Systematic error in an optical lever due to air/vacuum refractive index

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1 Overview

We estimate the influence of the in-air refraction onto an optical lever system. As discussed in this document, the KAGRA ETM optical levers would obtain a systematic error of about 9 μrad as the pressure in the chamber changes from the atmospheric to high vacuum or vice versa.

2 The layout

Figure 1 shows an simplified layout of the optical lever system used in the EYC chamber where the ETMY mirror is placed at the center of the chamber. The distance from the viewport is 1280 mm[1] and the test mass has a thickness of 150 mm [2].

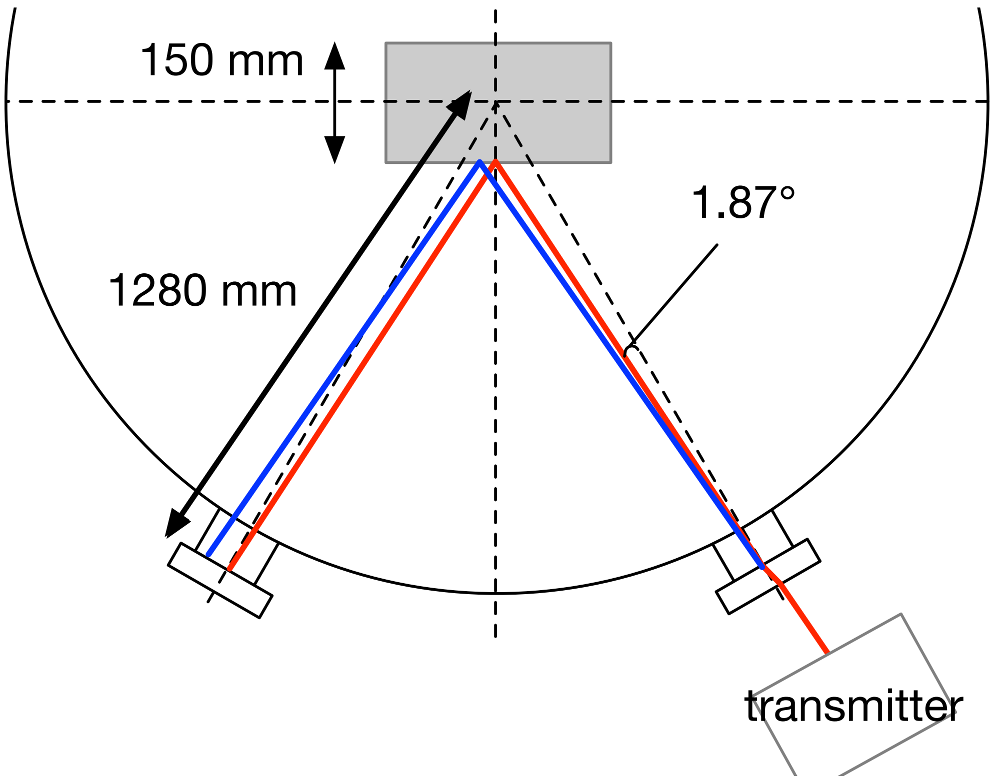

Figure 1: A simplified schematic of the layout used for the ETMY test mass optical lever. The dashed lines represent that joining the center of the chamber. The red lines represents the optical lever beam when the chamber is in atmospheric pressure, while the blue lines are those when the chamber is in high vacuum

We assume that the viewports are attached at the ±30 degree positions from the interferometer beam axis and the surfaces of each viewport is orthogonal to the line joining the chamber center. Due to the finite thickness of the test mass, the optical lever beam would be off from the line that joins the center of the mirror and the viewport center under the assumption that the optical lever beam goes through almost the center of the viewport. The angle between the optical lever beam and the viewport's normal vector (which joins the mirror center) can be geometrically computed to be 1.87 degrees when chamber is under atmospheric pressure.

3 Angle change induced by pump down/up

Using the ray transfer matrix, which properly includes the linearized Snell's law, one can simulate the effect of the viewport as

$$\hat{V} = \begin{pmatrix} 1 & 0 \\ 0 & n_v/n_c \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & n_a/n_v \end{pmatrix}, \tag{1}$$

where n_a, n_v and n_c are the refractive indices for air, the viewport medium and the medium in chamber (which should be either air or high vacuum), respectively. The rightmost matrix is the effect of the optical lever beam entering the viewport medium. The second matrix is the propagation in the viewport whose thickness is d. Finally the beam comes out to the medium in chamber via the leftmost matrix. Obviously we assume the viewport to have no wedge.

Performing the contraction of the above matrices, one can arrive at

$$\hat{V} = \begin{pmatrix} 1 & dn_a/n_v \\ 0 & n_a/n_c \end{pmatrix}. \tag{2}$$

Now, what we need to evaluate is the lower right element that changes the angle of the beam in chamber. More explicitly, the beam angle changes as it passes through the viewport by

$$\theta_o = \frac{n_a}{n_c}\theta_i, \tag{3}$$

where θ_i is the incident beam angle of 1.87 deg and θ_o is the output angle which should be identical to θ_i when the chamber is in air.

As the pressure in the chamber changes, the refractive index in chamber also changes by Δn_c. Consequently the output angle changes by

$$\begin{aligned} \Delta\theta_{\mathrm{o}} &= \frac{\partial\theta_{\mathrm{o}}}{\partial n_c}\Delta n_c. \\ &= -\left(\frac{n_a}{n_c}\right)\left(\frac{\Delta n_c}{n_c}\right)\theta_{\mathrm{i}}. \end{aligned} \tag{4}$$

Substituting the realistic vales of $n_a = n_c = 1.00027$ [3] for the 670 nm optical lever beam [4] and $\Delta n = -2.7 \times 10^{-4}$, one can obtain

$$\Delta\theta_{\mathrm{o}} = 8.8\,\mu\mathrm{rad}. \tag{5}$$

This means that the beam exiting the viewport goes further away from the line joining the center of the mirror and viewport as the chamber is pumped down. See Figure 1 for graphical representation.

4 Systematic error in optical lever readout

As derived in the previous section, the optical lever beam acquires another 8.8 μrad in its propagation angle. Therefore this results in a displacement at the quadrant photo detector by

$$\Delta x \approx 2L\Delta\theta_o, \tag{6}$$

where L is the one-way lever length of the optical lever system which should be something close to 1280 mm (neglecting the path length in the receiver module) as illustrated in Figure 1.

In a similar manner, the optical lever reads the displacement of the beam, Δx_m, caused by an angle change of the test mass $\Delta\theta_m$, so that

$$\Delta x_m = 2L\Delta\theta_m. \tag{7}$$

Therefore, the optical lever would acquire the systematic error in terms of the test mass angle as

$$\Delta\theta_m = \Delta\theta_o. \tag{8}$$

In summary, the optical lever used at EYC must have acquired a systematic error in the yaw direction by 9 μrad as seen by the optical lever when we pumped down the EY chambers.

References

[1] Y. Saito has a CAD diagram of the ETMY chamber made by Toshiba Inc.

[2] E. Hirose, “Drawing of Sapphire End Test Mass Mirror, ETMY”, JGW-D1503479-v4 (2016) https://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=3479&version=4

[3] S. Telada, “Technique for distance measure; dual-wave length interferometer for strain measurement” (in Japanese), http://www.icrr.u-tokyo.ac.jp/~telada/Chikyubuturi2005/DistanceMeasurements.pdf

[4] T. Akutsu, “Collimators and Focusers for Optical Levers (OpLevs) of KAGRA”, JGW-T1504478-v3 (2015) https://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=4478