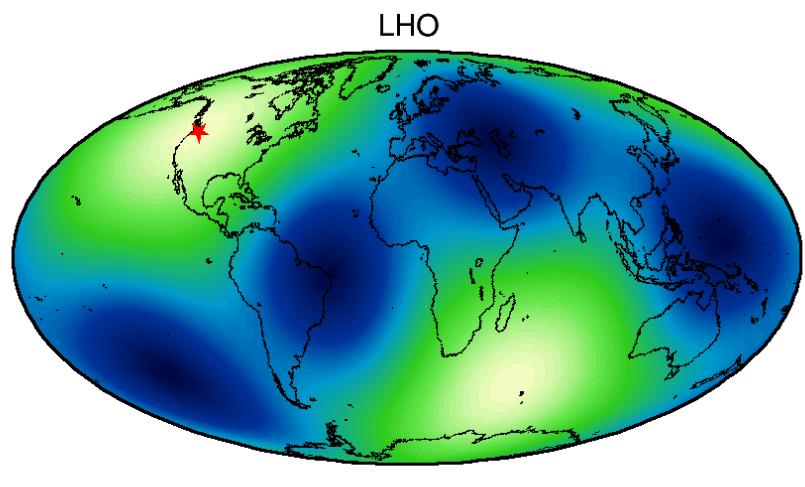
- LIGO 116 Mpc
- Virgo 62.9 Mpc
- KAGRA 9.5 Mpc



KAGRA's sensitivity curves: by Enomoto, Michimura  
LIGO, Virgo: taken from arXiv:1304.0670

# Detector antenna patterns

$$\text{Sqrt}(F_p^2 + F_x^2)$$



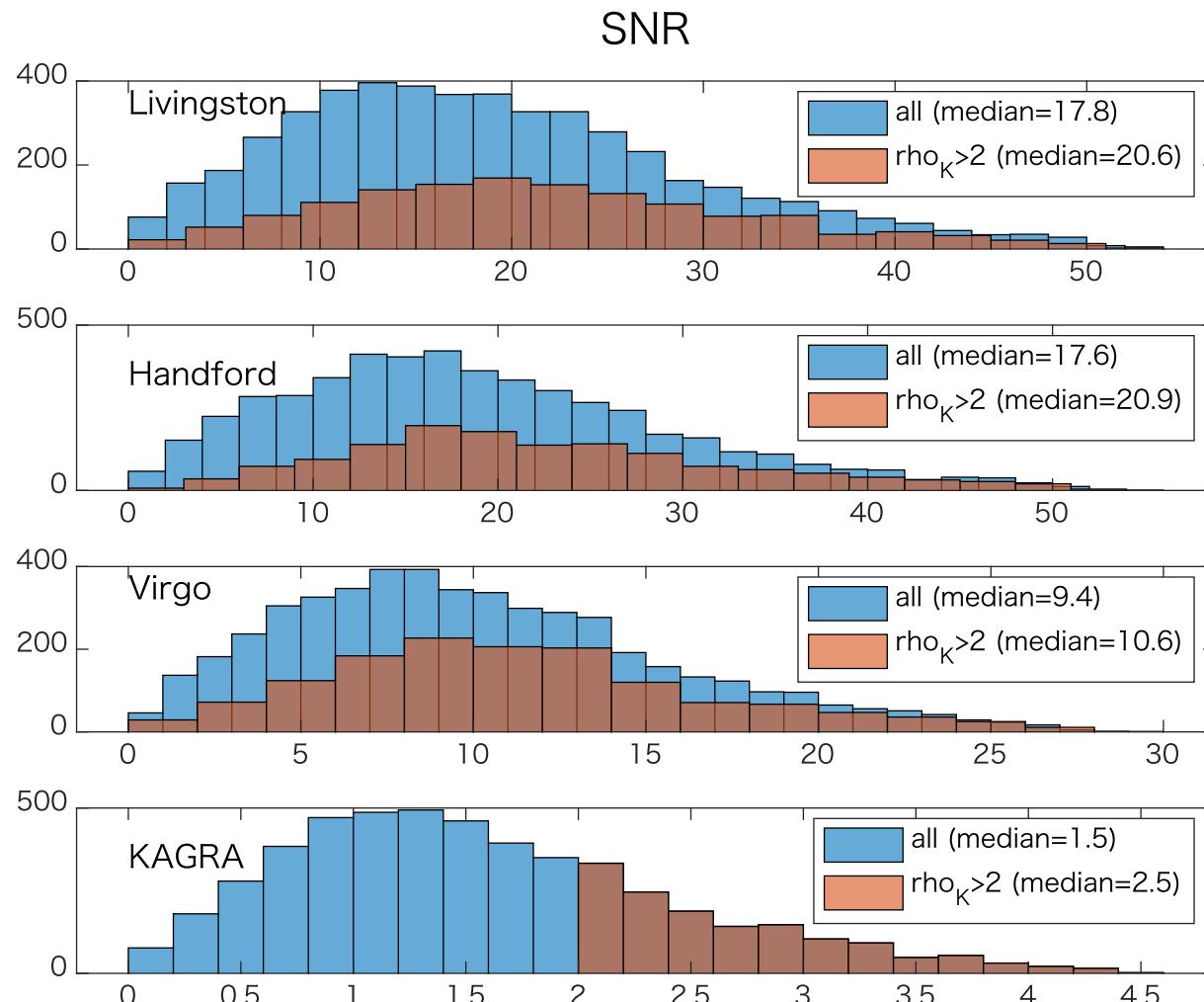
# O3 simulation

Fix distance of BNS at 40 Mpc, then ...

- Randomly distribute (BNS: 1.4-1.4)
  - inclination
  - polarization
  - sky location(25,000 = 5,000 events x5 detector cases)
- Fix: inclination (4) (BNS: 1.5-1.25)
  - and polarization (5)
  - Uniformly distribute source location (192)(15,360 = 192 x5 x4 x4 sensitivity cases)

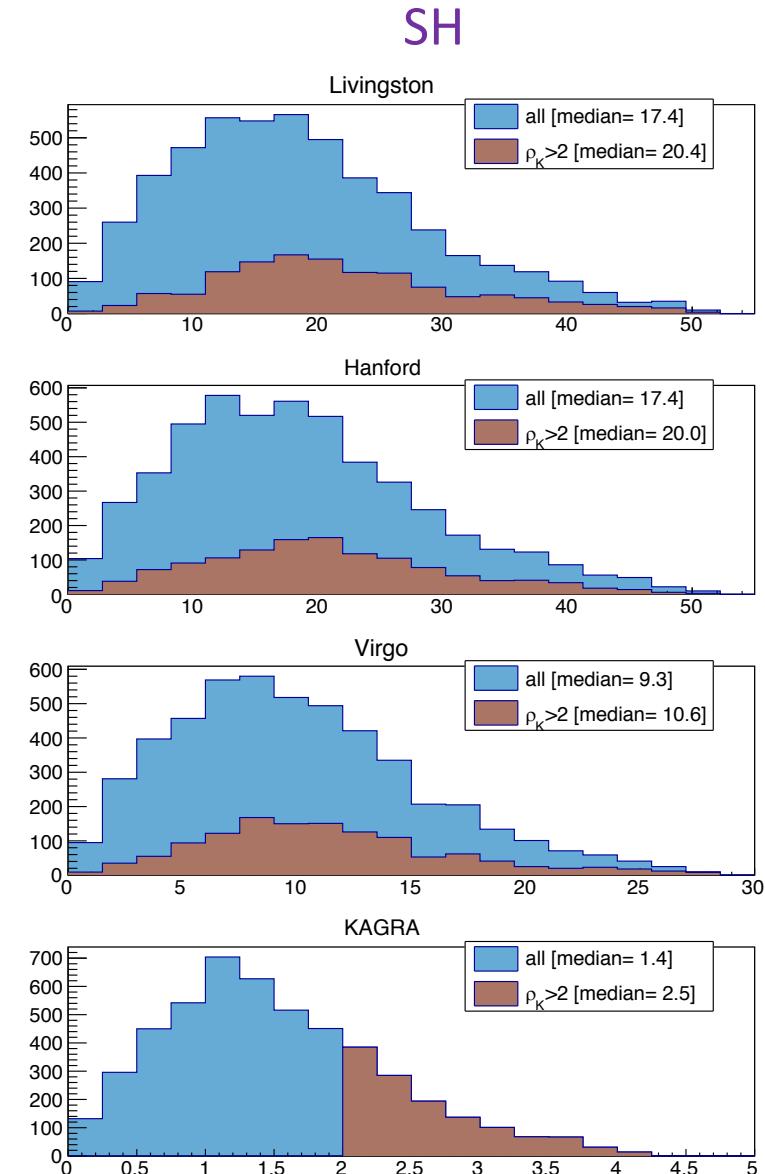
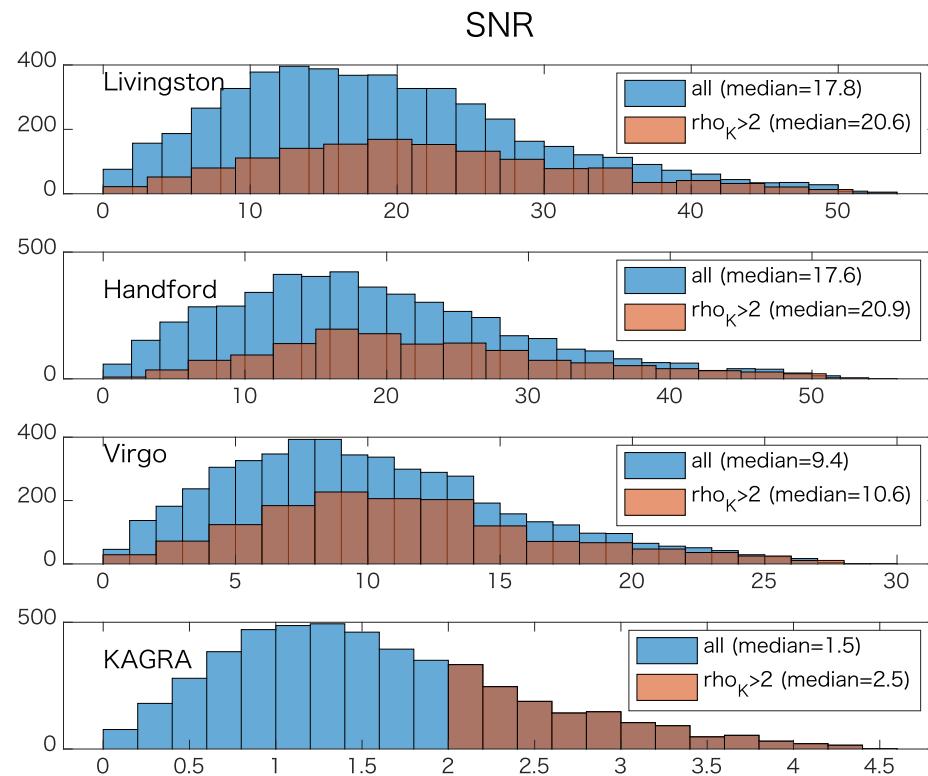
# Condition1 : Random inclination

Tagoshi-san's results



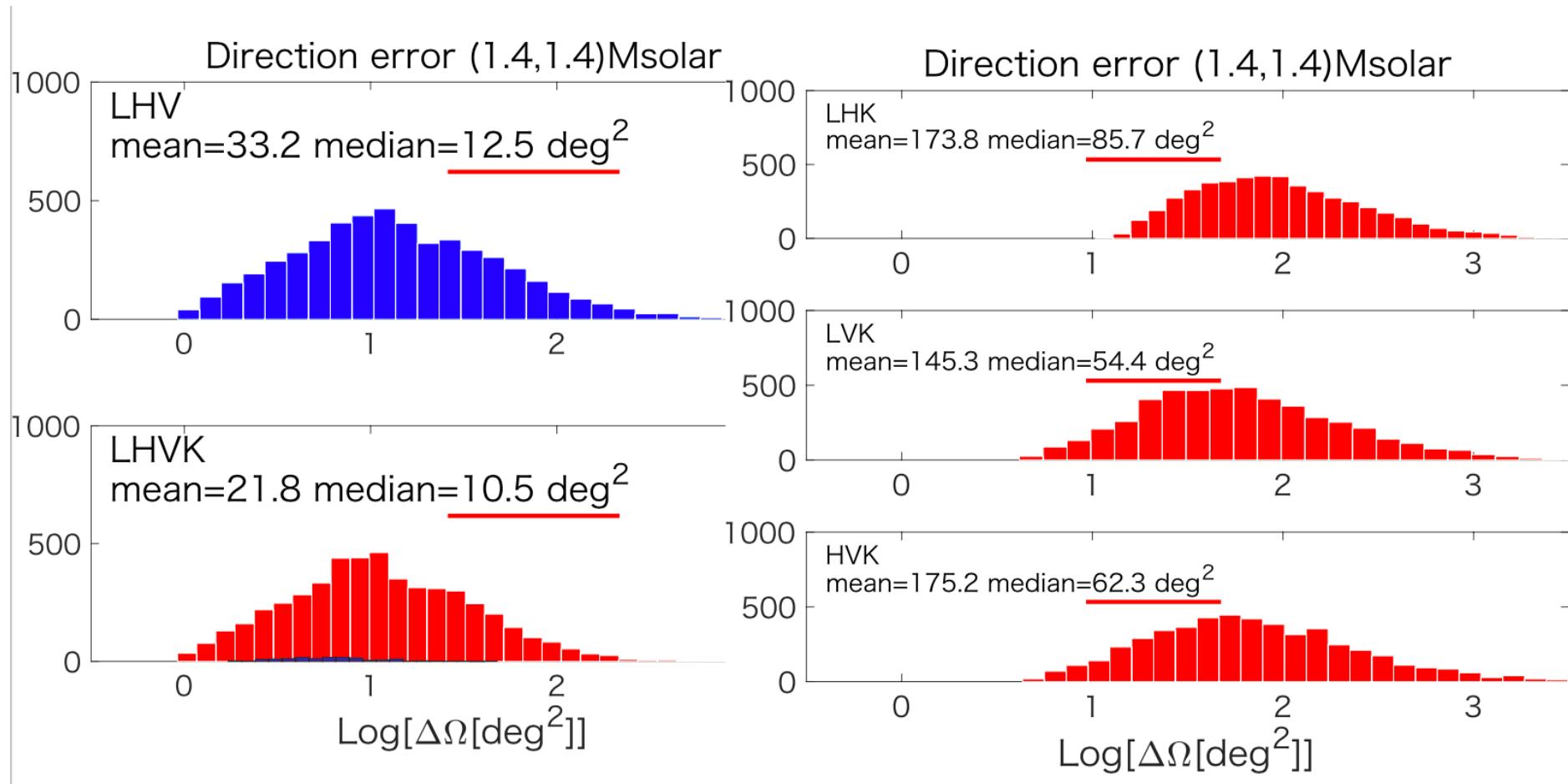
# Condition1 : Random inclination

Tagoshi-san's results



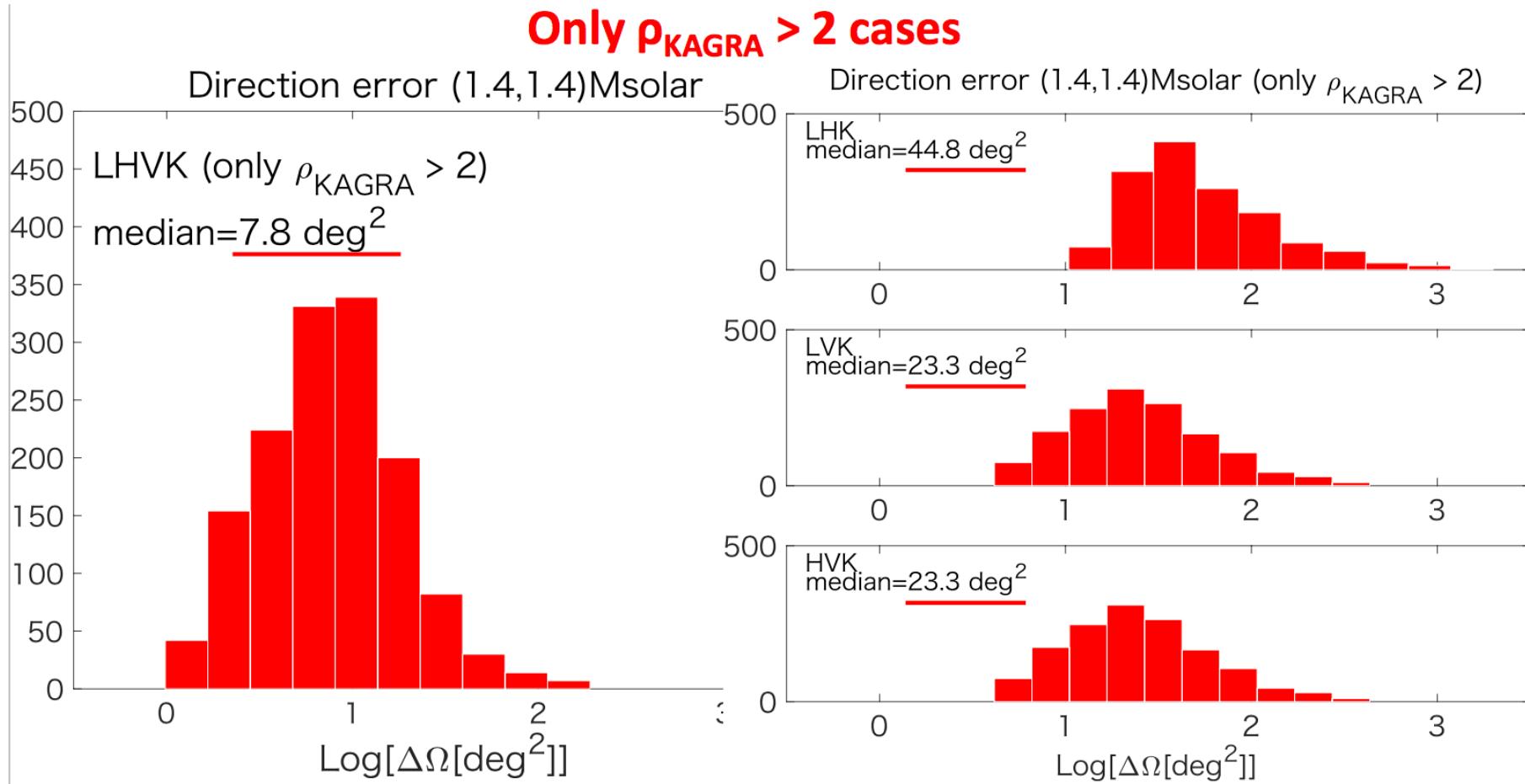
# Condition1 : Random inclination

Tagoshi-san's results

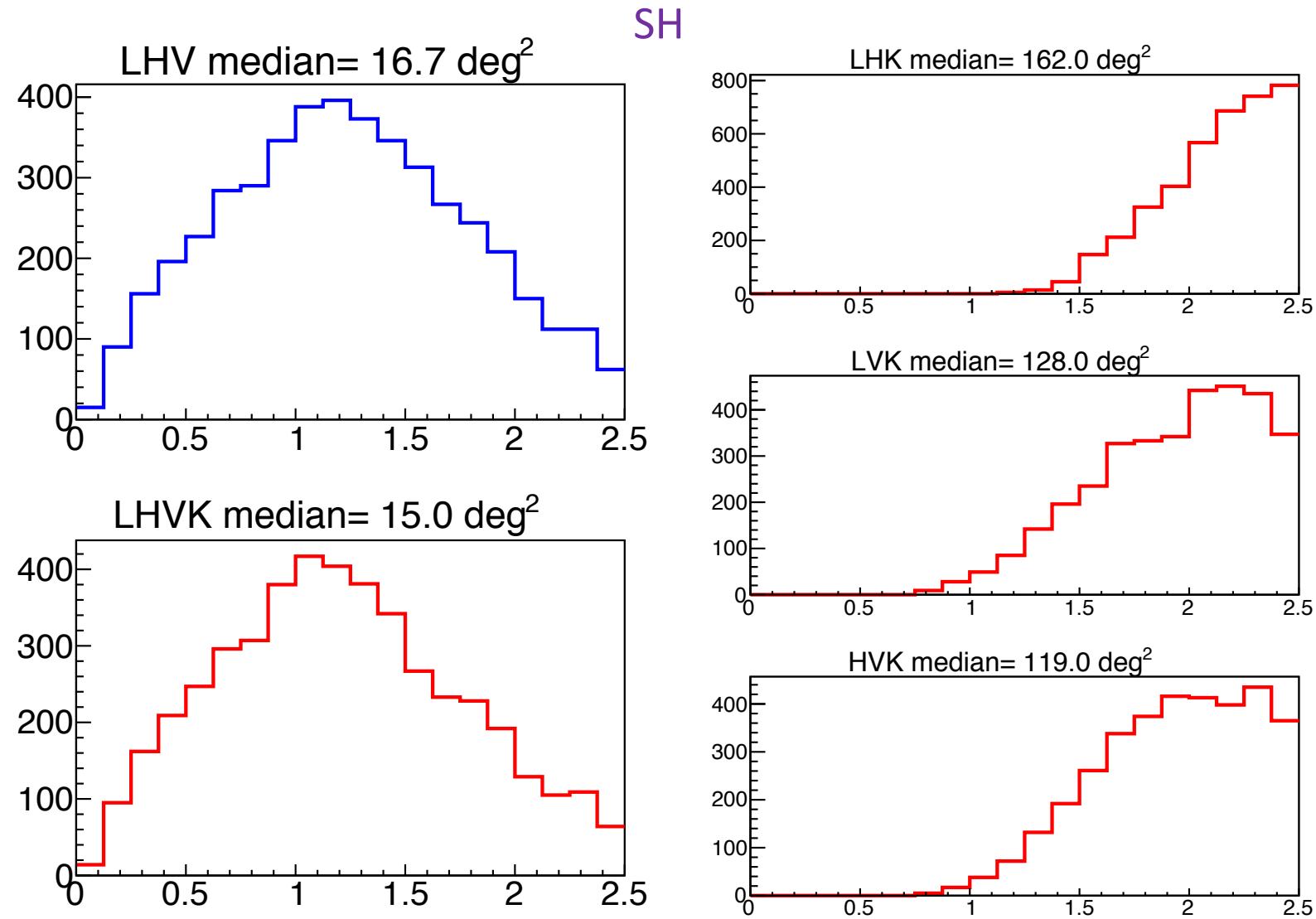


# Condition1 : Random inclination

Tagoshi-san's results

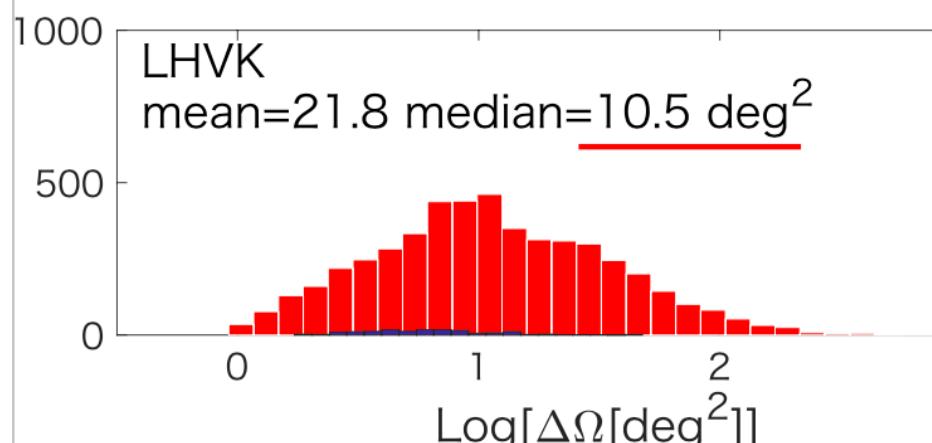
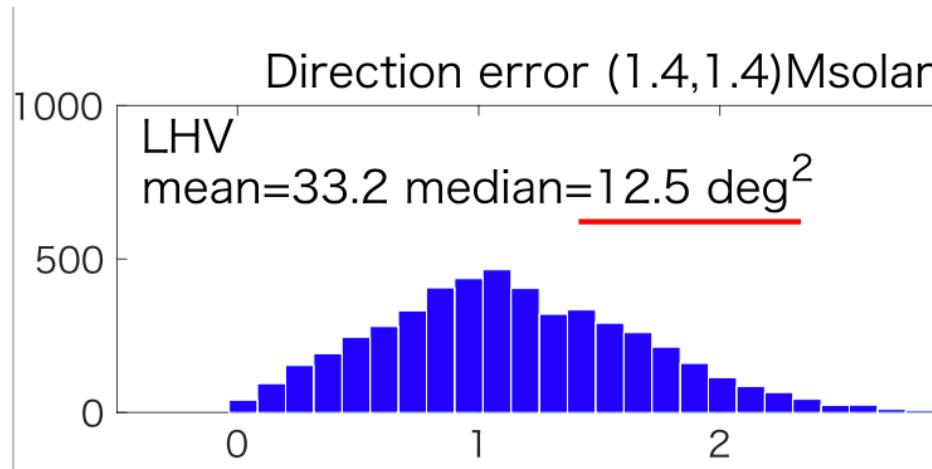


# Condition1 : Random inclination

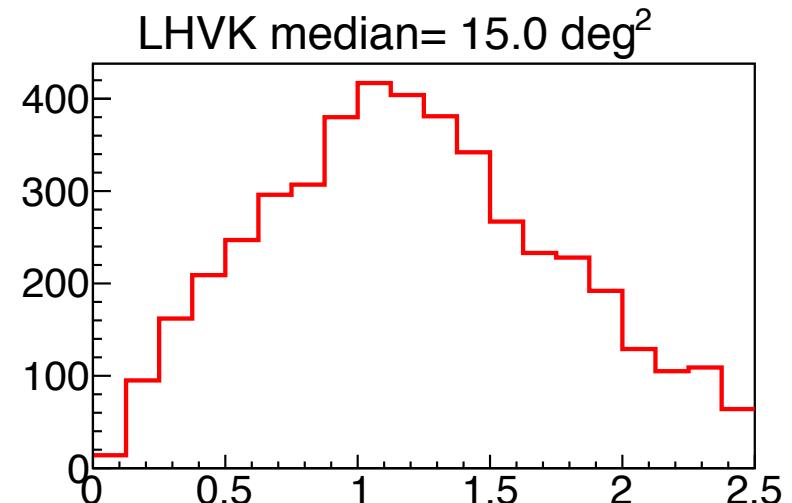
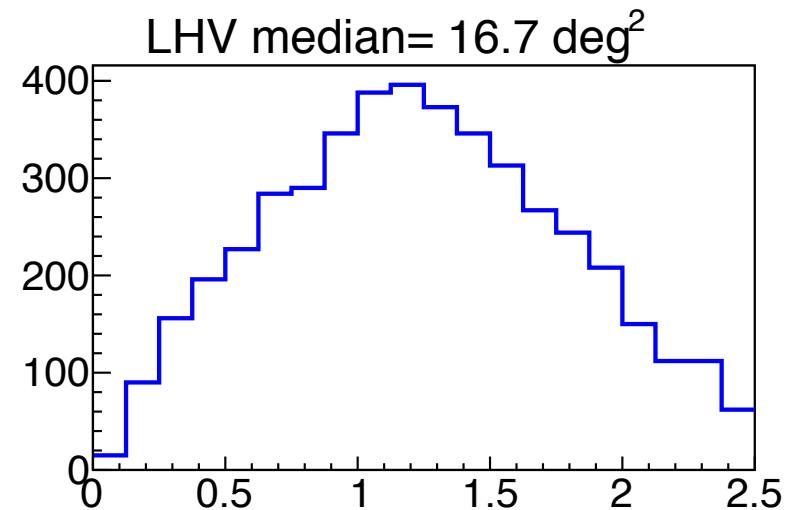


# Condition1 : Random inclination

Tagoshi-san's results

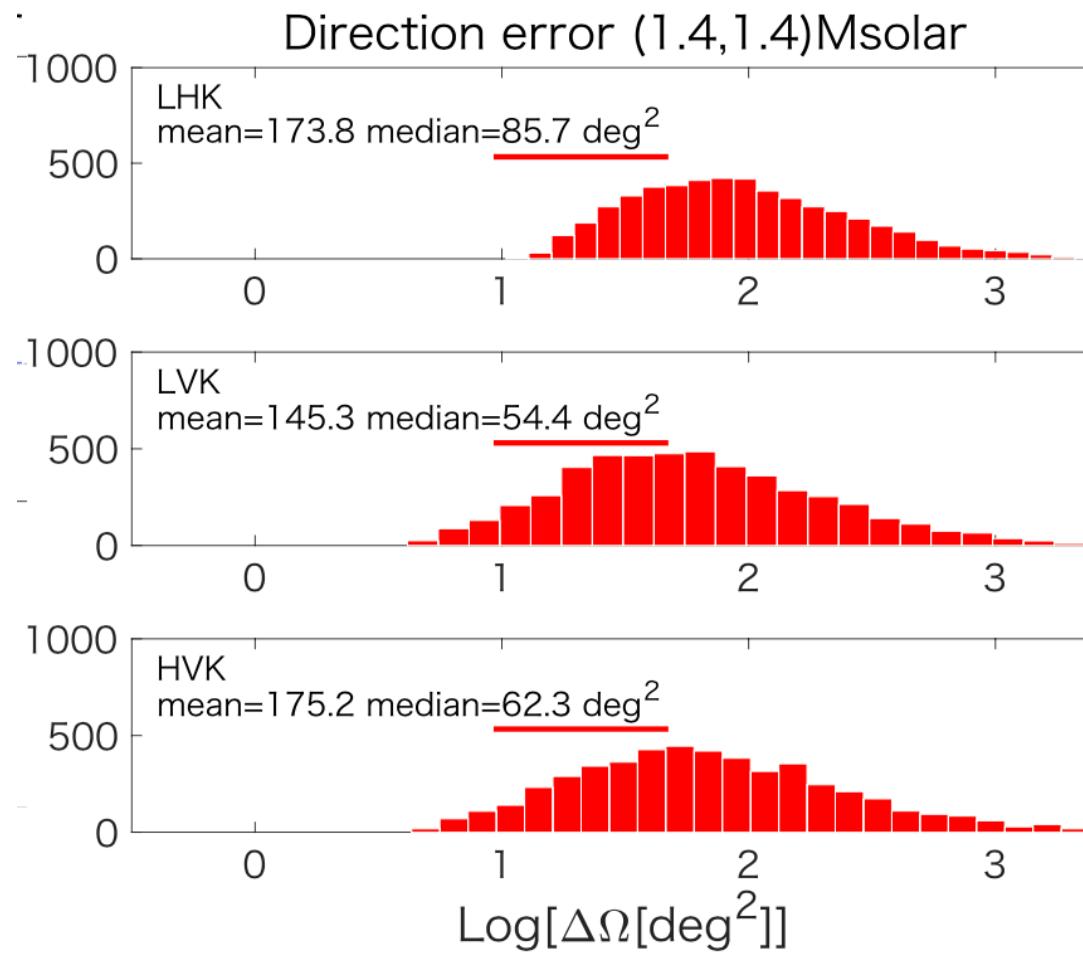


SH

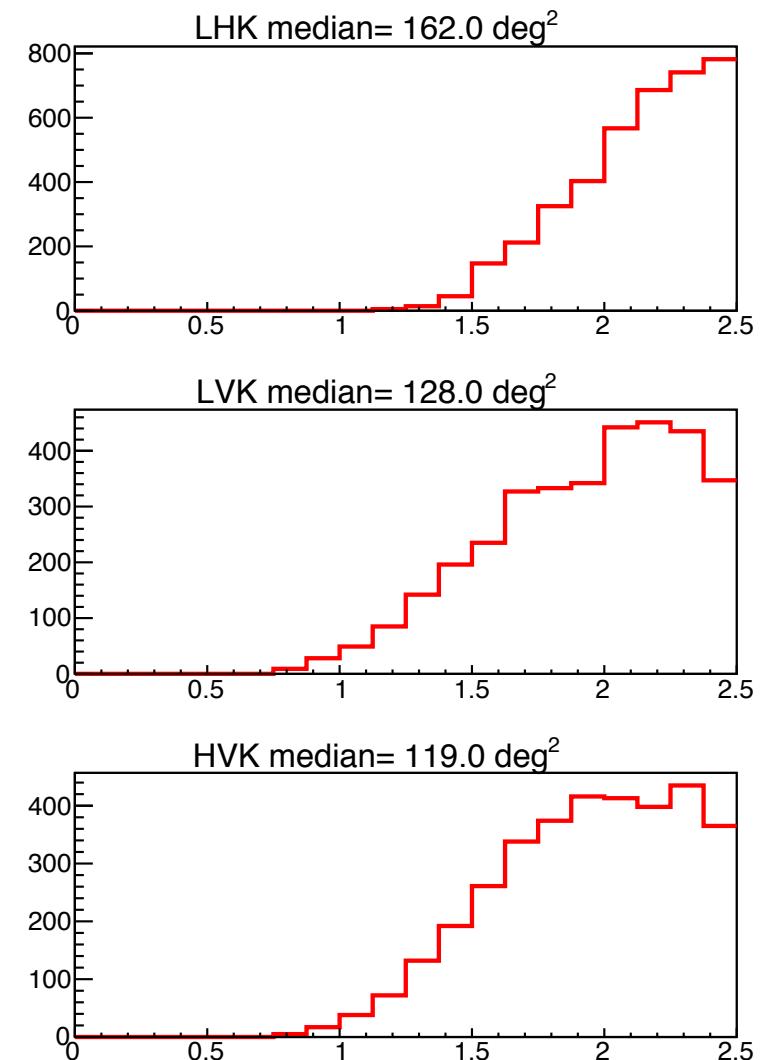


# Condition1 : Random inclination

Tagoshi-san's results

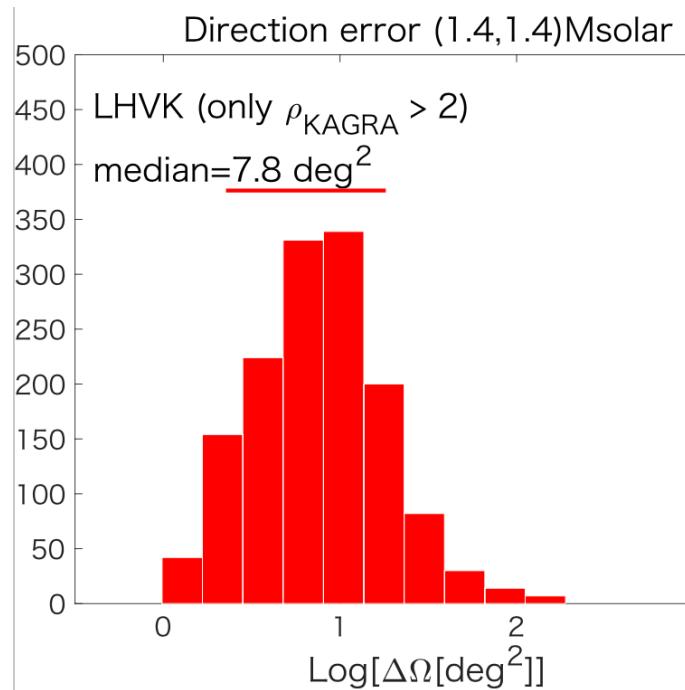
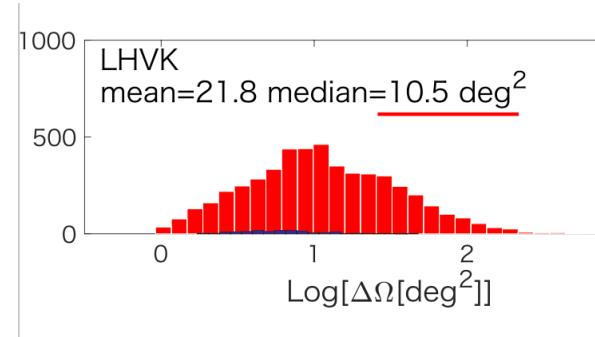


SH

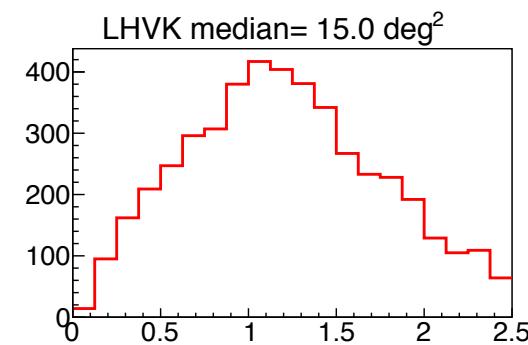


# Condition1 : Random inclination

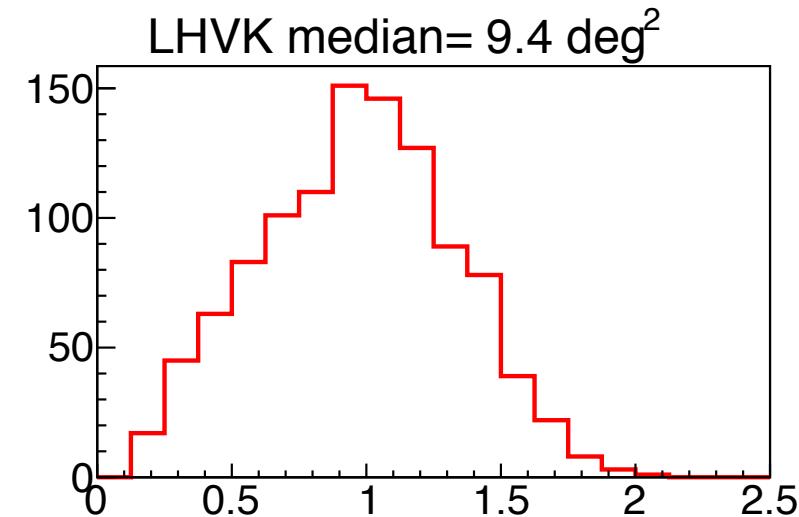
Tagoshi-san's results



SH



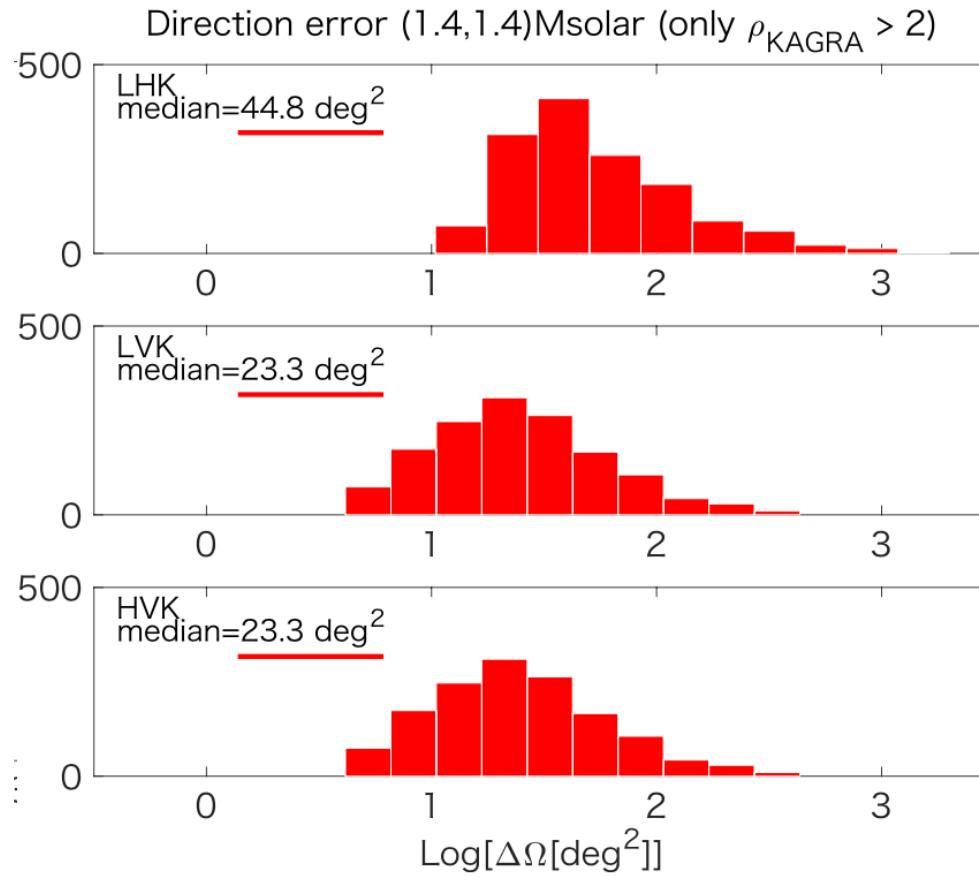
Only  $\rho_{\text{KAGRA}} > 2$  cases



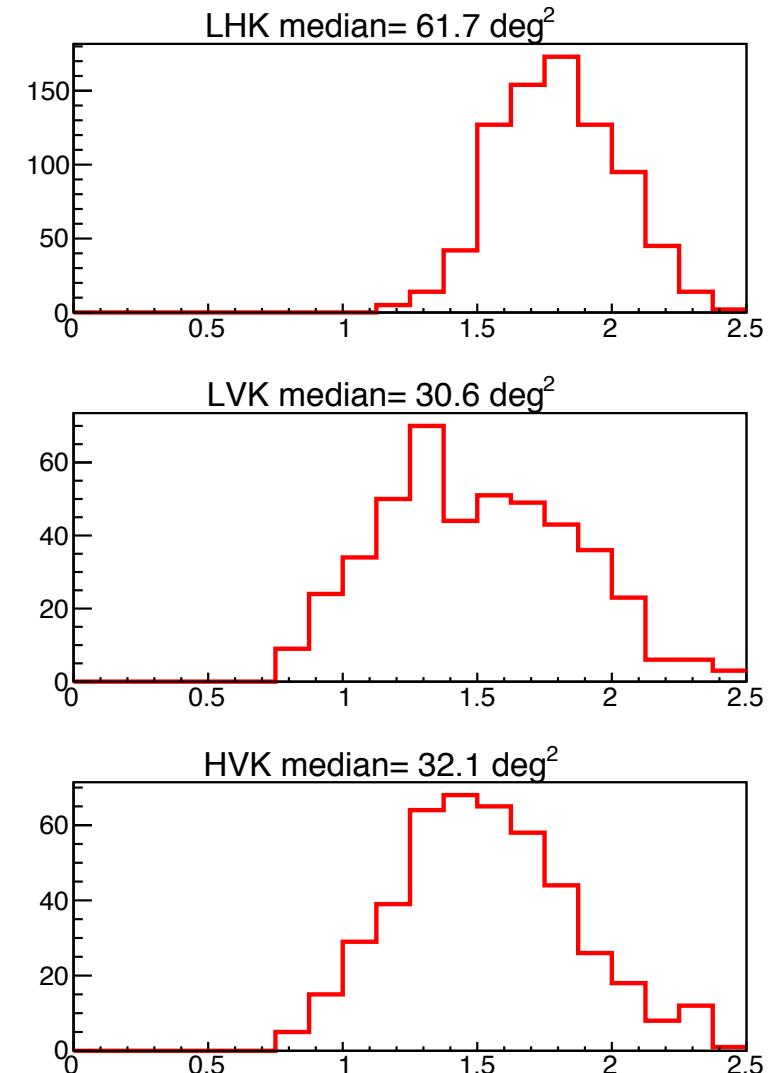
# Condition1 : Random inclination

Tagoshi-san's results

**Only  $\rho_{\text{KAGRA}} > 2$  cases**

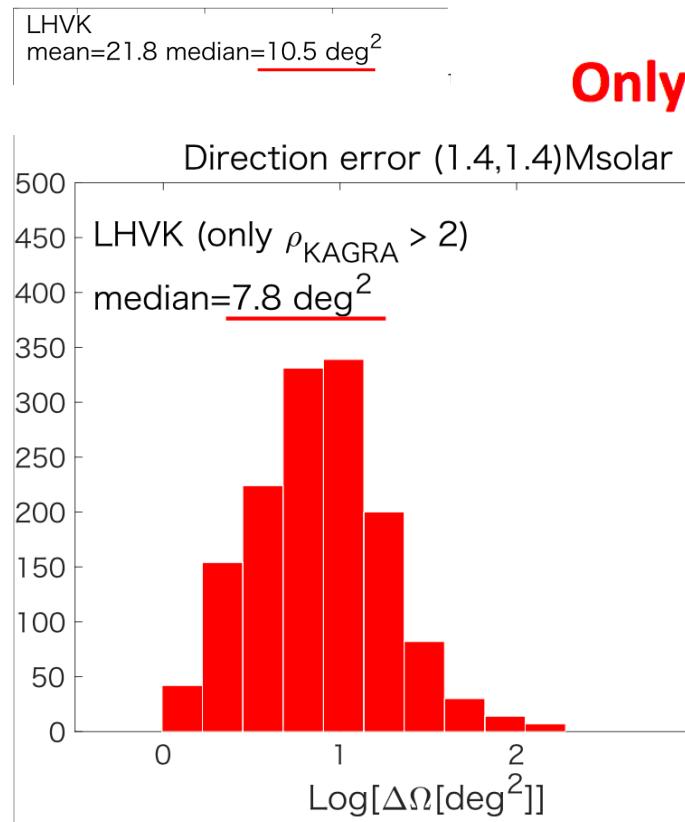


SH



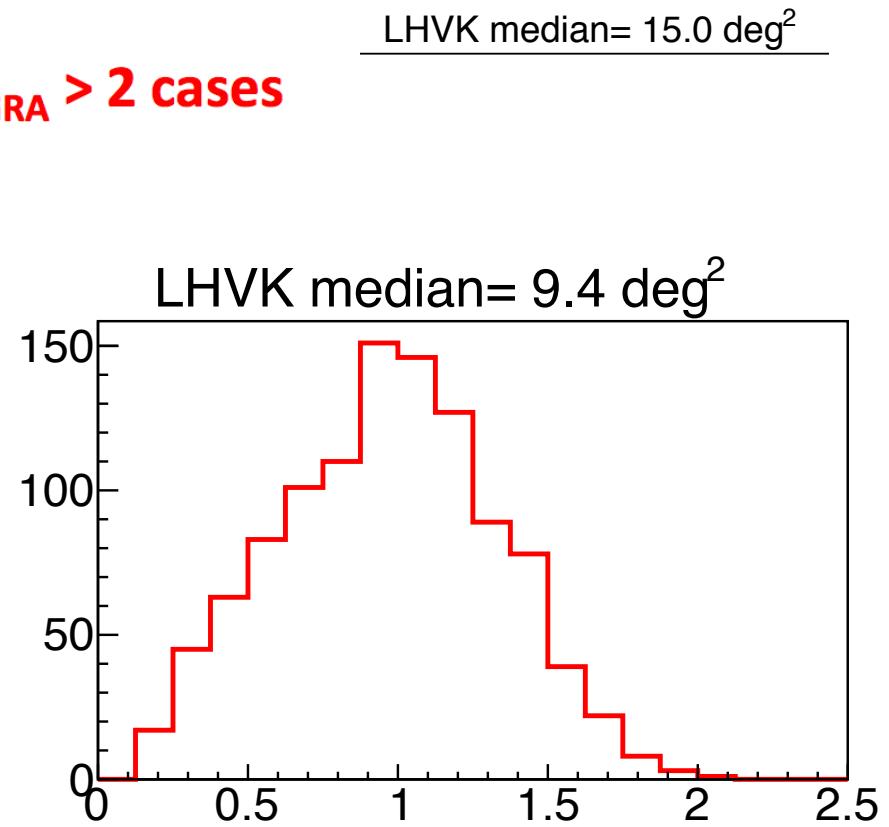
# Condition1 : Random inclination

Tagoshi-san's results



$$\frac{\langle \Delta\Omega_{LHVK}(\rho_{\text{KAGRA}} > 2) \rangle}{\langle \Delta\Omega_{\text{LHV}} \rangle} = 0.62$$

SH



$$\frac{\langle \Delta\Omega_{LHVK}(\rho_{\text{KAGRA}} > 2) \rangle}{\langle \Delta\Omega_{\text{LHV}} \rangle} = 0.63$$

# O3 simulation

Fix distance of BNS at 40 Mpc, then ...

- Randomly distribute (BNS: 1.4-1.4)
  - inclination
  - polarization
  - sky location(25,000 = 5,000 events x5 detector cases)
- Fix: inclination (4) (BNS: 1.5-1.25)  
and polarization (5)
  - Uniformly distribute source location (192)  
(15,360 = 192 x5 x4 x4 sensitivity cases)

# Simulation conditions

Signal injection: NS-NS (1.5-1.25) at 40 Mpc  
TaylorF2 waveform (no-spin, no-tidal)

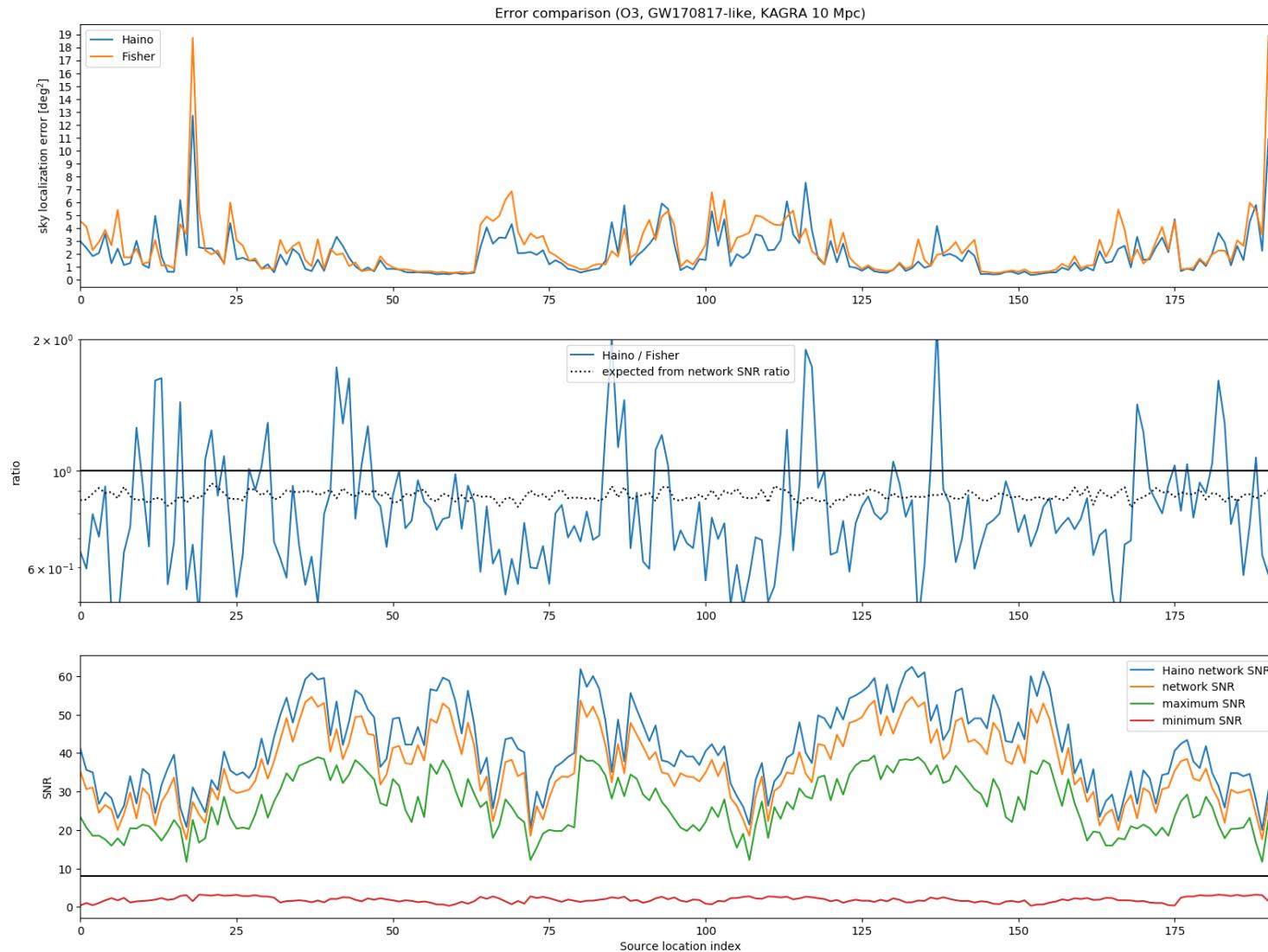
Injected with 15,360 (= 192 x5 x4 x4) conditions:

- 192 (Longitudes and Latitudes – *Healpix* uniform)
- 5 Polarization angles ( $\psi = 0, 0.2\pi, 0.4\pi, 0.6\pi, 0.8\pi$ )
- 4 Inclination angles ( $\theta_{JN} = -30^\circ, -42^\circ, 42^\circ, 30^\circ$ )
- 4 sensitivity scenarios

*computation time for each simulation is ~6 min.  
(with GeForce<sup>TM</sup> GTX 1080 Ti)*

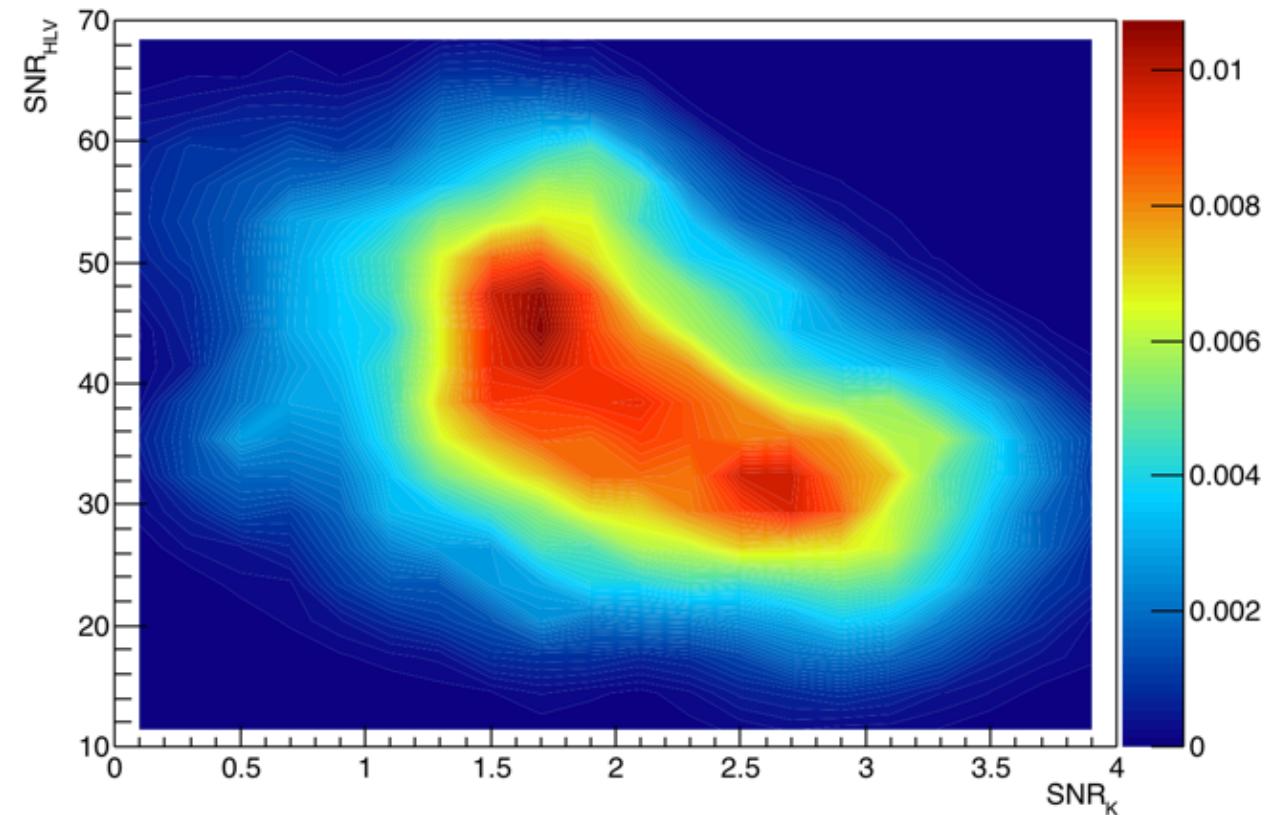
# Condition2 : Fix inclination

Comparison between Fisher (Y.Michimura et al.) and Nest (SH)



# Condition2 : Fix inclination

- LIGO 116 Mpc
- Virgo 62.9 Mpc
- KAGRA 9.5 Mpc



# Sensitivity scenarios

	LIGO <sup>*1</sup>	Virgo <sup>*1</sup>	KAGRA <sup>*2</sup>
Case 1	Late Low (116 Mpc)	Mid. Low (62.9 Mpc)	O3-40 (42.3 Mpc)
Case 2	Late Low (116 Mpc)	Late Low (83.1 Mpc)	O3-40 (42.3 Mpc)
Case 3	Late Low (116 Mpc)	Late Low (83.1 Mpc)	O3-20 <sup>*3</sup> (20.1 Mpc)
Case 4	Late Low (116 Mpc)	Mid. Low (62.9 Mpc)	O3-10 (9.5 Mpc)

<sup>\*1</sup> Sensitivity data from [arXiv:1304.0670](https://arxiv.org/abs/1304.0670) (*LRR* 19,1)

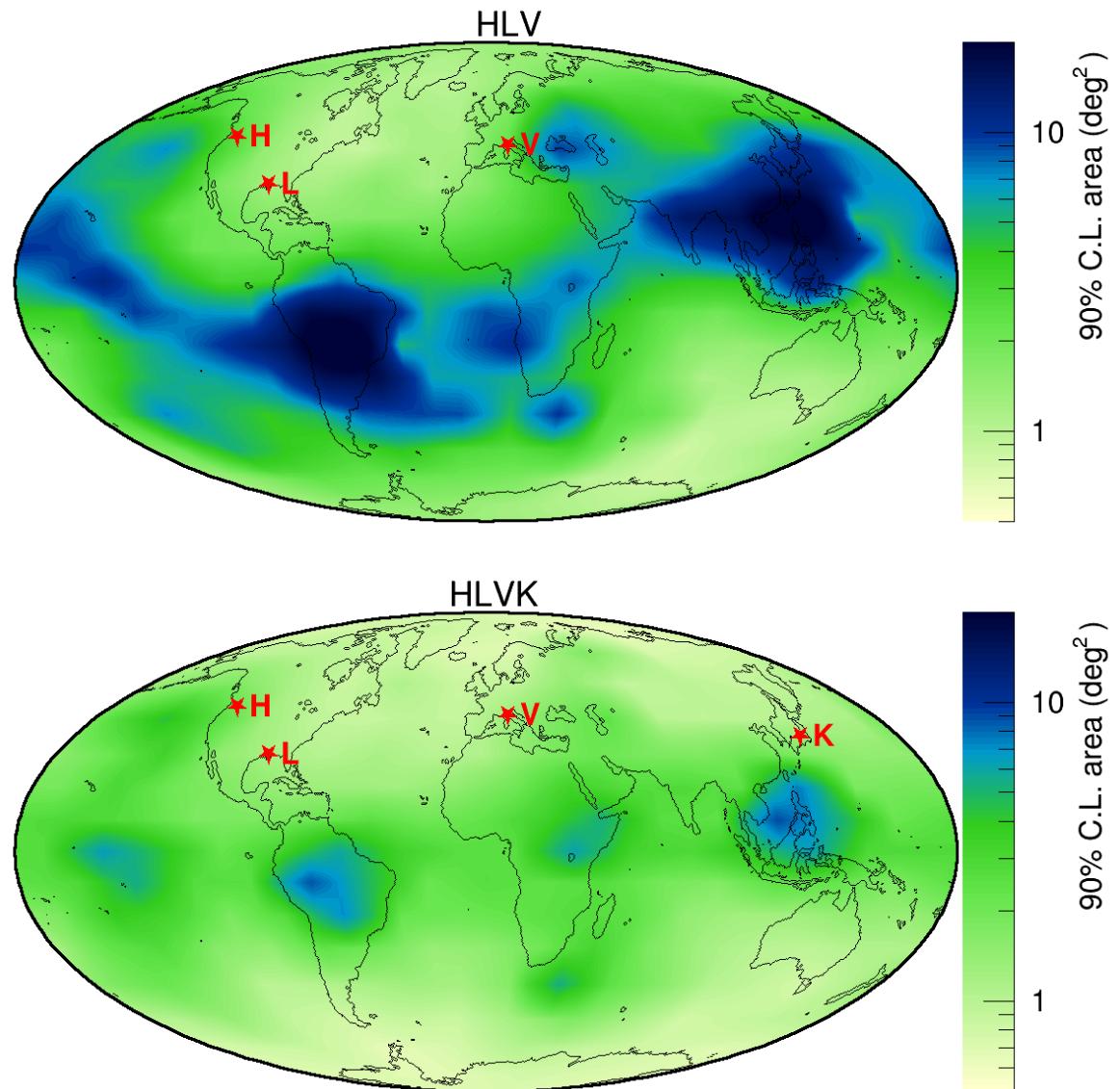
<sup>\*2</sup> Sensitivity data by Y. Enomoto [JGW-T1707556](https://jgw.nao.ac.jp/t1707556)

<sup>\*3</sup> Sensitivity data interpolated [JGW-T1707556](https://jgw.nao.ac.jp/t1707556)

# Sky confidence area comparison

Case 1: LVK= 120, 60, 40 M  
 $\psi = 0$   $\theta_{JN} = 30^\circ$

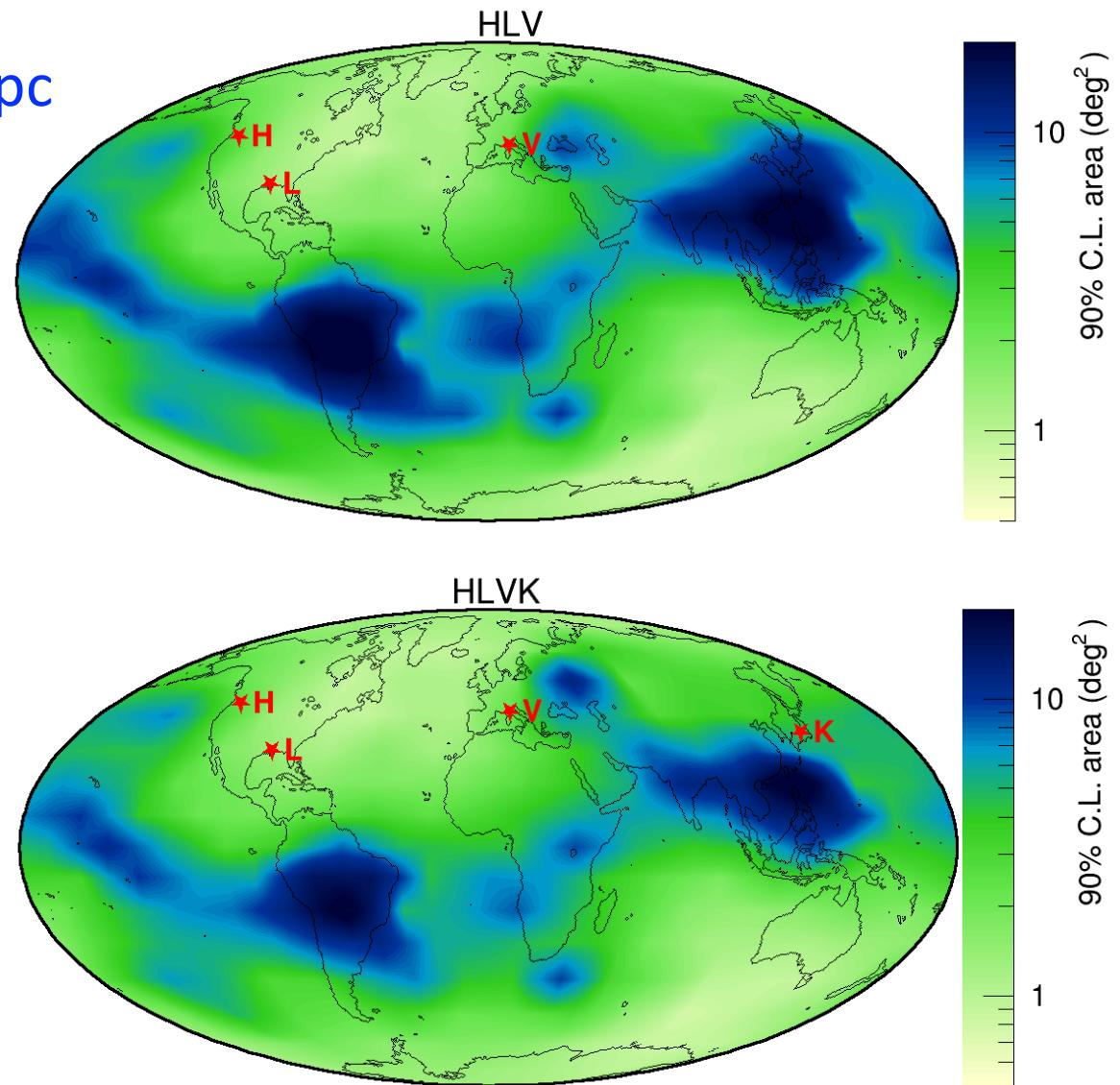
	LIGO <sup>*1</sup>	Virgo <sup>*1</sup>	KAGRA <sup>*2</sup>
Case 1		~60 Mpc	~40 Mpc
Case 2	~120 Mpc	~80 Mpc	~40 Mpc
Case 3			~20 Mpc
Case 4		~60 Mpc	~10 Mpc



# Sky confidence area comparison

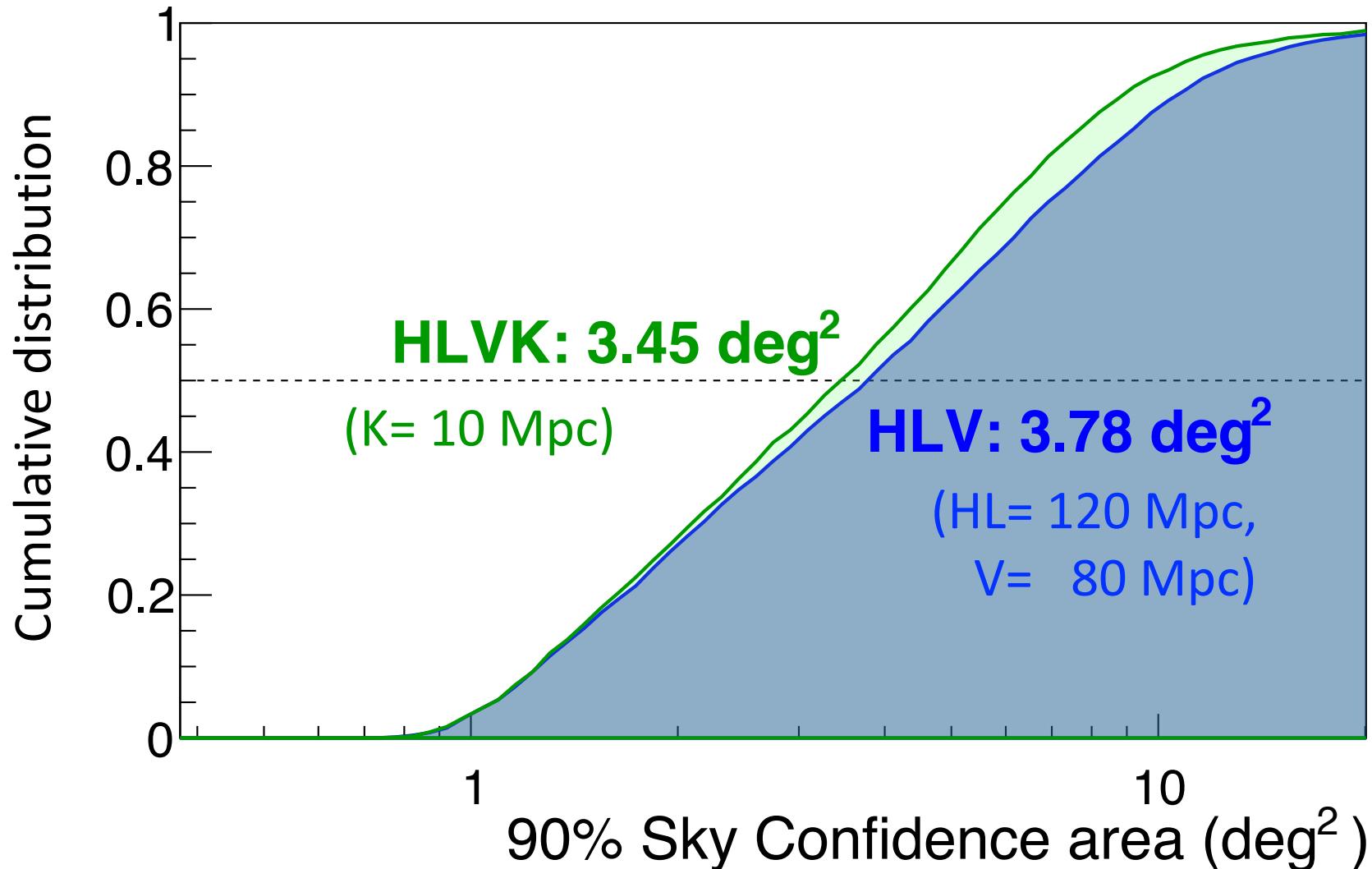
Case 1: LVK= 120, 60, 10 Mpc  
 $\psi = 0$   $\theta_{JN} = 30^\circ$

	LIGO <sup>*1</sup>	Virgo <sup>*1</sup>	KAGRA <sup>*2</sup>
Case 1		~60 Mpc	~40 Mpc
Case 2	~120 Mpc	~80 Mpc	~40 Mpc
Case 3			~20 Mpc
Case 4		~60 Mpc	~10 Mpc



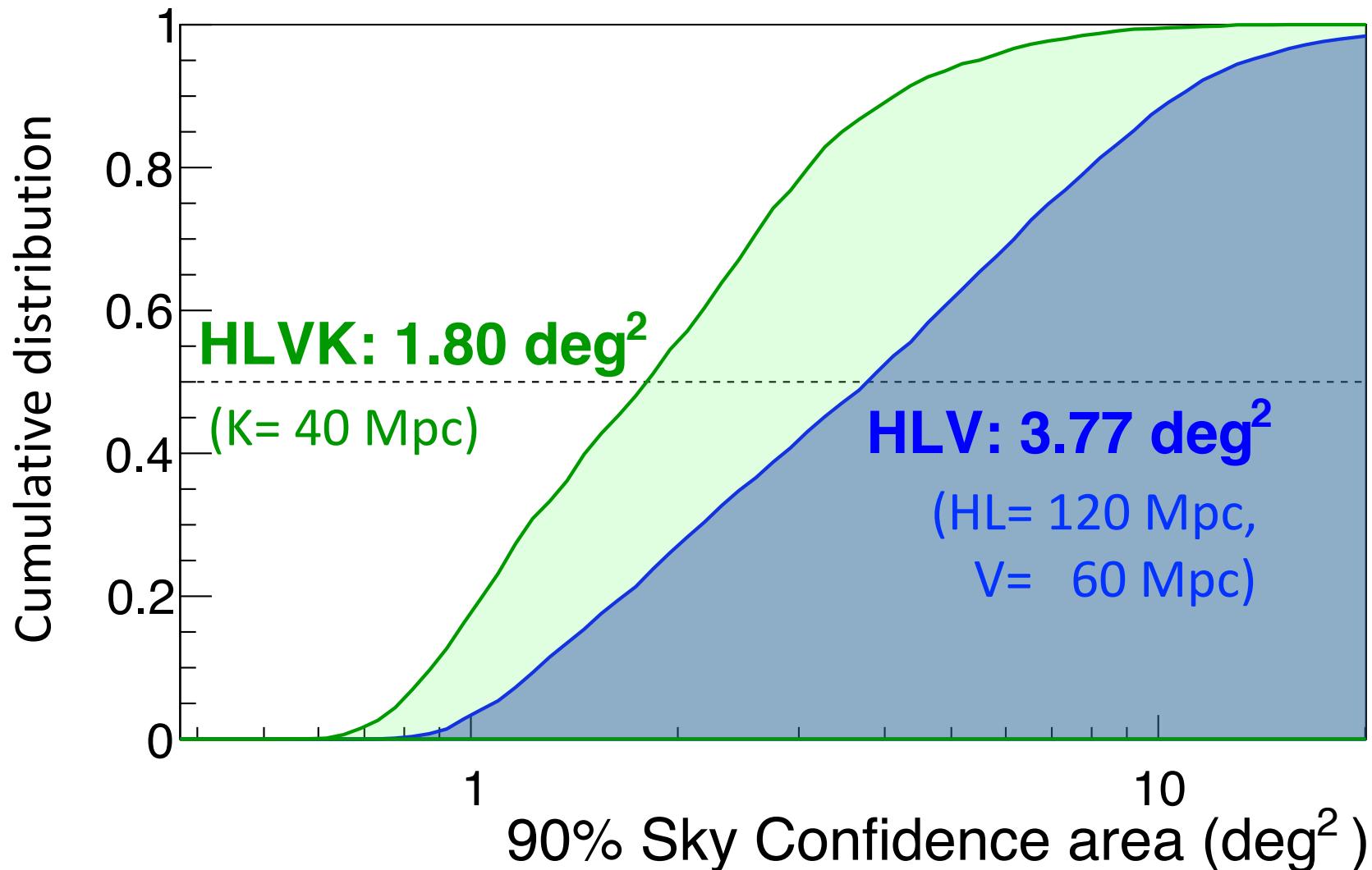
# Sky confidence area summary

Averaged over 3840 cases (192 long./lat. 5  $\psi$ , 4  $\theta_{JN}$ )



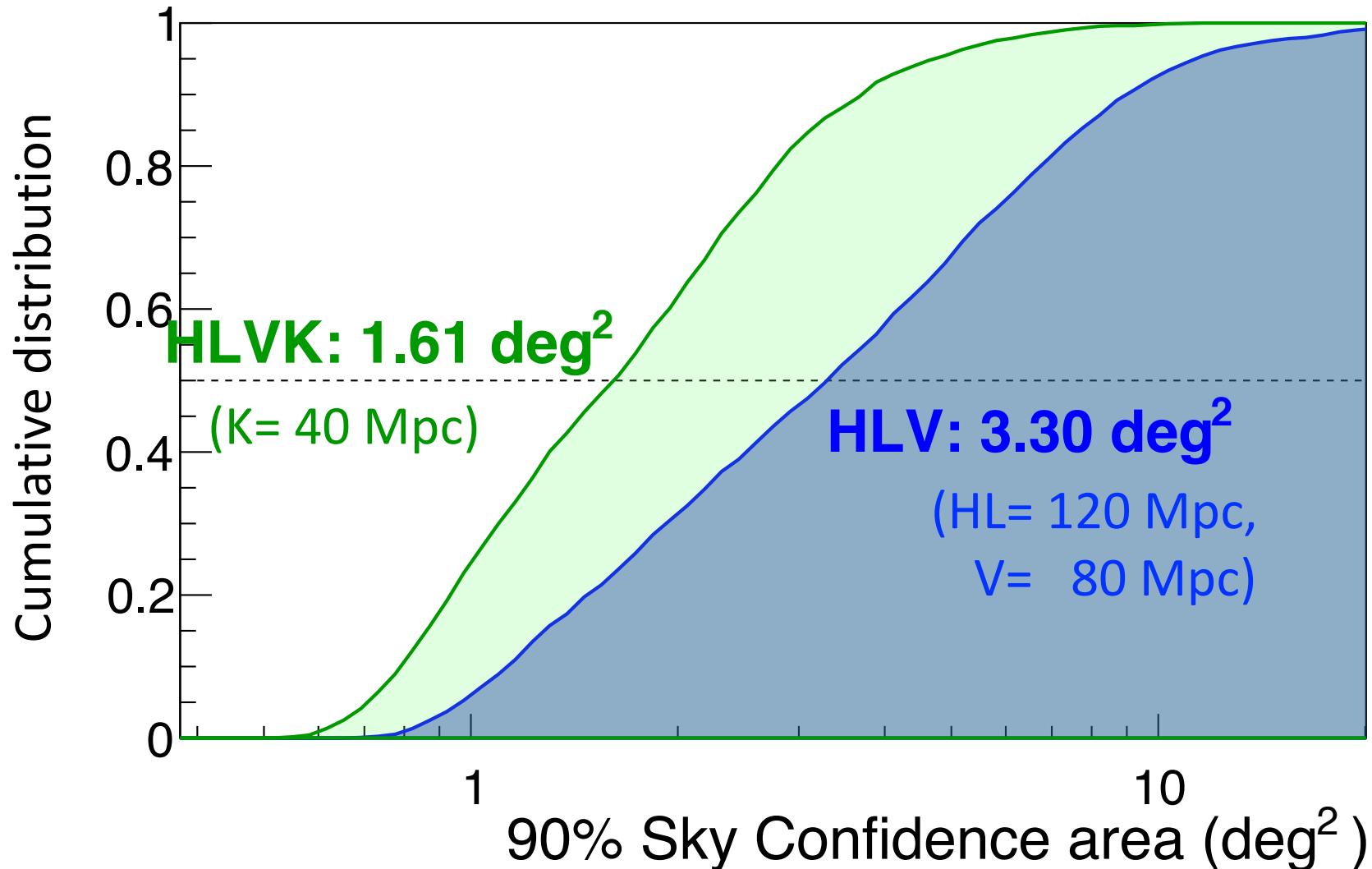
# Sky confidence area summary

Averaged over 3840 cases (192 long./lat. 5  $\psi$ , 4  $\theta_{JN}$ )



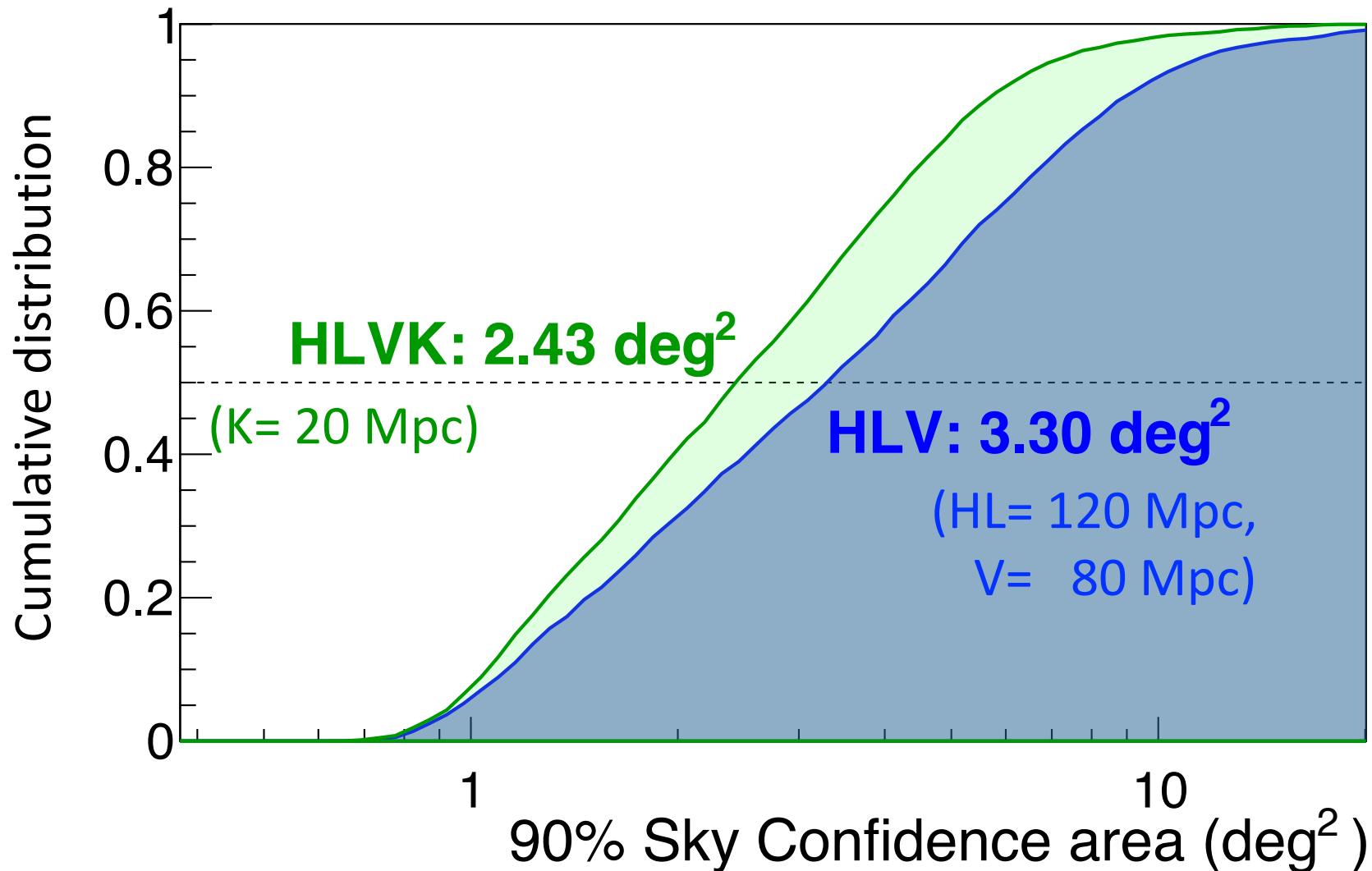
# Sky confidence area summary

Averaged over 3840 cases (192 long./lat. 5  $\psi$ , 4  $\theta_{JN}$ )



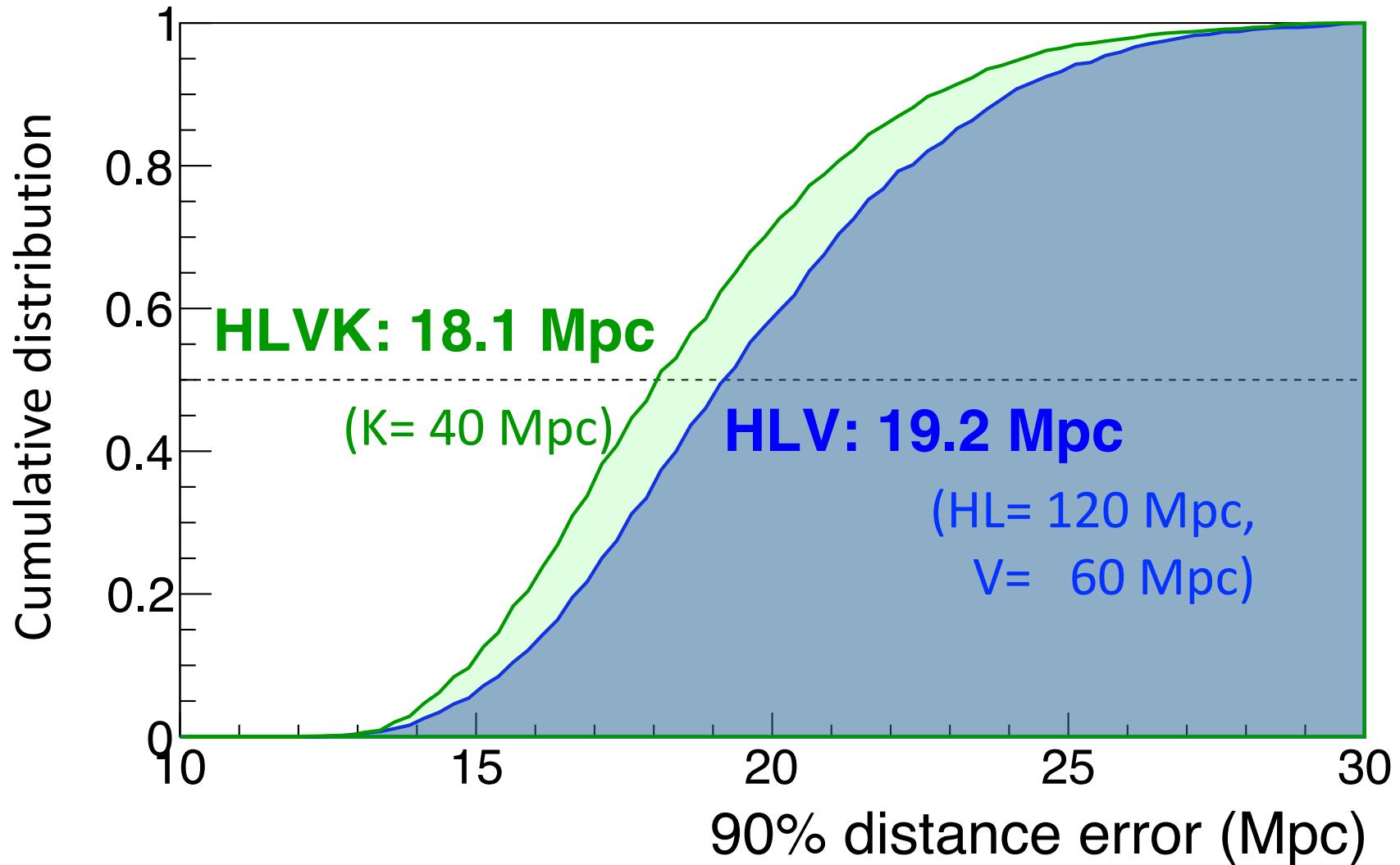
# Sky confidence area summary

Averaged over 3840 cases (192 long./lat. 5  $\psi$ , 4  $\theta_{JN}$ )



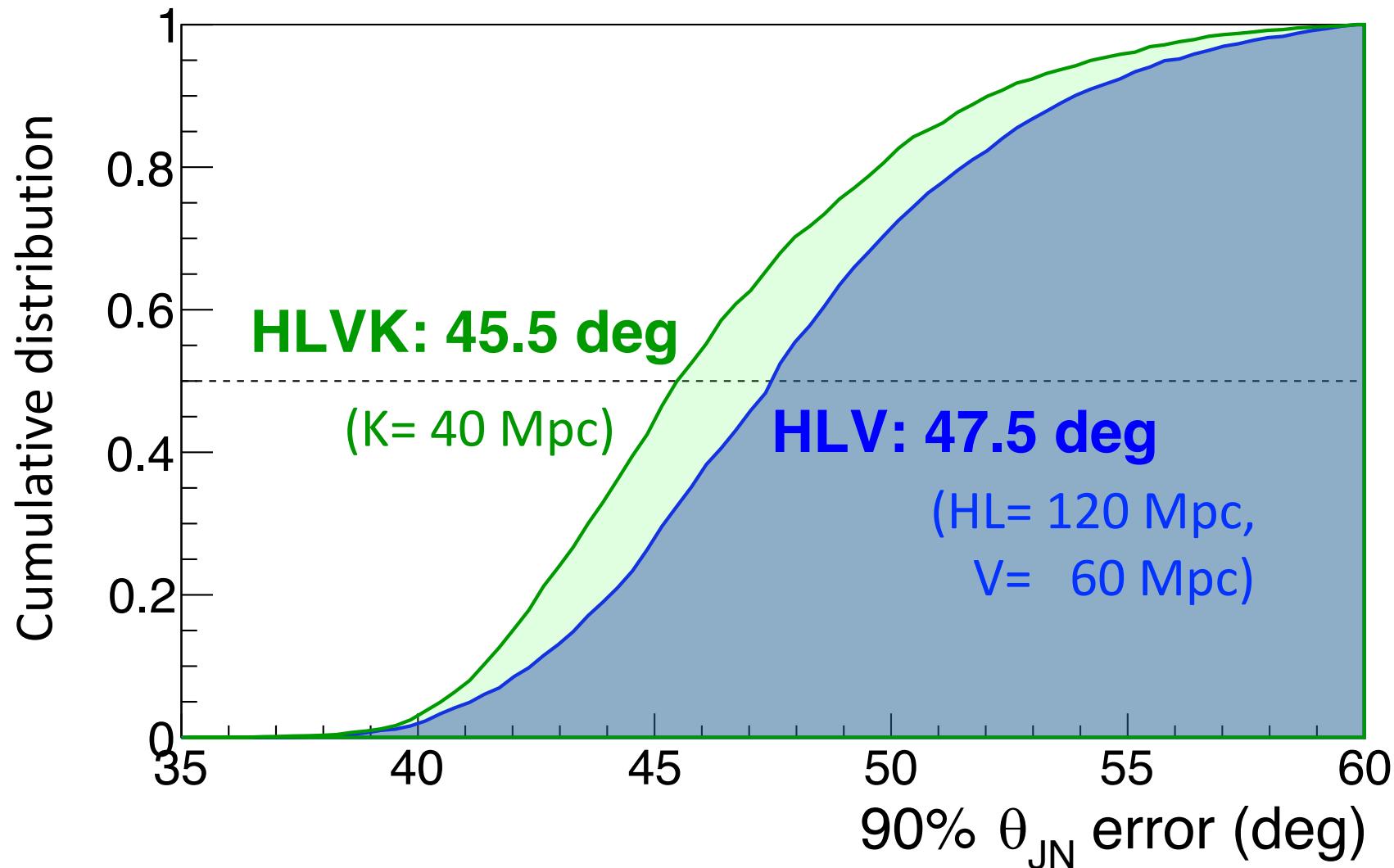
# Distance estimation error

Averaged over 3840 cases (192 long./lat. 5  $\psi$ , 4  $\theta_{JN}$ )



# Inclination angle estimation error

Averaged over 3840 cases (192 long./lat. 5  $\psi$ , 4  $\theta_{JN}$ )



# Simulation results

<http://www.icrr.u-tokyo.ac.jp/~haino/gsim/gsim.html>

## LVK (LIGO-Virgo-KAGRA) simulation results

Mass	Distance	LIGO	Virgo	KAGRA	Summary	Sky map
1.5-1.25 MS	40 Mpc	120 Mpc	60 Mpc	40 Mpc	[+] Expand	[+] Expand
1.5-1.25 MS	40 Mpc	120 Mpc	60 Mpc	10 Mpc	[+] Expand	[+] Expand
1.5-1.25 MS	40 Mpc	120 Mpc	80 Mpc	40 Mpc	[+] Expand	[+] Expand
1.5-1.25 MS	40 Mpc	120 Mpc	80 Mpc	20 Mpc	[+] Expand	[+] Expand
1.5-1.25 MS	40 Mpc	120 Mpc	80 Mpc	10 Mpc	[+] Expand	[+] Expand
1.5-1.25 MS	10 Mpc	120 Mpc	60 Mpc	40 Mpc		[+] Expand

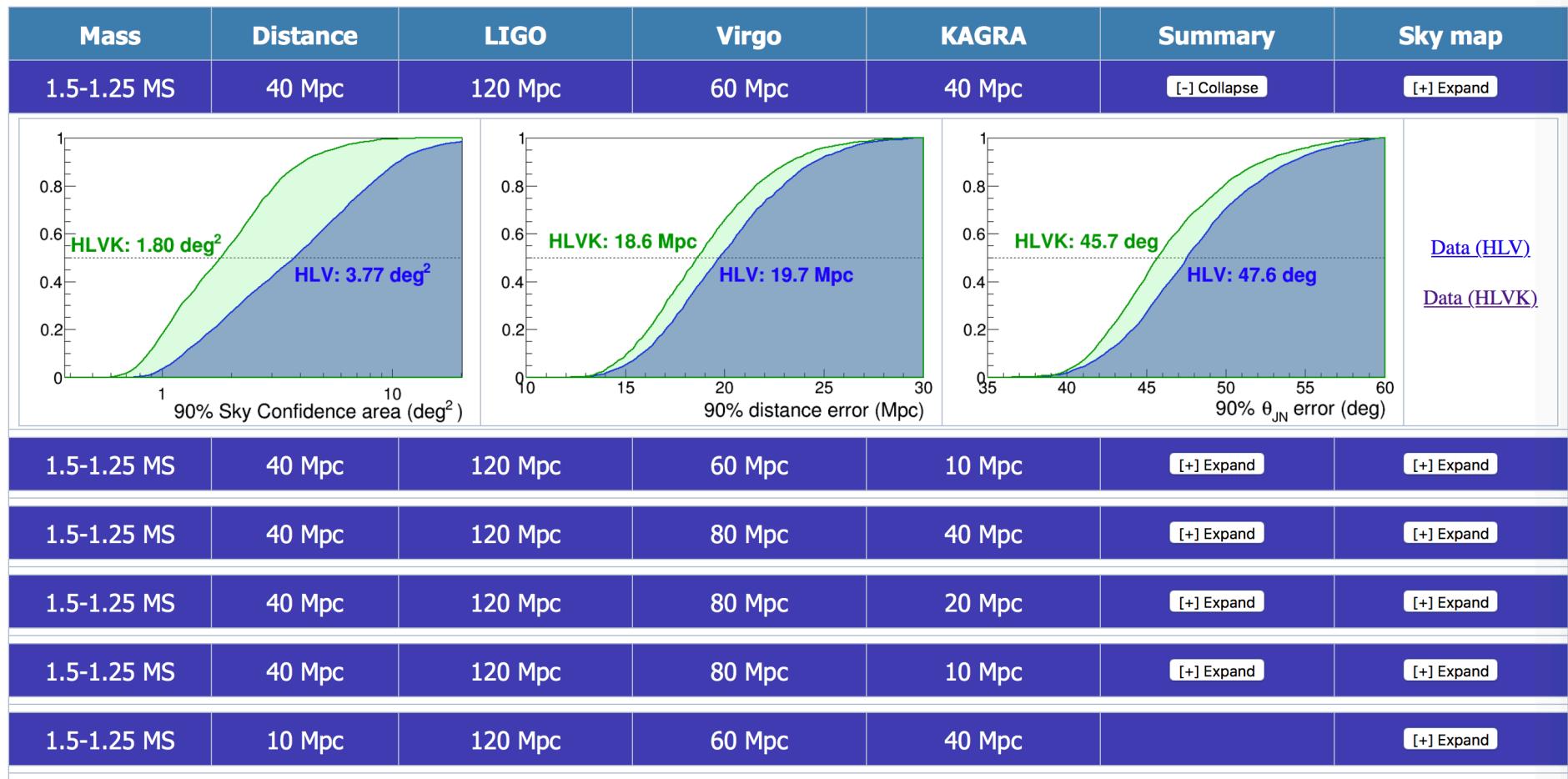
Updated by S.Haino on Mar 20, 2018

Refs: [JGW-G1807674](#) [JGW-G1808042](#)

# Simulation results

<http://www.icrr.u-tokyo.ac.jp/~haino/gsim/gsim.html>

## LVK (LIGO-Virgo-KAGRA) simulation results



# O3 simulation

Fix distance of BNS at 40 Mpc, then ...

- Randomly distribute

Significant improvement even with 10 Mpc  
(By selecting  $\rho_K > 2$ )

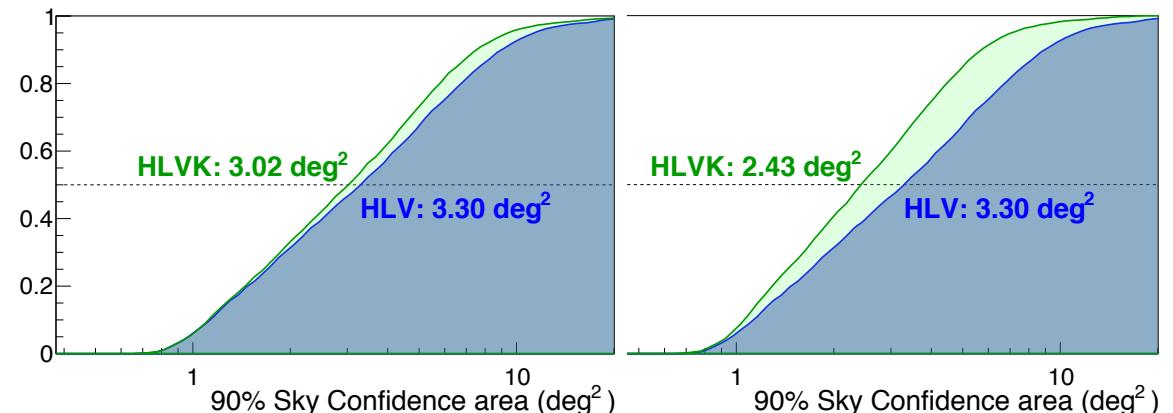
$$\frac{\langle \Delta\Omega_{LHV\bar{K}}(\rho_{KAGRA} > 2) \rangle}{\langle \Delta\Omega_{LHV} \rangle} = 0.62$$

- Fix: inclination and polarization

Only a small improvement with 10 Mpc

> 20 Mpc

is required



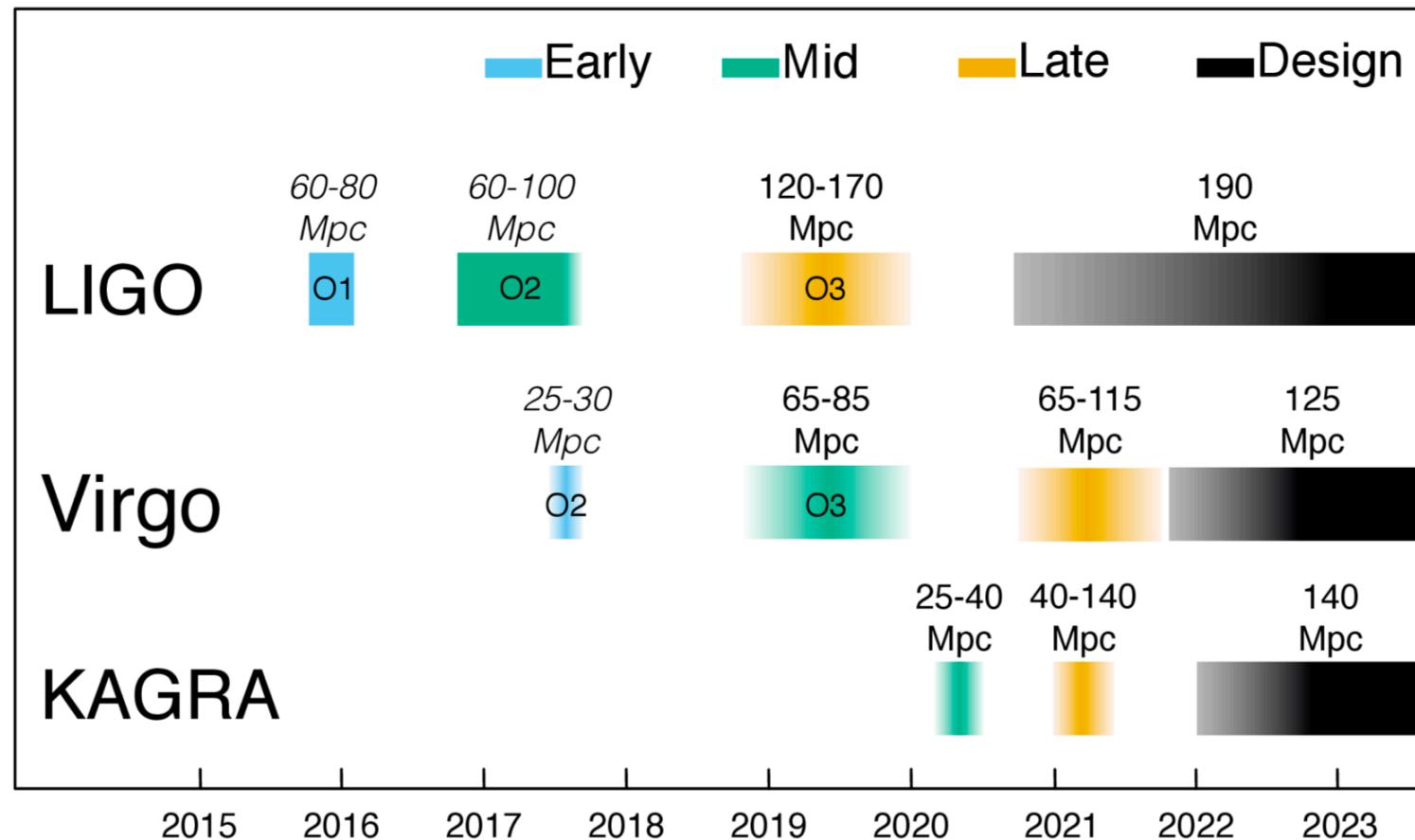
# Outline

- G.P.E. – GPU accelerated Parameter Estimation
- Comparison of results – GW events analysis
- Comparison of results – Simulation
- Immediate future – O3 simulation
- Physics targets – O5 simulation

# Outline

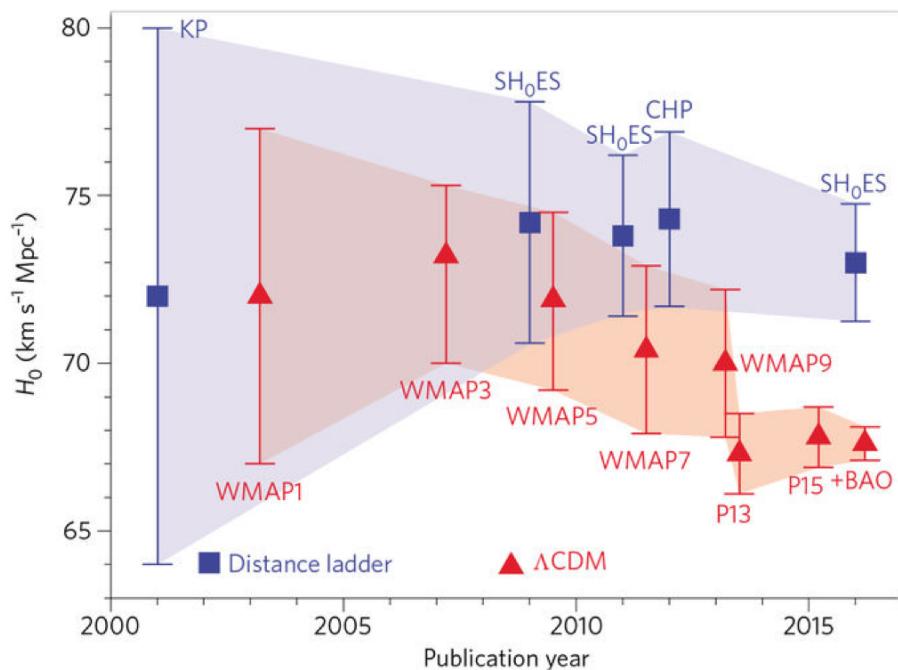
- G.P.E. – GPU accelerated Parameter Estimation
- Comparison of results – GW events analysis
- Comparison of results – Simulation
- Immediate future – O3 simulation
- Physics targets – O5 simulation

# KAGRA physics targets (O5)

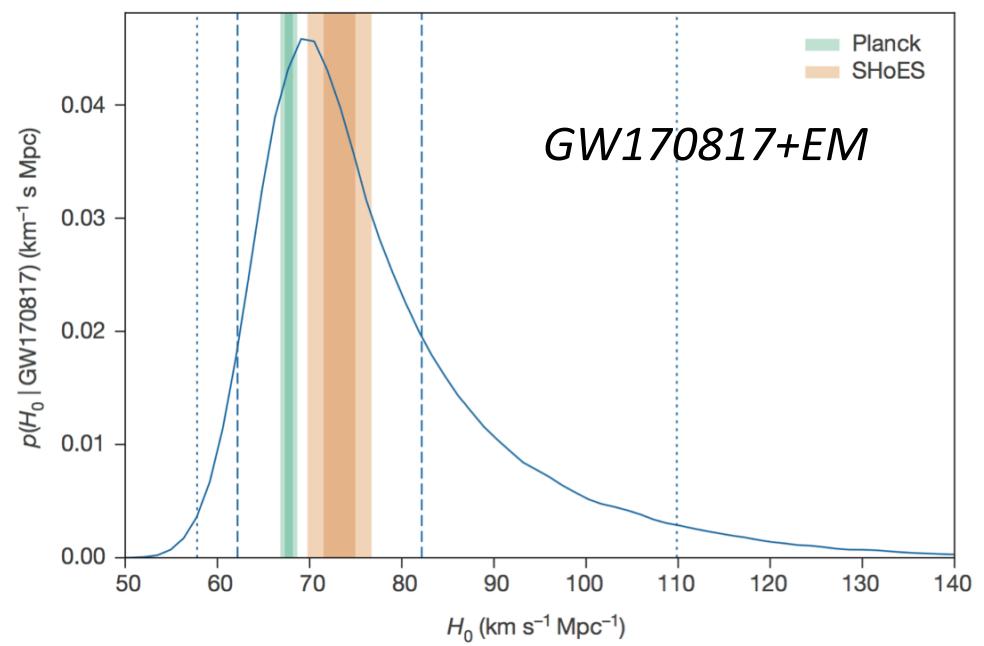


# Physics example:

## Hubble constant – an issue and a new hope



*Nature Astronomy* **1**, 0121 (2017)



*Nature* **551**, 85 (2017)

# GW standard siren (BNS, NSBH?)

- GW => Luminosity distance
- EM => redshift
- Good sky localization is needed to identify EM ( $N_{\text{det}} \geq 3$ )
- By accumulating many BNS events, distance error can be reduced with  $\sqrt{N}$

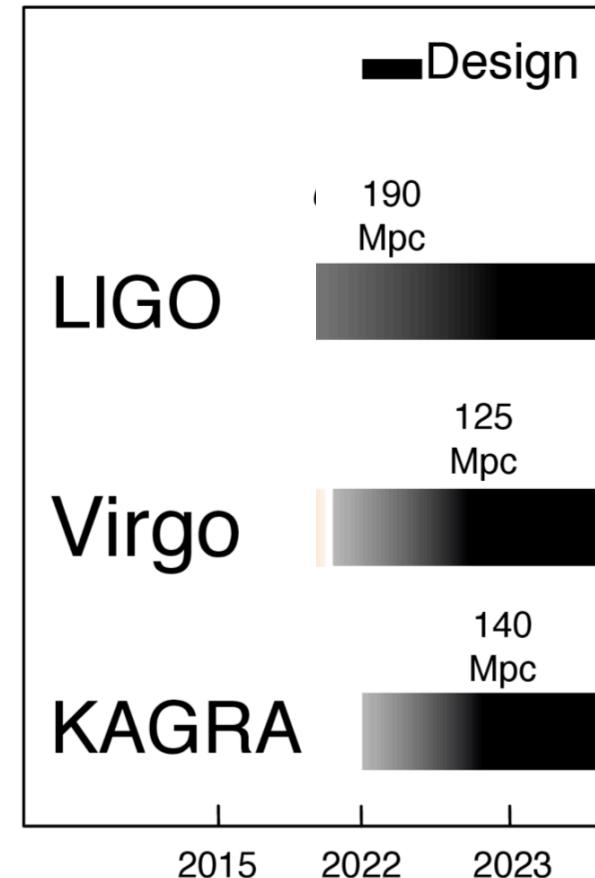
# KAGRA's contribution

KAGRA (with design sensitivity) will improve

- Network SNR
- Effective duty cycle ( $3/4 >> 3/3$ )
- Sky localization
- Distance/inclination estimation
- ...

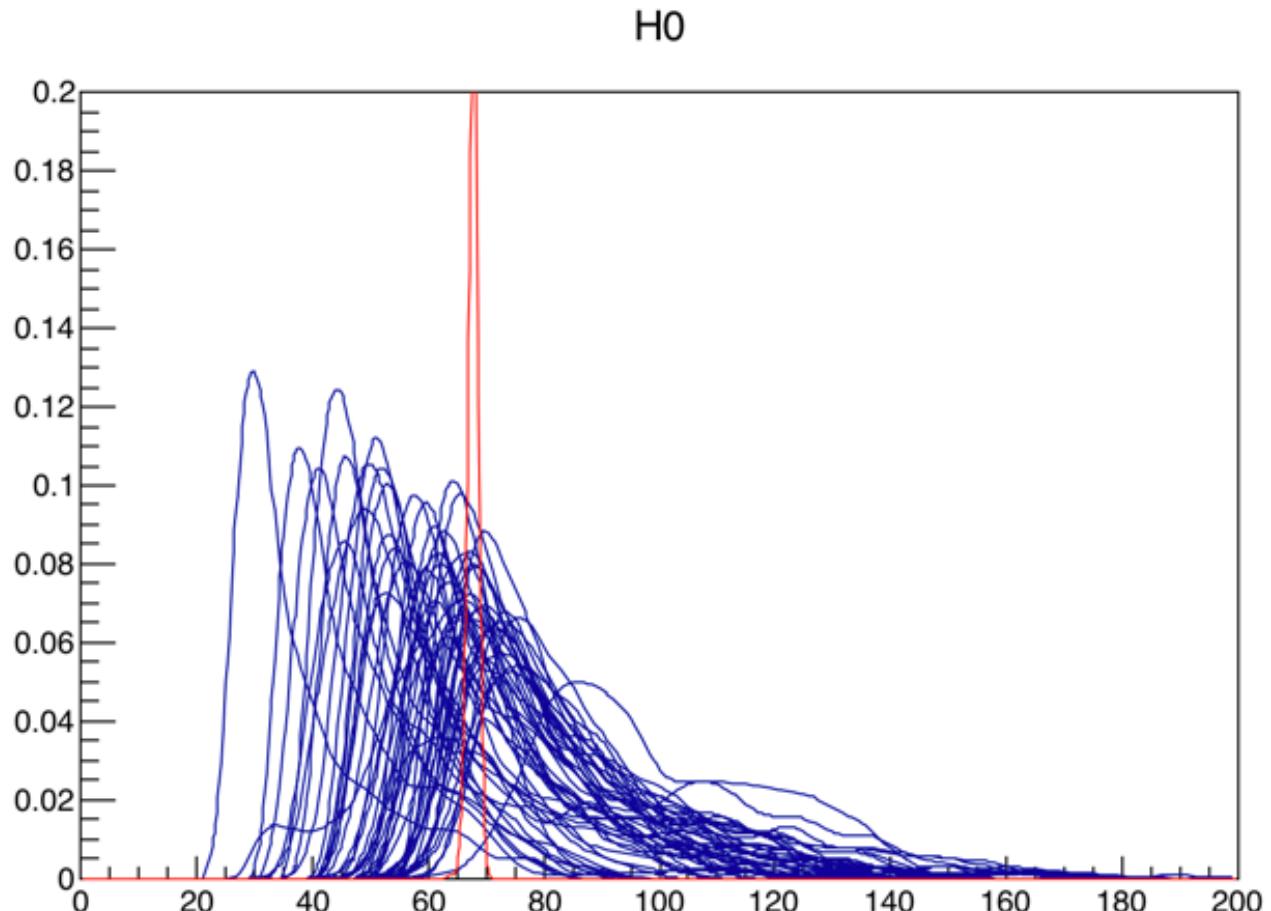
# BNS simulation

- 10,000 events with random distributions of
  - mass (1.0 – 2.0 Ms)
  - distance (by volume)
  - inclination, polarization, sky location
- Assuming design sensitivity

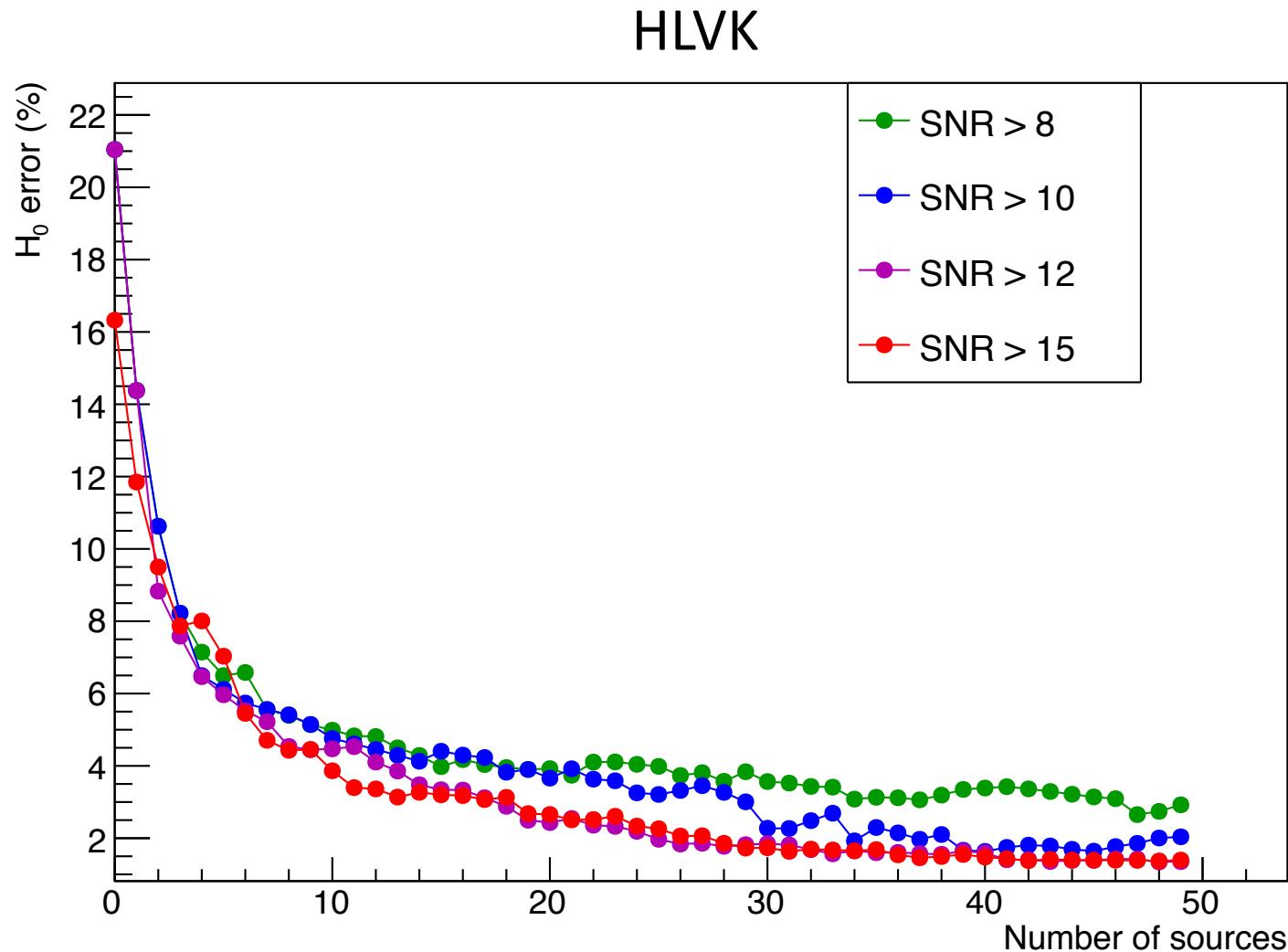


# Hubble constant from GW siren

$$c z = v^p + H_0 d.$$

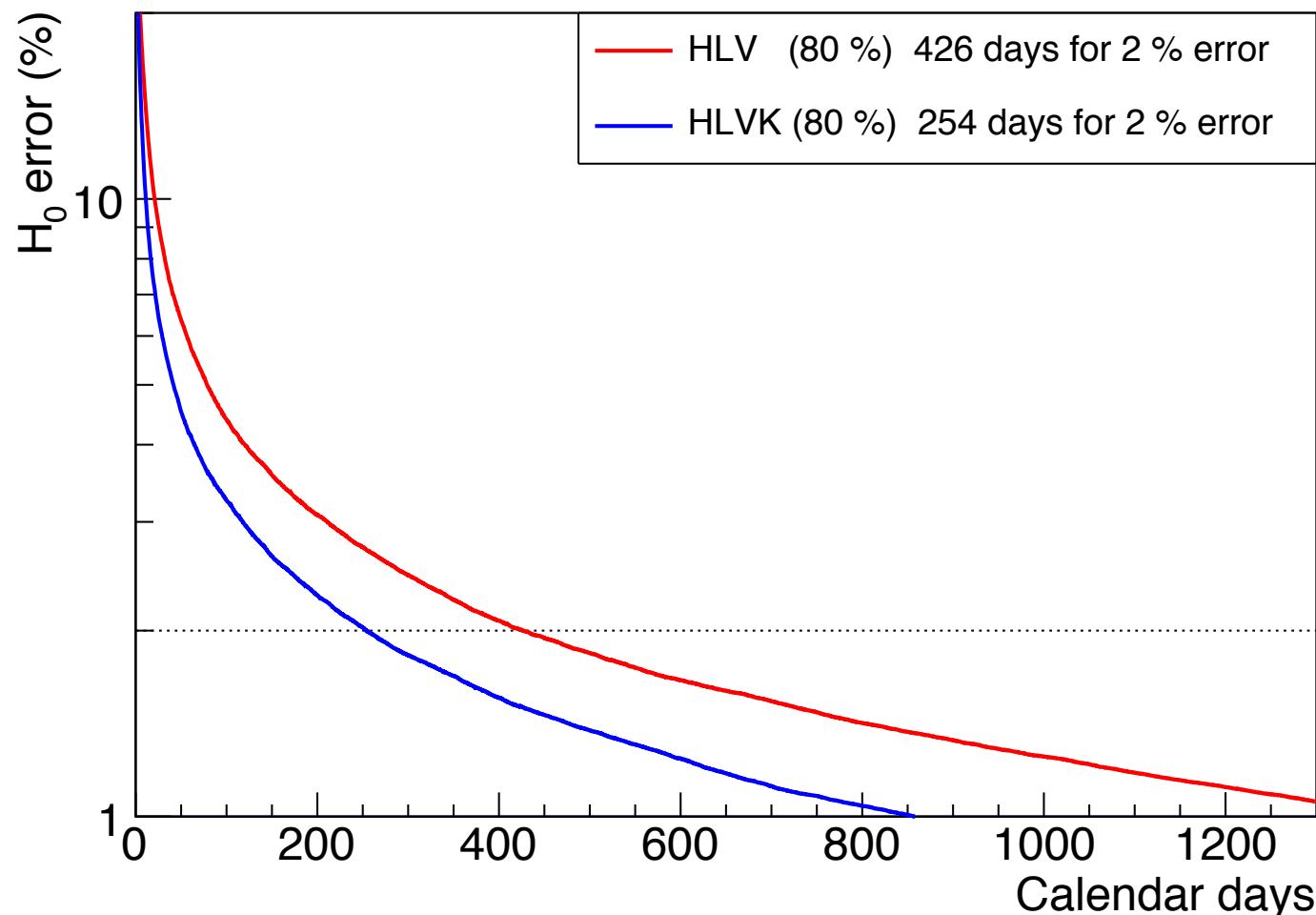


# Hubble constant VS events



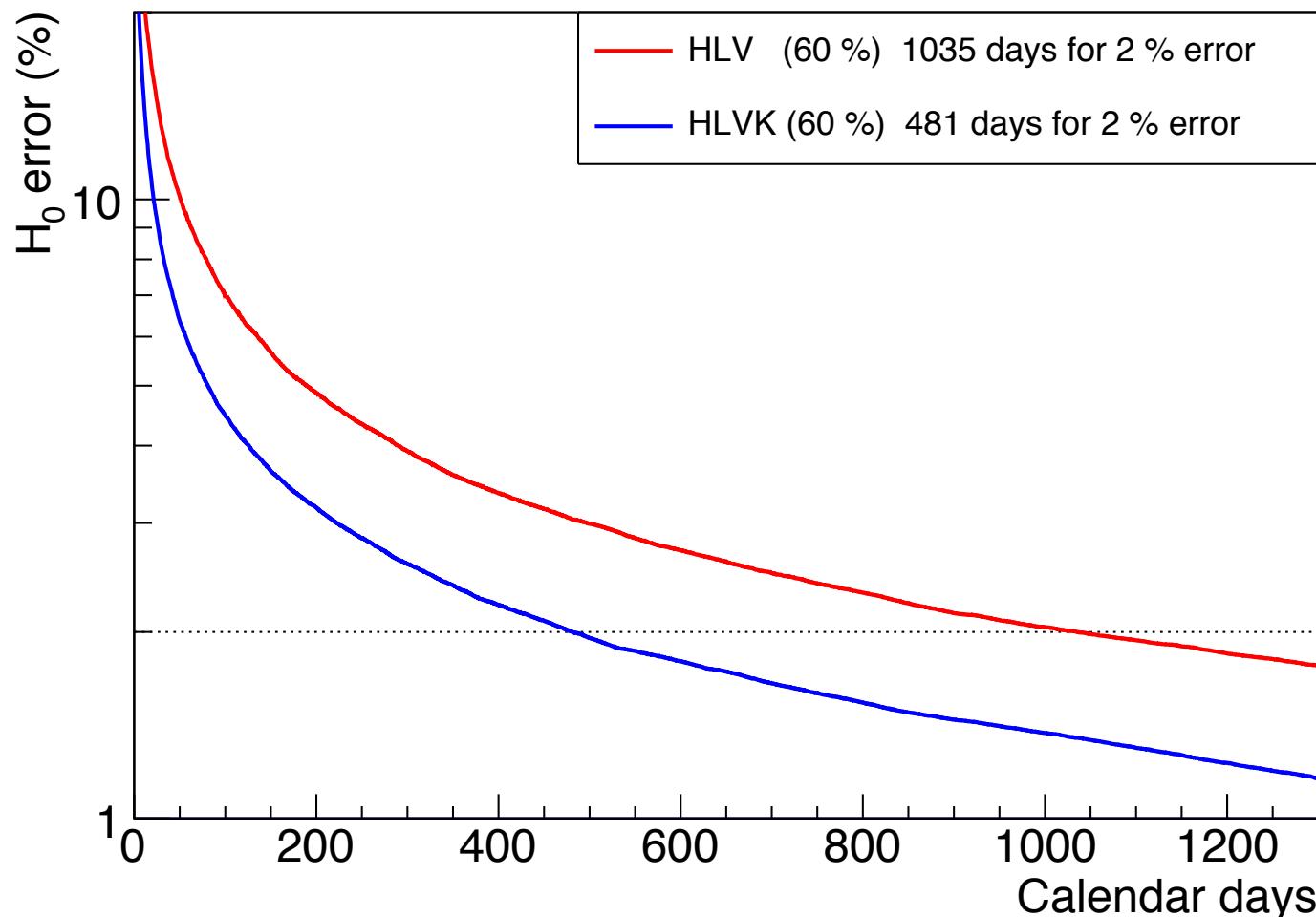
# Hubble constant error VS time

Assumed duty cycle : 80% for each detector



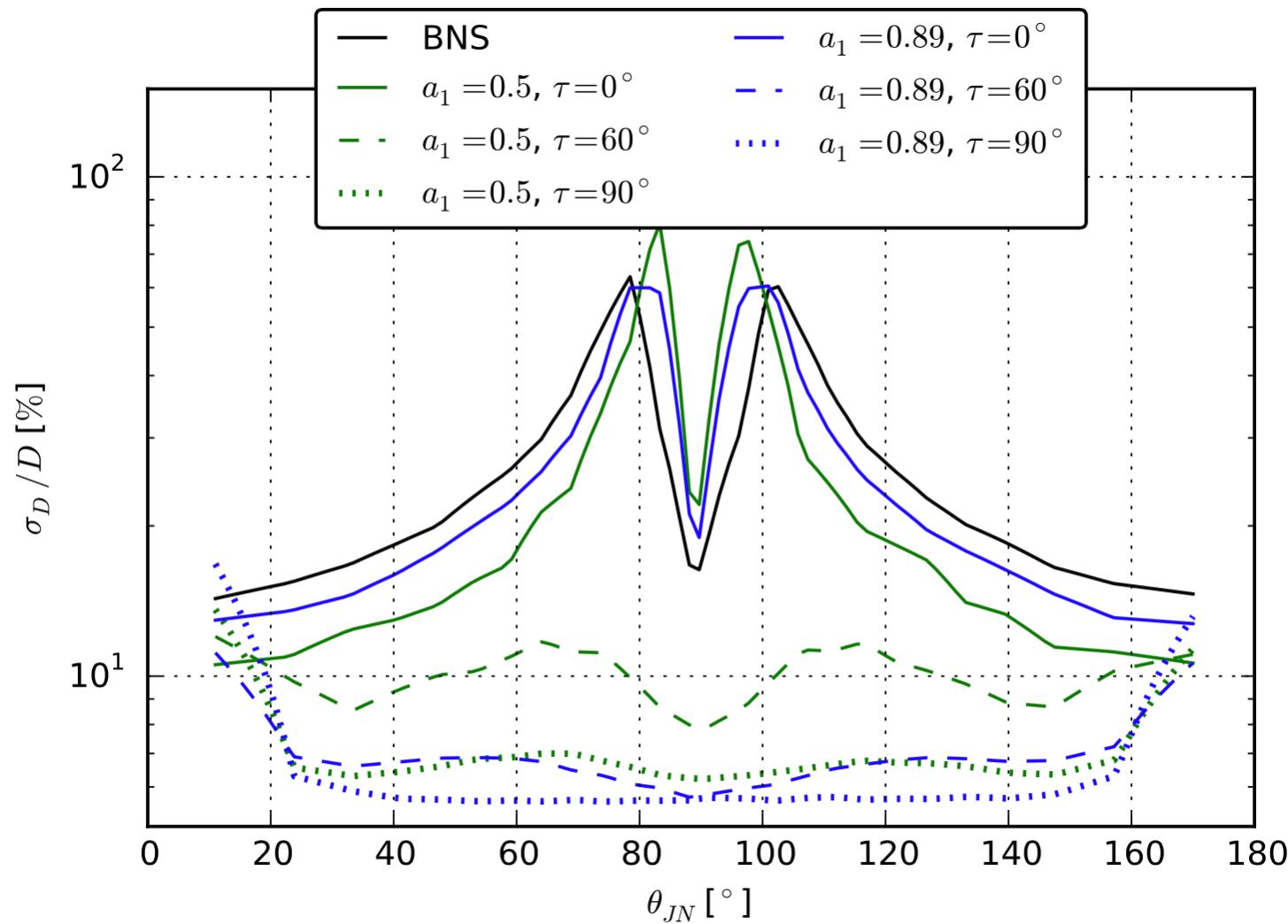
# Hubble constant error VS time

Assumed duty cycle : 60% for each detector



# NSBH

arXiv1804.07337



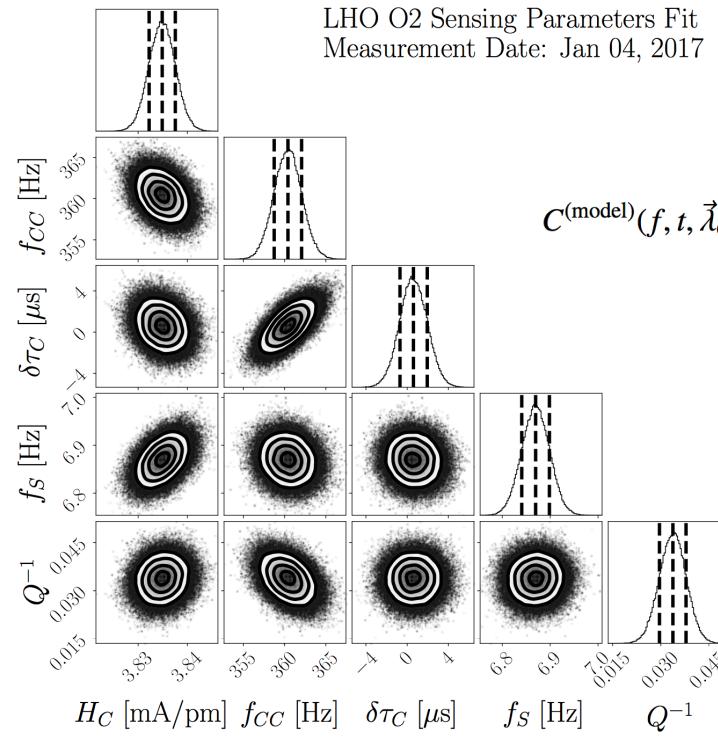
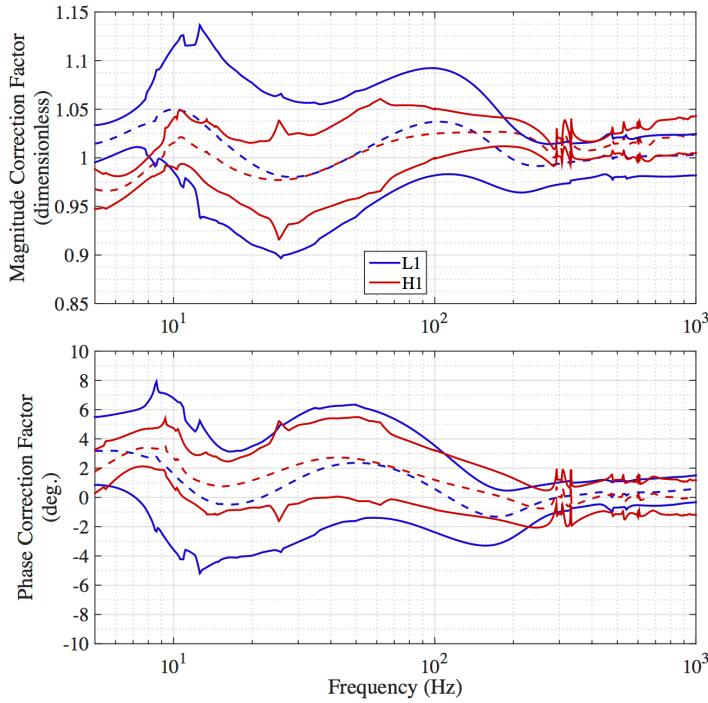
# KAGRA physics targets (O5)

- To be continued...
- More detailed discussions expected  
in KSC session of the coming f2f meeting at OCU

14:45-16:30 Session 15: <b>KSC and Collaborators session</b> (Chair: xxxx)	
R&D authorization ()	Yoichi Aso (NAOJ)
Phase-1 paper (15min)	??
KAGRA physics targets (20min)	Sadakazu Haino (Academia Sinica)
CPC (5 min)	??
Default Author-List 2017 (5 min)	Hisaoaki Shinkai (Osaka Inst. Tech.)
Document and Drawing ()	??

# Another on-going project

- Evaluating calibration error by implementing realistic IFO model and calibration error priors
- To be discussed in Kiban(s) meeting (May/21 OCU)



$$C^{(\text{model})}(f, t, \vec{\lambda}_C) = \frac{\kappa_C(t) H_C}{1 + if/f_{CC}} C_R(f) e^{2\pi if\tau_C} \times \frac{f^2}{f^2 + f_S^2 - if f_S Q_S^{-1}}$$