Notes on the beam layout without the ITMs

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Contents

1 Scope of this document

This document attempts to evaluate the effect of the absence of the input test masses (ITMs) in the context of the interferometer alignment. Since each ITM refracts the beam via the wedge, removing the ITMs results in a different interferometer beam layout. This consideration was brought up for the phase-1 KAGRA which will operate without the ITMs.

2 Summary

Without the ITMs, the interferometer beam will be displaced at various places from the nominal values as listed below.

- The beam is displaced by 14 mm at ITMX to the negative Y direction.
- The beam is displaced by 2.5 mm at ITMY to the positive X direction.
- The beam is displaced by 5.2 mm at BS to the negative Y direction.

3 Optical layout

Figure 1 shows a schematic layout of the corner interferometer with the ITMs in place. In this nominal configuration, the interferometer beam is supposed to hit the center of all the core optics at their HR surfaces.

The beams going through ITMs must be orthogonal to their HR surfaces because the ITMs are part of the Fabry-Perot cavities. The beam leaving each ITM toward the BS is bent by an angle θ due to the refraction at the wedge. The size of θ is expected to be

$$\theta = 329\,\mu\text{rad}. \tag{1}$$

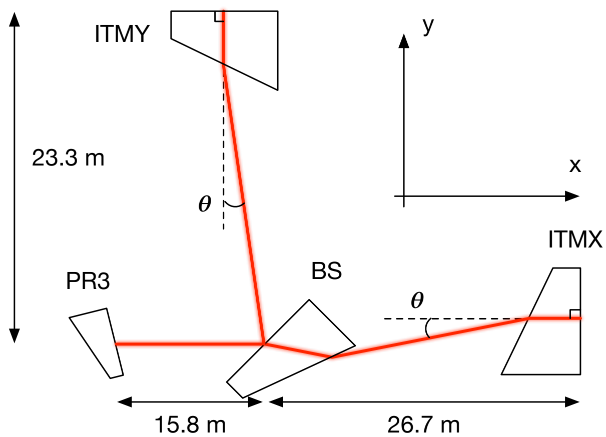

Figure 1: A schematic layout of the interferometer beam in the corner station. Not to scale.

See Appendix A for the derivation. BS is wedged as well. The wedge of all the optics are horizontal wedges as described in Appendix C.

4 Evaluating the effect of the absence of the ITMs

Our assumptions for the non-ITM interferometer are listed below.

- BS is approximated to be a non-wedged, virtually zero-thickness plate.
- The interferometer beam hits the center of PR3, ETMX and ETMY by rotating PR3 and BS appropriately.

While the first assumption makes us unable to compute the accurate refraction effect by the BS, it still allows us for computing the relative change in

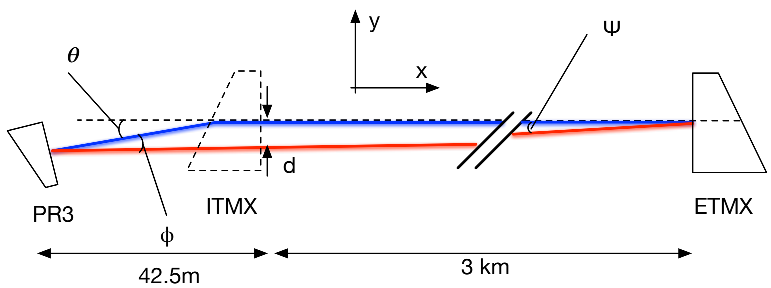

Figure 2: A simplified geometry for the inline beam with and without the ITMs. The blue lines represent that with ITM, and the red lines are that without ITM. BS is not shown.

the spot positions on the BS and other optics with respect to the nominal. Validation for the omission of the BS refraction is given in Appendix B.

As a first step, we evaluate the inline beam using the X arm only. The layout can be simplified to the one shown in figure 2. The incident angle of the beam onto ETMX changes by ψ as ITMX is removed. From some geometrical argument, one can easily show that ψ is approximately 4 μrad. This is so small that one can approximate the difference in the PR3 output angle ϕ to be

$$\phi \approx \theta. \tag{2}$$

Therefore, when there is no ITMX, the beam position at the location of the ITMX HR surface would be displaced by

$$d = \phi \times 42.5 \text{ m} = 14.0 \text{ mm}, \tag{3}$$

toward the dark port or the negative Y direction.

Similarly, the size of the displacement at the BS HR surface can be esti-

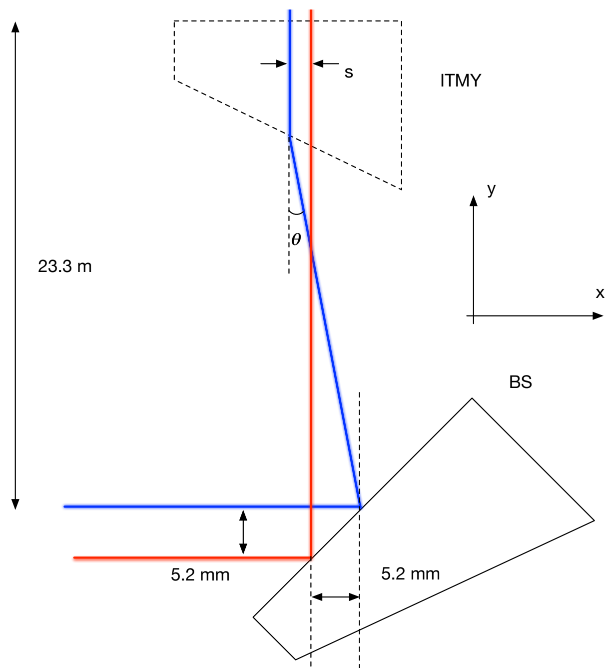

Figure 3: A simplified geometry for the perpendicular beam with and without the ITMs. The blue lines represent that with ITMs, and the red lines are without ITMs.

mated to be

$$d_{\mathrm{BS}} = \phi \times 15.8\mathrm{m} = 5.2 \text{ mm}, \tag{4}$$

toward the dark port or the negative Y direction.

Now, as for the perpendicular alignment to ETMY, the calculation is similar to that for the inline propagation except that one needs to take the position shift of the beam on the BS into account. Figure 3 shows the position shift due to the inline alignment by 5.2 mm in both X and Y directions. As evaluated below, the shift makes the displacement at ITMY smaller than that for ITMX.

One can estimate the displacement at ITMY as

$$s \approx \theta \times 23.3 \text{ m} - 5.2 \text{ mm} = 2.5 \text{ mm}, \tag{5}$$

toward the positive X direction.

5 Conclusion

The size of the beam spot displacements are estimated for the interferometer without the ITMs. The quantitative summary can be found in section 2.

A The refraction angle for ITMs

Using Snell's law, one can obtain

$$\theta = \arcsin\left(n \sin\theta_w\right) - \theta_w. \tag{6}$$

where θ_w is the wedge angle and n is the refractive index of the mirror substrate. The refractive index is set to $n = 1.754$ for the ITMs because they are made of sapphire [1]. The wedge angle is set to 0.025 deg according to the design specification [2]. Plugging these numbers to the above equation, one can obtain

$$\theta = 328.99 \times 10^{-6} \text{ rad}. \tag{7}$$

B The effect of the BS wedge

Consider a laser beam propagating through an wedged beam splitter as shown in figure 4. One can easily compute the relevant angles and relate the input and output angles as follows.

$$\begin{aligned}
\theta_o &= \arcsin\left(\frac{1}{n}\sin\theta_i\right) && (8)\\
\phi_i &= = \theta_o + \theta_w && (9)\\
\phi_o &= \arcsin\left(n \sin\phi_i\right) && (10)
\end{aligned}$$

Combining the above equations, one can relate the final output ϕ_o with

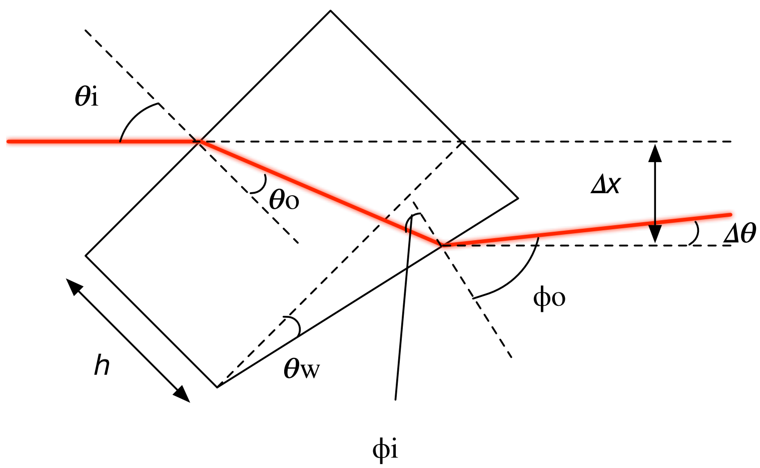

Figure 4: The refraction of the beam passing through the BS.

the first incident angle θ_i as

$$\phi_o = \arcsin\left\{n \sin\left(\arcsin\left(\frac{1}{n}\sin\theta_i\right) + \theta_w\right)\right\}. \tag{11}$$

Observe that this matches the ABCD matrix calculation for the small angle limit (θ_i, $\theta_w \ll 1$),

$$\phi_o \to \theta_i + n\theta_w \quad \text{(for small } \theta_i,\, \theta_w\text{)} \tag{12}$$

Now, from equation 11 and figure 4, one can find the change in the beam angle to be

$$\Delta\theta = \arcsin\left\{n \sin\left(\arcsin\left(\frac{1}{n}\sin\theta_i\right) + \theta_w\right)\right\} - \theta_i - \theta_w. \tag{13}$$

Similary, one can find the displacement to be

$$\begin{aligned}\Delta x &\approx h \sin\left(\theta_o - \theta_i\right), \\ &= h \sin\left[\arcsin\left(\frac{1}{n}\sin\theta_i\right) - \theta_i\right],\end{aligned} \tag{14}$$

where we have used equation 8 and assumed that θ_w is sufficiently small so that the thickness of the wedged part does not appreciably change the result.

Figure 5 shows the numerical evaluation of $\Delta\theta$ and Δx using equations 13 and 14 when the incident angle θ_i is at around $\pi/4$ or 45 deg. It is clear that as the incident angle changes by ± 1000 urad from $\pi/4$, the resulting $\Delta\theta$ varies by 3.5 urad only. Similarly, the displacement Δx varies by 0.07 mm only.

In conclusion, the BS refraction does not change the beam angle or position by considerable amount as the ITMs are removed. Therefore we can safely neglect the refraction effect of the BS and treat it as a non-wedged optic.

C Wedge directions

Figure 6 shows the direction of the wedges on all the core optics [3].

References

[1] The refractive index of sapphire is set identical to that used by Y. Aso who computed the interferometer layout

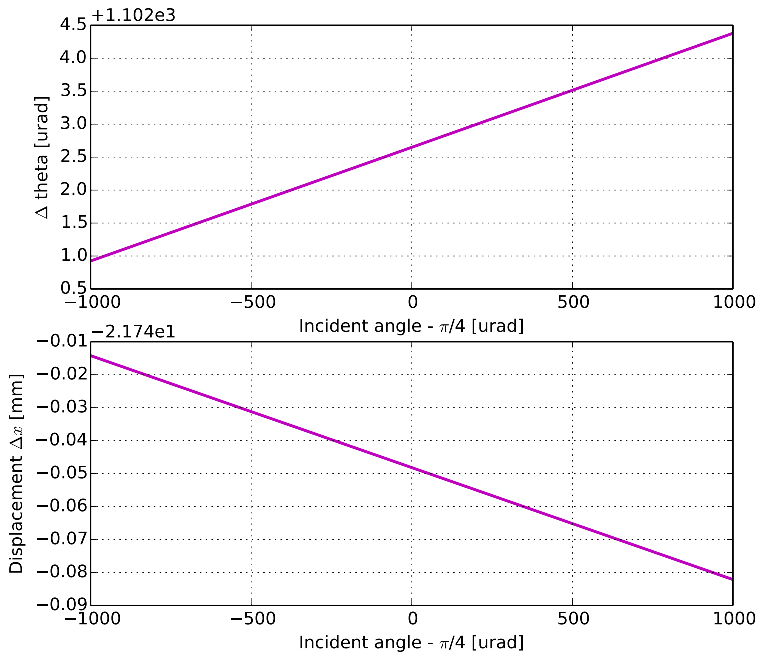

Figure 5: Changes in the output angle $\Delta\theta$ and displacement Δx as functions of the incident angle θ_i. The refractive index and the thickness are given as $n = 1.4496$ and $h = 0.08$, respectively.

https://granite.phys.s.u-tokyo.ac.jp/svn/LCGT/trunk/mif/OptLayout/Layout/

[2] The MIF wiki page. http://gwwiki.icrr.u-tokyo.ac.jp/JGWwiki/LCGT/subgroup/ifo/MIF/OptParam

[3] The figure was obtained from Y. Aso (2017)

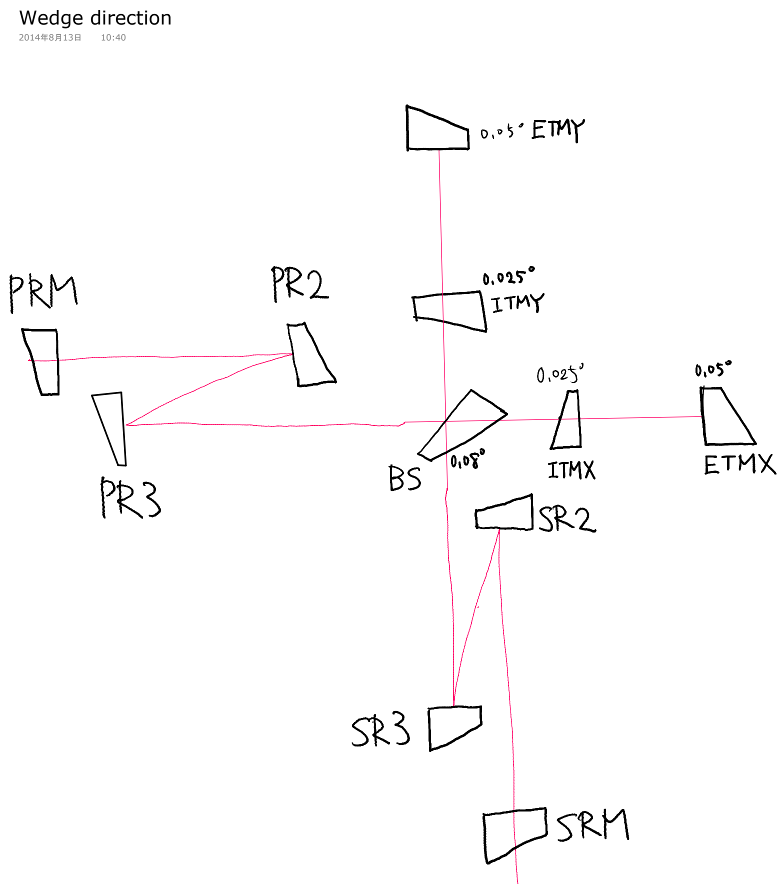

Figure 6: Wedge directions. Not to scale.