Length-Sensing OpLevs for KAGRA

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Basics

Length-Sensing Optical Levers are needed in order to measure the shift of mirrors along the optical path of the incident main-laser beam with time. The main idea behind this is to maintain a low-frequency control of the mirror's motions.

The principle for such a measurement is to use a second laser (or, as in case of KAGRA, a collimated monochromatic light source) besides the main-laser beam, and to direct it onto the central area of the mirror. The motions that a suspended mirror may have (it may yaw and swing) will then change the position of the reflected beam. Thus, the issue is to disentangle the two possible movements from the measured change of the position.

The most easiest way is to put a lens in the reflected beam and to use a small beam splitter together with two position-sensing detectors (PSD) to measure the swing along the main-laser beam and the yaw independently.

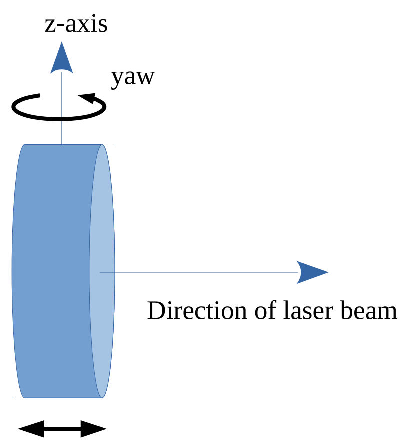

Figure 1: *Diagram of the basic concepts according to the movements of a mirror having a suspension like the mirrors used in KAGRA.*

The calculations for the disentanglement can easily be done by using the ABCD-matrix model. Thereby, it should be noted that a shift *d* along the main-beam's axis corresponds to a displacement (or shift) of the reflected beam of

$$X_1 = 2d \cdot \sin(\alpha)$$

where α is the incident angle on the mirror. In contrast, an angular displacement δ of the mirror (yaw, in our case) would lead also to an angular displacement θ :

$$\theta = 2\delta$$

In Figure 2, these basic relations are displayed graphically to give a better impression.

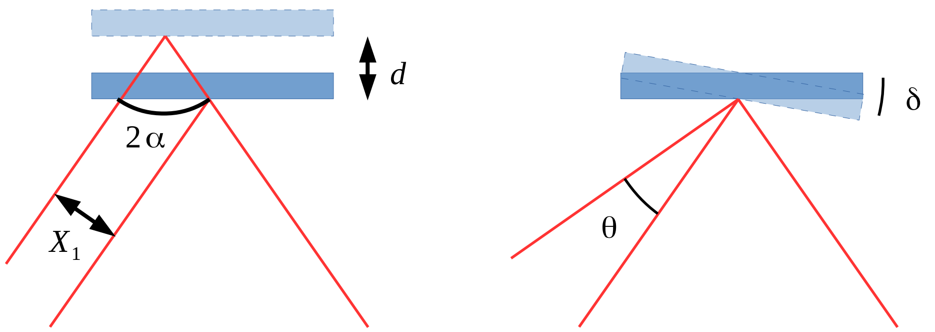

Figure 2: *View from top of the mirror in the cases of a horizontal shift (left) and a yaw around the z-axis (right). The paths of the OpLev-beam are given in red.*

It should be noted that, strictly speaking, only the second case (yaw) is related to an optical lever. However, in order to keep things simple, I will use “optical lever” also for the first case.

If we put a lens in the path of the reflected beam, there will be a displacement measurable also in the image and the focal plane of the lens which are due to an actual shift of these planes along the optical axis. But while a horizontal shift of the mirror will not change the focal plane of the lens, an angular displacement will do. Conversely, an angular displacement will not change the localization of the image plane but a horizontal shift of the mirror will do.

Using the ABCD-matrix model, we can write

$$\begin{pmatrix} X_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 1 & D \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -f^{-1} & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X_1 \\ \theta_1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 - D/f & D + L(1 - D/f) \\ -f^{-1} & 1 - f^{-1} \end{pmatrix} \cdot \begin{pmatrix} X_1 \\ \theta_1 \end{pmatrix}$$

where D and L are the distance from the sensor plane to the lens and from the lens to the mirror, respectively (see also Figure 3).

θ_2 θ_1

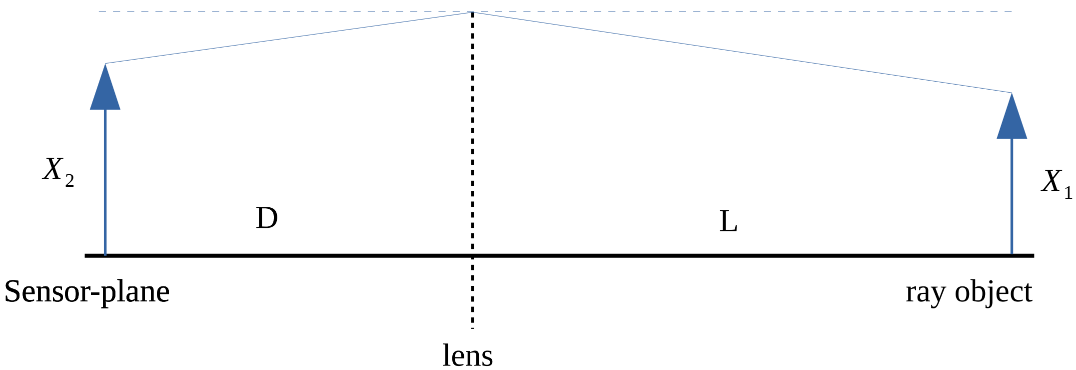

Figure 3: *Simplified diagram to visualize the basic parameters used in the calculation.*

The displacement visible on a PSD is thus

$$X_2 = (1 - D/f) X_1 + (D + L(1 - D/f)) \theta_1$$

Obviously, when the sensor plane (the PSD) lies on the focal plane of the lens ($D = f$), then X_2 is sensitive only to a yaw of the mirror (θ_1):

$$X_2 = f \theta_1$$

When the sensor plane lies on the image plane of the mirror ($D = Lf/(L - f)$), then X_2 is sensitive only to the displacement X_1 :

$$X_2 = \frac{-f}{L - f} \cdot X_1$$

In reality, however, a PSD can never be put 100%ly in the correct position to match with the focal or the image plane. Thus, there will be always a misplacement δD in the positioning of the PSD. If we put this misplacement in the equations above, we will get a more realistic expression for X_2 in the image and the focal plane:

$$X_2 = \left(\frac{-f}{L - f} - \frac{\delta D}{f} \right) \cdot X_1 + \delta D \left(1 - \frac{L}{f} \right) \cdot \theta_1 \qquad \rightarrow \text{image plane}$$

$$X_2 = \delta D \cdot X_1 + (f + \delta D (1 + L)) \cdot \theta_1 \qquad \rightarrow \text{focal plane}$$

It is possible to calculate the misplacement, of course, when all other parameters are known. However, the main issue is that these other parameters are hard to specify without significant errors in the real conditions of KAGRA.
The typical amplitude of a yaw and a shift are in the order of several μrad and μm, respectively. Assuming a misplacement of 5 mm, a shift of the mirror would have a ca. 10 times higher impact on the QPD in the image plane than a yaw.

Although it has been mentioned that the sensor can also lie in the focal plane of the lens to measure the yaw of the mirror, in KAGRA the respective QPD will lie in the direct optical path of the OpLev beam without being manipulated by any optical device (see Figure 4). In this configuration, the sensitivity due to a yaw of the mirror is bigger than with a lens ($X_2=(L+f)\theta_1>f\theta_1$) and we would not have any additional influence (vibrations, etc.) from optical devices. The only drawback is that the QPD would sense both a yaw and a shift from the mirror. However, we assume that any effect related to a shift would be negligible on the QPD compared to the yaw of the mirror. In addition to that, the effect of a once measured shift of the mirror on that QPD (X_1) can easily be calculated with above equations and thus subtracted from the overall signal so that only the effect due to a yaw of the mirror is left.

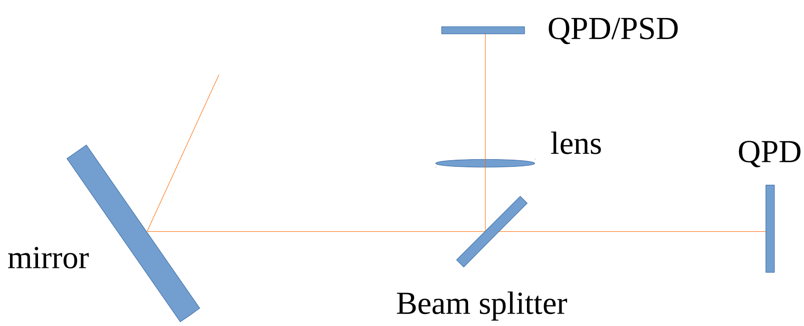

Figure 4: *Basic structure of the OpLev in KAGRA*

Beam Profile

The beam profile of the collimator (plus focuser) used for the beam splitter has been measured in the laboratory and is given as a basic example in Figure 5. Though every mirror will (basically) have a different collimator and focuser, the most important principles can be understood already from one profile measurement. In Figure 5 the diameter of the beam (assumed as Gaussian) is given as a function of the distance to the collimator together with photographs of the beam's profile taken with a CCD camera. Although difficult to see, the profile appears to be quite unsymmetrical and misshaped especially at distances before the beam's waist. This behavior has been observed also for the other collimator (see the report by Akutsu-san: http://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=4478). However, with a lens relatively close to the waist's distance and the QPD in the image plane, I consider that the beam will be in distances where the profile is much more Gaussian-like as it is the case already for the yaw-QPDs.

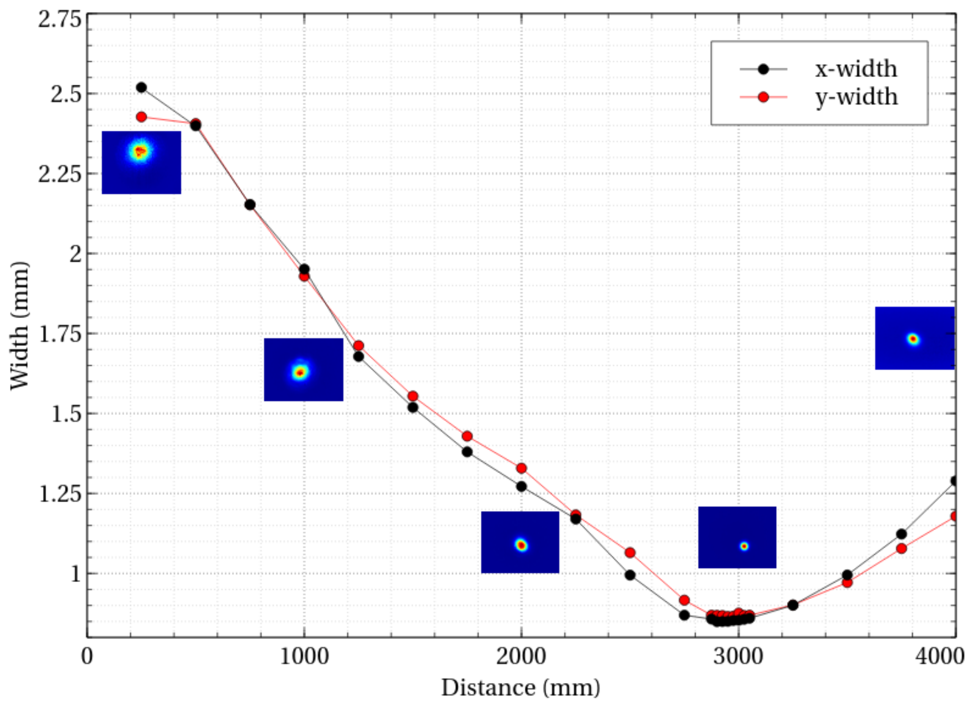

Figure 5: *Development of the beam profile of the used collimeter in two dimensions. The small pictures are photographs of the actual profile. The width means the actual diameter of the beam.*

By putting a lens in the beam, the profile will change according to the focal length of the lens and its position. If we assume the beam as being approximately Gaussian, the changed profile can be calculated by using the beam-profile parameter q:

$$q = z + i z_R$$

with z being the distance to the waist of the beam and $z_R = \pi w_0^2 / \lambda$ where w_0 is the half diameter of the beam's waist.

By taking the ABCD-matrix model of a lens (see above), the new q-parameter q_f becomes

$$q_f = \frac{q}{1 - \frac{q}{f}} = z_f + i\pi \frac{w_{0f}^2}{\lambda}$$

which, after some calculation, is turning to

$$q_f = \frac{z - \frac{z^2}{f} + \frac{z_R}{f}}{\left(1 - \frac{z}{f}\right)^2 + \frac{z_R^{\ 2}}{f^2}} + i \frac{z_R}{\left(1 - \frac{z}{f}\right)^2 + \frac{z_R^{\ 2}}{f^2}}$$

In this matter, z appears to be the position of the lens in terms of its distance to the beam's waist.

Beam Splitter

Configuration of the Length-Sensing OpLevs for the beam splitter in KAGRA

In theory, the angle of incidence of the collimated beam onto the BS (beam splitter) mirror is 37º and the distance from either viewport to the center of the chamber is 988.8 mm and thus 972.8 mm to the mirror's surface (viewports and mirror should be symmetrically aligned; see Figure 6). **The overall distance from mirror to the lens (*L*) is (right now) difficult to specify. But, it should be around 1230 mm!**

Note: the beam should not enter (and leave) the viewports right in their center but with a vertical offset of around 12 mm (bottom viewport upwards; top viewport downwards). This is because of the 40 mm thickness of the beam splitter and the alignment of the viewports toward the center of the chamber. It is further assumed that the beam splitter is centered in the chamber.

Additionally, the beam height with respect to the optical table should be around 86 mm (this can be assumed to be a default value for all the other OpLevs too).

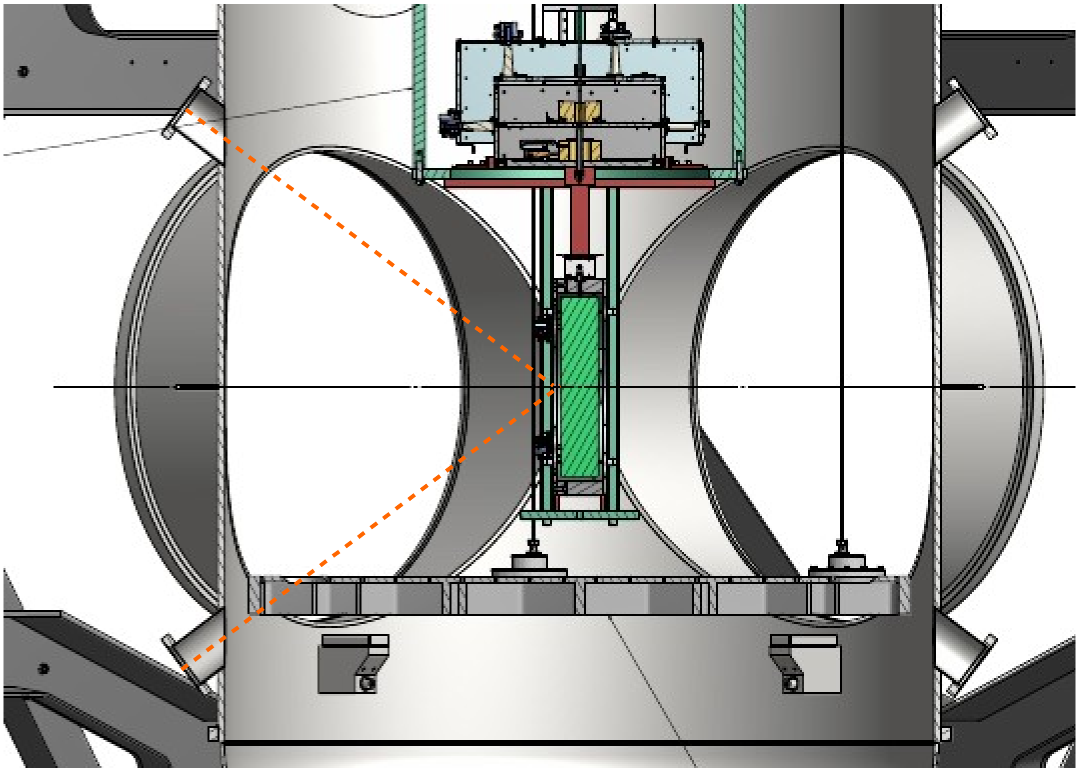

Figure 6: *Drawing of the beam-splitter chamber and the path of the OpLev beam (Orange). The total length of this path is 2 x 972.8 mm.*

With these basic parameters we can estimate the profile of the beam behind the lens (considering $\delta D=0$). According to Figure 5, the waist of the collimated beam is around 3000 mm away from the collimator, so that *z* can be approximated with -500 mm. λ , the wavelength, is ca. 670 nm, and w_0 is 0.43 mm (see Figure 5):

f	$\frac{Lf}{L-f}$	z_f	w_{0f}	w_{image}	$X_2=\frac{f\cdot 2d\cdot\sin(\alpha)}{L-f}$
100	108.85	-94	0.034	0.097	0.107 *d*
200	238.83	-177	0.065	0.212	0.234 *d*
300	396.77	-250	0.092	0.353	0.388 *d*
400	592.77	-312	0.116	0.527	0.58 *d*
500	842.47	-367	0.138	0.749	0.824 *d*
600	1172.43	-414	0.157	1.041	1.146 *d*
700	1624.53	-455	0.174	1.444	1.59 *d*
800	2288.37	-490	0.189	2.034	2.239 *d*
900	3354.55	-522	0.203	2.982	3.283 *d*

Table 1: *List of all important parameters and their development when using different lenses positioned 1230 mm away from the mirror. All parameters, except the last column, are given in mm!*

For the actual measurements, the most important parameters are the width of the beam in the image plane and the sensitivity (last two columns in Table 1). Basically, as *L* would decrease, both values will increase for all *f*. The minimum value *L* could have is thus ~1000 mm (right behind the viewport) and the respective parameters become:

f	$\frac{Lf}{L-f}$	z_f	w_{0f}	w_{image}	$X_2=\frac{f\cdot 2d\cdot\sin(\alpha)}{L-f}$
100	111.1	-94	0.03	0.122	0.134 *d*
200	250	-178	0.057	0.273	0.301 *d*
300	428.57	-252	0.081	0.469	0.516 *d*
400	666.67	-318	0.103	0.729	0.802 *d*
500	1000	-376	0.122	1.094	1.204 *d*
600	1500	-428	0.14	1.64	1.805 *d*
700	2333.33	-474	0.156	2.552	2.808 *d*
800	4000	-514	0.17	4.374	4.815 *d*
900	9000	-550	0.183	9.842	10.833 *d*

Table 2: *The same as Table 1 but with the lenses positioned 1000 mm away from the mirror. All parameters, except the last column, are given in mm!*

The reason why the width of the beam is important is its influence on the choice of the detector. A QPD (Quadropole Photo-Diode) would need a beam with at least ~0.4 mm diameter while a PSD can deal with smaller diameters. Therefore, a QPD requires a configuration with $f\geq 400$ mm in the first case and $f\geq 300$ mm in the second.

PR3

The PR mirrors will not have a vertical OpLev alignment but a horizontal. **The angle of incidence of the OpLev beam will be likely in the range of 56° while the distance from mirror to the out-going viewport is 761.8 mm**. The distance from mirror to the yaw-sensing QPD would be around 1062 mm, approximately the same distance as to the lens (pylons with a mounted optical table are placed already in front of the viewports of the PR3 vacuum chamber leaving very little freedom to change the parameters). Additionally, the collimator used for the PR chambers is different from the BS with a "waist" being located around 2 m away from the source. Measurements were taken by Akutsu-san for these collimators (see: http://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/private/DocDB/ShowDocument?docid=4478).

All lenses with a focal length $f>300$ mm would lead us to image planes that are too far away for a reasonable setup of the QPDs (with respect to the available size on the optical tables).

f	$\frac{Lf}{L-f}$	z_f	w_{0f}	w_{image}	$X_2=\frac{f\cdot 2d\cdot\sin(\alpha)}{L-f}$
100	110.4	-98	0.049	0.074	0.172 *d*
200	246.41	-186	0.09	0.165	0.385 *d*
300	418.14	-262	0.136	0.279	0.653 *d*
400	641.76	-322	0.174	0.429	1.002 *d*
500	945	-367	0.207	0.632	1.476 *d*
600	1379.56	-399	0.235	0.922	2.154 *d*
700	2054.34	-419	0.259	1.373	3.208 *d*
800	3244.61	-431	0.279	2.168	5.067 *d*
900	5906.18	-436	0.296	3.947	9.223 *d*

Table 3: *Parameter table for the PR3 OpLev. All parameters, except the last column, are given in mm!*

PR2

The PR2 chamber is very similar to the PR3 chamber. However, the difficulty for the OpLev is the fact that only one viewport can be used for the incoming and out-going beam. This issue may be solved by either putting the beam collimator inside the chamber on the side where the missing viewport should be or by using a mirror on this specific location which either reflects the beam back to the PR2 mirror or directly toward the viewport (pick-off). The latter configuration may be used also for focusing the beam on the PR2 mirror which reflects the light toward the viewport again.

Here, we will give the calculations for these two latter cases. If the beam is picked-off by the inner mirror, L will be ~1866 mm. If the inner mirror is focusing the beam toward the PR2 mirror, L will be similar to PR3 (~970 mm). In both cases the angle of incidence is ~50°. The respective parameters in both cases are listed in Table 4 and Table 5. In both cases we would need a collimator equal to that of the beam splitter to reach a beam-diameter small enough for the QPDs.

f	$\frac{Lf}{L-f}$	z_f	w_{0f}	w_{image}	$X_2=\frac{f\cdot 2d\cdot\sin(\alpha)}{L-f}$
100	105.66	-95	0.043	0.068	0.087 *d*
200	224.01	-179	0.082	0.144	0.184 *d*
300	357.47	-249	0.117	0.229	0.294 *d*
400	509.14	-308	0.15	0.327	0.418 *d*
500	683.02	-356	0.173	0.438	0.561 *d*
600	884.36	-394	0.197	0.567	0.726 *d*
700	1120.24	-424	0.217	0.719	0.92 *d*
800	1400.38	-448	0.234	0.898	1.15 d
900	1738.51	-467	0.249	1.115	1.427 *d*

Table 4: *Parameter table for the PR2 OpLev in case the beam is picked off by an additional mirror. All parameters, except the last column, are given in mm!*

f	$\frac{Lf}{L-f}$	z_f	w_{0f}	w_{image}	$X_2=\frac{f\cdot 2d\cdot\sin(\alpha)}{L-f}$
100	111.49	-95	0.043	0.092	0.176 *d*
200	251.95	-179	0.082	0.208	0.398 *d*
300	434.33	-249	0.117	0.358	0.686 *d*
400	680.7	-308	0.147	0.561	1.075 *d*
500	1031.91	-356	0.173	0.85	1.63 *d*
600	1572.97	-394	0.196	1.296	2.484 *d*
700	2514.81	-425	0.216	2.072	3.972 *d*
800	4564.71	-448	0.234	3.76	7.21 *d*
900	12471.43	-467	0.249	10.274	19.698 *d*

Table 5: *Parameter table for the PR2 OpLev in case the beam is directed to the PR2 mirror by an additional mirror. All parameters, except the last column, are given in mm!*

PRM

The case of the PRM mirror will be similar to the PR3 mirror. **The distance between the viewports and the mirror is again approximately 765 mm while the yaw-QPD is located ~900 mm away**

from the mirror leaving ~850 mm distance for the lens. The angle of incidence is slightly different, though. We can say that it is around 45° according to the principle drawings of the vacuum chamber. Hence, the parameters are the same as for the PR3 mirror except the corresponding precision which is a function of the angle of incidence.

f	$\frac{Lf}{L-f}$	z_f	w_{0f}	w_{image}	$X_2=\frac{f\cdot 2d\cdot\sin(\alpha)}{L-f}$
100	113.33	-93	0.047	0.106	0.1886 *d*
200	261.54	-169	0.086	0.244	0.4351 *d*
300	463.64	-230	0.12	0.433	0.7714 *d*
400	755.56	-277	0.148	0.706	1.2571 *d*
500	1214.29	-314	0.171	1.135	2.0203 *d*
600	2040	-342	0.191	1.907	3.3941 *d*
700	3966.67	-363	0.208	3.708	6.5997 *d*
800	13600	-380	0.222	12.714	22.6274 *d*
900	-15300	-392	0.234	[14.303]	-

Table 6: *Parameter table for the PRM OpLev. All parameters, except the last column, are given in mm!*

ETMY

For the end test-masses, there are basically two different setups to be built. First, a regular (angular) OpLev and a LS (length-sensing) OpLev for the Sapphire test-mass itself. And second, a regular OpLev for the marionette of the payload.

The radius of the vacuum chamber is 1210 mm but the Sapphire mirror is ~100 mm long. **So, the distance from the viewport opening in the chamber walls to the surface of the mirror is ~1174 mm (assuming 45° incident angle). The distance to the OpLev lens is about 1674 mm!**

Assuming 1674 mm for the distance to the lens, the properties' table of the LS OpLev is as follows:

f	$\frac{Lf}{L-f}$	z_f	w_{0f}	w_{image}	$X_2=\frac{f\cdot 2d\cdot\sin(\alpha)}{L-f}$
100	106.35	-105	0.044	0.045	0.09 *d*
200	227.14	-215	0.092	0.096	0.192 *d*

300	365.5	-326	0.144	0.155	0.309 *d*
400	525.59	-428	0.196	0.223	0.444 *d*
500	712.95	-511	0.248	0.302	0.602 *d*
600	935.2	-569	0.297	0.396	0.79 *d*
700	1203.08	-597	0.341	0.51	1.016 *d*
800	1532.27	-594	0.38	0.65	1.294 *d*
900	1946.51	-566	0.412	0.825	1.644 *d*