

Thermal noise

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Lecture about gravitational wave detector for fresh persons

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References

Peter R. Saulson,

Thermal noise in mechanical experiments,

Physical Review D 42 (1990) 2437.

Good review before revolution

(on the end of 20th century)

S. Rowan, J. Hough, and D.R.M. Crooks,

Thermal noise and material issues for

gravitational wave detectors

Physics Letters A 347 (2005) 25.

**One of the special articles for 100th anniversary of
Annus Mirabilis (year of miracle) of A. Einstein**

References

Erick D. Black

Notes on thermal noise, with a bibliography

LIGO-T030142- 01- R (2004)

<http://www.ligo.caltech.edu/~blacke/T030142-01.pdf>

From Brown and Einstein to Yamamoto

G.M. Harry, T. Bodiya, and R. DeSalvo (Editors)

***Optical Coatings and Thermal Noise in Precision
Measurements***

Cambridge University Press, Cambridge (2012)

0. Abstract

I would like to explain ...

(1) History until **Fluctuation-Dissipation Theorem**

What is the Fluctuation-Dissipation Theorem ?

(2) Thermal noise of **interferometric** gravitational wave **detector before revolution** (drastic progress in research of thermal noise) on the end of 20th century

(3) Thermal noise of **interferometric** gravitational wave **detector after revolution** on the end of 20th century

Contents

- 1. Until Fluctuation-Dissipation Theorem***
- 2. Interferometer before revolution***
- 3. Interferometer after revolution***
- 4. Summary***

1. *Until Fluctuation-Dissipation Theorem*

Robert **Brown** investigated **random motion** of **small particles** ($\sim 1 \mu\text{m}$) **in water**.

R. Brown, Philosophical Magazine 4 (**1828**) 161.

At first, he thought that this motion of particles from pollens stems from **activity of life**. However, he discovered that particles from **inorganic** materials also move at random.

Trivia : R. Brown observed motion of small particles from pollens, **not pollens themselves** ! Since pollens are too large ($25 \mu\text{m} \sim 100 \mu\text{m}$), it is difficult to observe Brownian motion.

1. *Until Fluctuation-Dissipation Theorem*

Robert **Brown** investigated **random motion** of **small particles** ($\sim 1 \mu\text{m}$) **in water**.

R. Brown, Philosophical Magazine 4 (**1828**) 161.

Mechanism was **unknown**.

Many **ideas** were proposed and **rejected**.

Random **collisions with atoms** of water ?

For example,

G. Cantoni, Nuovo Ciment 27 (**1867**) 156.

J. Delsaulx

They are **not proof** but guesses.

1. *Until Fluctuation-Dissipation Theorem*

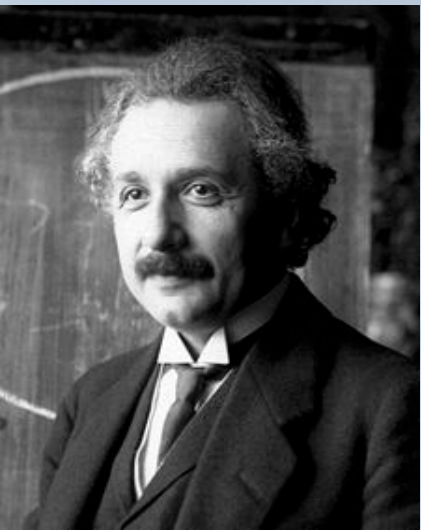
Albert **Einstein** showed **theory** of Brownian motion.

A. Einstein, Annalen der Physik 17 (**1905**) 549.

Why is so important this result ?

(1) Evidence of existence of atom

Avogadro constant derived from observation and his theory is consistent with those from other methods.



1. Until Fluctuation-Dissipation Theorem

(2) **Relation** between **diffusion** (thermal motion) of particles and **viscosity** (dissipation) of water

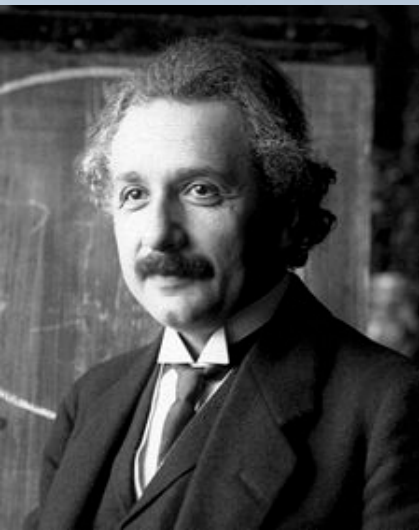
He assumed that the law of physics of **macroscopic** body is the **same** as that of **microscopic** one.

Einstein's relation **diffusion**

$$D = \frac{RT}{N} \frac{1}{6\pi\eta a}$$

Avogadro constant

viscosity



1. Until Fluctuation-Dissipation Theorem

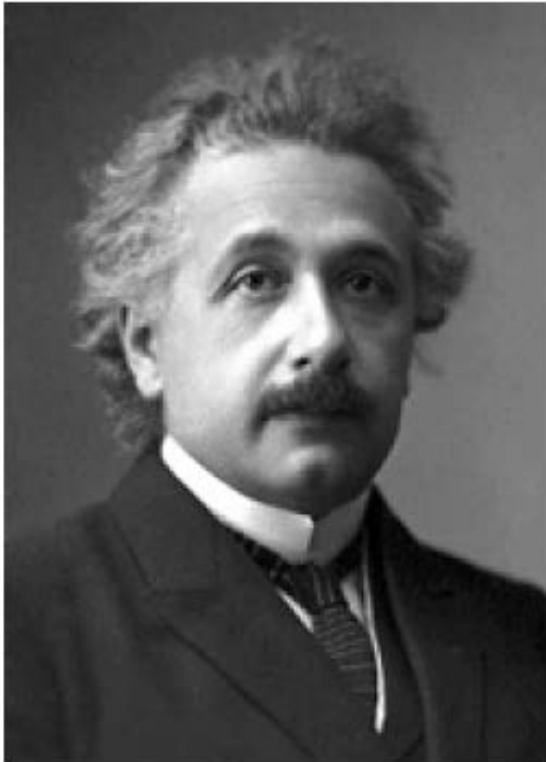
Jean Baptiste **Perrin**'s **experiment** proved that **Einstein**'s theory is **correct**.

J. Perrin, Ann. Chim. Phys. 18 (**1909**) 1.

Perrin checked and **confirmed Einstein's assumption** (the law of physics of **macroscopic** body is the **same** as that of **microscopic** one) experimentally.

Perrin observed Brownian motion and derived Avogadro constant using Einstein's theory. The result is consistent with those of other methods.

1. *Until Fluctuation-Dissipation Theorem*



Albert Einstein

The Nobel Prize in Physics 1921 was awarded to Albert Einstein *"for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect"*.

1. Until Fluctuation-Dissipation Theorem

Presentation speech of **Nobel** prize in Physics **1921**

(Laureate is A. Einstein)

Throughout the first decade of this century the so-called **Brownian movement** stimulated the keenest interest. In 1905 **Einstein founded a kinetic theory to account for this movement** by means of which he derived the chief properties of suspensions, i.e. liquids with solid particles suspended in them. This theory, based on classical mechanics, helps to explain the behaviour of what are known as colloidal solutions, a behaviour which has been studied by Svedberg, **Perrin**, Zsigmondy and countless other scientists within the context of what has grown into a large branch of science, colloid chemistry.

1. *Until Fluctuation-Dissipation Theorem*



The Nobel Prize in Physics 1926

"for his work on the discontinuous structure of matter, and especially for his discovery of sedimentation equilibrium"



Jean Baptiste Perrin

Web site of Nobel foundation

1. *Until Fluctuation-Dissipation Theorem*

Thermal fluctuation of **electrical voltage** (or **current**)

J.B. Johnson, Physical Review 32 (**1928**) 97.

Measurement

H. Nyquist, Physical Review 32 (**1928**) 110.

Theory

Nyquist's theorem

$$G_V = 4k_B T R$$



Relation between **electrical voltage fluctuation** and **resistance**

1. *Until Fluctuation-Dissipation Theorem*

Thermal fluctuation of **electrical voltage** (or **current**)

J.B. Johnson, Physical Review 32 (1928) 97.

He measured thermal current of resistance using (resonant type or band pass type) amplifier.

The significance of the mathematical expression for the effect will be developed with the aid of the generalized circuit diagram of Fig. 1. Z is the conductor under investigation, A the amplifier to which it is connected, J the thermocouple ammeter. The amplifier A is characterized by a complex

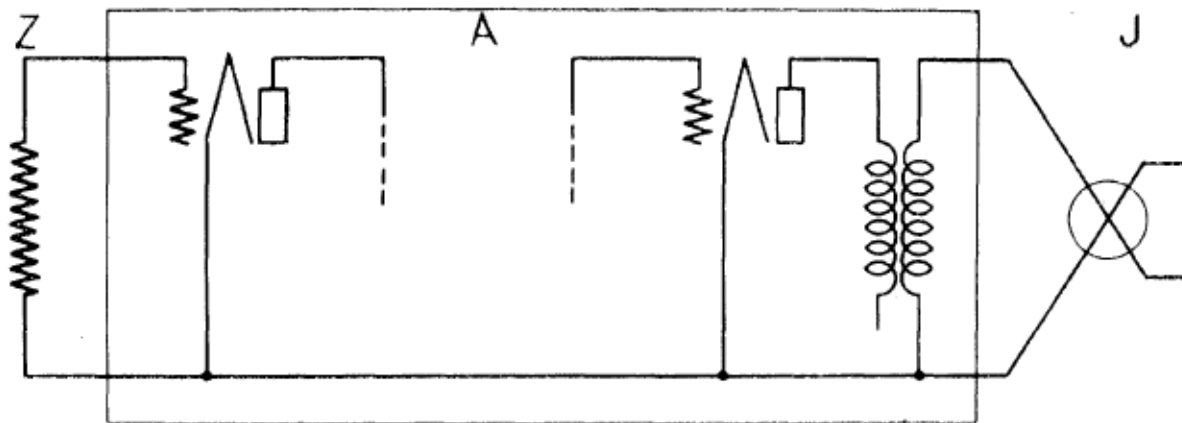


Fig. 1. Simplified diagram of the circuit.

1. Until Fluctuation-Dissipation Theorem

Thermal fluctuation of **electrical voltage** (or **current**)

J.B. Johnson, Physical Review 32 (1928) 97.

He confirmed formula for thermal fluctuation.

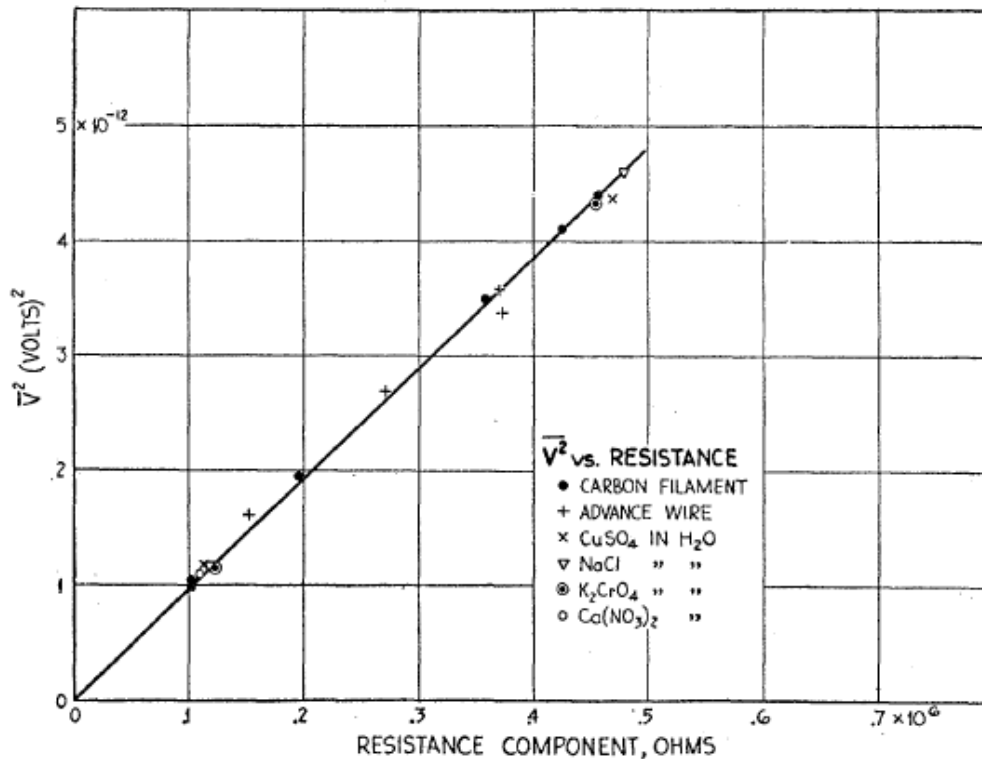


Fig. 4. Voltage-squared vs. resistance component for various kinds of conductors.

1. Until Fluctuation-Dissipation Theorem

Typical thermal voltage fluctuation (100 ohm, 300 K)

$$1.3 \text{ nV} / \sqrt{\text{Hz}}$$

Typical thermal current fluctuation (100 ohm, 300 K)

$$13 \text{ pA} / \sqrt{\text{Hz}}$$

1. *Until Fluctuation-Dissipation Theorem*

Thermal fluctuation of **electrical voltage** (or **current**)

H. Nyquist, Physical Review 32 (**1928**) 110.

His theory is based on

(1) Principle of energy equipartition

(2) Assumption that **ohm law is correct**

even if we consider **voltage (current) fluctuation**.

(Law for **small fluctuation**

is the **same** as that of **macroscopic** voltage or current)

This assumption is **similar to Einstein's**.

1. Until Fluctuation-Dissipation Theorem

Trivia

We can find three technical terms named after Nyquist.

Nyquist's Theorem : Thermal noise

Nyquist criterion of stability : Stability of control

Nyquist sampling theorem : Sampling rate of measurement



Are all of them work by same person ?

The answer is yes !

1. Until Fluctuation-Dissipation Theorem

Thermal fluctuation of **mechanical harmonic oscillator**

Many people measured and analyzed fluctuation of angle of **torsion pendulum** using optical lever **around 1925**.

W. Einthoven et al., Physica 5 (**1925**) 358.

J. Tinbergen, Physica 5 (**1925**) 361.

W.J.H. Moll et al., Philosophical Magazine 50 (**1925**) 626.

G. Ising, Philosophical Magazine 1 (**1926**) 827.

F. Zernike, Zeitschrift fuer Physik 40 (**1926**) 628.

A.V. Hill, Journal of Scientific Instruments 4 (**1926**) 72.

Probably, this is **not perfect** list.

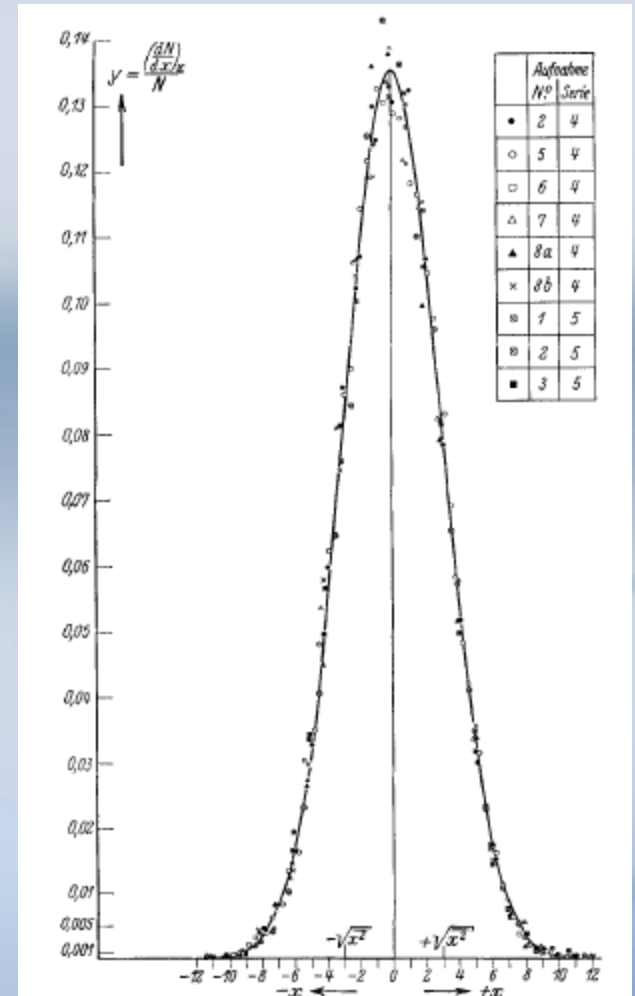
1. Until Fluctuation-Dissipation Theorem

Thermal fluctuation of **mechanical harmonic oscillator**

E. Kappler, Annalen der Physik
11-3 (1931) 233.

Torsion pendulum

He evaluated Avogadro constant and it is consistent with those of other experiment.



Experimentelle Verteilungskurve aus 9 Schwankungsaufnahmen von durchschnittlich je 11-stündiger Beobachtungsdauer

1. Until Fluctuation-Dissipation Theorem

Onsager reciprocity theorem

L. Onsager, Physical Review 37 (1931) 405.

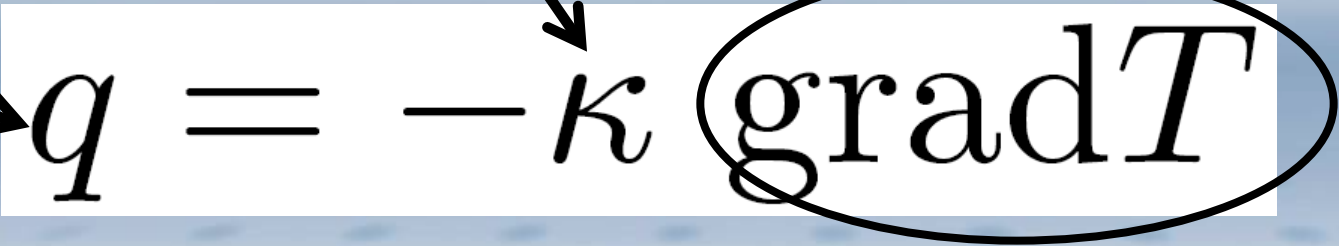
Fourier's law

Thermal conductivity

Heat flow \rightarrow

$$q = -\kappa \text{ grad} T$$

Temperature gradient



In general case, κ is tensor.

According to Onsager reciprocity theorem, this tensor **should be symmetric** even if the **material is not isotropic** (like sapphire!).

1. *Until Fluctuation-Dissipation Theorem*

Onsager reciprocity theorem

L. Onsager, Physical Review 37 (1931) 405.

Onsager's assumption

(1) Microscopic reversibility

Symmetry of cross correlation function

in time reflection

$$\langle \alpha_1(t) \alpha_2(t + \tau) \rangle = \langle \alpha_1(t) \alpha_2(t - \tau) \rangle$$

$$\langle \alpha_1(t) \alpha_2(t + \tau) \rangle = \langle \alpha_2(t) \alpha_1(t + \tau) \rangle$$

1. *Until Fluctuation-Dissipation Theorem*

Onsager reciprocity theorem

L. Onsager, Physical Review 37 (1931) 405.

Onsager's assumption

(2) The **average decay of fluctuations** will obey the **ordinary laws**.

Law for **average decay of small fluctuation** is the **same** as that of **macroscopic** motion (ordinary law).

This assumption is **similar to Einstein's and Nyquist's**.

1. *Until Fluctuation-Dissipation Theorem*



Lars Onsager

The Nobel Prize in Chemistry 1968 was awarded to Lars Onsager *"for the discovery of the reciprocal relations bearing his name, which are fundamental for the thermodynamics of irreversible processes"*.

Web site of Nobel foundation

1. *Until Fluctuation-Dissipation Theorem*

Finally, **general theorem** appeared.

Fluctuation-Dissipation Theorem (FDT)

H.B. Callen and R.F. Greene, Physical Review 86 (**1952**) 702.

R.F. Greene and H.B. Callen, Physical Review 88 (**1952**) 1387.

Relation between **thermal fluctuation** and **dissipation**

Fluctuation : Energy **from** heat bath

Dissipation : Energy **to** heat bath

Interaction between system and heat bath

1. Until Fluctuation-Dissipation Theorem

Fluctuation-Dissipation Theorem is **valid if**

(1) system is **linear**.

(2) system is in **thermal equilibrium**.

$$G(f) = -\frac{4k_B T}{\omega} \text{Im}[H(\omega)]$$

Power spectrum
of thermal **fluctuation**

Imaginary part of susceptibility
(**dissipation**)

1. *Until Fluctuation-Dissipation Theorem*

Fluctuation-Dissipation Theorem is **valid if**

(1) system is **linear**.

(2) system is in **thermal equilibrium**.

$$G_{ij}(f) = -\frac{4k_B T}{\omega} \text{Im}[H_{ij}(\omega)]$$

↑
Cross correlation spectrum
of thermal **fluctuation**

1. *Until Fluctuation-Dissipation Theorem*

(a) Einstein's relation

Relation between **Brownian motion (fluctuation)**
and **viscosity (dissipation)** of water

FDT in the case with free mass
with viscous damping at low frequency

(b) Nyquist's theorem

Relation between **thermal voltage fluctuation**
and **resistance (dissipation)**

FDT in electric circuit

(c) Onsager reciprocity theorem

Cross correlation spectrum at low frequency in FDT

All these formulae are examples of FDT !

1. *Until Fluctuation-Dissipation Theorem*

Assumption of fluctuation dissipation theorem

Onsager's assumption

The **average decay** of **fluctuations** will obey the **ordinary laws**.

Law for **average** of **small fluctuation** is the **same** as that of **macroscopic** motion with **dissipation**.

Relation between fluctuation and dissipation is **assumed**, not proved !

1. *Until Fluctuation-Dissipation Theorem*

Fluctuation : Energy **from** heat bath

Dissipation : Energy **to** heat bath

Interaction between system and heat bath

In the case of Brownian motion

Fluctuation : **Random collision** of atoms

Dissipation : Collision of atoms

Even **in dissipation**, collision is at **random**.

In some cases of dissipation process, atoms give particle energy. However, in **average**, particle gives atoms energy.

Therefore, the **dissipation** process is the **average of fluctuation**.



Onsager's assumption

1. *Until Fluctuation-Dissipation Theorem*

How to derive fluctuation dissipation theorem ?

Onsager's assumption implies that **time development of auto (or cross) correlation function of thermal fluctuation is the same** as that of **step function response** which is the **decay motion to new equilibrium position** after applied constant force vanished.

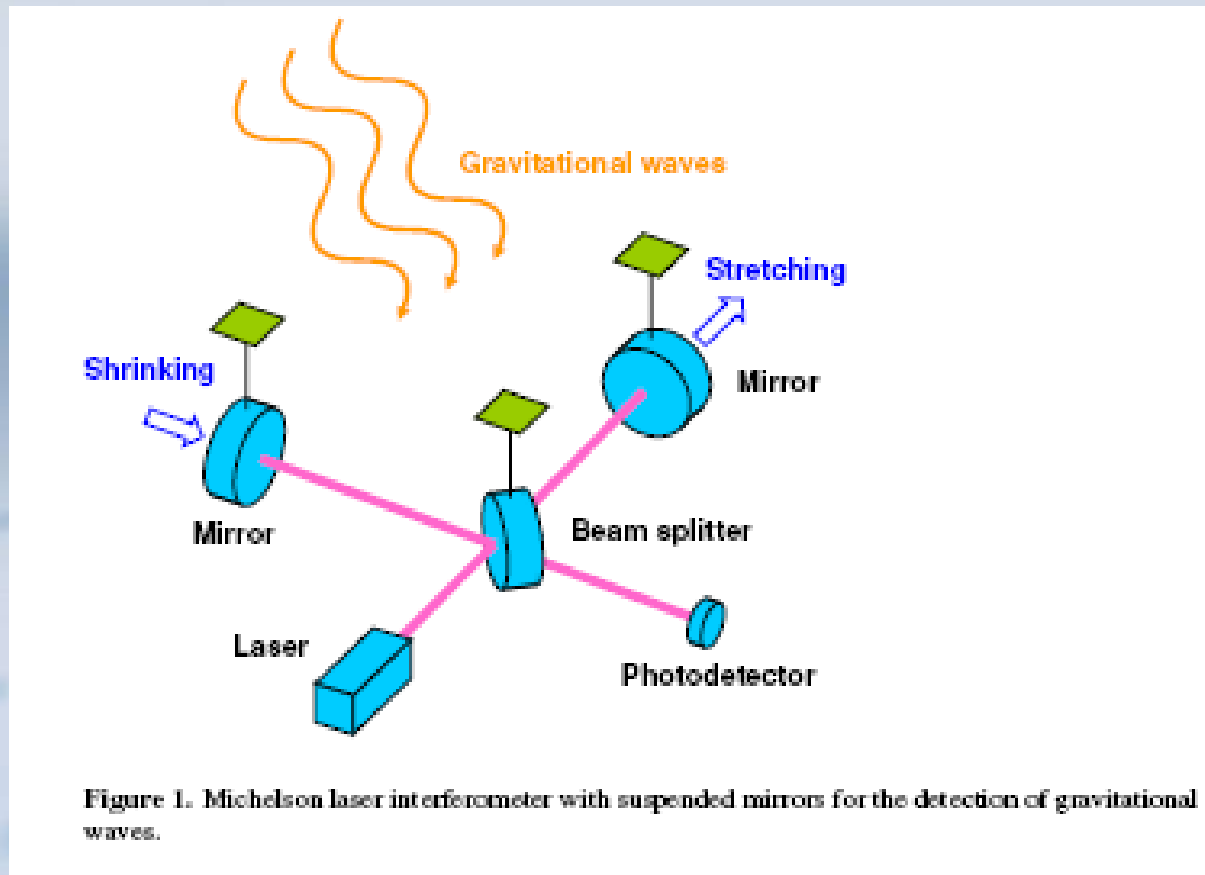
The **amplitude** of auto correlation function is derived from principle of **energy equipartition**.

Power (or cross correlation) spectrum is Fourier transform of auto (or cross) correlation function.
Wiener-Khinchin relation

2. Interferometer before revolution

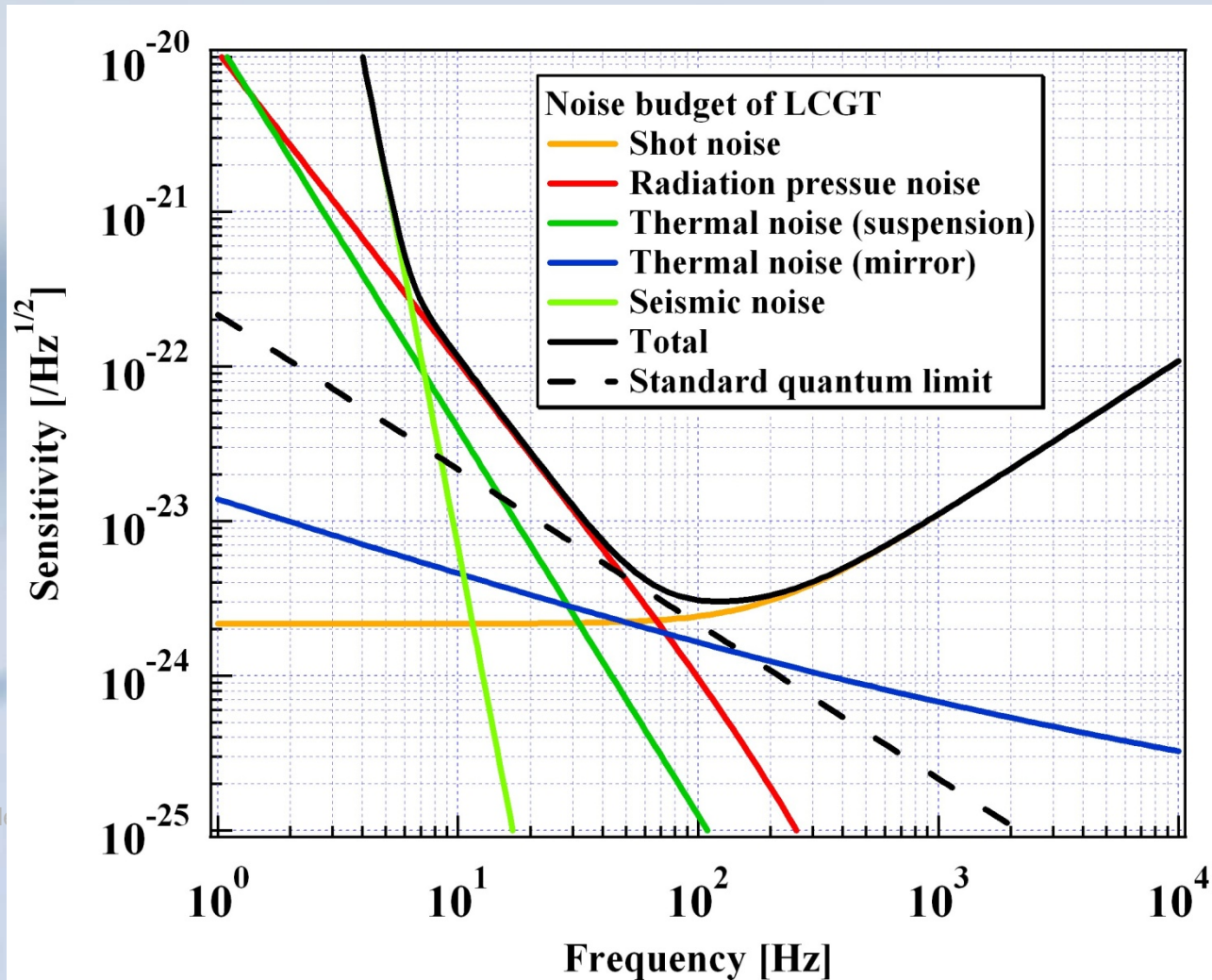
Interferometric gravitational wave detector

Mirrors must be **free** and are **suspended**.



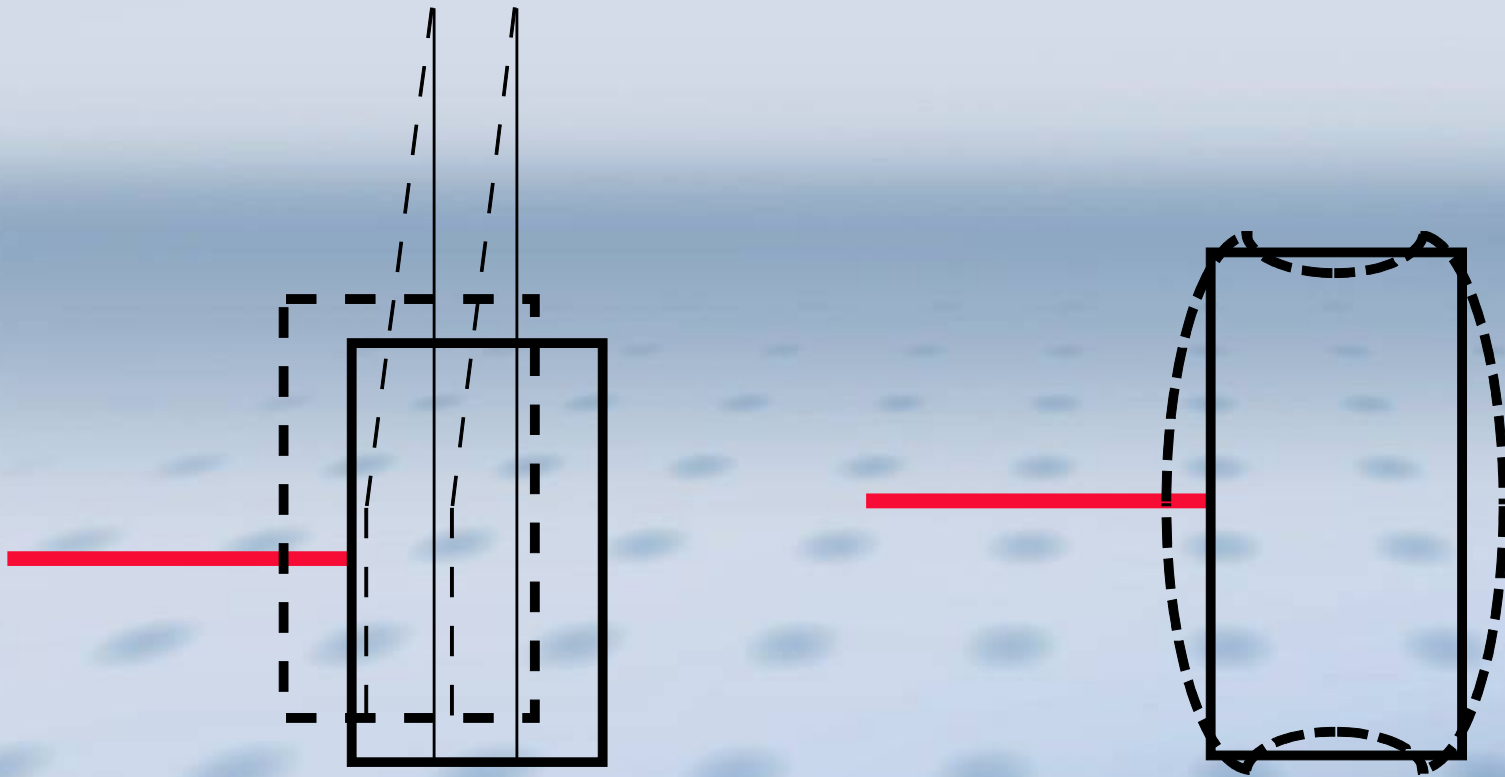
2. Interferometer before revolution

Typical example of sensitivity of interferometer
(Old version of KAGRA(LCGT))



2. Interferometer before revolution

Thermal noise of **suspension** and **mirror**



2. Interferometer before revolution

Suspension and mirror : Mechanical harmonic oscillator

Resonant frequency : suspension : ~ 1 Hz

mirror : > 10 kHz

Target frequency of gravitational wave : ~ 100 Hz

Off resonance thermal fluctuation of displacement

Residual gas damping is not a problem because interferometer in vacuum ($< 10^{-7}$ mbar).

Mechanical loss in suspension and mirror is crucial.

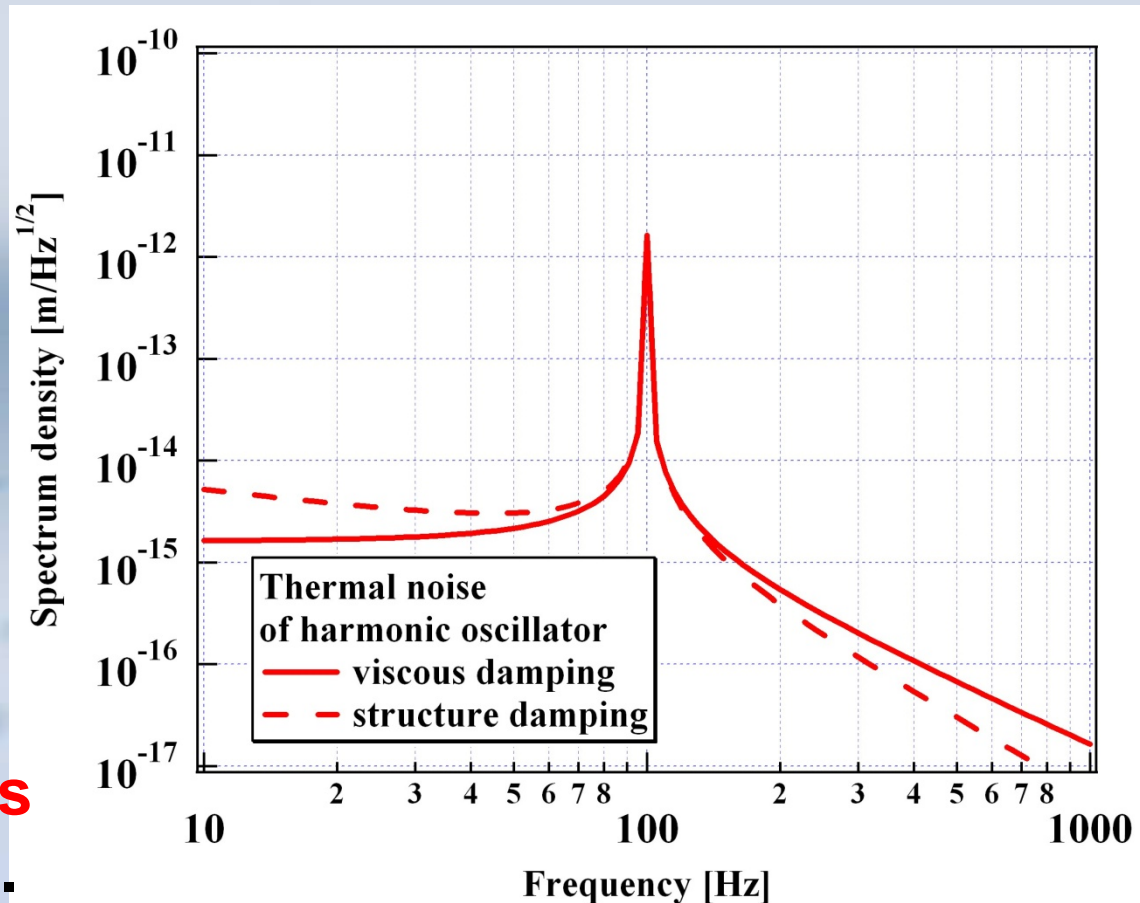
2. Interferometer before revolution

Spectrum density of **thermal noise** of **harmonic oscillator**

Viscous damping :
Friction force is
proportional
to **velocity**.

Structure damping :
Loss is
independent
of **frequency**.

Loss in many materials
are **structure** damping.



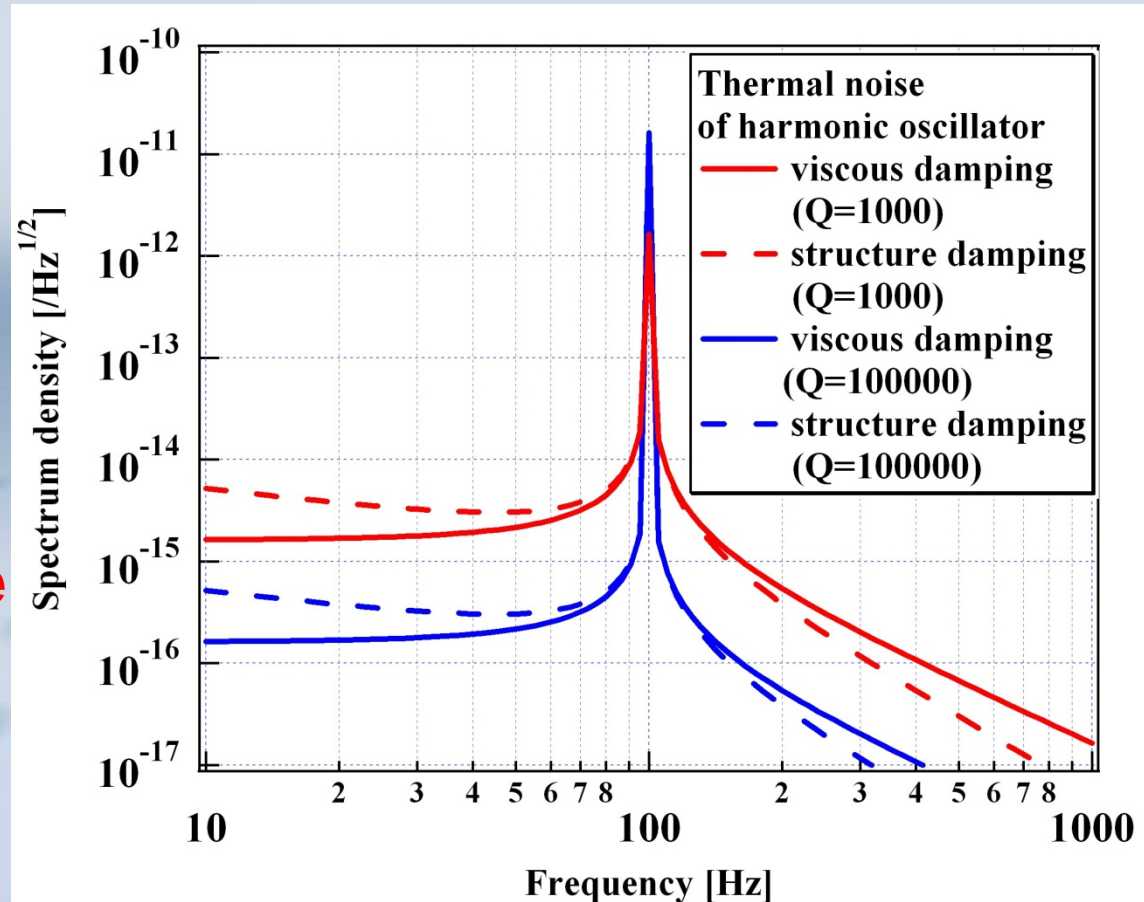
2. Interferometer before revolution

Spectrum density of **thermal noise** of **harmonic oscillator**

Q-value :
Magnitude of loss

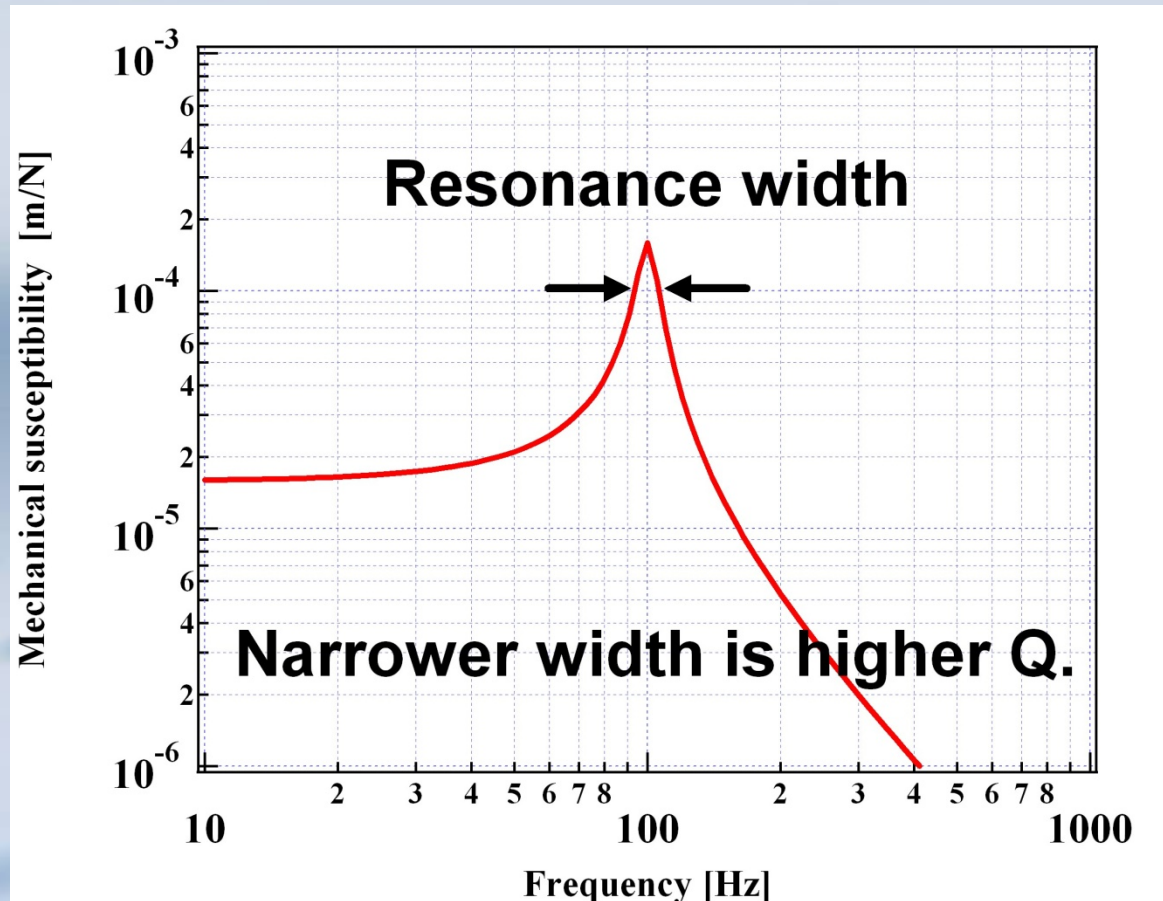
Higher Q implies
smaller loss.

Higher Q means
**smaller off resonance
thermal noise
and better.**



2. Interferometer before revolution

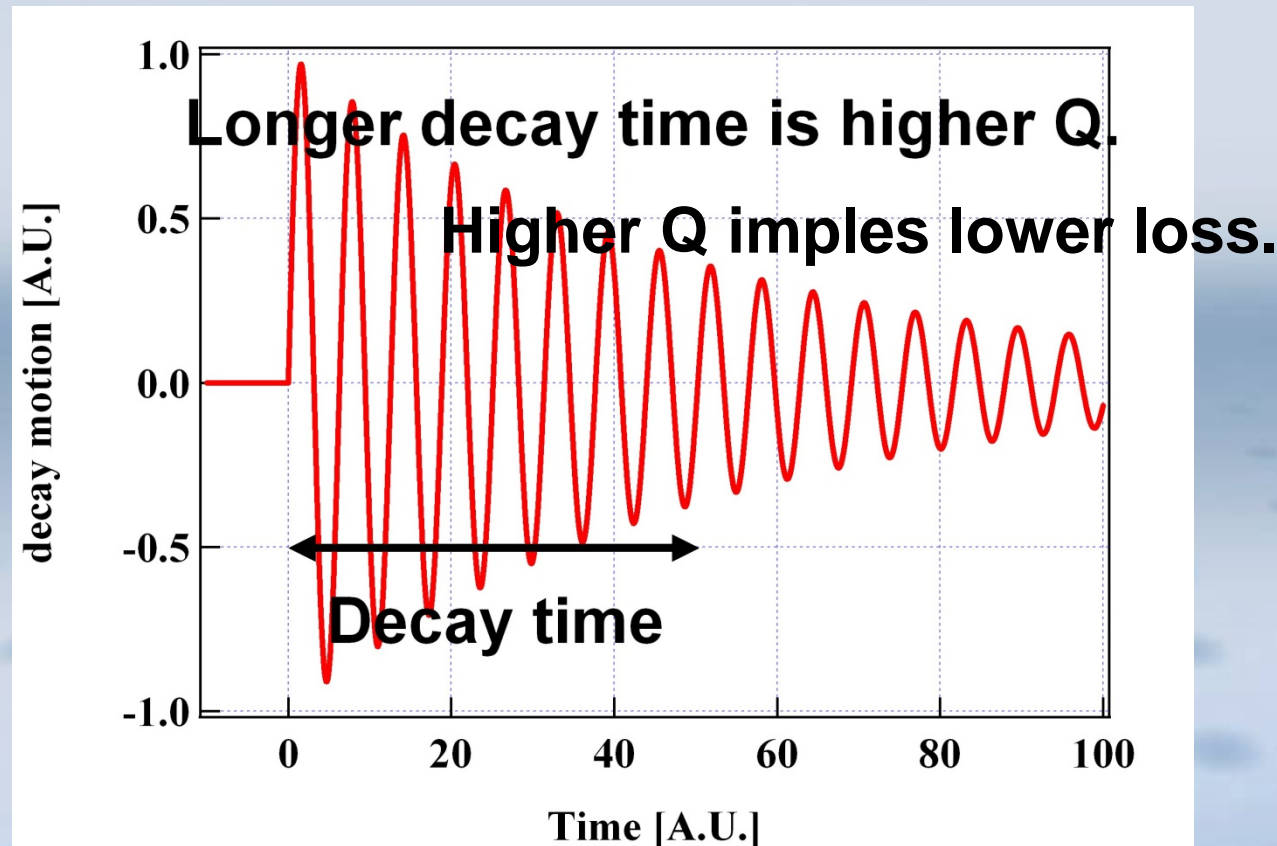
Measurement of Q-value (**Width** of resonance peak)



If Q-value is too **high**, measurement is **difficult**.

2. Interferometer before revolution

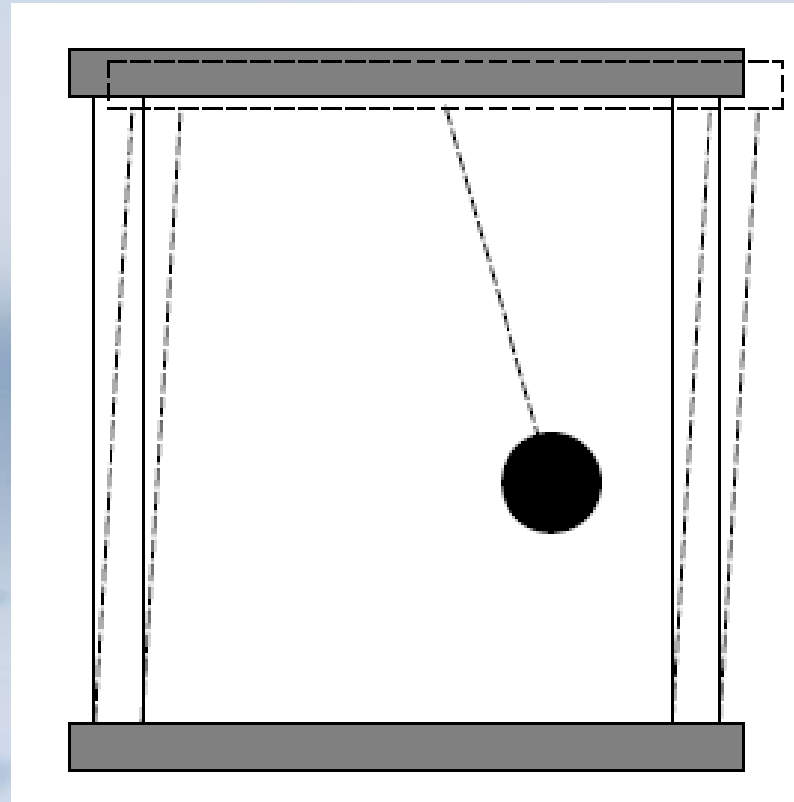
Measurement of Q-value (**Decay time** of resonance motion)



In (our) usual cases, we **adopt** this method.

2. Interferometer before revolution

Recoil loss (problem in measurement of **decay time**)



Contamination of **loss in support** system

Suspension : Rigid and heavy support system

2. Interferometer before revolution

Measurement of Q-value of pendulum
(monolithic fused silica)

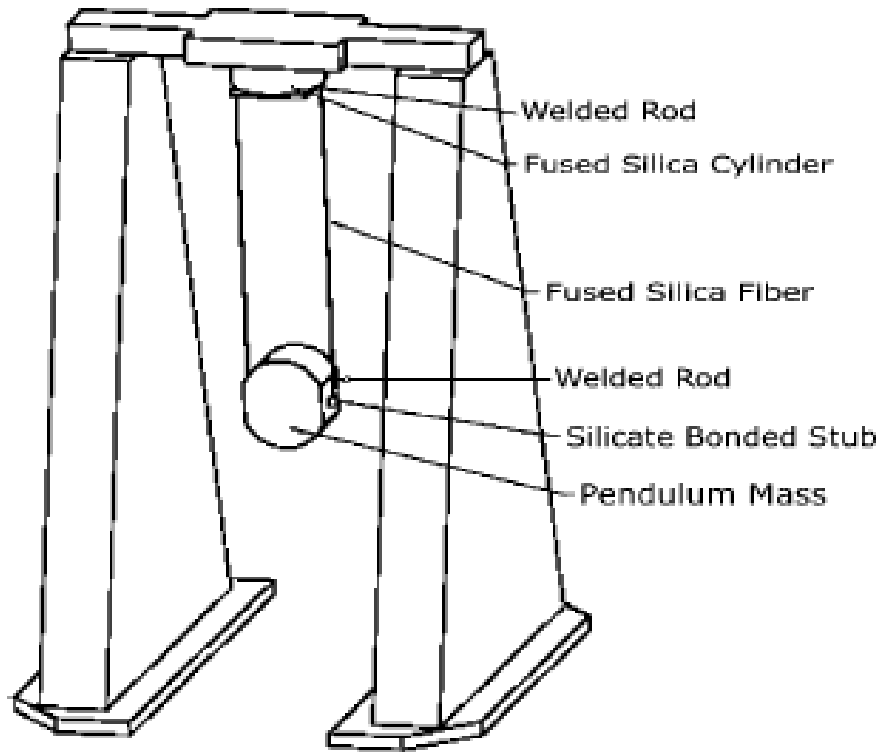


FIG. 1. Sketch of experimental setup.

$$Q = 2.3 * 10^7$$

2. Interferometer before revolution

Measurement of Q-value of **fiber** itself

It is easier, but this Q-value does **not include loss in clamp** and so on. Moreover, ...

Potential energy of total pendulum consists of

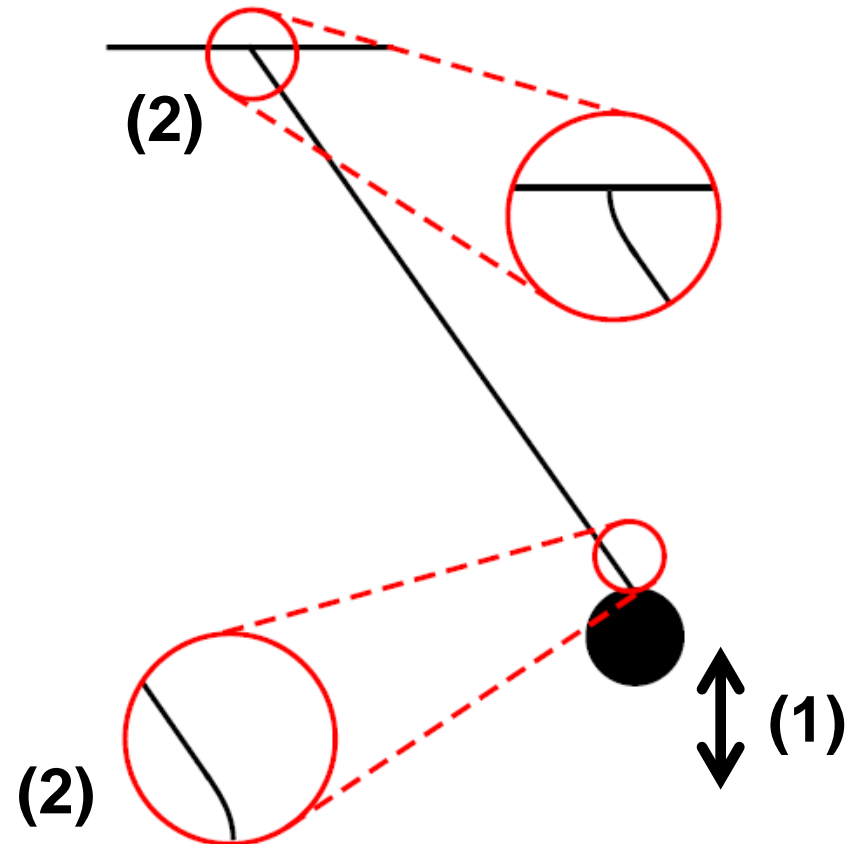
(1) **gravitational** one

(2) **elastic** one

(1) is much **larger** than (2).

But (1) has **no loss**. Only

(2) has loss.



2. Interferometer before revolution

Measurement of Q-value of **fiber** itself

$$\phi_{\text{Pen}} = \frac{E_{\text{elas}}}{E_{\text{grav}} + E_{\text{elas}}} \phi_{\text{elas}} \approx \frac{E_{\text{elas}}}{E_{\text{grav}}} \phi_{\text{elas}}$$

Loss of total pendulum
(Inverse number of Q)

Loss of elastic energy of wire
(Inverse number of Q)

Dilution factor : 10^{-3} or 10^{-4} in typical cases
(Almost unity in KAGRA).

Q-values of **pendulum** is much **higher** than Q-values of wires.

2. Interferometer before revolution

Recoil loss (problem in measurement of **decay time**)

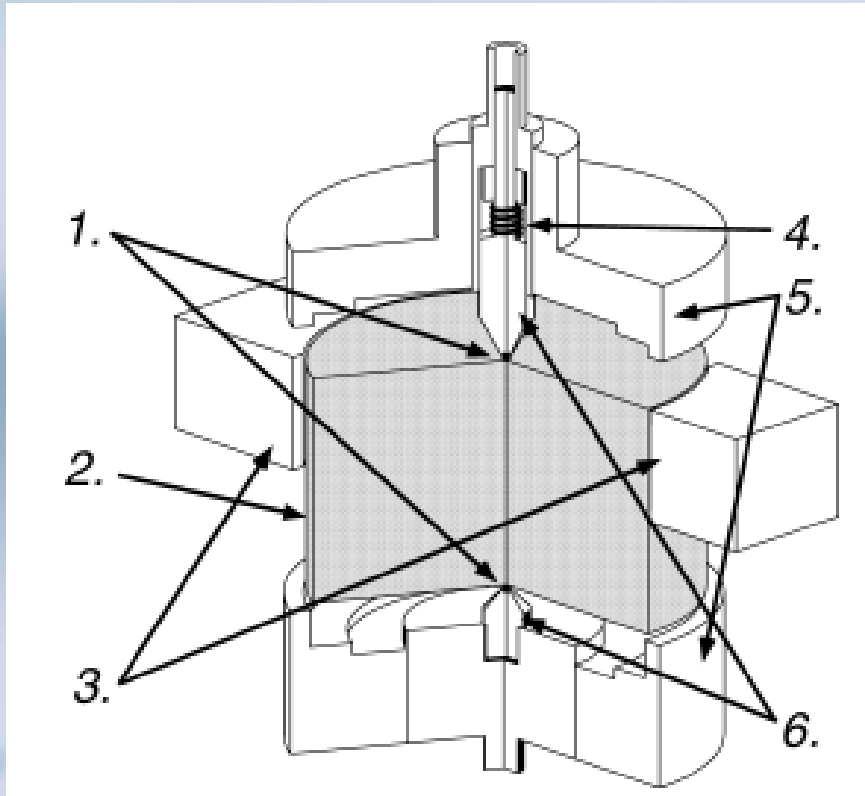


Fig. 1. Cross-section of our support system: samples were supported by two ruby balls at the center of the top and bottom surfaces. 1. *Ruby balls* (diameter 2 mm). 2. *Sample* (diameter 10 cm, height 6 cm). 3. *Electrodes*, sometimes replaced by piezoelectric actuators. 4. *Spring* to provide weak force (about 1N). 5. *Adjusters*. 6. *Supporting rods* (stainless steel).

Mirror : **Nodal support** system

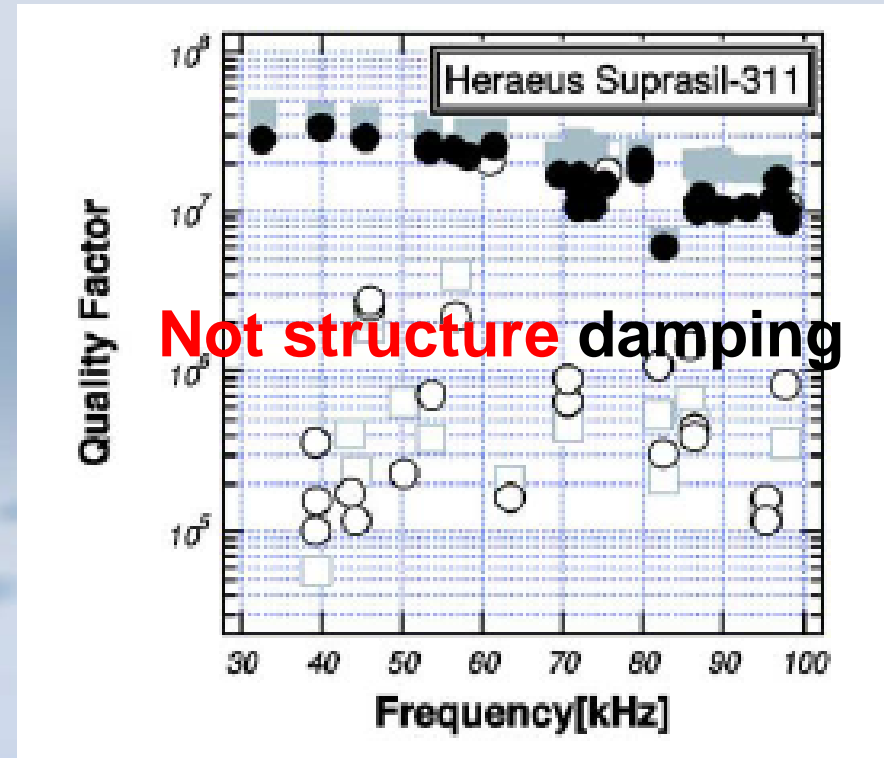
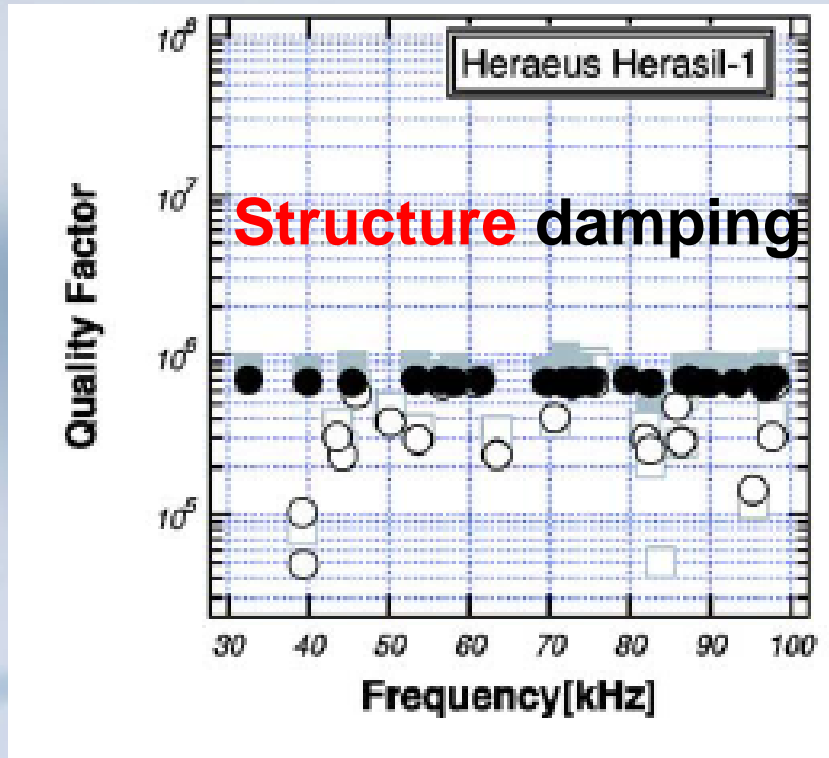
(**Center of flat surface is node** for many modes)

K. Numata *et al.*, Physics Letters A 276 (2000) 37.

2. Interferometer before revolution

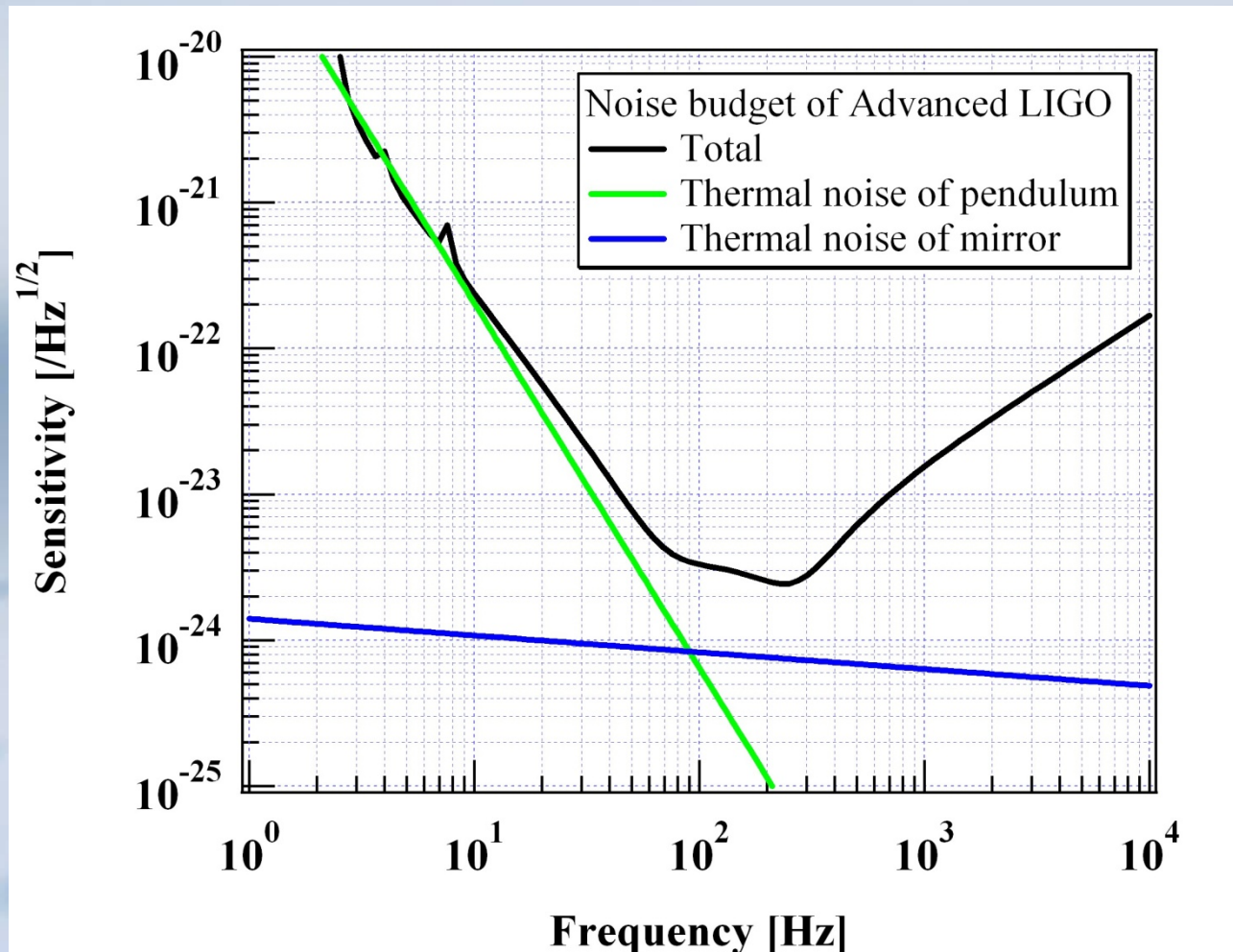
Measurement of Q-value of fused silica mirror

$$Q = 4 * 10^7$$



2. Interferometer before revolution

Evaluated thermal noise based on **Q-value measurement**



This sensitivity is **not latest** one.

3. Interferometer after revolution

(1) Thermoelastic noise of mirror

(2) Thermal noise caused by coating mechanical loss

(3) Reduction of thermal noise

(4) Impact on other fields

3. Interferometer after revolution

(1) Thermoelastic noise of mirror

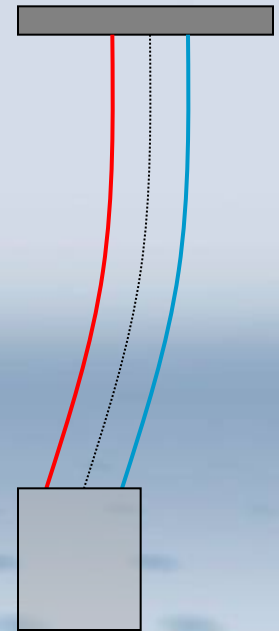
Thermoelastic damping : a kind of loss
Inhomogeneous strain

Thermal expansion coefficient

Temperature gradient

Heat flow

We can **calculate** thermoelastic noise
using **only material properties.**



Drawing by
Tobias Westphal

3. Interferometer after revolution

(1) Thermoelastic noise of mirror

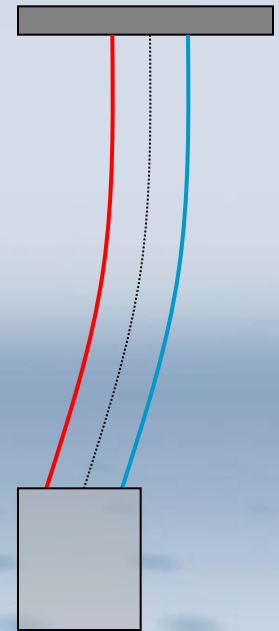
Thermoelastic noise :
thermal noise by thermoelastic damping

Other interpretation

Temperature **fluctuation**

Thermal expansion coefficient

Deformation of elastic body



3. Interferometer after revolution

(1) Thermoelastic noise of mirror

Long, long history of research of thermoelastic damping

1 dimension (bar, ribbon)

C. Zener, Physical Review 52 (1937) 230; 53 (1938) 90.

2 dimension (disk)

L.D. Landau and E.M. Lifshitz

***“The Theory of Elasticity”* , 1953.**

A.S. Nowick and B. Berry

***“Anelastic Relaxation in Crystalline Solids”* , 1972.**

3 dimension (mirror) 1999

3. Interferometer after revolution

(1) Thermoelastic noise of mirror

Thermoelastic noise of 3 dimension mirror without coating

V. B. Braginsky *et al.*, Physics Letters A 264 (1999) 1.

**We can calculate thermoelastic noise
using only substrate material properties.**

This noise is larger than expectation !

3. Interferometer after revolution

(1) Thermoelastic noise of mirror

Fused silica vs. **Sapphire**

Current interferometric gravitational wave detector

Fused silica mirror

Future interferometer

Sapphire was a candidate.

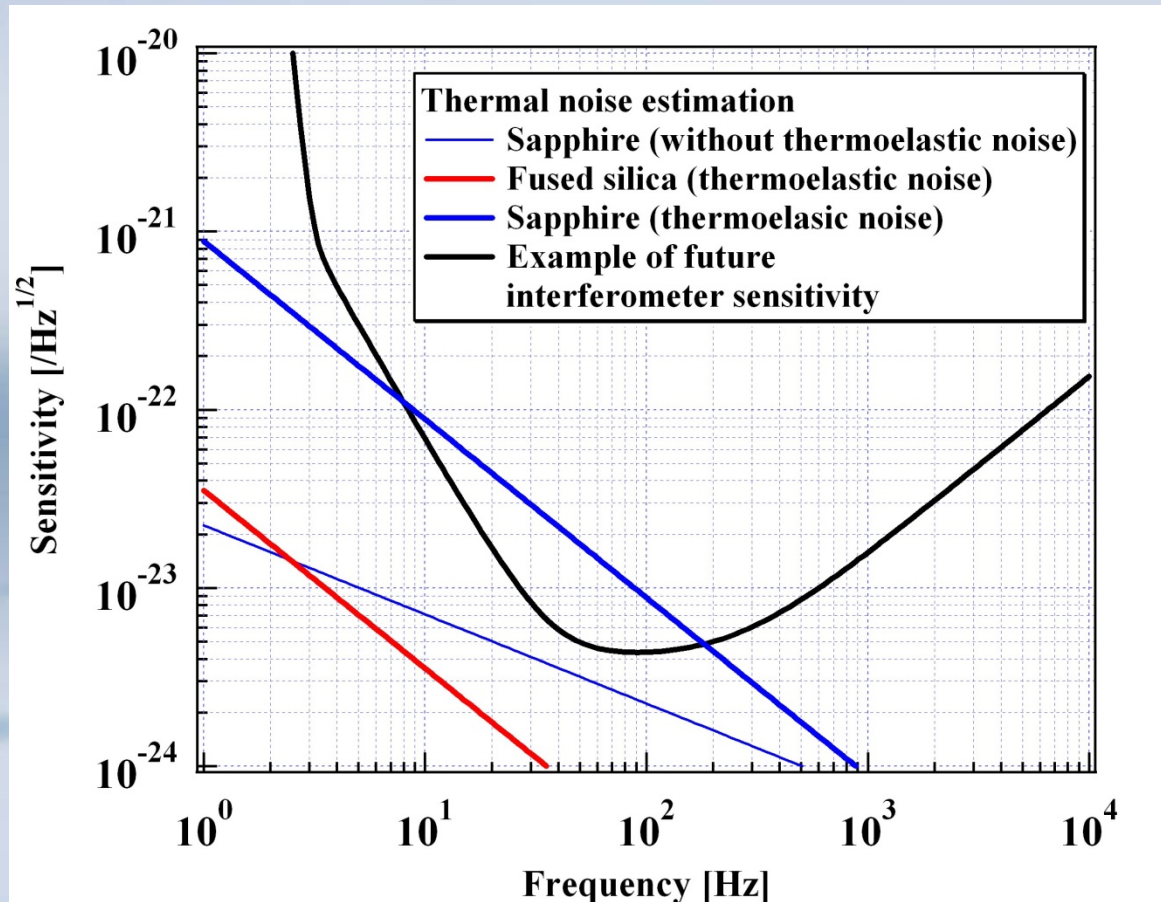
Optical properties of **fused silica** is **better**.

It was expected that thermal noise of **sapphire** is **smaller**.

However ...

3. Interferometer after revolution

(1) Thermoelastic noise of mirror



One of **advantages** of **sapphire** was **lost**.

Future (room temperature) interferometer

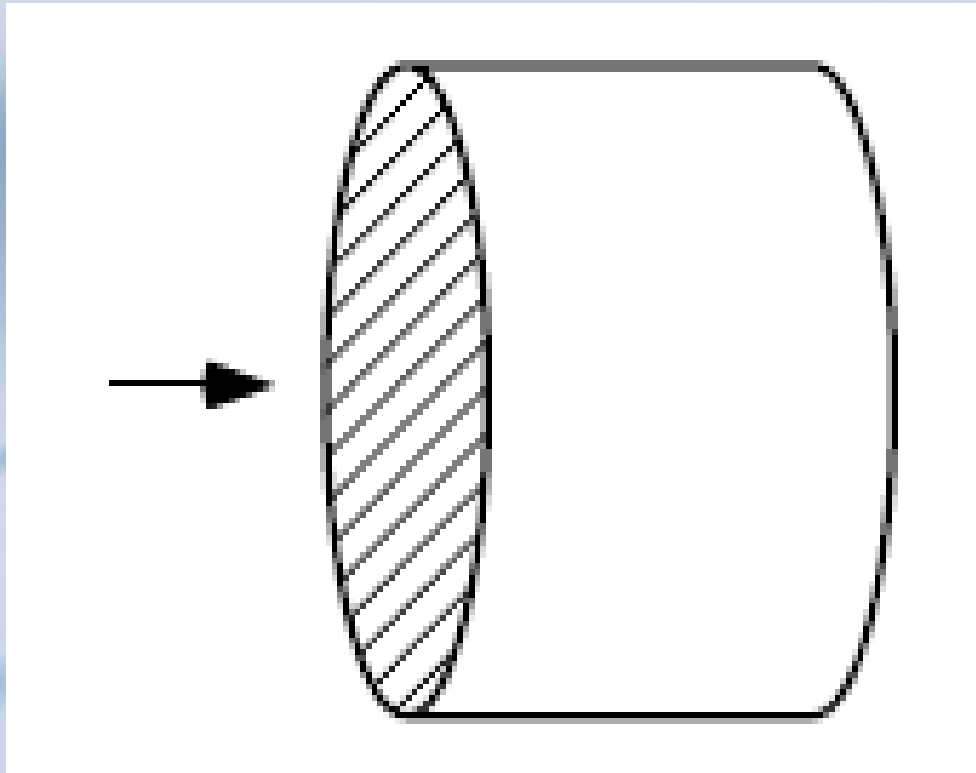
will use **fused silica** mirror. 54

3. Interferometer after revolution

(2) Thermal noise caused by coating mechanical loss

Mirror consists of **not only bulk !**

Reflective coating



3. Interferometer after revolution

(2) Thermal noise caused by coating mechanical loss

Reflective coating : Dielectric multilayer (amorphous) coating for **high reflectivity**

Two kinds of layers

($\text{Ta}_2\text{O}_5/\text{SiO}_2$)

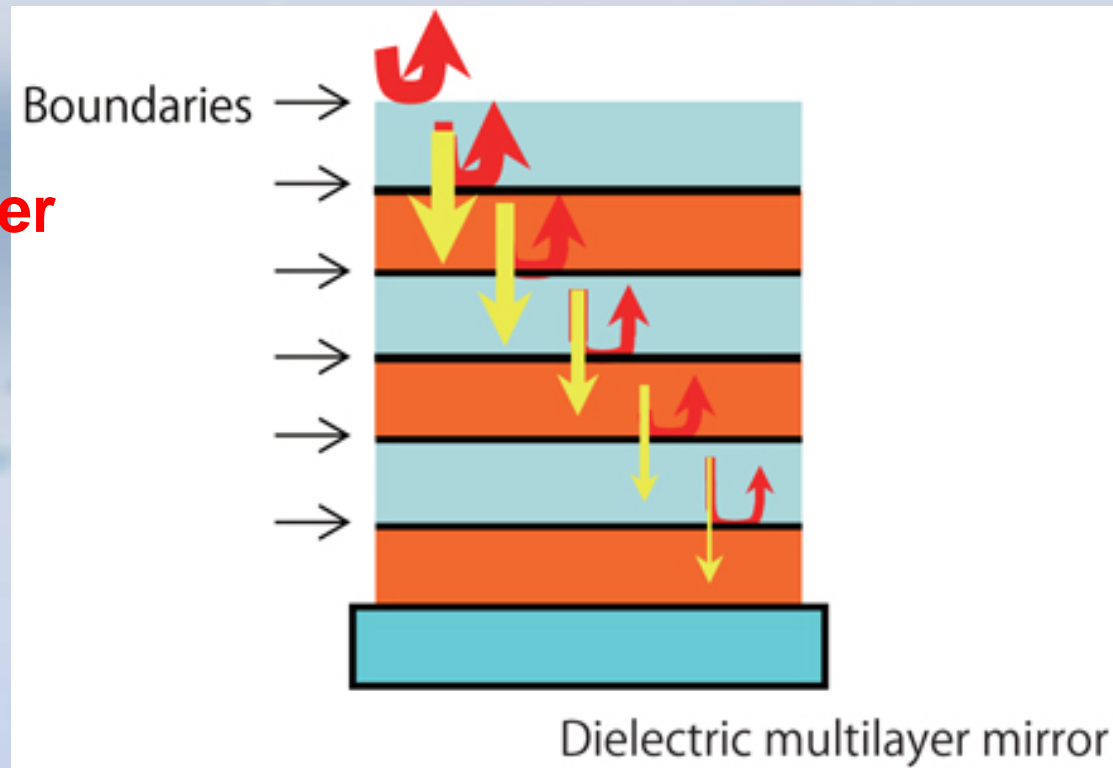
Optical thickness is **quarter of wavelength**

Reflected light :

Constructive interference

Transmitted light :

Destructive interference



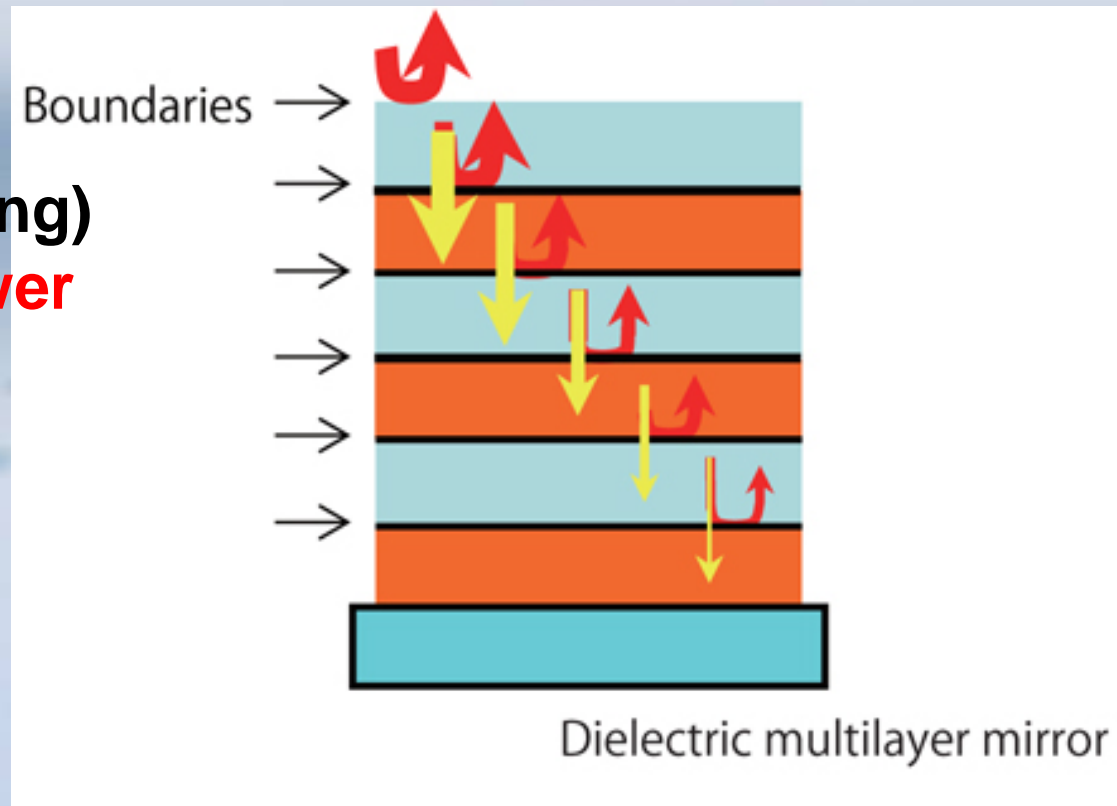
3. Interferometer after revolution

(2) Thermal noise caused by coating mechanical loss

Reflective coating : Dielectric multilayer coating
for **high reflectivity**

Advantages

- (1) **Low optical loss**
(Absorption and scattering)
- (2) **No break with high power**



3. Interferometer after revolution

(2) Thermal noise caused by coating mechanical loss

Mirror consists of **not only bulk** !

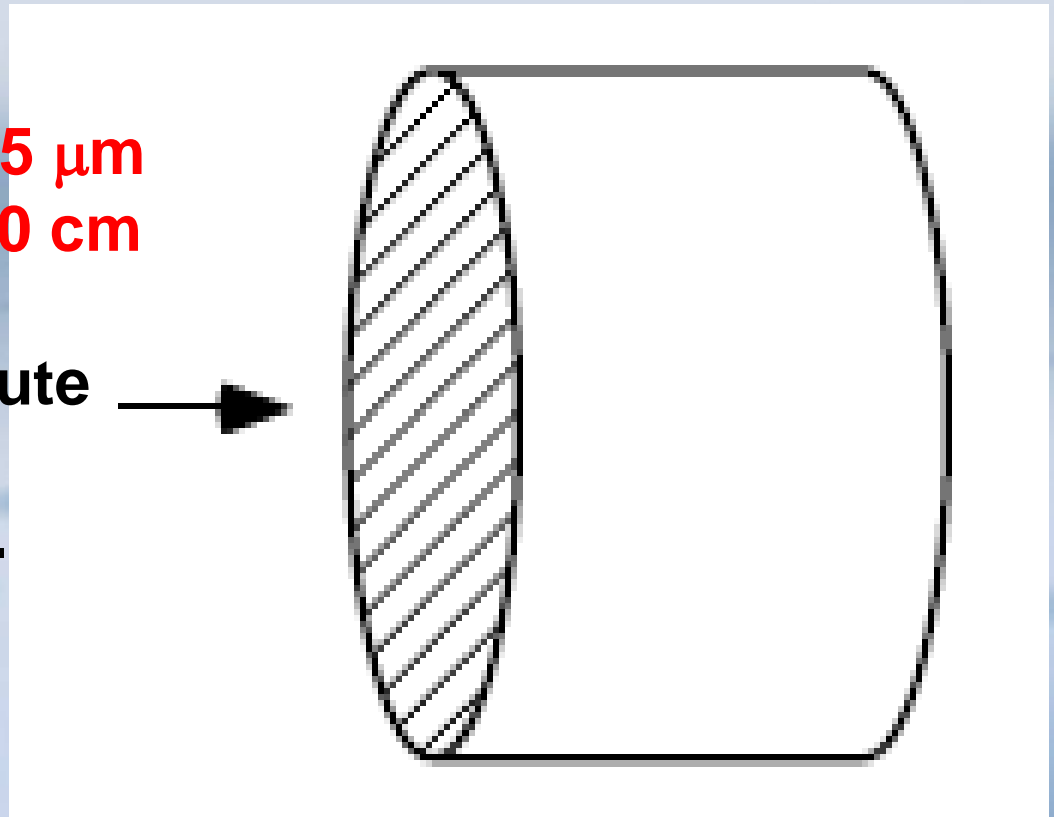
Reflective coating

Thickness of **coating** : ~ 5 μm

Thickness of **mirror** : ~ 10 cm

Why can coating contribute thermal noise ?

Nobody cared until 1998.



3. Interferometer after revolution

(2) Thermal noise caused by coating mechanical loss

Mirror consists of **not only bulk** !

Reflective coating

Thickness of **coating** : ~ **5 μm**

Thickness of **mirror** : ~ **10 cm**

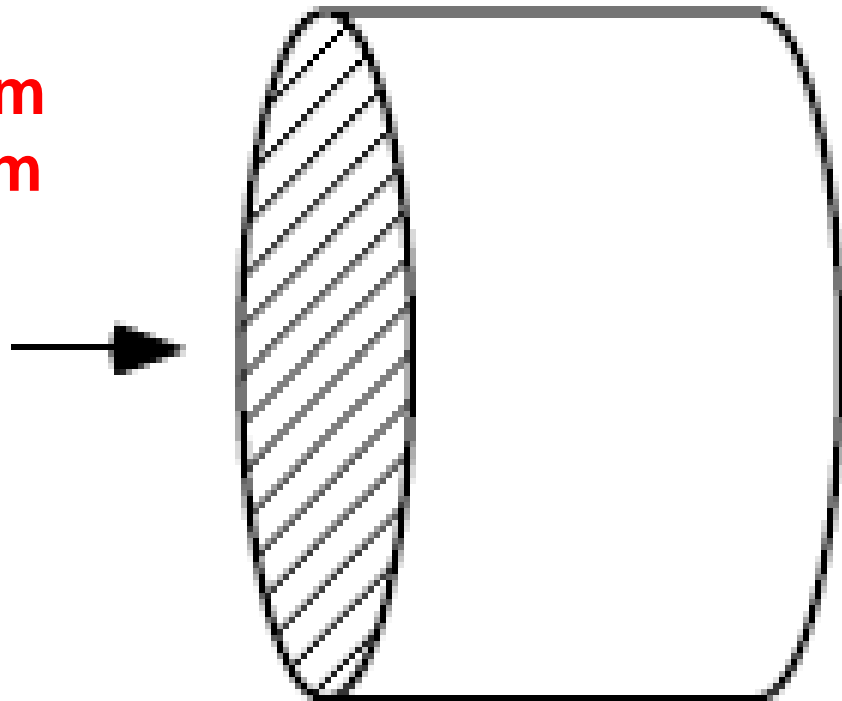
Loss angle ϕ ($1/Q$)

Bulk : 10^{-8}

Coating : 10^{-4}

Comparable ?

This is not end of story ...



3. Interferometer after revolution

(2) Thermal noise caused by coating mechanical loss

Y. Levin, Physical Review D 57 (1998) 659.



This is **qualitative** discussion, but conclusion is amazing.

Q-value of loss at **A** is the **same** as that at **B**.

Loss at A can shake **illuminated surface** more **largely** owing to **conservation of momentum**.

Contribution of loss **near beam spot** is **quite larger** !

3. Interferometer after revolution

(2) Thermal noise caused by coating mechanical loss

Quantitative evaluation

Analytical formula about **coating thermal noise**

G. Harry *et al.*, Classical and Quantum Gravity 19 (2002) 897.

N. Nakagawa *et al.*, Physical Review D 65 (2002) 102001.

Numerical evaluation (Finite Element Method)

K. Yamamoto *et al.*, Physics Letters A 305 (2002) 18.

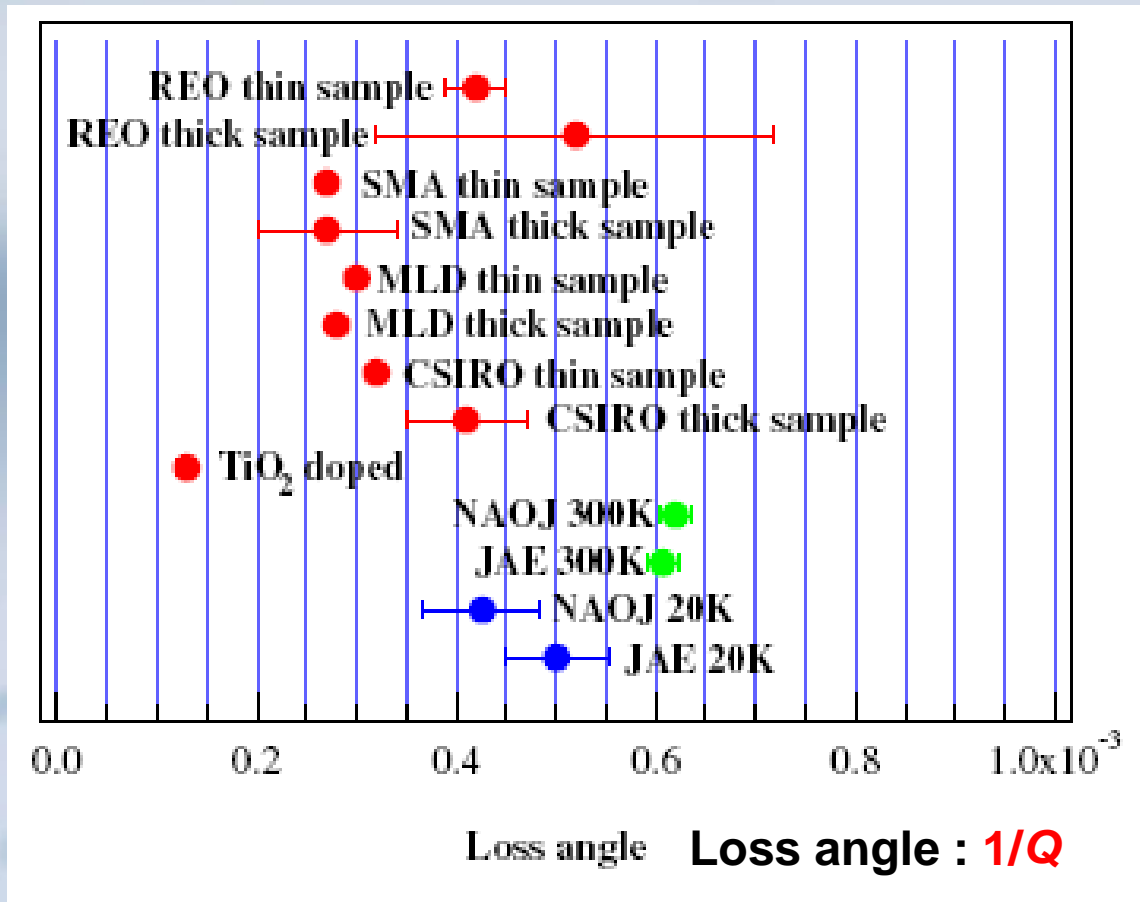
Conclusion

Coating thermal noise is **larger** than bulk thermal noise.

3. Interferometer after revolution

(2) Thermal noise caused by coating mechanical loss

Old summary of coating mechanical loss



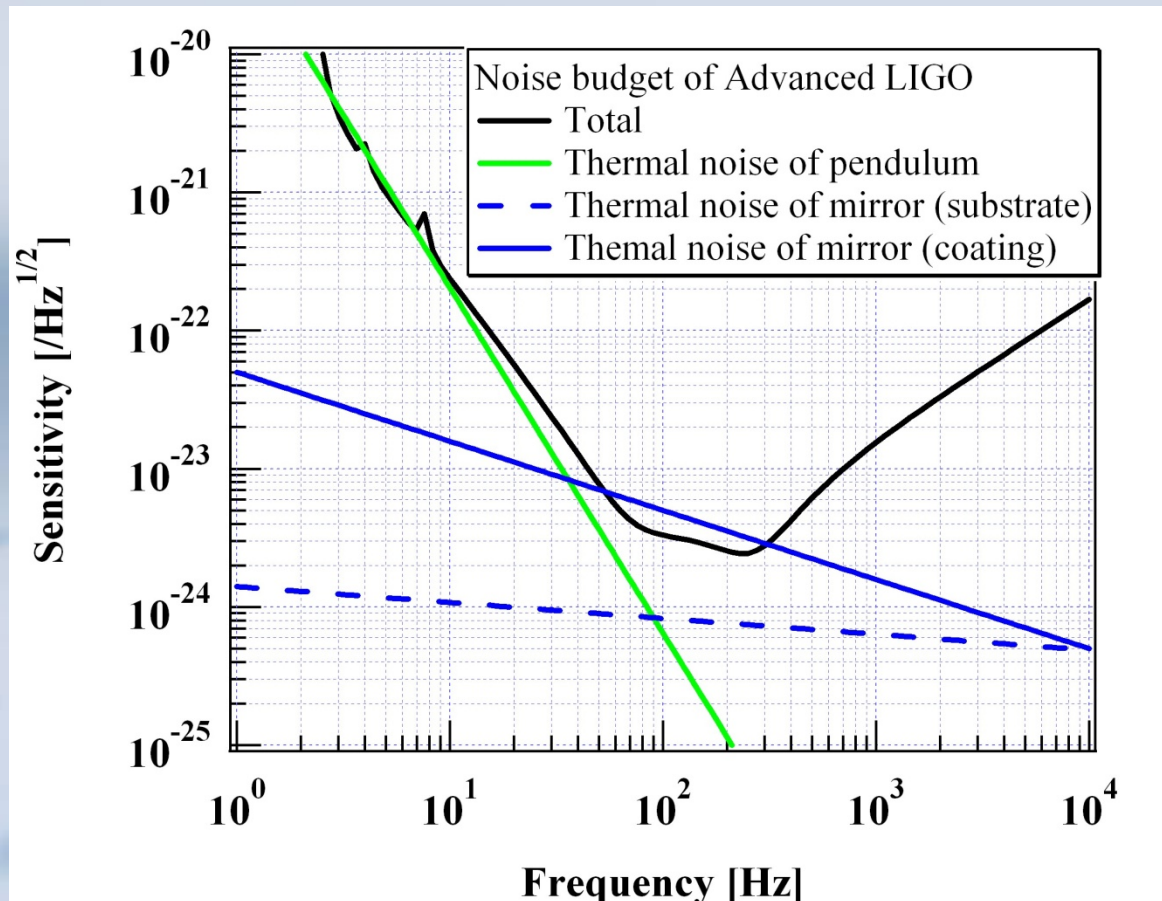
Similar results
(same order
of magnitude)

Structure damping

3. Interferometer after revolution

(2) Thermal noise caused by coating mechanical loss

Evaluated thermal noise based on **measurement**



Coating thermal noise is the most **serious** problem in future! ₆₃

3. Interferometer after revolution

(2) Thermal noise caused by coating mechanical loss

How to reduce coating mechanical loss

So far, no silver bullet ...

(We have to consider not only mechanical but also optical properties of coating)

Two promising candidates with low mechanical loss

(Somebody could give other candidates ...)

(a) AlGaAs/AlGaP : Crystal coating

Problem : Is it feasible for large mirror

for long arm interferometer ?

(b) Amorphous Silicon

Problem : Large absorption

3. Interferometer after revolution

(3) Reduction of thermal noise

Coating thermal noise is the most **serious** problem !
Coating loss reduction is not so easy ...

Second generation interferometer

Advanced LIGO (U.S.A.)

and Virgo (Italy and France): **Larger beam**

-> Beyond AdLIGO : Voyager (**Cooled silicon mirror**)

KAGRA (Japan) : **Cooled sapphire mirror**

Third generation interferometer (10 times better sensitivity)

Einstein Telescope (ET, Europe) :

Cooled silicon or sapphire mirror and **larger beam**

3. Interferometer after revolution

(3) Reduction of thermal noise

Larger beam

Correlation between thermal fluctuation at two points is **smaller** when **distance** between these points is **larger**.

When we observe **wider** mirror surface, integral of fluctuation of surface is **smaller** owing to **weaker** correlation.

Mirror diameter limits beam size.

Larger mirror or **other shape** beam (not Gaussian beam) ?

3. Interferometer after revolution

(3) Reduction of thermal noise

Larger beam

Mirror radius should be **3 times larger** than
Gaussian beam radius to avoid large clipping loss.

How about other kinds of beam shape ?

Mesa hat, higher modes...

**The details are in Chapter 13 (A. Freise) of
G.M. Harry, T. Bodiya, and R. DeSalvo (Editors)**

Optical Coatings and Thermal Noise

in Precision Measurements

Cambridge University Press, Cambridge (in press)

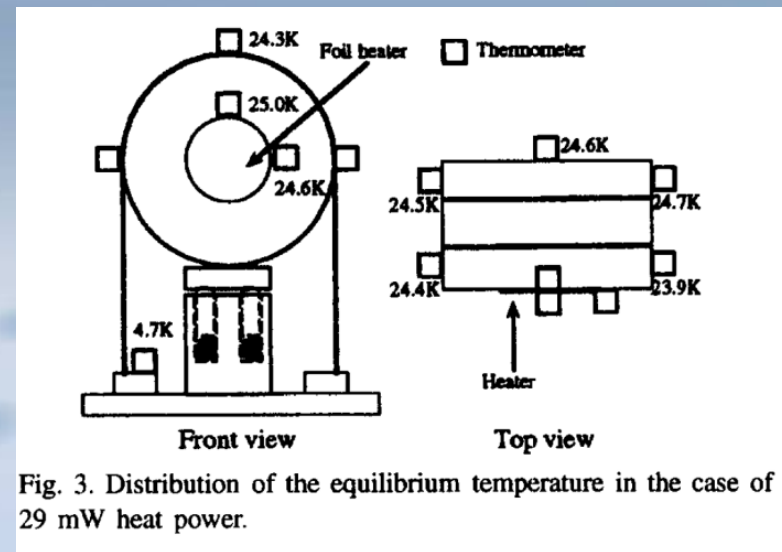
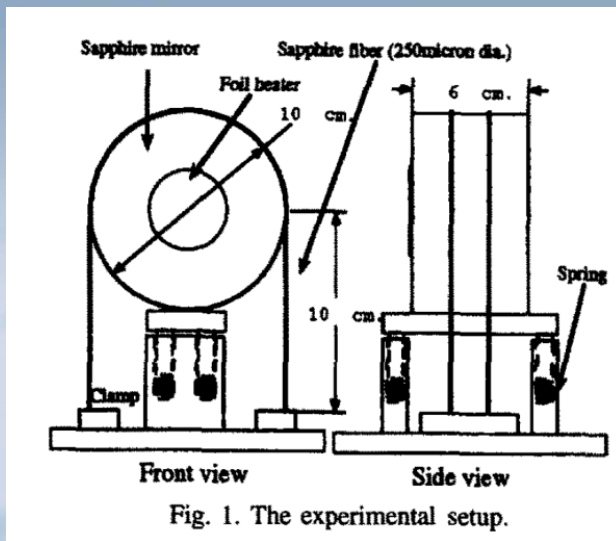
3. Interferometer after revolution

(3) Reduction of thermal noise

One of the **simplest solutions** : **Cooling mirrors**

First feasibility study

T. Uchiyama *et al.*, Physics Letters A 242 (1998) 211.



3. Interferometer after revolution

(3) Reduction of thermal noise

One of the **simplest solutions** : **Cooling mirrors**

In KAGRA, **sapphire mirror** is suspended by **sapphire fibers**.

Small mechanical loss (small thermal noise)

T. Uchiyama *et al.*, Physics Letters A 273 (**2000**) 310.

Large thermal conductivity (effective cooling)

T. Tomaru *et al.*, Physics Letters A 301 (**2002**) 215.

In ET(Voyager), **silicon** is also (only) candidate.

S. Hild *et al.*,

Classical and Quantum Gravity 28 (**2011**) 094013.

R. Nawrodt *et al.*,

General Relativity and Gravitation 43 (**2011**) 593.

3. Interferometer after revolution

(3) Reduction of thermal noise

Amplitude of thermal noise is proportional to

$$\mathbf{(T/Q)^{1/2}}$$

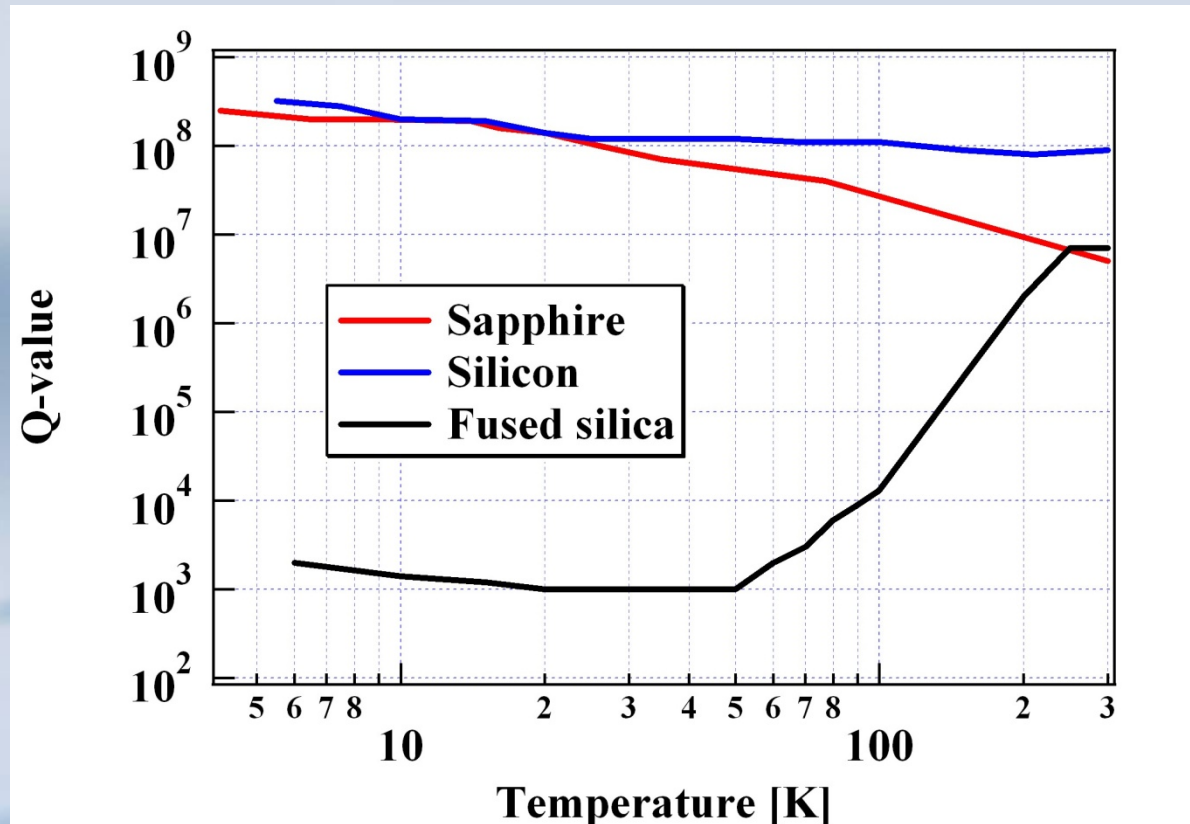
In general, Q-value depends on T (temperature).

We must investigate how dissipation depends on temperature in cryogenic region.

3. Interferometer after revolution

(3) Reduction of thermal noise

Structure damping in substrate



T. Uchiyama *et al.*, Physics Letters A 261 (1999) 5-11.

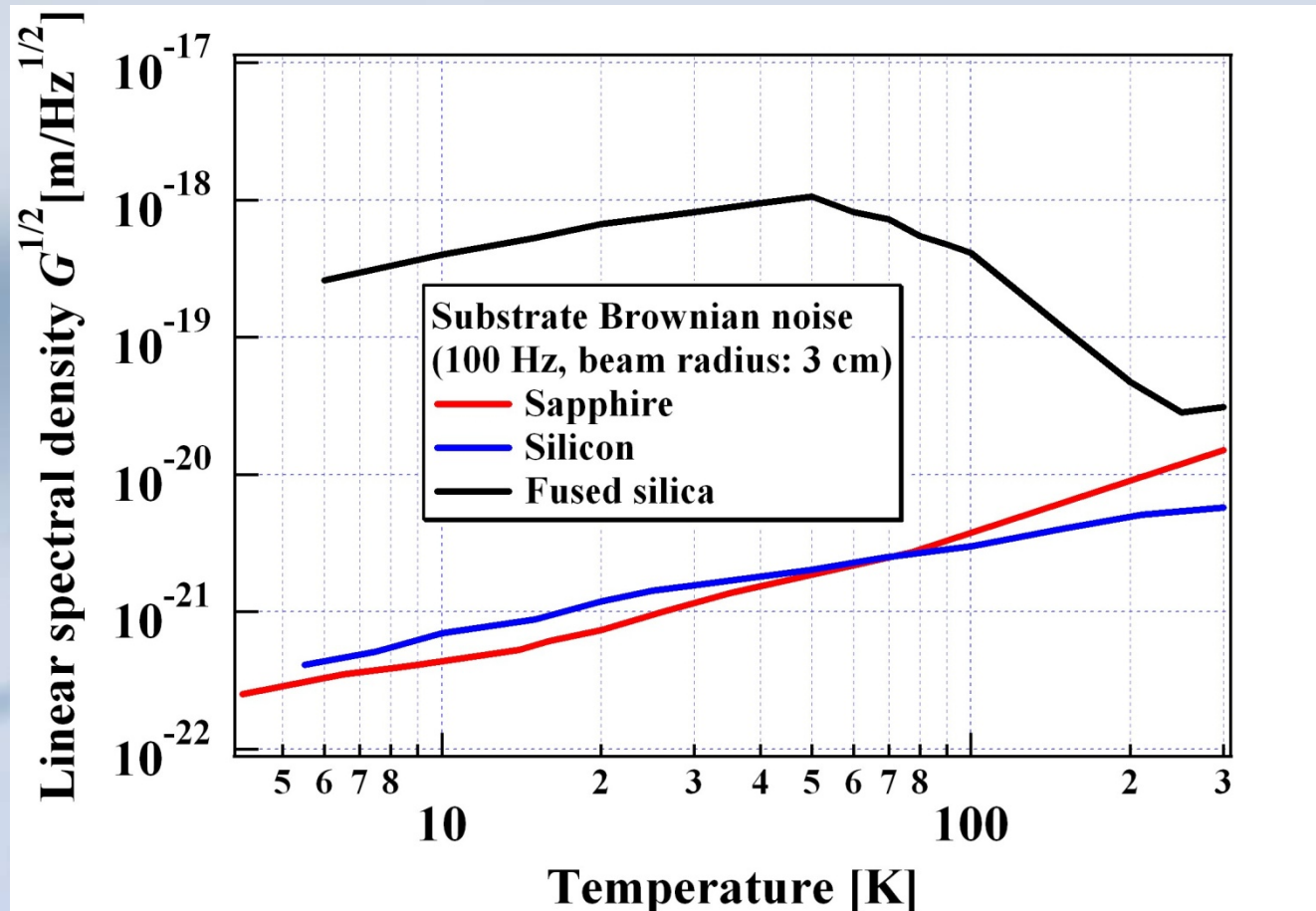
R. Nawrodt *et al.*, Journal of Physics: Conference Series 122 (2008) 012008.

C. Schwarz *et al.*, 2009 Proceedings of ICEC22-ICMC2008.

3. Interferometer after revolution

(3) Reduction of thermal noise

Structure damping in substrate

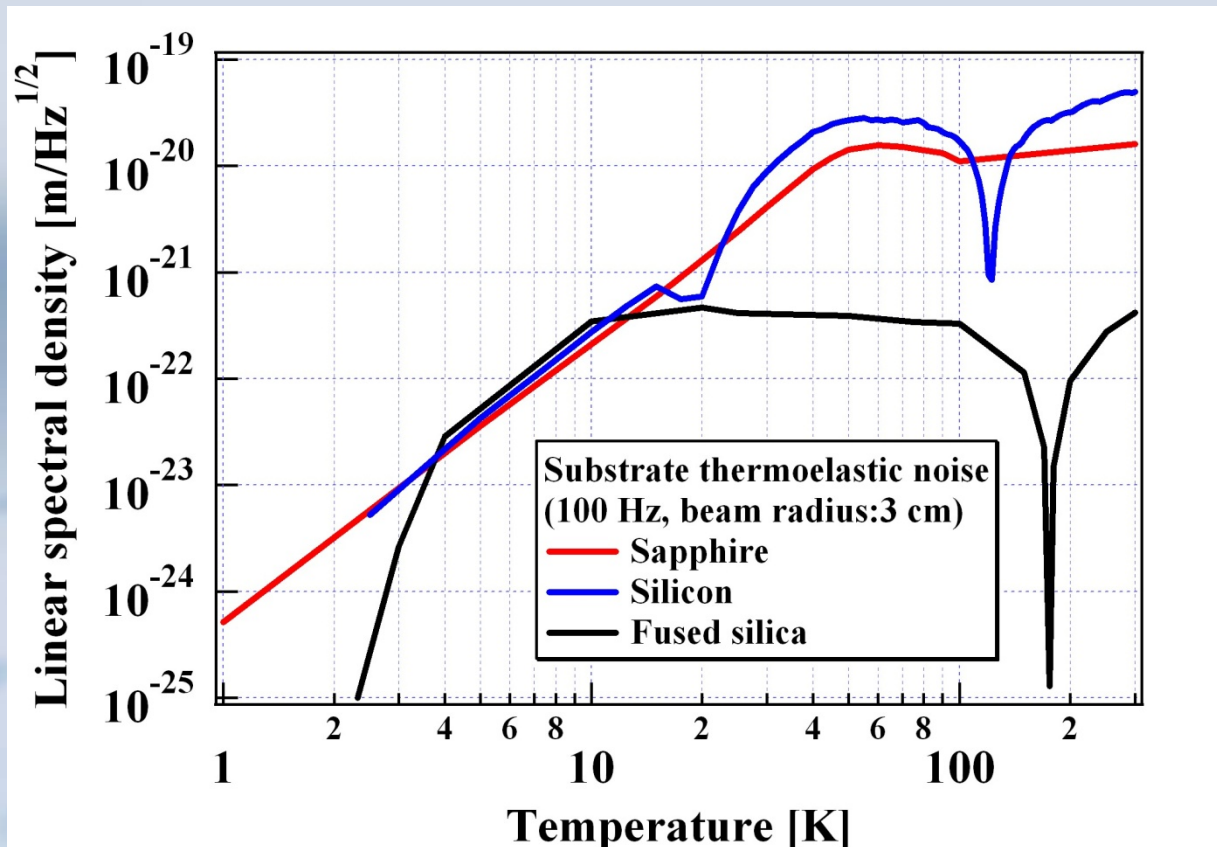


Fused silica can not be used !

3. Interferometer after revolution

(3) Reduction of thermal noise

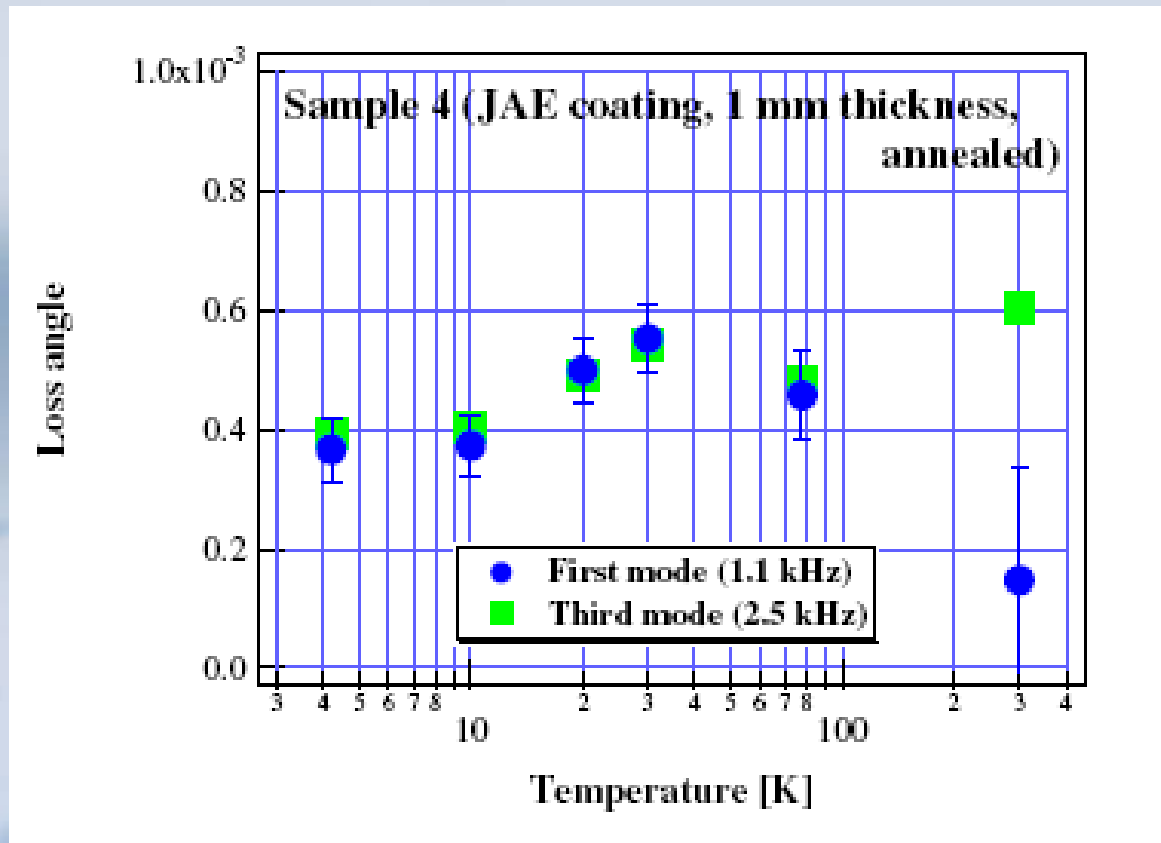
Thermoelastic damping in substrate



3. Interferometer after revolution

(3) Reduction of thermal noise

Structure damping in coating

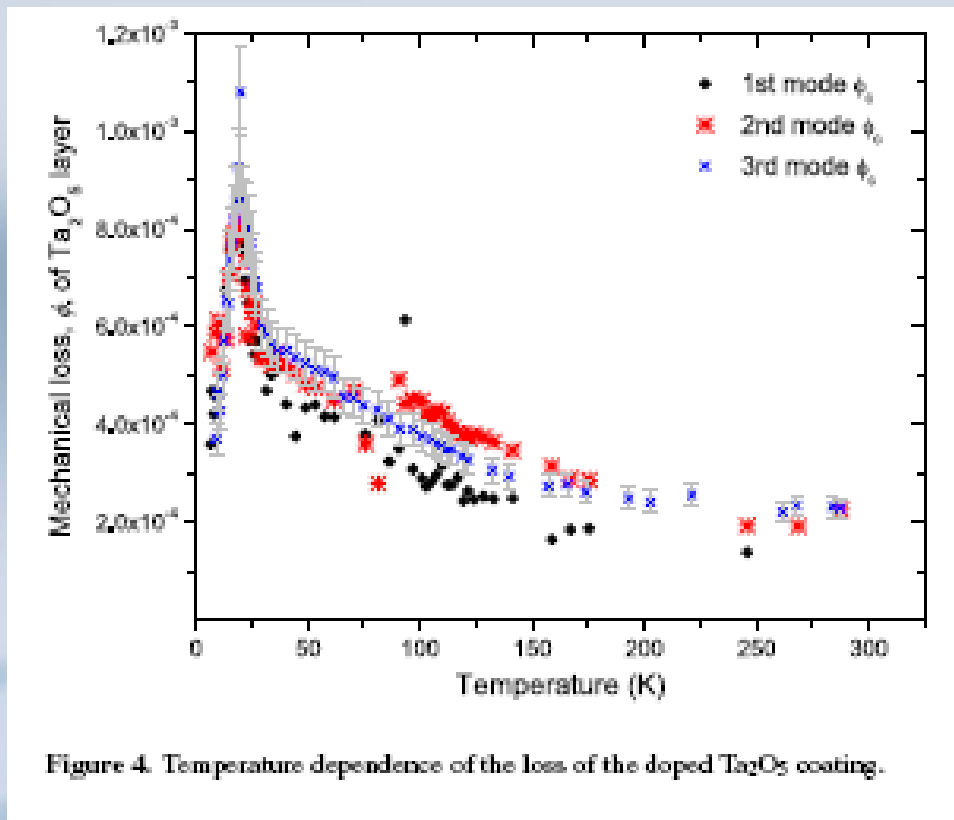


Loss angle is almost **independent of temperature.**

3. Interferometer after revolution

(3) Reduction of thermal noise

Structure damping in coating

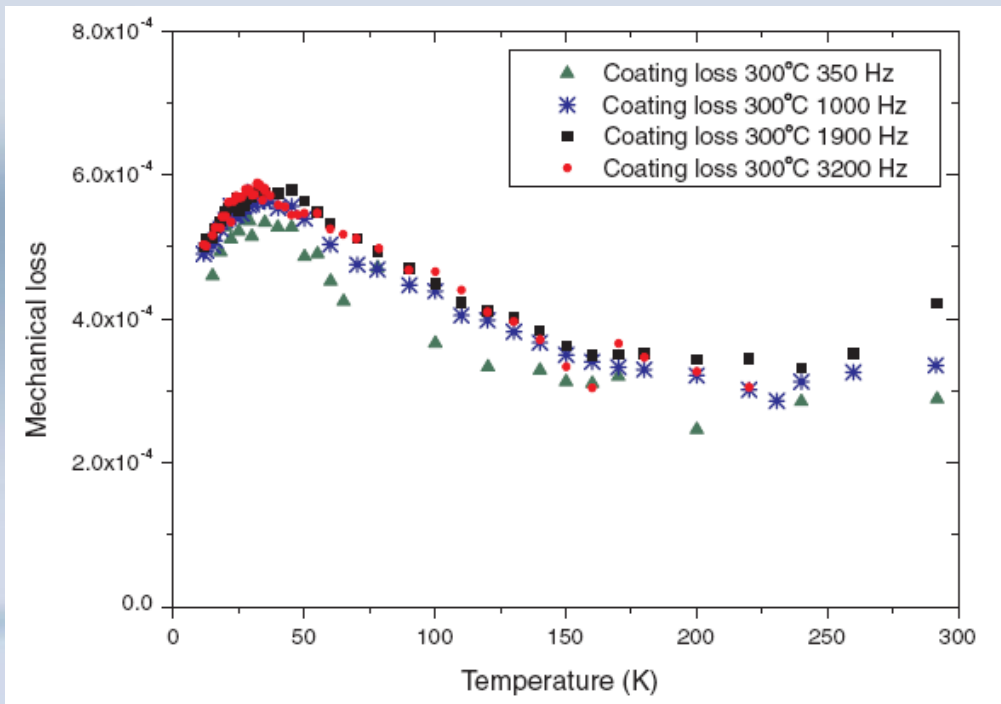


Peak at 20 K ?

3. Interferometer after revolution

(3) Reduction of thermal noise

Structure damping in coating



Peak at 20 K ?

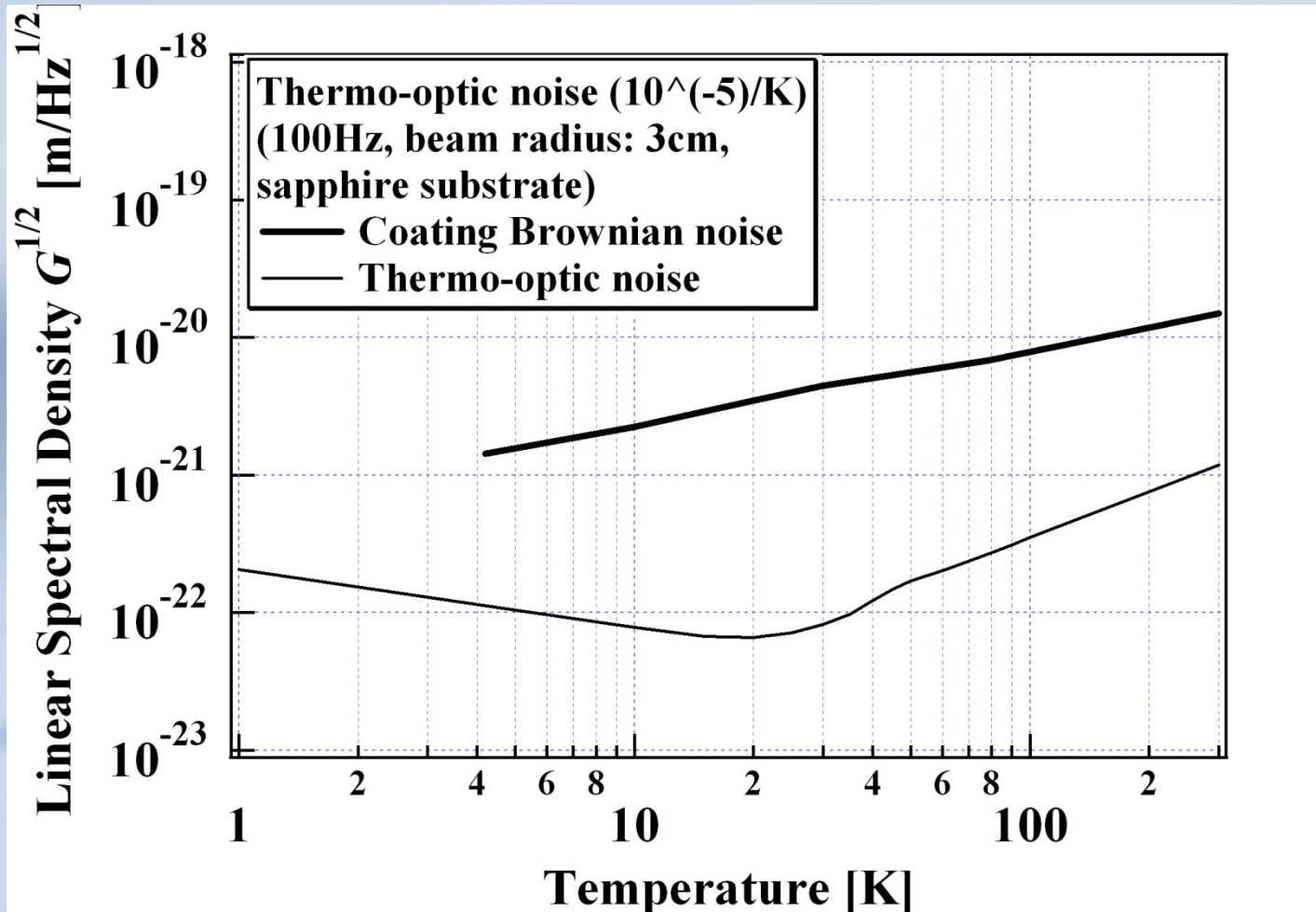
Annealing suppresses peak (not perfectly).

It is assumed that loss is constant.

Kazuhiro's comment :
Mystery still remains.

3. Interferometer after revolution

(3) Reduction of thermal noise

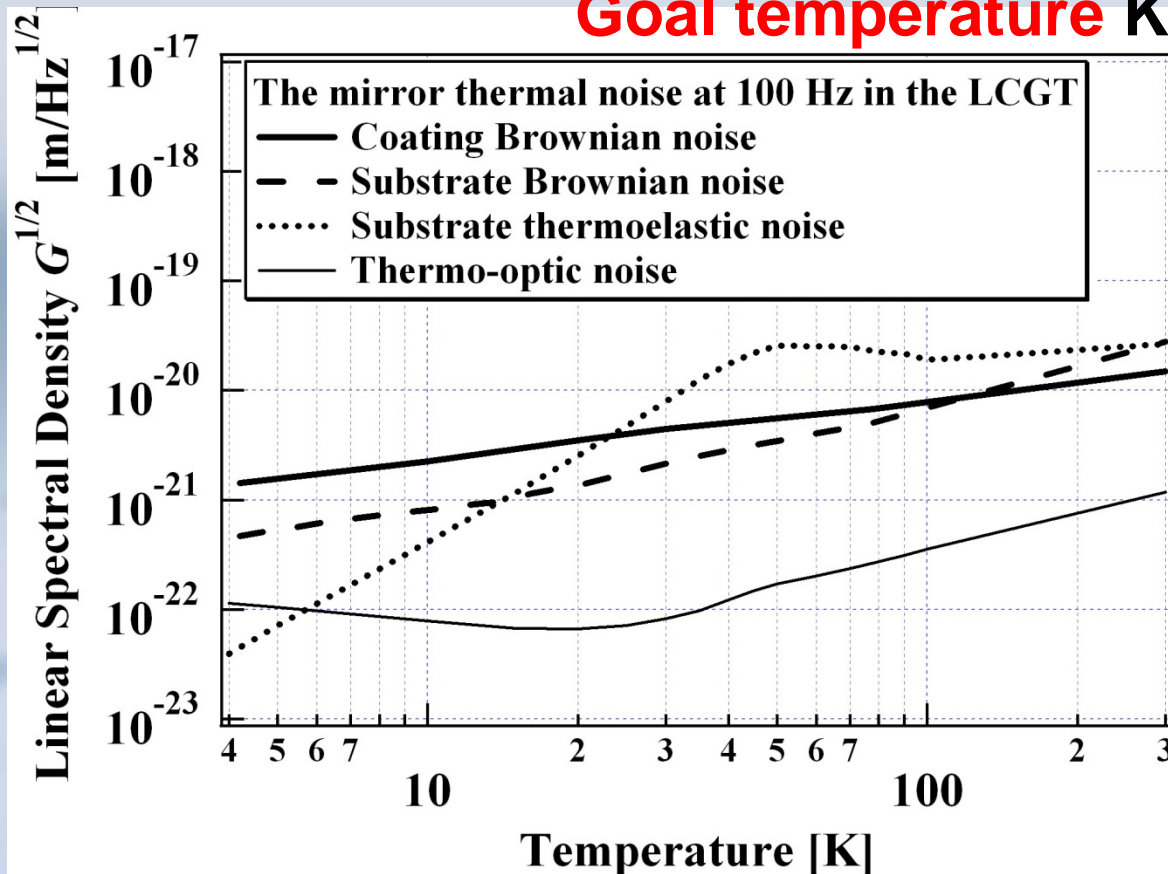


3. Interferometer after revolution

(3) Reduction of thermal noise

Structure damping in **coating**

Goal temperature KAGRA : 20 K



Coating thermal noise is the most **serious** problem !

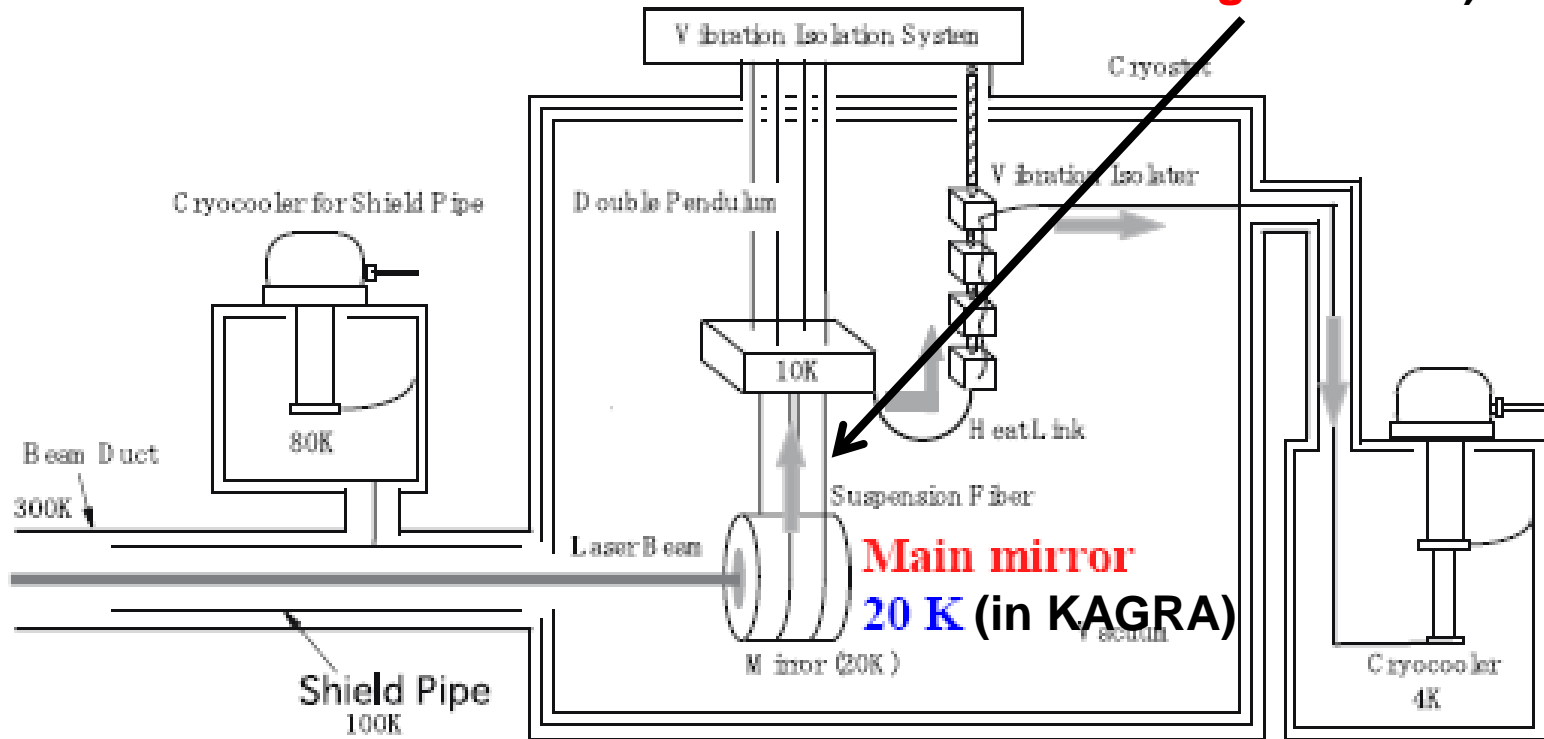
3. Interferometer after revolution

(3) Reduction of thermal noise

How can we **cool mirrors** ?

Sapphire or **silicon** fiber
(**high thermal conductivity**
and **high Q-value**)

Schematic view of cryostat (by T. Tomaru)



3. Interferometer after revolution

(3) Reduction of thermal noise

Cryocooler

Some kinds of **heat engine**

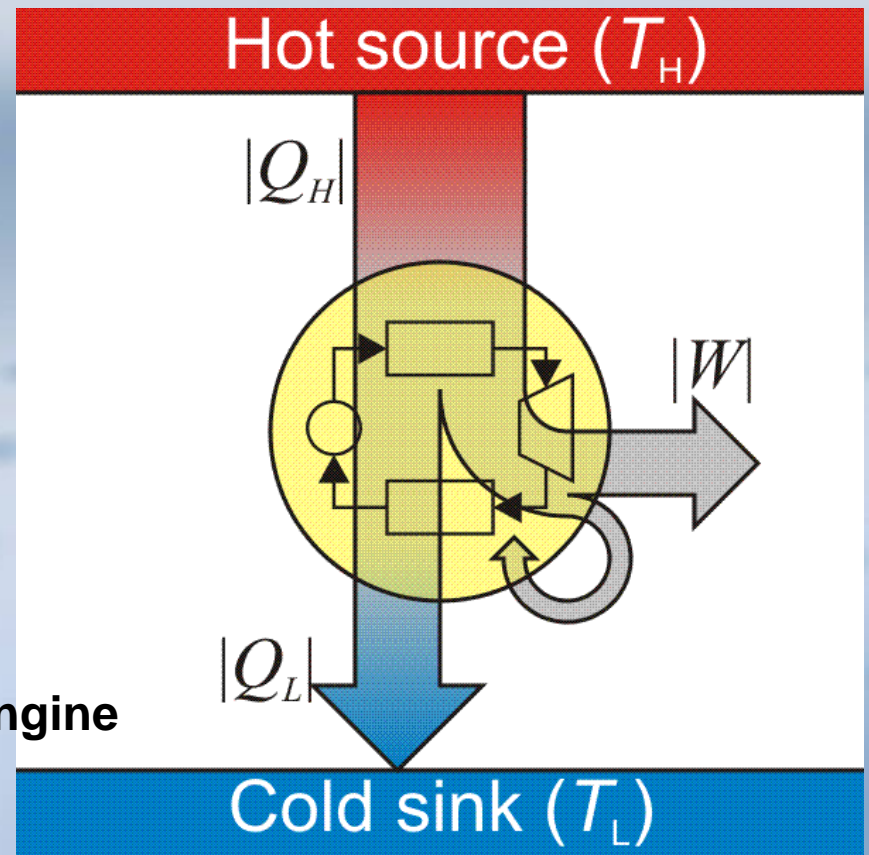
According to thermodynamics ...

(a) Hot source gives engine heat (Q_H).

(b) Part of heat (Q_L) is give to cold sink.

(c) Other part of heat is converted to work (W).

Wikipedia : heat engine



3. Interferometer after revolution

(3) Reduction of thermal noise

Cryocooler

Some kinds of **heat engine**

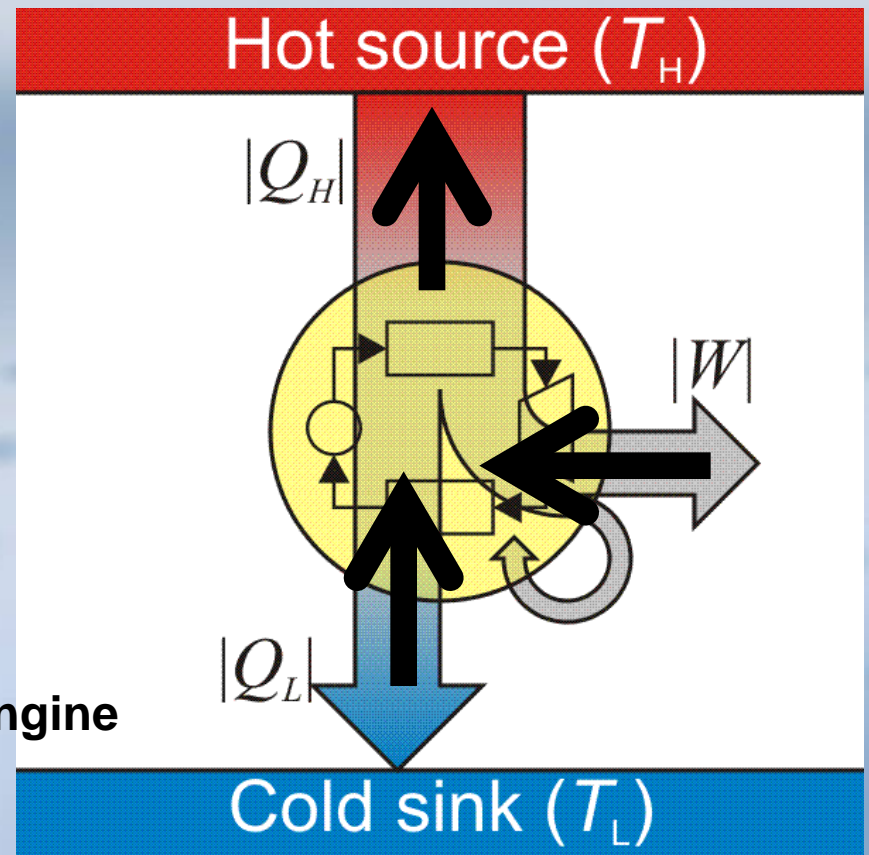
In the case of **cryocooler** ...

(a) We **apply work** (W) to engine.

(b) **Cold sink gives heat** (Q_L) to engine.

(c) Engine gives heat (Q_H) to Hot source.

Wikipedia : heat engine



3. Interferometer after revolution

(3) Reduction of thermal noise

Cryocooler

Why ?

Usual case : **Liquid nitrogen** and **helium**

Safety and **maintenance** in **mine**



Cryocooler

Usual cryocooler : **Gifford-McMahon** cryocooler

Large vibration (Heat engine has piston !)



Pulse-tube cryocooler (without solid piston)

But, vibration of **commercial one** is **not enough small**.

3. Interferometer after revolution

(3) Reduction of thermal noise

Schematic view of **silent pulse-tube cryocooler**

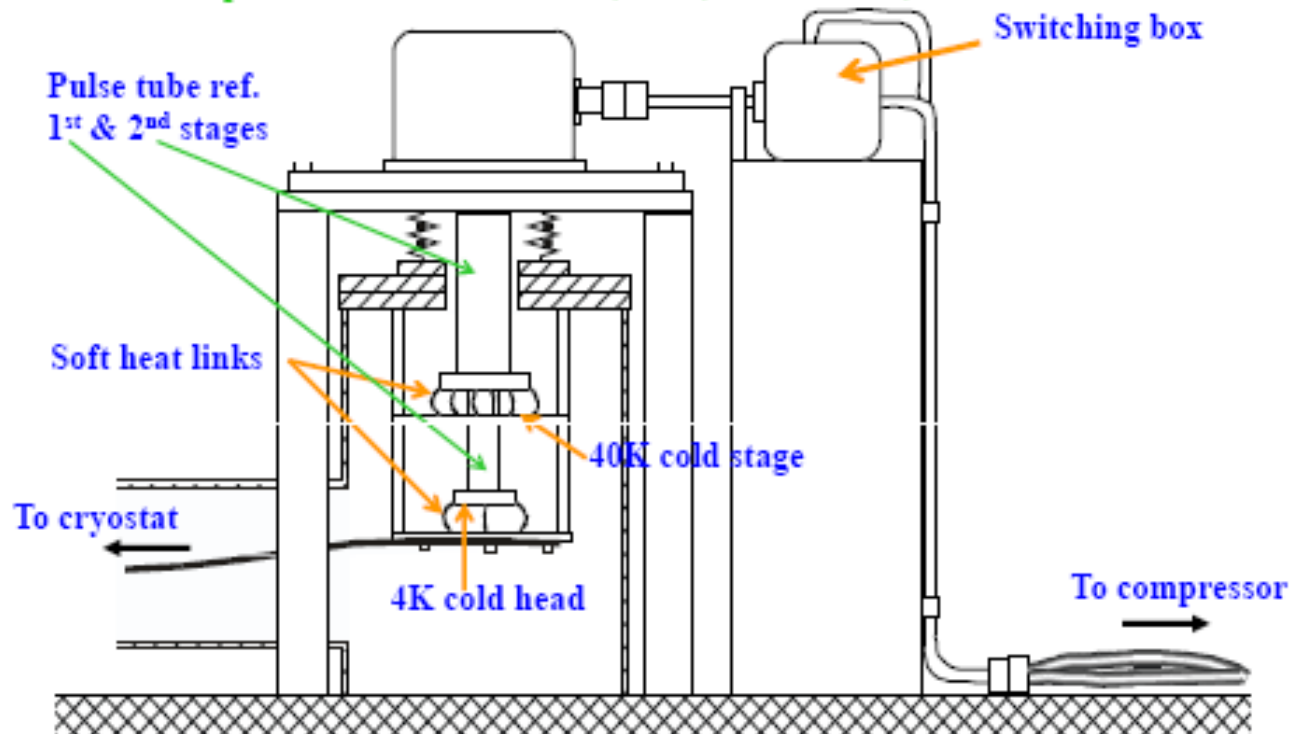
Quiet refrigerator was developed (design in 2003)

T. Tomaru *et al.*, *Classical Quantum Gravity* 21 (2004) S1005.

T. Tomaru *et al.*, *Cryocoolers* 13 (2005) 695.

R. Li. *et al.*, *Cryocoolers* 13 (2005) 703.

patent: Pa-3 Tomaru *et al.*, 2003; Suzuki *et al.*, 2003.



3. Interferometer after revolution

(4) Impact on other fields

Contents of G.M. Harry, T. Bodiya, and R. DeSalvo (Editors)

Optical Coatings and Thermal Noise in Precision Measurements

Cambridge University Press, Cambridge (in press)

14	Gravitational Wave Detection	<i>D. J. Ottaway and S. D. Penn</i>	256
15	High-Precision Laser Stabilization via Optical Cavities	<i>M. J. Martin and J. Ye</i>	281
16	Quantum Optomechanics	<i>G. D. Cole and M. Aspelmeyer</i>	308
17	Cavity Quantum Electrodynamics	<i>T. E. Northup</i>	335

Thermal noise is also sensitivity limit
on the **other fields** in precision measurement.

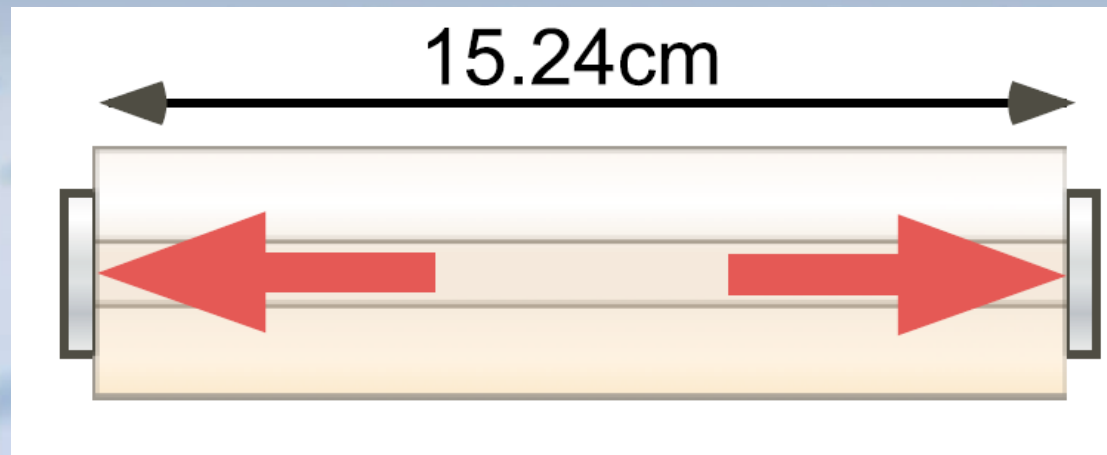
3. Interferometer after revolution

(4) Impact on other fields

For example ...

Cavity as reference for laser **frequency stabilization**

Current **best** laser frequency stabilization
with rigid cavity **at room temperature**
is **limited** by **thermal noise** of mirrors.

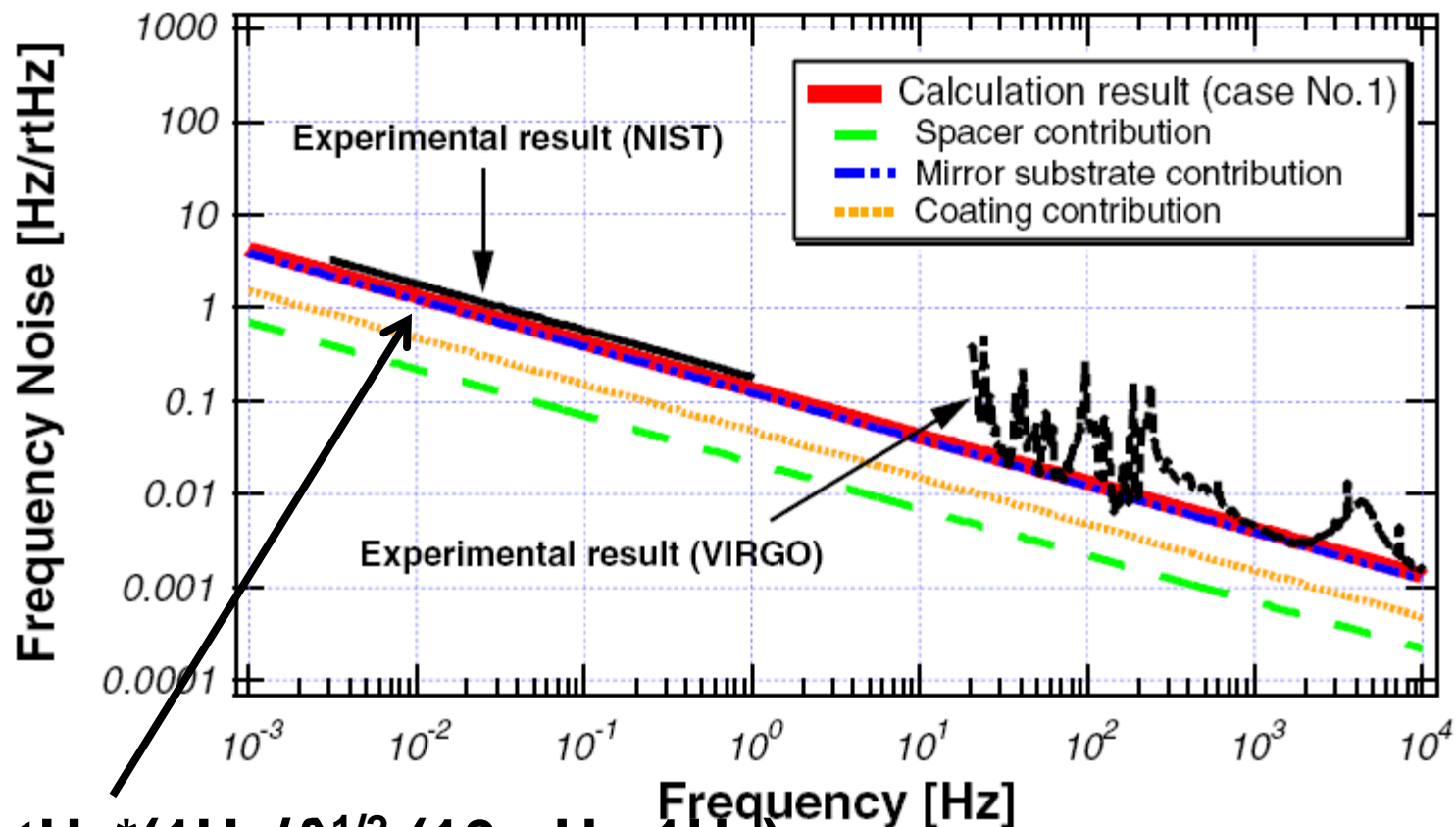


3. Interferometer after revolution

(4) Impact on other fields

PRL 93, 250602 (2004)

PHYSICAL REVIEW LETTERS



0.1 Hz/rtHz*(1Hz/f)^{1/2} (10mHz-1Hz)

Allan deviation : 4*10⁻¹⁶ (10mHz-1Hz)

3. Interferometer after revolution

(4) Impact on other fields

Hot paper (ISI Web of Knowledge)

Thermal-noise limit in the frequency stabilization of lasers with rigid cavities

著者名: Numata, K (Numata, K); Kemery, A (Kemery, A); Camp, J (Camp, J)

PHYSICAL REVIEW LETTERS
巻: 93 号: 25
記事番号: 250602
DOI: 10.1103/PhysRevLett.93.250602
発行: DEC 17 2004

引用ネットワ
263 被引用数
24 引用文献
Related Records
引用アラ
(Web of Science Co

1 referred journal paper every 2 weeks ! (until 11 July 2017)

4. Summary

(1) **Long history** of research of **thermal noise (200 years !)**
General theorem for thermal noise, Fluctuation-
Dissipation Theorem, appears only **60 years ago**.

(2) Interferometric detector

It is essential to **measure Q-values**.

Pendulum : Rigid and heavy support system

Mirror : Nodal support system

4. Summary

(3) Interferometric detector

Drastic progress of research on the **end of 20th century**

New kinds of thermal noise

Thermoelastic noise and coating thermal noise

Reduction of thermal noise

(larger beam and **cryogenic techniques**)

Impact on **other fields**

There are open questions and this field will be hot in future.

Thank you for your attention !

1. Until Fluctuation-Dissipation Theorem

FDT in quantum mechanics

H.B. Callen and T.A. Welton, Physical Review 83 (1951) 34.

When we should take quantum mechanics into account ?

$$\hbar\omega \gg k_{\text{B}}T$$

Smallest energy
in **quantum** mechanics

Average energy
in **classical** statistical mechanics

At room temperature, if the frequency is more than **$6 \cdot 10^{12}$ Hz**, we should consider quantum mechanics.

1. Until Fluctuation-Dissipation Theorem

Fluctuation Dissipation Theorem in Quantum mechanics

Kubo formula

R. Kubo,

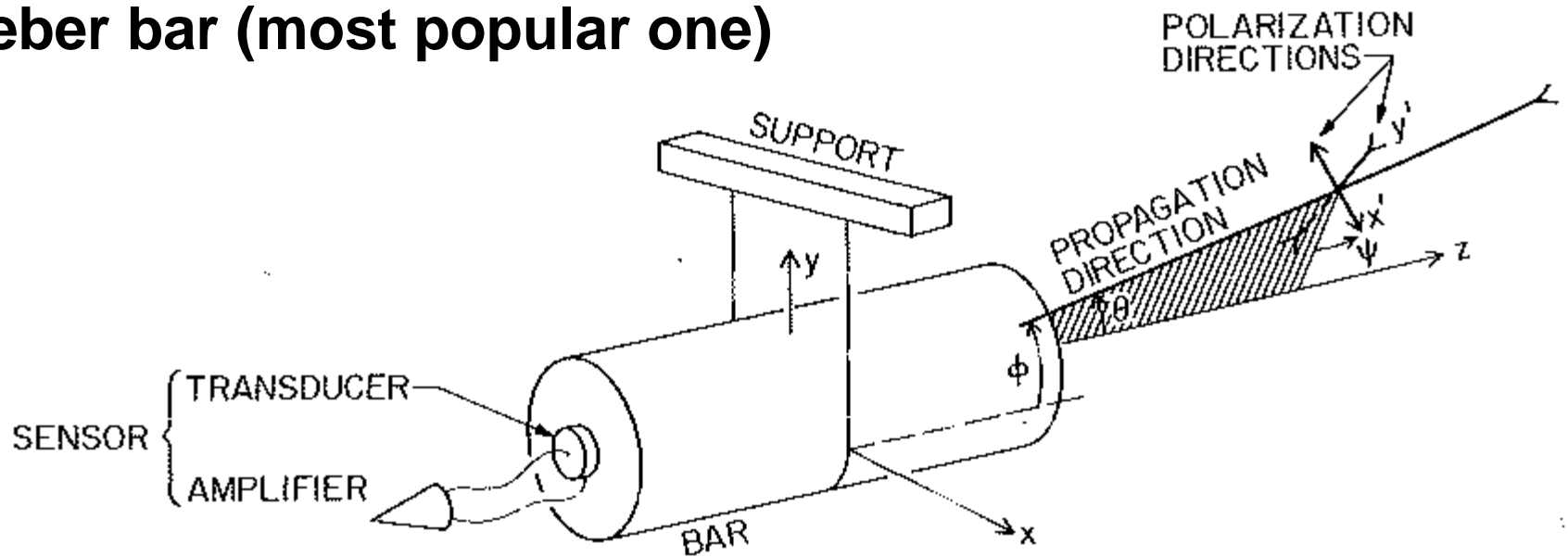
Journal of the Physical Society of Japan 12 (1957) 570.

2. Resonant detector

Resonant detector

Gravitational wave **excites resonant motion** of elastic body.

Weber bar (most popular one)



“300 years of gravitation”

(1987) Cambridge University Press

Fig. 9.8

Diameter : several tens cm

Length : a few meters

Resonant frequency : about 1 kHz

2. Resonant detector

Resonator : **tidal force** of gravitational wave
Thermal **fluctuation force** must be considered.

Observation of thermal fluctuation of torsion pendulum
Displacement, not force, was monitored.

Formula of thermal fluctuation **force** (on resonance)

$$G_F(f_0) = \frac{4k_B T m \omega_0}{Q}$$

T/Q should be small.

Low temperature (low T), Small mechanical loss (large Q)

2. Resonant detector

First generation (room temperature)

Weber bar (University of Maryland, U.S.A.) ...

Second generation (4 K) Liquid helium

Explorer (Italy, CERN), Allegro (U.S.A.),

Niobe (Australia), Crab (Japan) ...

Third generation (< 100 mK) Dilution refrigerator

Nautilus (Italy), Auriga (Italy),

Mini-Grail (Netherlands),

Mario Schenberg (Brazil) ...

This is not a perfect list !

NAUTILUS
INFN - LNF

G. Pizzella, ET first general meeting (2008)

2. Resonant detector

High Q-value (low mechanical loss) material

Small dissipation at **low temperature**

Sapphire and Silicon (Moscow)

Niobium (Australia)

CuAl6% (Mini-Grail (Netherlands), Mario Schenberg (Brazil))

K.S. Thorne, Chapter 9 of “*300 years of gravitation*”(1987)

Cambridge University Press p409.

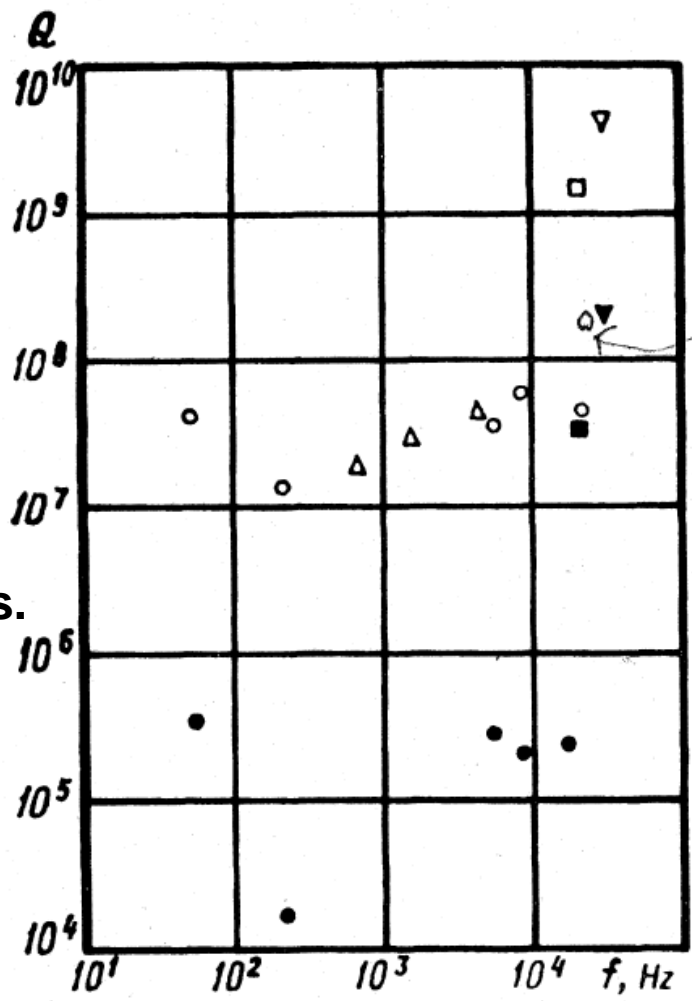
A de Waard *et al.*, *Classical and Quantum Gravity* 21 (2004) S465.

O.D. Aguiar *et al.*, *Classical and Quantum Gravity* 25 (2008) 114042.

Almost all resonators (Weber bar also) are made from aluminum.

Large bulk, low cost ...

V.B. Braginsky,
 V.P. Mitrofanov,
 K.S. Thorne
 "Systems
 with Small Dissipation"
 (1986)
 University of Chicago Press.



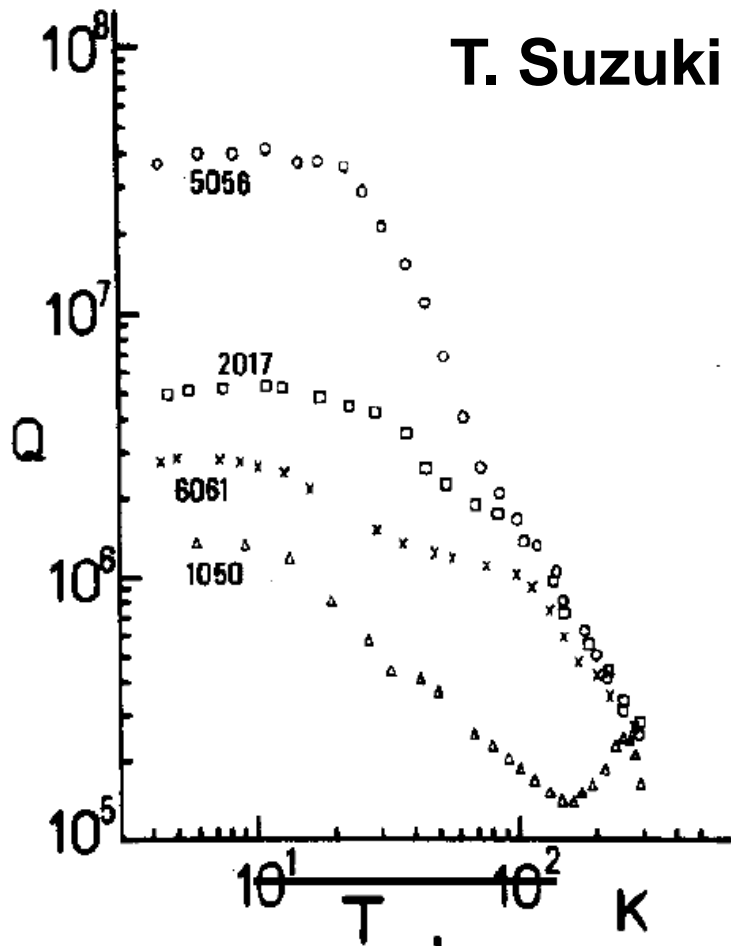
Coccia et al
 PLAZI 219 (19
 35 kHz
 40mK

Fig. 6 Maximum quality factors for mechanical resonators made of different materials. \circ = Al-5056 (Oide, Tsubono, and Hirakawa 1981), Δ = Nb (Blair *et al.* 1980b), \square = Si (McGuigan *et al.* 1978), ∇ = A (Bagdasarov *et al.* 1977). Solid symbols correspond to $T = 300\text{K}$, open symbols to $T = 4.2\text{K}$. Note added in press: Recently Veitch *et al.* (1985) have reported a Q of 2×10^8 at 4K in a niobium bar.

2. Resonant detector

What is kinds of **aluminum alloy best** ?

T. Suzuki *et al.* discovered that **Al5056** has high Q-value. Almost all resonators are made from Al5056.



T. Suzuki *et al.*, Physics Letters 67A (1978) 2.

Alloy designation (AA)	Nominal composition (%) (balance Al)	Q
1050	< 0.5	1.3×10^6
2017	4.0 Cu, 0.7 Mn, 0.5 Mg	5.3×10^6
5056	5.1 Mg, 0.12 Mn, 0.12 Cr	4.0×10^7
6061	1.0 Mg, 0.6 Si, 0.27 Cu, 0.20 Cr	2.9×10^6

4. Interferometer after revolution

(1) Thermoelastic noise of mirror

Thermo-optic noise

Temperature fluctuation **in reflective coating**



Thermal **expansion** (α)

Temperature coefficient of **refractive index** (β)



Fluctuation of **phase of reflected light**

Material properties of reflective coating are important issues.

V. B. Braginsky *et al.*, Physics Letters A 312 (2003) 244.

V. B. Braginsky *et al.*, Physics Letters A 271 (2000) 303.

M. Evans *et al.*, Physical Review D 78 (2008) 102003.

4. Interferometer after revolution

(2) Thermal noise caused by inhomogeneous loss

Mirror is **not a** harmonic oscillator.

It has **a lot of resonant modes.**

Modal expansion

Thermal noise of mirror is the **summation** of those of **resonant modes.**

Thermal noise of resonant mode is the same as that of a harmonic oscillator .

Peter R. Saulson, Physical Review D 42 (**1990**) 2437.

Same Q-values implies same thermal noise.

4. Interferometer after revolution

(3) Direct measurement of thermal noise

Are formulae of thermal noise **correct** ?

Direct measurement of thermal noise of **mirror**

University of Tokyo, California Institute of Technology

Small beam radius (about **0.1 mm**)

to enhance thermal noise

Fabry Perot cavity length ~ 1 cm

Direct measurement of thermal noise of **suspension**

University of Tokyo

Underground site with **small seismic motion**

Cavity length ~ 100 m

4. Interferometer after revolution

(2) Thermal noise caused by inhomogeneous loss

Modal expansion predicts that **thermal noise** is the **same** if **Q-values** are **same**. But Levin's discussion proved that it is **invalid**.

What is wrong ?

Although some people investigated, K. Yamamoto's four papers provide clear solution for this problem.

4. *Interferometer after revolution*

(2) Thermal noise caused by inhomogeneous loss

What is wrong in modal expansion ?

Inhomogeneous loss causes **couplings between modes**.
Inhomogeneous loss destroys the shape of a mode.
Other modes appear.

These couplings generate **correlations** between thermal fluctuations of resonant modes. This **correlation is not taken into account** in modal expansion.

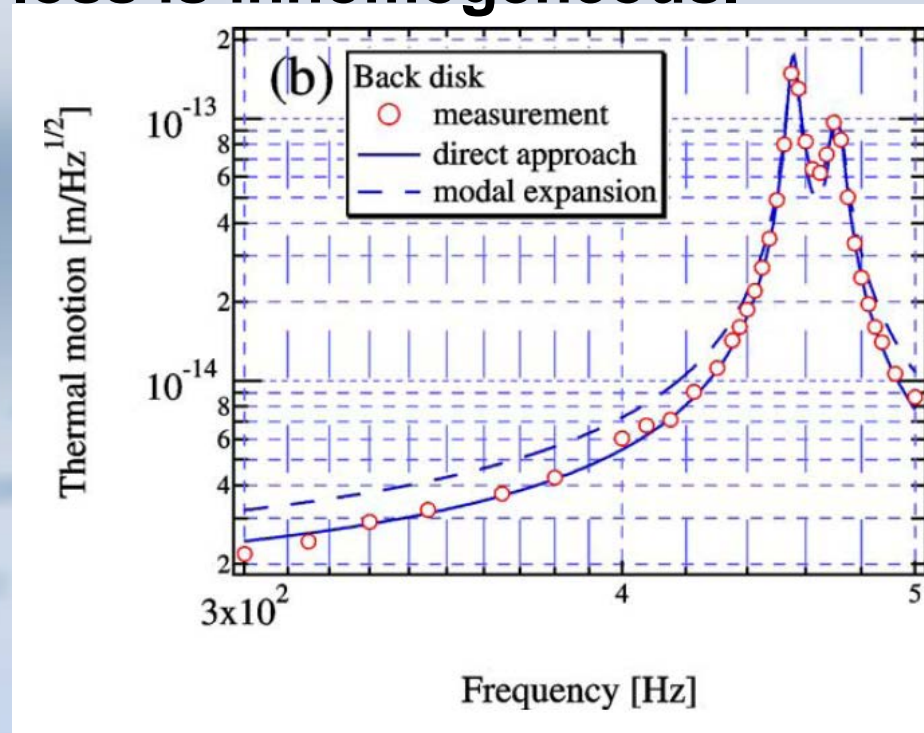
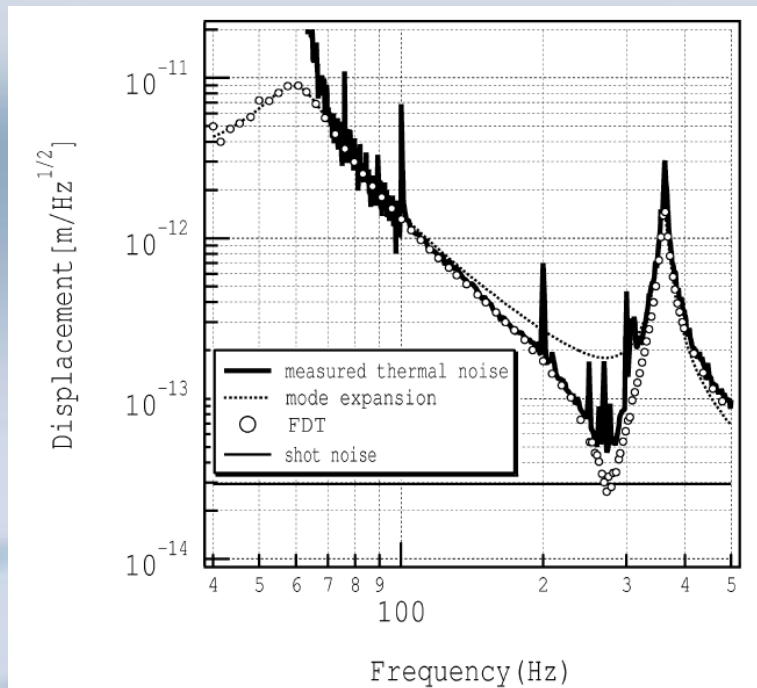
If we consider these correlations, modal expansion can provide correct evaluation (advanced modal expansion).

K. Yamamoto *et al.*, Physical Review D 75 (**2007**) 082002.

4. Interferometer after revolution

(2) Thermal noise caused by inhomogeneous loss

Experimental checks : (**Traditional**) modal expansion **breaks down** and **advanced** modal expansion and **Levin's method** are **correct** if loss is inhomogeneous.



K. Yamamoto *et al.*, Physics Letters A 280 (2001) 289.

K. Yamamoto *et al.*, Physics Letters A 321 (2004) 79.

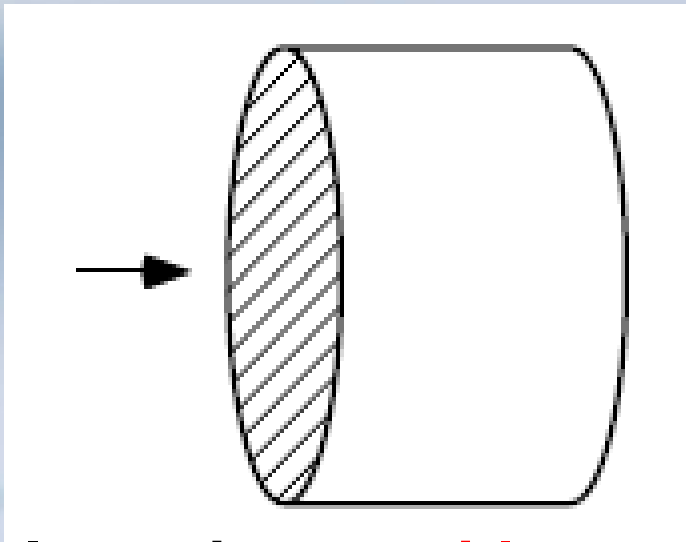
4. Interferometer after revolution

(2) Thermal noise caused by inhomogeneous loss

Quantitative discussion

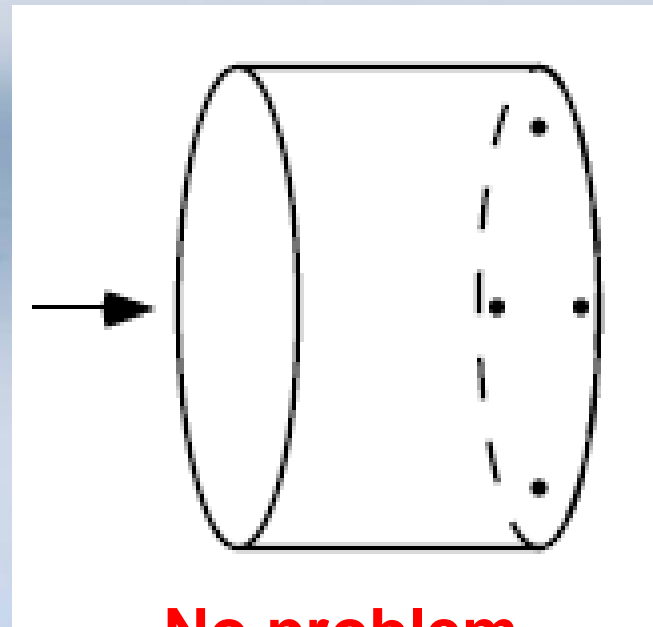
K. Yamamoto *et al.*, Physics Letters A 305 (2002) 18.

Reflective coating



It can be a **problem**.

Magnet



No problem

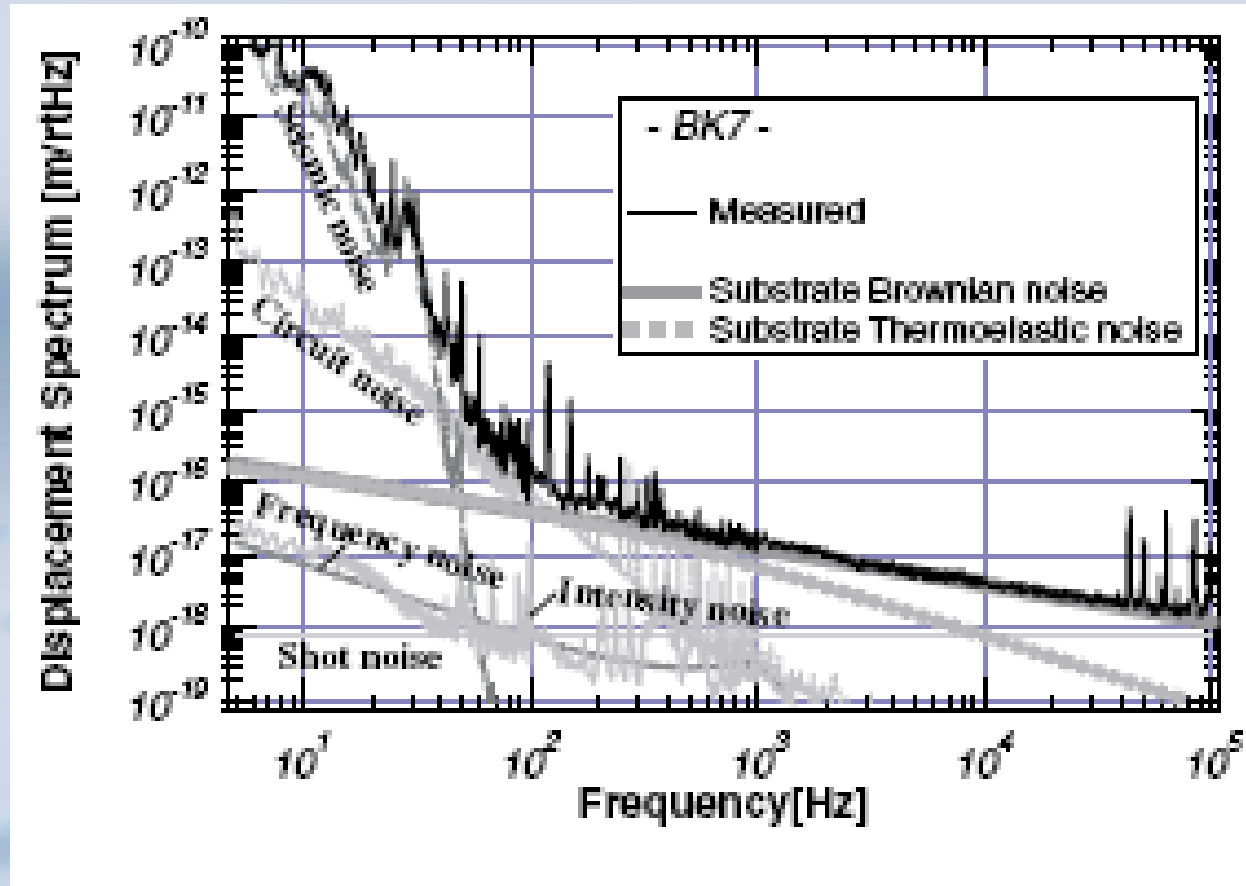
The **previous** expectation is perfectly **wrong**.

The strategy of research of thermal noise **must be changed**.¹⁰⁶

4. Interferometer after revolution

(3) Direct measurement of thermal noise

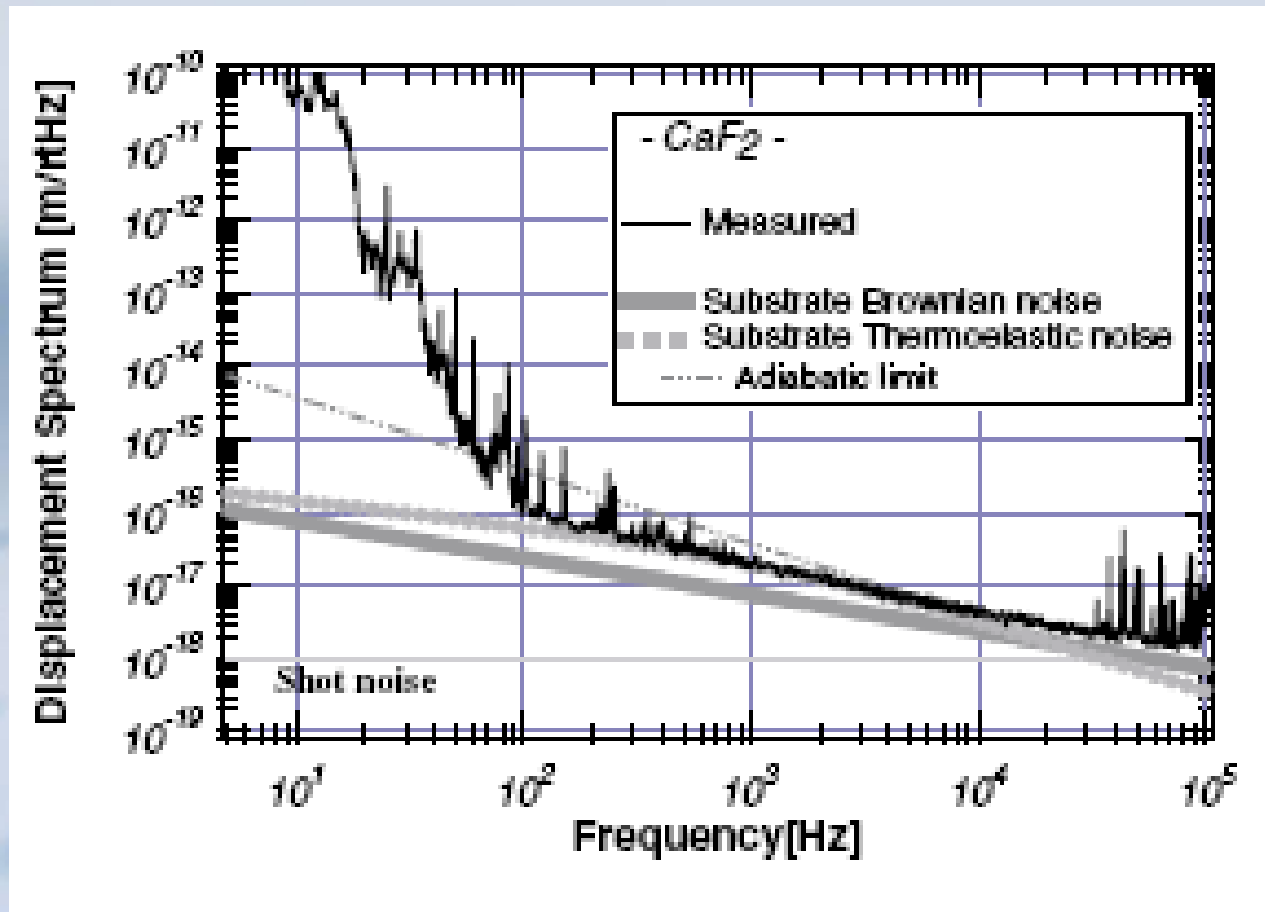
BK7 : **Structure** damping in **substrate**



4. Interferometer after revolution

(3) Direct measurement of thermal noise

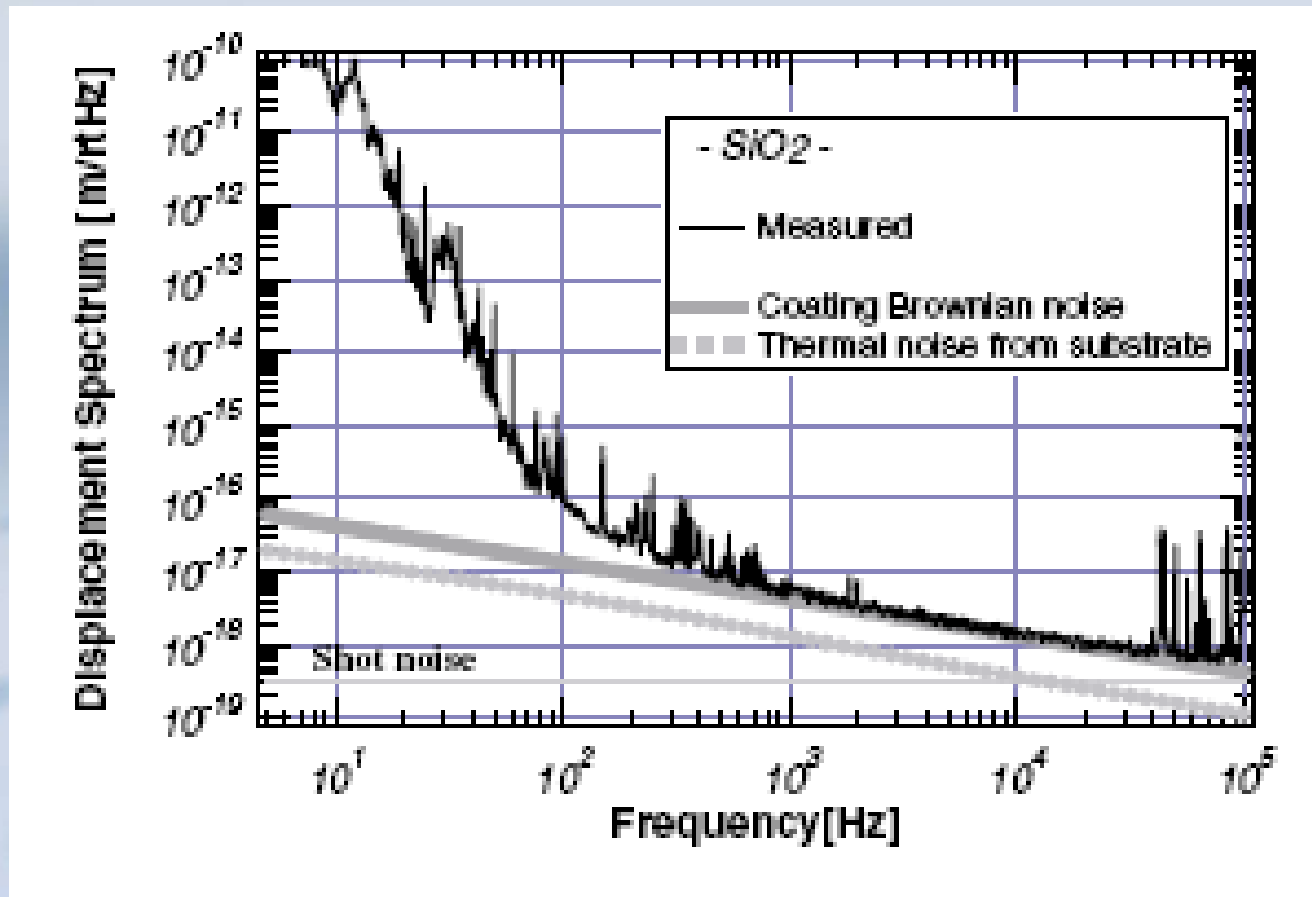
Calcium fluoride : **Thermoelastic** damping in **substrate**



4. Interferometer after revolution

(3) Direct measurement of thermal noise

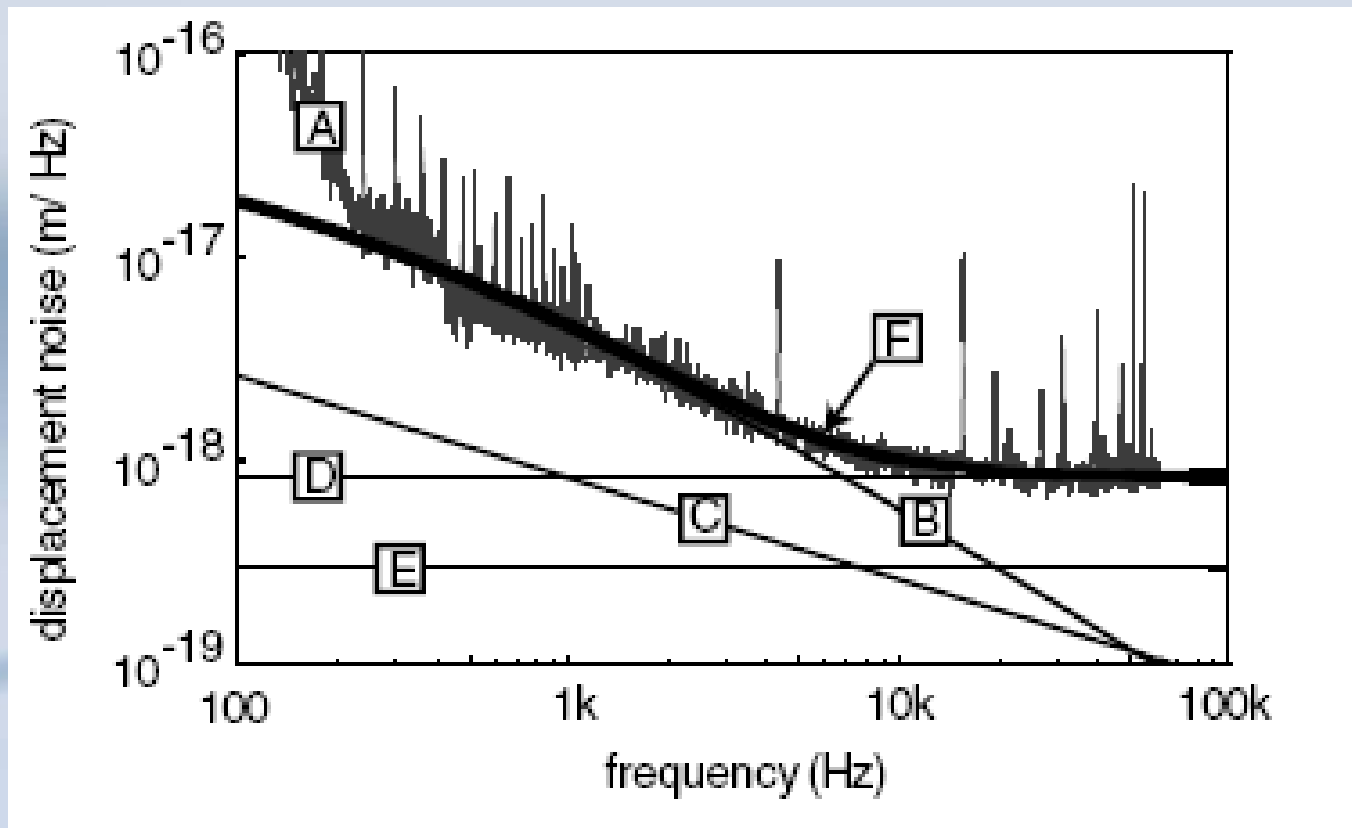
Fused silica : **Structure** damping in **coating**



4. Interferometer after revolution

(3) Direct measurement of thermal noise

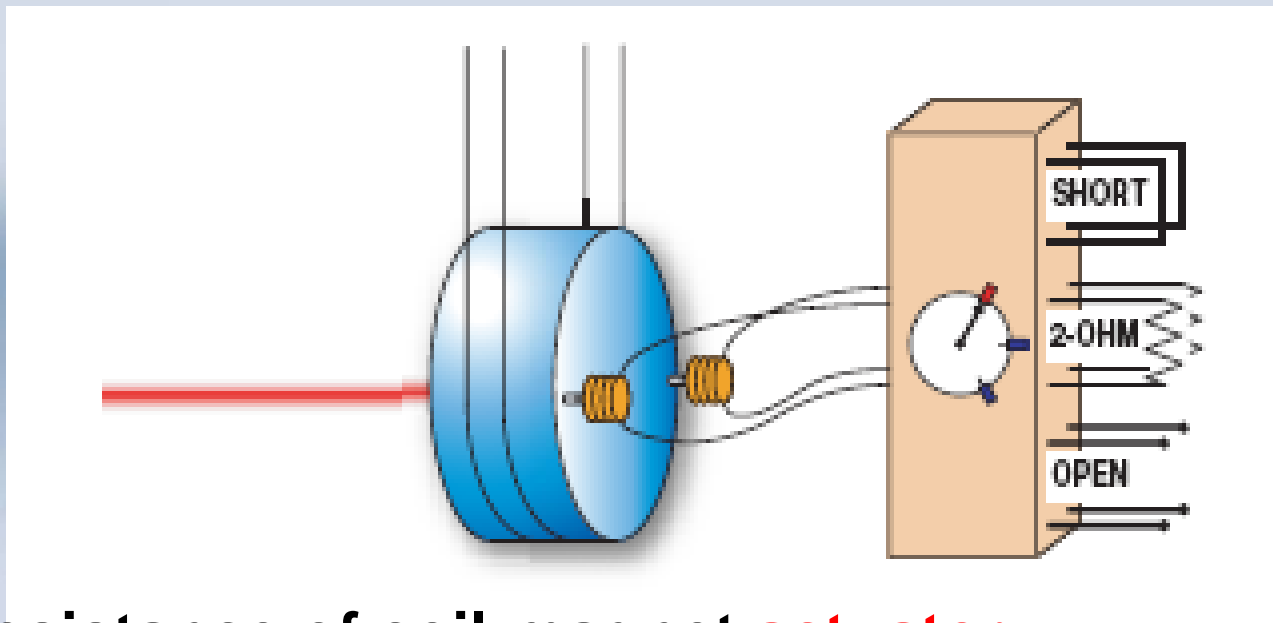
Sapphire : **Thermoelastic** damping in **substrate**



4. Interferometer after revolution

(3) Direct measurement of thermal noise

Direct measurement of thermal noise of **suspension**



Loss : Resistance of coil-magnet **actuator**
(**not material or clamp loss**)

We can **change** this resistance.

Q-value is still enough **high** ($\sim 10^5$).

K. Agatsuma *et al.*, Physical Review Letters 104 (2010) 040602.

4. Interferometer after revolution

(3) Direct measurement of thermal noise

Cryogenic Laser Interferometer Observatory (CLIO, Japan)
100 m, Kamioka (Japan)

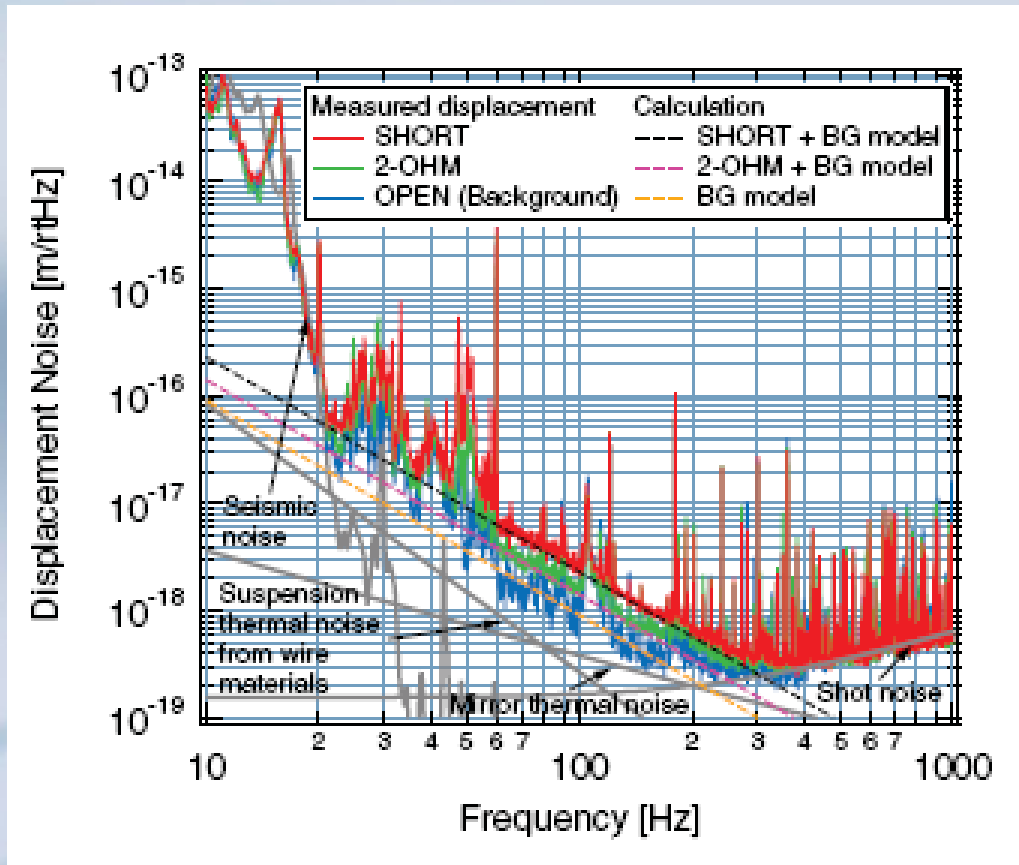


Prototype for LCGT ; cryogenic interferometer
in underground site with small seismic motion

S. Kawamura, Classical and Quantum Gravity 27 (2010) 084001.

4. Interferometer after revolution

(3) Direct measurement of thermal noise



Change of **resistance**



Change of **observed** sensitivity as theoretical prediction

Next step : Thermal noise by **loss in suspension itself**

4. Interferometer after revolution

(3) Direct measurement of thermal noise

I expect that the sensitivities of LIGO(U.S.A.), Virgo (Italy and France) and GEO (Germany and U.K.) are comparable with suspension thermal noise.

However, I can not find refereed papers which claim observation of suspension thermal noise.

4. Interferometer after revolution

(4) Direct measurement of off resonance dissipation

Now we observe thermal fluctuation directly.

Fluctuation dissipation theorem

$$G(f) = \frac{4k_B T}{\omega} \text{Im}[H(\omega)]$$

Power spectrum
of thermal **fluctuation**

Imaginary part of susceptibility
(**dissipation**)

How about direct measurement of **dissipation**
(imaginary part of susceptibility) ?

4. Interferometer after revolution

(4) Direct measurement of off resonance dissipation

How about dissipation (imaginary part of susceptibility) ?
It is not so easy.

Imaginary part is much **smaller** than real part.

$$\frac{\text{Im}[H(\omega)]}{\text{Re}[H(\omega)]} \sim \frac{1}{Q} < 10^{-6}$$

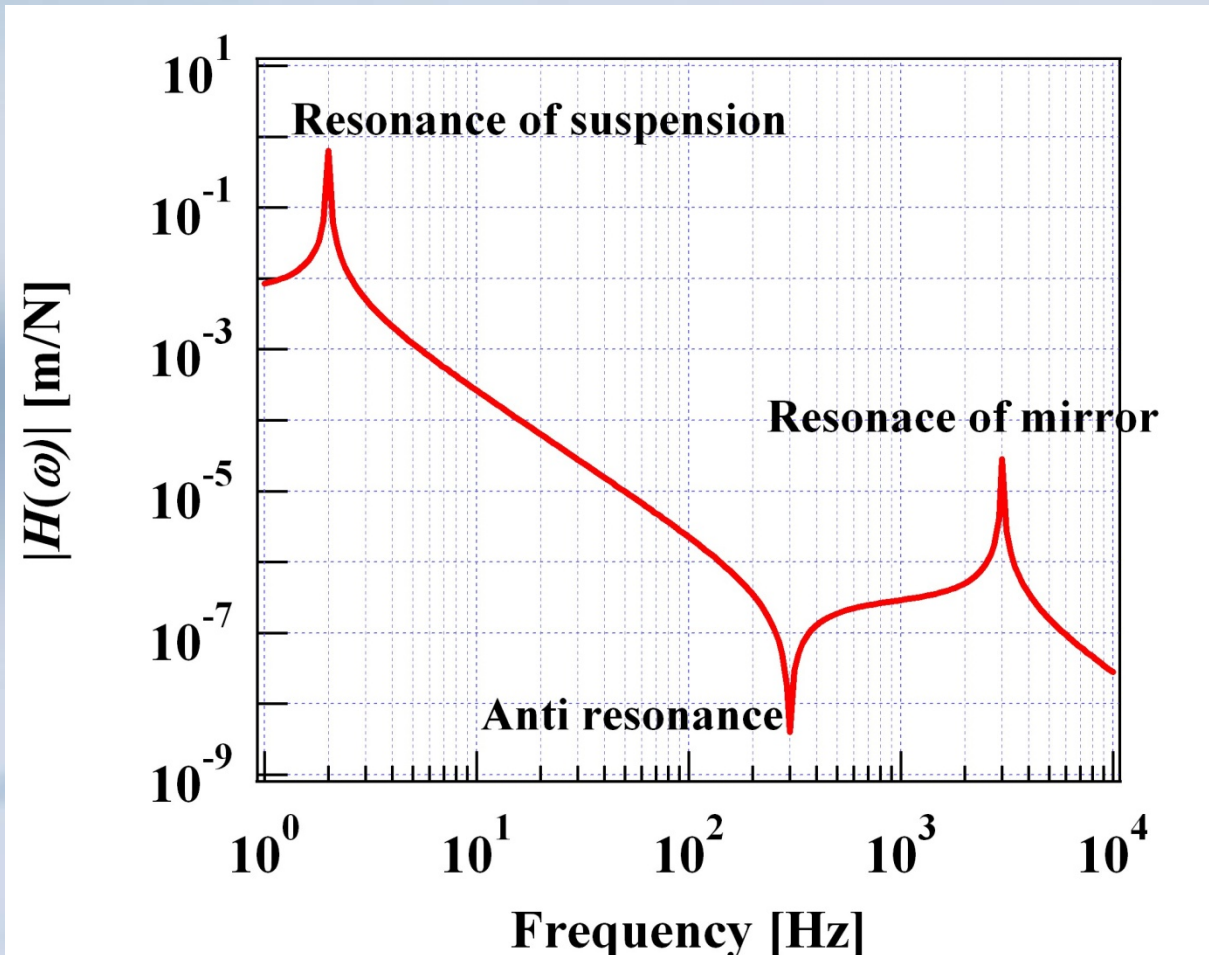
Off resonance region

Extreme precise measurement is necessary.

Relative error should be **smaller than 10^{-6}** !

4. Interferometer after revolution

(4) Direct measurement of off resonance dissipation

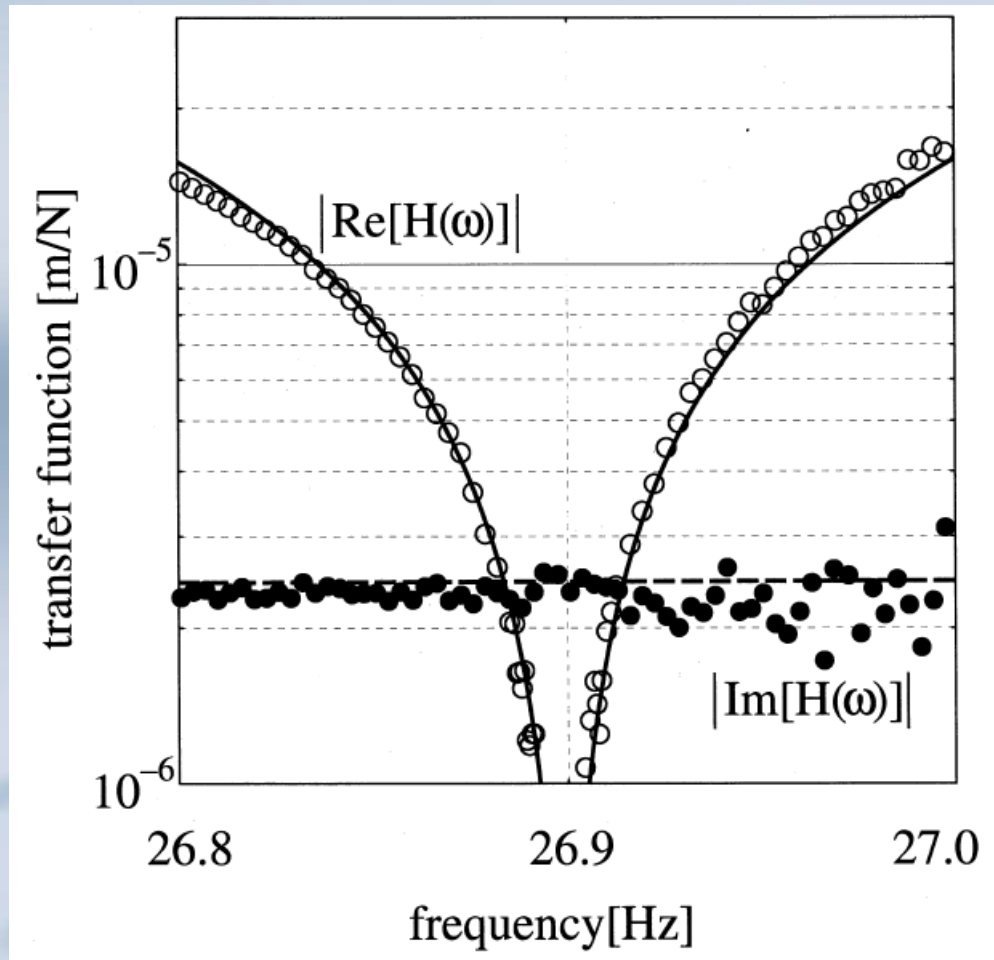


On **anti resonance**
 $\text{Re}[H]$ vanishes.
 $|H|$ is $|\text{Im}[H]|$
(dissipation).

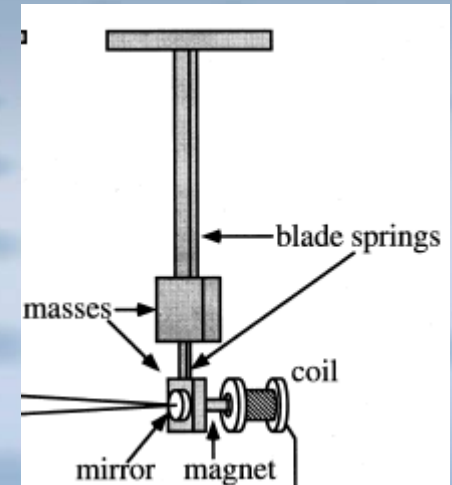
Requirement of
relative error is not
so serious.

4. Interferometer after revolution

(4) Direct measurement of off resonance dissipation



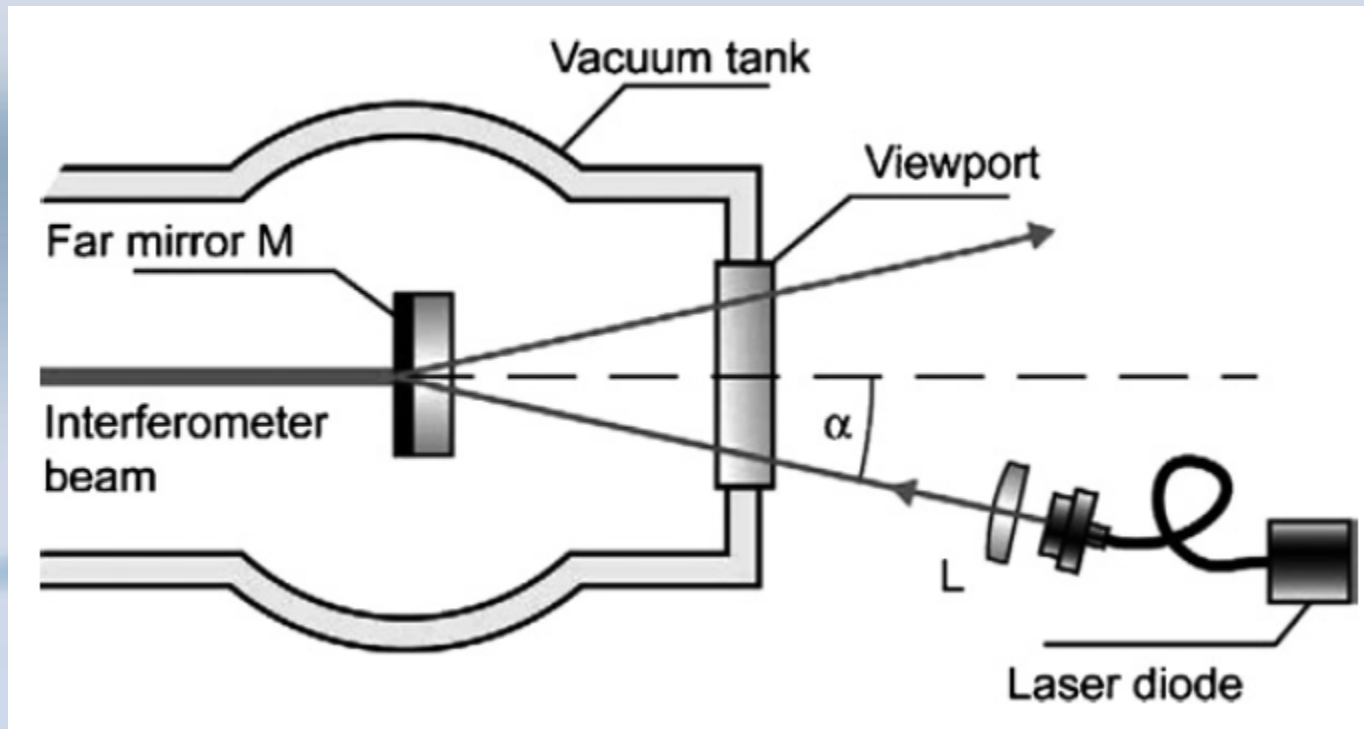
Ohishi confirmed that **this method works well** using small 2 modes oscillator.



4. Interferometer after revolution

(4) Direct measurement of off resonance dissipation

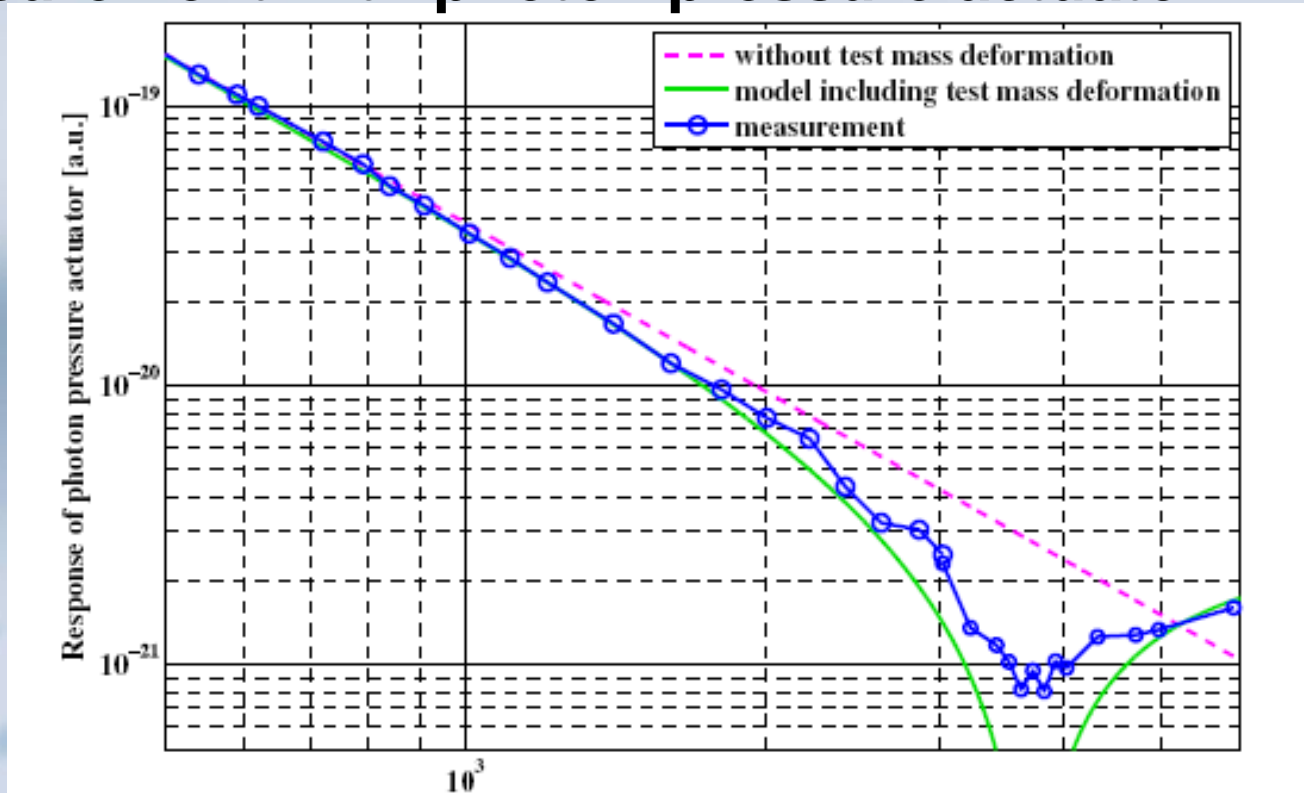
In gravitational wave detector, actuator is a problem. Probably, **photon pressure actuator** is the best one.



4. Interferometer after revolution

(4) Direct measurement of off resonance dissipation

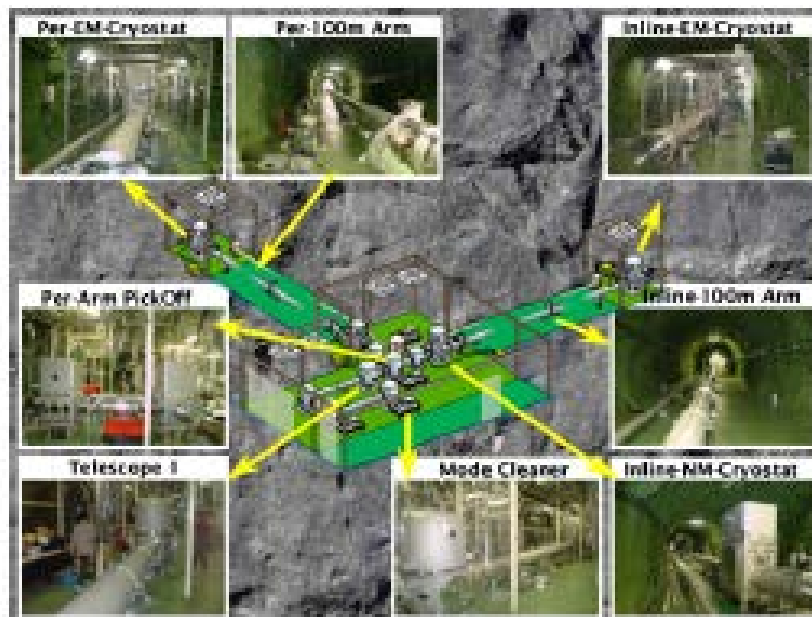
Measurement with photon pressure actuator in GEO600



Susceptibility could not be measured at anti resonance.
The progress is necessary.

4. Interferometer after revolution

(5) Reduction of thermal noise **100 m**, Kamioka (Japan)
Cryogenic Laser Interferometer Observatory (CLIO, Japan)

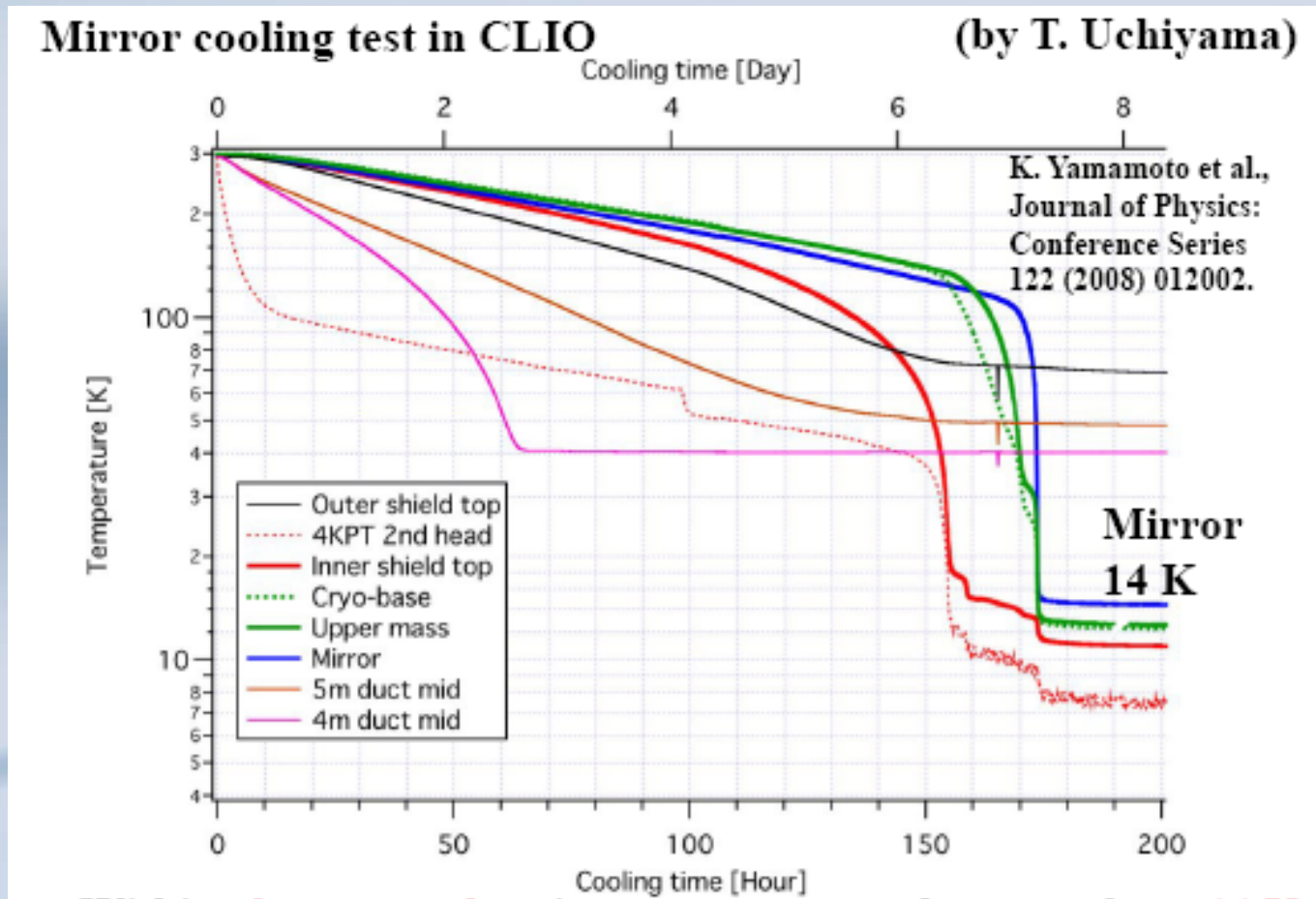


Result of CLIO for cryogenic techniques are introduced.

CLIO cryostat have already been **installed**
(**different scale** from those of **LCGT** and **ET**).

4. Interferometer after revolution

(5) Reduction of thermal noise



Within **about a week**, mirror temperature became about **14 K** (mirror temperature must be **below 20 K**).

4. Interferometer after revolution

(5) Reduction of thermal noise

Measurement of **vibration in cryostat**

with **silent pulse-tube cryocooler** at **silent site**

Cryocoolers do **not increase** vibration even if they are put on **site** with **extremely small seismic motion**.

K. Yamamoto *et al.*,
Journal of Physics:
Conference Series 32 (2006)
418.

