Report: latest estimated sensitivity curve of bKAGRA

Sep. 07, 2017
KSC board

Abstract & Background

In the KSC session at Toyama f2f (Aug. 28, 2017), the correction of the latest estimated sensitivity curve of bKAGRA was proposed. Three volunteer referees checked the code and submitted a report to the KSC board. KSC board members checked their report and hereby pass them to EO with this report.

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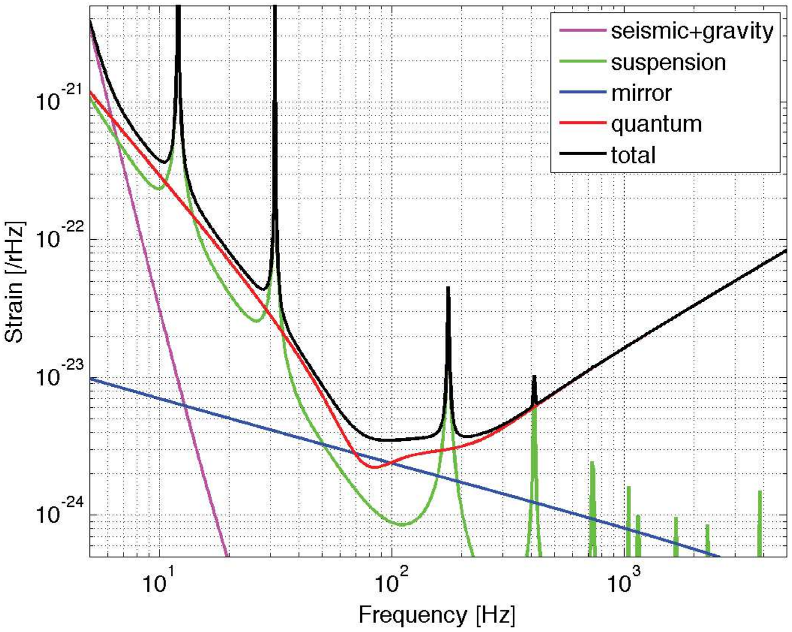
proposed sensitivity curve



Code and parameter list available from JGW-T1707038.

[1] Summary of KSC board

At this moment, three referees report that the current code has no problem. We can never prove this code has no error, but we suggest to update the latest estimated sensitivity curve according to the code proposed by K. Komori, Y. Michimura and K. Somiya at JGW-T1707038.

The report by Kazuhiro Yamamoto. He mainly checked the thermal noise calculations. The report says the code uses right formula.

The report by Takahiro Yamamoto. He mainly checked the inspiral range calculations and parameters. He requests the references of some parameters which are used without citations. The main one is

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%sky average constant given by Kanda-san
averageconst=0.442478;
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which is consistent with the widely used factor, 2.26(=1/0.442478), given by Finn-Chernoff (1993). This part, however, does not directly relate with the sensitivity curve of the detector itself. Also, the relevant references are added to the code.

The report by Yutaro Enomoto. He mainly checked the quantum noise calculations. The report says no errors.

Therefore, <u>we, KSC Boards, suggest to update the latest estimated sensitivity curve.</u>

Thermal noise report

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1 Introduction

Thermal noise is one of fundamental noise sources of interferometric gravitational wave detectors. This is measurement limit by statistical mechanics. There are “two kinds” of thermal noise; mirror itself and suspension. The former is mirror reflective surface deformation by thermal elastic vibration of mirror. The latter is fluctuation of center of suspended mirror.

According to Fluctuation Dissipation Theorem, thermal noise depends on the mechanical loss [1, 2] in mirror and suspension. Information of dissipation is absolutely necessary to evaluate thermal noise.

Code to calculate thermal noise was developed by Kentaro Somiya (probably, he referred Kazuhiro Yamamoto's document and advices). Kentaro Komori also developed code. Although he referred Somiya's report, Komori himself checked many papers and documents and developed his code. Therefore, this is some kinds of “cross check”. Now, Kazuhiro Yamamoto checks Kentaro Komori's code.

2 Thermal noise of mirror

Many kinds of thermal noise were“discovered” and formulae were derived. In usual cases, these three kinds of mirror thermal noise dominate.

- Substrate Brownian noise
- Substrate thermoelastic noise
- Coating Brownian noise

The Komiri code only takes these three ones into account.

Levin's [4] and Nakagawa's [5] approaches are the common method to derive formulae of these thermal noise amplitudes. In usual case (and in Komiri code), when we derived thermal noise formulae from equation of motion and these approaches, we assume that

- Mirror is half-infinite elastic body

- Frequency is lower than mirror elastic modes.

The first assumption is appropriate because laser beam radius is (2.5 times at least) smaller than mirror radius to suppress diffraction loss and thickness of mirror is comparable with mirrror diameter[6]. Since the lowest mirror elastic mode (about 10kHz) is higher than interferometer frequency range (below about 1kHz), the second assumption has no problem.

Details of three kinds of mirror thermal noise are described below.

2.1 Substrate Brownian noise

Mechanical dissipation in mirror substrate must be considered. "Brownian noise" implies thermal noise caused by "structure damping", which is almost independent of frequency. The formula of thermal noise caused by structure damping in substate was derived by F. Bondu et al[7]. Kazuhiro Yamamoto confirmed formula in Komori code is same as Eq. (14) of Ref. [7].

2.2 Substrate thermoelastic noise

Thermoelastic damping is relaxation process of temperature gradient caused by inhomogeneous elastic deformation. M. Cerdonio et al. derived exact formula which is correct in all frequency band [8]. Howevevr, K. Numata et al., pointed out missing factor, $1/\pi$ [9]. K. Somiya et al. showed simplified integral expression to reduce calculation cost of this thermelastic noise formula[10][1]. They take what Numata et al., pointed out into account. Kazuhiro Yamamoto found that formula in Komori code is consistent with Eqs.(3) and (7) of Ref. [10].

2.3 Coating Brownian noise

Nevertheless reflective coating is four orders of magnitude thinner than substrate, contribution to thermal noise is even larger than that of substrate. Because dissipation of coating is four orders magnitude larger [3] and, as Levin pointed out [4], dissipation near beam spot generates larger thermal noise. G. Harry et al. reported coating Brownian noise formula, Eq.(22) of Ref. [11][2]. Komori code adapts this formula. Kazuhiro Yamamoto confirmed the formula in Komori code is same as Eq.(22) of Ref. [11][3].

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[1] Komori and Haino confirmed calculated simplified integral is same as that of original one.

[2] N. Nakagawa et al. also derived formula. However, it is assumed that Young's modulus of coating is same as that of substrate. Since sapphire bulk is harder than coating, this formula gives underestimated value (roughly speaking, 30% in the case of KAGRA)

[3] In Komori code, it is supposed that $\phi_{\parallel}$ and $\phi_{\perp}$ in Eq.(22) of [11] are equal. This assumption is adapted because measurement of $\phi_{\perp}$ is quite difficult. Since coating consists of two material, Eq.(22) is applied to each material (Ta_2O_5 and SiO_2). Their power spectrum densities (in m^2/Hz) are summed.

3 Thermal noise of suspension

Sapphire mirror is a part of cryogenic payload. This payload is suspended from vibration isolation system (14m in height) at room temperature. In order to simplify discussion, in this Komori code, only sapphire mirror and intermediate mass are taken into account. In other words, we treat cryogenic payload and vibration isolation system as "double pendulum". This "brave" approximation works well (Refs. [13, 14] and Section 4.3 of Ref. [15]) because thermal vibration caused by dissipation in other parts cannot be transmitted to sapphire mirror in observation band (above 10 Hz). In short, vibration isolation system and cryogenic payload act as mechanical filter. Komori codes revealed that the thermal noise of "double pendulum model" is almost same as that of "single pendulum model"(in this model, only sapphire mirror and fibers, blade springs to suspend mirror are considered) above 10 Hz. Even dissipation in the stage to suspend intermediate mass does not matter.

Details of "double pendulum model" are as follows [16]. Sapphire mirror is suspended from intermediate mass by sapphire fibers. Intermediate mass is suspended by CuBe wire. Top end of these CuBe wires are fixed on heavy and rigid body. In order to compensate sapphire fiber length difference and make vertical bounce mode frequency lower, sapphire blade springs are inserted between sapphire fibers and intermediate mass. This Komori code includes this spring as a simple harmonic oscillator[4]. Therefore, in actuality, "double pendulum model" consists of three harmonic oscillators.

Komori code takes not only horizontal but also vertical motion into account (vertical motion is leaked to horizontal motion because of gradient of interferometer baseline). Since equations of motion of these degrees of freedom are independent from each other in Komori code, there is no correlation between these motions.

When horizontal motion is calculated, tension by weight of mass (in other words, by gravity) in fibers must be considered as restoring force. Even if wire is enough thin, we have to take elasticity of wires into account because tension by gravity has no mechanical loss. Imaginary part of Young's modulus represents loss of elastic energy in wire. There are higher order modes of wires, violin modes. In Komori code, effective spring constant of fibers ($K_{\rm susp}$, this includes even violin modes) from Ref. [17][5] is adopted. Kazuhiro checked this effective spring constant in Komori code. It is consistent with Eq.(14) of Ref. [17][6]. Frequency of (two) pendulum modes and first violin mode are around 1 Hz

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[4] Mass is equivalent to mass of blade spring itself. Spring constant is derived from displacement under load by sapphire mirror or resonant frequency of blade spring without load

[5] Boundary condition of Ref. [17] is that top end of fibers are fixed rigidly. However, in actuality, intermediate is not fixed. Kazuhiro Yamamoto expects that it does not bring any serious difference

[6] In Ref. [17], mass is suspended by a wire and it can be rotated. In our case, mirror is suspended by four fibers. Therefore, mirror cannot be rotated. $K_{\rm susp}$ in Komori code is four times larger than Eq.(14) of Ref. [17] because of four fibers. Since mirror can not be rotated, it is assumed that moment of inertia

and 170Hz, respectively. Horizontal mode of sapphire blade spring is 2 kHz. Kamori code does not consider violin modes of CuBe wires. The violin modes of these wire does not matter because of vibration isolation of sapphire fiber stage (Section 4.9.1 of Ref. [15]). The effective spring constant wire without violin modes can be derived from Eq.(14) of Ref. [17][7]. Kazuhiro confirmed that this effective spring constant of CuBe wires in Komori mode is same as that derived from Eq.(14) of Ref. [17][8].

In equation of the vertical motion, wires are treated as simple springs (higher modes are not included). Their spring constant is ratio of product of Young ' s modulus and cross section of wire to wire length. There are three vertical modes; blade spring (around 14Hz), CuBe wires (around 30Hz), sapphire fibers (1.15 kHz).

Temperature is not uniform in the cryogenic payload. In Komori code, local thermal equilibrium is supposed. Under this assumption, thermal fluctuation force in a local region depends on product of dissipation and temperature in that local region. Therefore, difference of temperature can be converted to difference of dissipation. "Double pendulum model " has uniform " average " temperature.

Based on above assumption and consideration, equations of (horizontal and vertical) motion are described as 3×3 matrixes. Thermal noise is evaluated from these matrixes and fluctuation dissipation theorem as Section 3.2 of Ref. [19][9].

Kazuhiro Yamamoto checked suspension thermal noise in Komori code and totally agrees.

Kazuhiro also calculated some values without Komori code and compared with result of Komori code.

- Resonant frequency
  - High horizontal mode : Since blade spring is hard, we need to consider only blade spring, its mass, and intermediate mass. Moreover, blade spring is lighter

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J of mirror is infinite. Terms which do NOT include J in Eq.(14) of Ref. [17] are neglected. After you consider these above points, you find $K_{\rm susp}$ in Komori code is same as Eq.(14) of Ref. [17]

[7] Necessary assumptions are as follows. (1)Low frequency($\omega \approx 0$). Approximations of k and $k_{\rm e}$ in Ref. [17] are adopted. (2)$k_{\rm e}L \ll 1$(Wire is thin. Then, bending length is much shorter than wire itself).

[8] Dilution factor in this Komori code is twice times larger than famous formula, Eq. (23) of Ref. [13]. In Ref. [13], it is supposed that mass has no momentum of inertia (or is suspended by one wire). In such a case, wires are never bended at bottom side. On the contrary, in Komori code, mass is suspended by four wires. Then mass can not be rotated. Wires are bended at both top and bottom sides. This is the reason why dilution factor of Komori code is twice times larger. If you would like to study details, you read Refs. [17, 18].

[9] In previous version of this code, there is a serious issue. Obviously, suspension thermal noise below 20Hz seemed to be overestimated. Finally, this is caused in the calculation of inverse matrix in this process. In previous version, function of codes to calculate inverse matrix was not used. Components of inverse matrix are calculated manually and result is written on code. This causes loss of significant digits. In current version, code function is used and such a problem is never found.

than intermediate mass, angular resonant frequency is square root of ratio of blade spring constant to mass of spring itself. This value (resonant frequency) is 2kHz. Komori code also shows 2kHz peak.

  - Violin mode of sapphire stage
  - First vertical mode : Softest part is blade spring. Sapphire mirror is fixed on blade spring rigidly. Moreover intermediate mass does not move because of stiffness of CuBe wires. So, mode angular frequency is square root of ratio of blade spring constant to sapphire mirror mass. Kazuhiro ’ s calculation is 14.5Hz. Komori code shows 13Hz peak. Since intermediate mass stage is not so much hard, resonant mode might be lower than Kazuhiro ’ s result.
  - Second vertical mode : Second softest part is CuBe. We take only intermediate mass and CuBe wires. Kazuhiro ’ s calculation shows that this resonant frequency is 26.5Hz. Komori code shows that 30Hz peak. Since sapphire stage is not so much soft, resonant mode might be higher than Kazuhiro ’ s result.
  - Third vertical mode : Hardest part is sapphire fiber. On both ends, there is sapphire mirror and blade spring. Since blade spring is lighter, angular resonant frequency is square root of ratio of fiber spring constant and blade spring mass. Kazuhiro ’ s calculation shows that 1.0 kHz. Komori code shows 1.15 kHz peak.

- Thermal noise
  - Horizontal motion (Around 50Hz) : We can assume that blade spring is rigid (not deformed) and intermediate mass is fixed. Dilution factor must be considered. But it is almost unity. So, here, this factor is assumed to be one. Kazuhiro ’ s calculation answer is $h = 2.7 \times 10^{-23}/\sqrt{\mathrm{Hz}}$. According to Komori code, this value looks $h = 1 \times 10^{-23}/\sqrt{\mathrm{Hz}}$.
  - Vertical motion (Around 100Hz) : We can assume that sapphire mirror is fixed on blade spring and that intermediate mass is fixed. Kazuhiro ’ s calculation answer is $h = 4.7 \times 10^{-25}/\sqrt{\mathrm{Hz}}$. According to Komori code, this value looks $h = 2 \times 10^{-25}/\sqrt{\mathrm{Hz}}$.

Comparison with Kazuhiro ’ s old documtnt

4 Parameters

Kazuhiro Yamamoto checked these parameters. Here, Kazuhiro ’ s comments are summarized.

- mirror parameters

  - Sapphire density, Young ’ s modulus, Poisson ratio : Ref. [21]
  - Sapphire Q is based on measurement shown by Ref. [24]
  - Temperature of mirror is 22K. Komori considered balance between quantum noise and thermal noise. When power at BS is larger (smaller), quantum noise is smaller (larger) and thermal noise is larger (smaller). Optimum temperature is 22K in KAGRA design.
  - Some material properties strongly depends on temperature. In our case, thermal conductivity, specific heat and thermal expansion must be considered. We refer Ref. [26] for sapphire specific heat and thermal conductivity and Ref. [27] for thermal expansion. Then fitting functions were generated. Although we find some discrepancies from measured material properties, fitting functions agree well with measurement around 20 K. In Ref. [28], these values are summarized

- Coating
  - Young ’ s modulus and Poisson ratio of coating : Ref. [20]
  - Coating refractive index : Ref. [22]. In this reference, refractive index of Ta_2O_5 is 2.1. This index of SiO_2 in this reference is same as that in Komori code.
  - Formula of coating thickness : Number of SiO_2 layer is larger than that Ta_2O_5. This is because top SiO_2 layer which faces air has twice times larger thickness.
  - Loss angle coating : Ref. [23]

- Suspension parameters
  - CuBe
    - Young ’ s modulus
    - Q
  - Sapphire
    - Sapphire fiber Q : This is based on measurement shown by Ref. [25]
    - Thermal conductivity of sapphire fiber is different from sapphire bulk because of size effect [29]. Fitting function for sapphire fiber is also made based on measurement in Ref. [30].
    - Sapphire blade spring Q (requirement) : Table 4.1 of [31]

5 Quantum noise

Sadakazu Haino and Yutaro Enomoto checked quantum noise part. Therefore, Kazuhiro is quite sure that this part never includes any serious or fatal mistakes.

6 Summary

Kazuhiro Yamamoto checked Komori code to evaluate sensitivity of KAGRA (mainly, thermal noise part) and found no mistakes or issues. Kazuhiro thinks that this code is appropriate for KAGRA and must be released widely.

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Code Review for KAGRA latest sensitivity

Inspiral range part

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September 2, 2017

Comment

- I checked the code for computing inspiral range with [1] as a reference.

  There are no difference between matlab code and Eq. (7.181-182) in [1]. I was not able to find the mistakes in the code. I requested to added references for equations, parameters and so on in the matlab code and confirmed modification.

- I checked computational error caused by coarse frequency resolution in integration.

  I recalculated in more fine resolution and checked that error is less than 0.1% ($\sim$ 0.1Mpc at 1.4-1.4$M_\odot$). Higher order harmonics of violin mode vanishes on the new sensitivity because of coarse frequency resolution. However computational cost becomes too large for calculating with fine resolution. We decide to keep current frequency resolution and add comment about accuracy of violin peak in a document.

- I calculated inspiral range using another two codes.

  One is the c-code written by T. Yamamoto and the other is haskell-code written by KAGRA DetChar group [2]. Three independent results are consistent within $\sim$ 0.12% (0.15Mpc at 1.4-1.4$M_\odot$). Difference in the results is caused by integration method.

- In original matlab code, approximate values was used as physical constants and some constant was used without citations. I requested to use correct values in [3–5] and confirmed modification.

- The redshift z was not considered in the matlab code. However I concluded that we need not to consider z because the difference in inspiral range whether to consider z or not is within a few percent. When we want to compute the inspiral range of the future detector such as KAGRA+, current code should be updated because the effect of z on computing inspiral range increases.

- I think this code should be approved.

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Code Review for KAGRA latest sensitivity

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September 2, 2017

Comment

It can be said that this new code just corrects two "errors" of the previous one; one error is in the calculation of suspension thermal noise and the correction for the other "error" is to use a required value for ITM heat absorption. A detailed review report is the following.

Quantum noise

I mainly reviewed the quantum noise part. The calculation code follows Buonanno and Chen (2001; PRD 64 042006). I checked twice the code one term by one term, and I could not find any mistakes. In addition, the quantum noise part of the previous code has been reviewed independently by Haino-san. Therefore I think this part is correct.

Mirror thermal noise

I checked this part one line by one line, and I could not find any errors in equations compared with the references listed in the code. I asked the proposers to put the reference for the thermoelastic noise calculation. Thus I judge this part to be correct.

Suspension thermal noise

I checked also this part one line by one line, and I could not find any errors in the model and equations used here; the model was explained by the proposers. Since I am convinced that the error that existed in the previous code was largely due to an improper treatment of temperature difference among stages of the test mass suspension and the treatment used in the new one mitigates this problem, I judge this part to be correct.