Measuring the Schnupp asymmetry in a simple Michelson interferometer

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Kiwamu Izumi and Yuta Michimura

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1 Overview

In this document, we describe two possible methods to measure the Schnupp asymmetry in a simple Michelson interferometer. One of the proposed methods relies on the phase measurement whereas the other relies on the reflectivity/transmissivity measurement. Both of them seem applicable to the KAGRA phase 1 configuration – the 3 km Michelson configuration.

2 Mathematical preparation

A Michelson interferometer shows the reflectivity for the laser of angular frequency ω,

$$r(\omega, L_x, L_y) = \frac{1}{2}\left[r_x \exp\left(-2i\omega L_x/c\right) + r_y \exp\left(-2i\omega L_y/c\right),\right] \tag{1}$$

where r_x and r_y represent the amplitude reflectivity of ETMX and ETMY, respectively, L_x and L_y are the single trip lengths for the X and Y arms, respectively and where c is the speed of light. Any imbalance or optical losses on the beam splitter can be absorbed into r_x and r_y and therefore we retain the factor of 1/2 i.e., the beam splitter virtually preserves the ideal 50/50 splitting ratio.

Introducing the common and differential reflectivity

$$\begin{aligned} r_c &= \frac{r_x + r_y}{2}, \\ r_d &= \frac{r_x - r_y}{2}, \end{aligned} \tag{2}$$

one can rewrite equation (1) and convert it to the power reflectivity as

$$R = r_d^2 + \left(r_c^2 - r_d^2\right)\cos^2\left[\frac{\omega(L_x - L_y)}{c}\right], \tag{3}$$

where the first term represents the contrast defect and the second term is the Michelson fringe.

3 Method 1: Phase measurement

3.1 Working principle

In this method, one needs to lock the Michelson interferometer at a middle fringe and measure the optical phase by observing changes in the reflected power with the laser frequency intentionally excited. In addition, combining another measurement with the Michelson length excited should allows us for partially getting rid of systematic errors in the estimation.

The response of the interferometer for a frequency change can be derived by taking the derivative of equation (3) with respect to ω so that

$$\begin{aligned} H_\omega &= \frac{\partial R}{\partial \omega}, \\ &= -\left(r_c^2 - r_d^2\right) \frac{L_x - L_y}{c} \sin\left(2\omega(L_x - L_y)/c\right). \end{aligned} \tag{4}$$

When it is locked to a mid fringe (say $\omega(L_x - L_y)/c = \pi/4$), the response should be

$$H_\omega \bigg|_{\text{m.f.}} = -\left(r_c^2 - r_d^2\right) \frac{L_x - L_y}{c}. \tag{5}$$

Therefore the response is proportional to the Schnupp asymmetry, $L_x - L_y$. Even if the reflectivity r_c and r_d are not known to a good precision, one can still use this measurement to estimate the asymmetry by combining a different measurement as follows.

Similarly to the response to the frequency, we now derive the one to displacements,

$$\begin{aligned} H_L &= \frac{\partial R}{\partial (L_x - L_y)}, \\ &= -\left(r_c^2 - r_d^2\right) \frac{\omega}{c} \sin\left(2\omega(L_x - L_y)/c\right). \end{aligned} \tag{6}$$

When the Michelson is held at a mid fringe, this should become

$$H_L\bigg|_{\text{m.f.}} = -\left(r_c^2 - r_d^2\right) \frac{\omega}{c}. \tag{7}$$

Finally, dividing equation (5) by equation (7), one can arrive at

$$H_\omega / H_L = \frac{L_x - L_y}{\omega}. \tag{8}$$

Since we know the angular frequency of the laser relatively precisely, we can estimate the Schnupp asymmetry. As shown in the equation, this technique should be robust against the systematic error due to the uncertainties in the mirror's reflectivity.

3.2 Practical implementaions

Assuming an effective input power of 10 mW (1 W $\times T_p$ where $T_p = 10\%$ is the PRM transmissivity),

$$H_\omega \sim 2 \times 10^{-12} \text{ W/Hz} \tag{9}$$

$$H_L \sim 6 \times 10^{4} \text{ W/m} \tag{10}$$

To change the laser frequency, one needs to actuate (through the IMC control loop) on the VCO that drives the AOM for the reference cavity. Assuming that the practical VCO range is ± 10 MHz [1], one can get

$$\Delta P = H_\omega \times 10 \text{ MHz} = 2 \times 10^{-5} \text{ W} \tag{11}$$

In terms of RIN, this value is translated to 2×10^{-3} which seems reasonably high enough to measure.

The accuracy and precision of this method will be influenced by the following factors.

- Calibration uncertainty of the VCO
- Calibration uncertainty of the Michelson length response

Assuming a typical accuracy/precision of a few %, this method would probably provide us with an accuracy and precision of several %.

4 Method 2: Reflectivity measurement

4.1 Working principle

In this method, the Michelson needs to be locked at either bright fringe or dark fringe using an RF signal. While holding the Michelson to the fringe, one can simply measure the transmissivity or reflectivity of the Michelson interferometer for the RF sidebands perhaps with the use of an optical spectrum analyzer.

4.2 Implementation

TBW.

References

[1] M. Nakano, private communication (2018).