Length-Sensing OpLevs for KAGRA

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Basics

Length-Sensing Optical Levers are needed in order to measure the shift of mirrors along the optical path of the incident main-laser beam with time. The main idea behind this is to maintain a low-frequency control of the mirror’s motions.

The principle for such a measurement is to use a second laser (or, as in case of KAGRA, a collimated monochromatic light source) besides the main-laser beam, and to direct it onto the central area of the mirror. The motions that a suspended mirror may have (it may yaw and swing) will then change the position of the reflected beam. Thus, the issue is to disentangle the two possible movements from the measured change of the position.

The most easiest way is to put a lens in the reflected beam and to use a small beam splitter together with two position-sensing detectors (PSD) to measure the swing along the main-laser beam and the yaw independently.

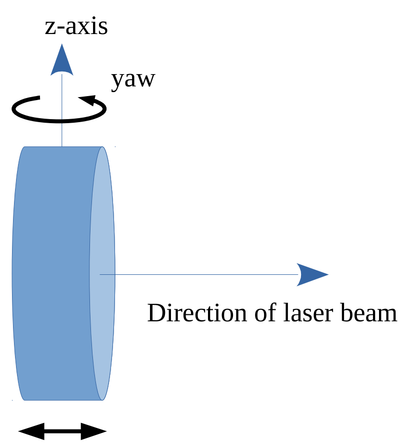

Figure 1: *Diagram of the basic concepts according to the movements of a mirror having a suspension like the mirrors used in KAGRA.*

The calculations for the disentanglement can easily be done by using the ABCD-matrix model. Thereby, it should be noted that a shift d along the main-beam’s axis corresponds to a displacement (or shift) of the reflected beam of

$$X_1 = 2d \cdot \sin(\alpha)$$

where α is the incident angle on the mirror. In contrast, an angular displacement δ of the mirror (yaw, in our case) would lead also to an angular displacement θ :

$$\theta = 2\delta$$

In Figure 2, these basic relations are displayed graphically to give a better impression.

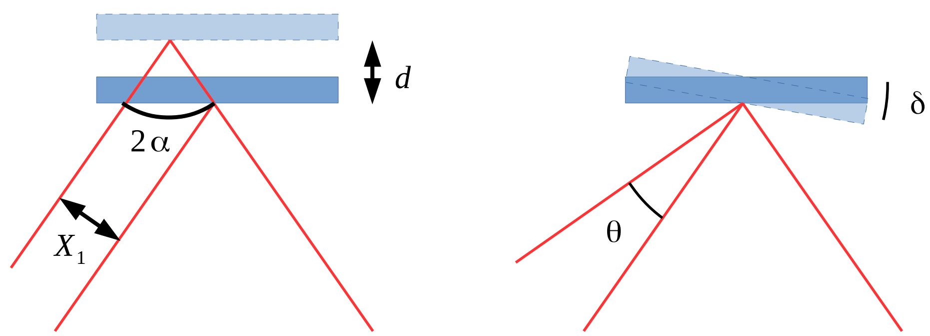

Figure 2: *View from top of the mirror in the cases of a horizontal shift (left) and a yaw around the z-axis (right). The paths of the OpLev-beam are given in red.*

It should be noted that, strictly speaking, only the second case (yaw) is related to an optical lever. However, in order to keep things simple, I will use “optical lever” also for the first case.

If we put a lens in the path of the reflected beam, there will be a displacement measurable also in the image and the focal plane of the lens which are due to an actual shift of these planes along the optical axis. But while a horizontal shift of the mirror will not change the focal plane of the lens, an angular displacement will do. Conversely, an angular displacement will not change the localization of the image plane but a horizontal shift of the mirror will do.

Using the ABCD-matrix model, we can write

$$\begin{pmatrix} X_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 1 & D \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -f^{-1} & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X_1 \\ \theta_1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 - D/f & D + L(1 - D/f) \\ -f^{-1} & 1 - f^{-1} \end{pmatrix} \cdot \begin{pmatrix} X_1 \\ \theta_1 \end{pmatrix}$$

where *D* and *L* are the distance from the sensor plane to the lens and from the lens to the mirror, respectively (see also Figure 3).

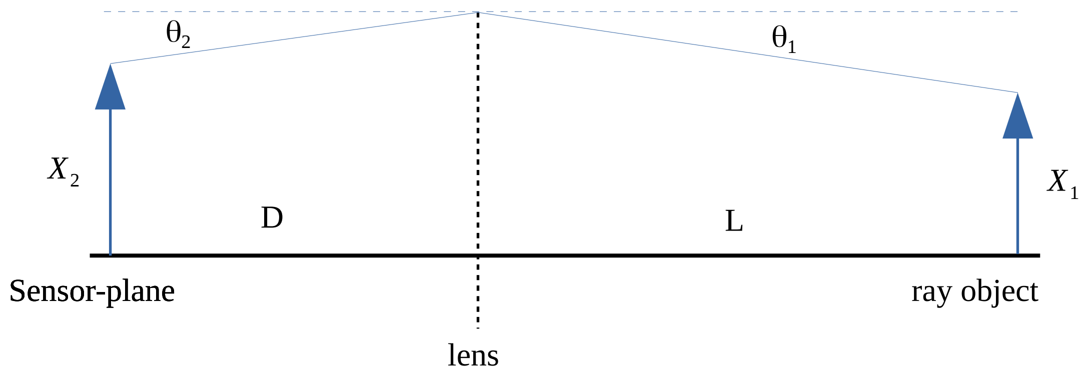

Figure 3: *Simplified diagram to visualize the basic parameters used in the calculation.*

The displacement visible on a PSD is thus

$$X_2 = (1 - D/f)\,X_1 + (D + L(1 - D/f))\,\theta_1$$

Obviously, when the sensor plane (the PSD) lies on the focal plane of the lens ($D = f$), then X_2 is sensitive only to a yaw of the mirror (θ_1):

$$X_2 = f\,\theta_1$$

When the sensor plane lies on the image plane of the mirror ($D = Lf/(L - f)$), then X_2 is sensitive only to the displacement X_1 :

$$X_2 = \frac{-f}{L - f} \cdot X_1$$

In reality, however, a PSD can never be put 100%ly in the correct position to match with the focal or the image plane. Thus, there will be always a misplacement δD in the positioning of the PSD. If we put this misplacement in the equations above, we will get a more realistic expression for X_2 in the image and the focal plane:

$$X_2 = \left(\frac{-f}{L - f} - \frac{\delta D}{f}\right) \cdot X_1 + \delta D\left(1 - \frac{L}{f}\right) \cdot \theta_1$$ → image plane

$$X_2 = \delta D \cdot X_1 + (f + \delta D(1 + L)) \cdot \theta_1$$ → focal plane

It is possible to calculate the misplacement, of course, when all other parameters are known. However, the main issue is that these other parameters are hard to specify without significant errors in the real conditions of KAGRA.
The typical amplitude of a yaw and a shift are in the order of several µrad and µm, respectively. Assuming a misplacement of 5 mm, a shift of the mirror would have a ca. 10 times higher impact on the QPD in the image plane than a yaw.

Although it has been mentioned that the sensor can also lie in the focal plane of the lens to measure the yaw of the mirror, in KAGRA the respective QPD will lie in the direct optical path of the OpLev beam without being manipulated by any optical device (see Figure 4). In this configuration, the sensitivity due to a yaw of the mirror is bigger than with a lens ($X_2=(L+f)\theta_1>f\theta_1$) and we would not have any additional influence (vibrations, etc.) from optical devices. The only drawback is that the QPD would sense both a yaw and a shift from the mirror. However, we assume that any effect related to a shift would be negligible on the QPD compared to the yaw of the mirror. In addition to that, the effect of a once measured shift of the mirror on that QPD (X_1) can easily be calculated with above equations and thus subtracted from the overall signal so that only the effect due to a yaw of the mirror is left.

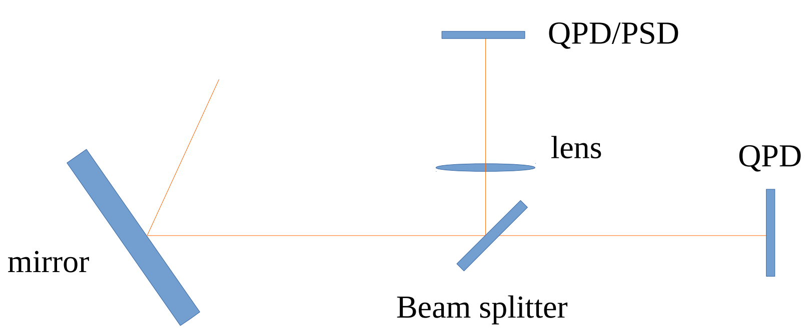

Figure 4: *Basic structure of the OpLev in KAGRA*

Beam Profile

The beam profile of the used collimeter has been measured in the laboratory and is given in Figure 5.

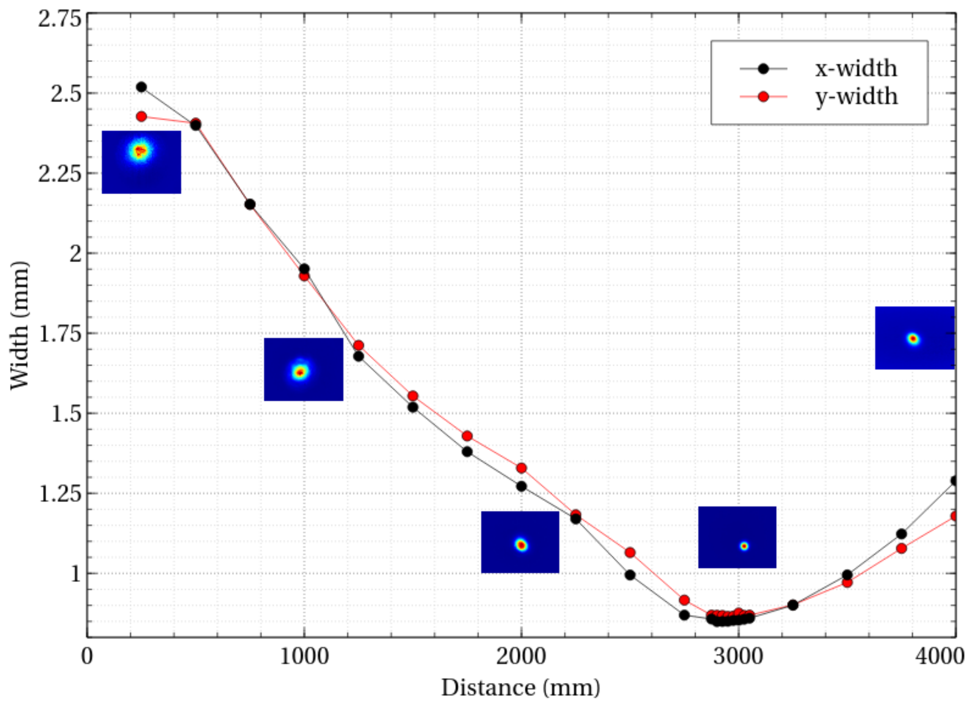

Figure 5: *Development of the beam profile of the used collimeter in two dimensions. The small pictures are photographs of the actual profile. The width means the actual diameter of the beam.*

By putting a lens in the beam, the profile will change according to the focal length of the lens and its position. If we assume the beam as being approximately Gaussian, the changed profile can be calculated by using the beam-profile parameter q:

$$q = z + i z_R$$

with z being the distance to the waist of the beam and $z_R = \pi w_0^2 / \lambda$ where w_0 is the half diameter of the beam's waist.

By taking the ABCD-matrix model of a lens (see above), the new q-parameter q_f becomes

$$q_f = \frac{q}{1 - \frac{q}{f}} = z_f + i\pi \frac{w_{0f}^2}{\lambda}$$

which, after some calculation, is turning to

$$q_f = \frac{z - \frac{z^2}{f} + \frac{z_R}{f}}{\left(1-\frac{z}{f}\right)^2 + \frac{z_R^2}{f^2}} + i\frac{z_R}{\left(1-\frac{z}{f}\right)^2 + \frac{z_R^2}{f^2}}$$

In this matter, *z* appears to be the position of the lens in terms of its distance to the beam's waist.

Beam Splitter

Configuration of the Length-Sensing OpLevs for the beam splitter in KAGRA

In theory, the angle of incidence of the collimated beam onto the BS (beam splitter) mirror is 33° and the distance from either viewport to the mirror is 988.8 mm (viewports and mirror should be symmetrically aligned; see Figure 6). **The overall distance from mirror to the lens (*L*) is (right now) difficult to specify. But, it should be around 1500 mm!**

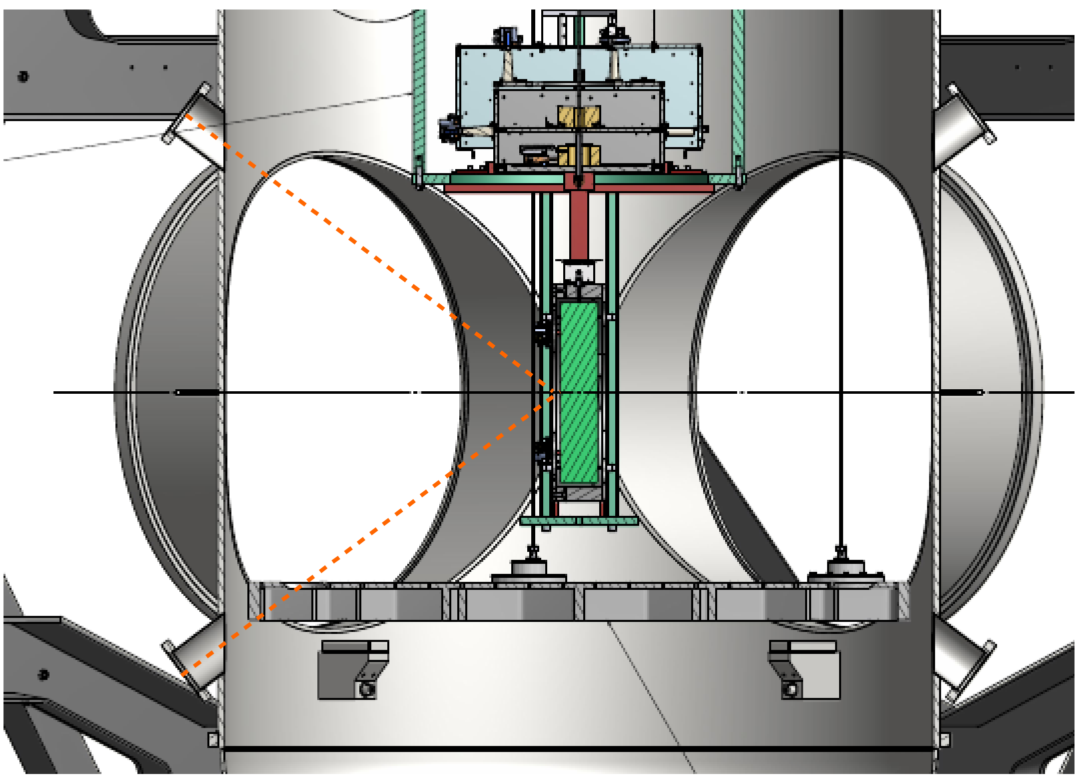

Figure 6: *Drawing of the beam-splitter chamber and the path of the OpLev beam (Orange). The total length of this path is 2 x 988.8 mm.*

With these basic parameters we can estimate the profile of the beam behind the lens (considering $\delta D = 0$). According to Figure 5, the waist of the collimated beam is at around 3000 mm away from the collimeter, so that z can be approximated with -500 mm. λ , the wavelength, is ca. 630 nm, and w_0 is 0.43 mm (see Figure 5):

f	$\frac{Lf}{L-f}$	z_f	w_{0f}	w_{image}	X_2 in terms of d
100	107.14	-95	0.039	0.073	0.0778
200	230.77	-179	0.074	0.158	0.1676
300	375	-252	0.106	0.257	0.2723
400	545.45	-313	0.133	0.373	0.3961
500	750	-365	0.158	0.514	0.5446
600	1000	-408	0.18	0.685	0.7262
700	1312.5	-443	0.2	0.899	0.9531
800	1714.29	-472	0.216	1.174	1.2449
900	2250	-496	0.231	1.541	1.6339

Table 1: *List of all important parameters and their development when using different lenses positioned 1500 mm away from the mirror. All parameters are given in mm!*

For the actual measurements, the most important parameters are the width of the beam in the image plane and the sensitivity (last two columns in Table 1). Basically, as L would decrease, both values will increase for all f. The minimum value L could have is thus ~1000 mm (right behind the viewport) and the respective parameters become:

f	$\frac{Lf}{L-f}$	z_f	w_{0f}	w_{image}	X_2 in terms of d
100	111.1	-95	0.03	0.091	0.121
200	250	-179	0.057	0.205	0.2723
300	428.57	-254	0.081	0.352	0.4668
400	666.67	-320	0.103	0.547	0.7262
500	1000	-379	0.122	0.821	1.0893
600	1500	-431	0.14	1.232	1.6339
700	2333.33	-477	0.156	1.916	2.5416
800	4000	-518	0.17	3.284	4.3571
900	9000	-555	0.183	7.39	9.8035

Table 2: *The same as Table 1 but with the lenses positioned 1000 mm away from the mirror. All parameters are given in mm!*

The reason why the width of the beam is important is its influence on the choice of detector. A QPD (Quadropole Photo-Diode) would need a beam with at least ~0.4 mm diameter while a PSD can deal with smaller diameters. Therefore, a QPD requires a configuration with $f \geq 300$ mm in case of $L = 1500$ mm ($f \geq 200$ mm for $L = 1000$ mm).

PR3

The PR mirrors will not have a vertical OpLev alignment but a horizontal. **The angle of incidence of the OpLev beam will be likely in the range of 52° while the distance from mirror to the out-going viewport is around 700 mm**. The distance from mirror to the yaw-sensing QPD would be around 900 mm while the distance to lens can be assumed to be at around 850 mm (pylons with a mounted optical table are placed already in front of the viewports of the PR3 vacuum chamber leaving very little freedom to change the parameters). With these assumptions, a lens with a focal length of more than 850 mm would not create an image anymore and as can be seen from the table below (Table 3), all lenses with a focal length $f > 300$ mm would lead us to image planes that are too far away for a reasonable setup of the QPDs (with respect to the available size on the optical tables).

f	$\frac{Lf}{L-f}$	z_f	w_{0f}	w_{image}	X_2 in terms of d
100	113.33	-95	0.023	0.157	0.2101
200	261.54	-182	0.044	0.362	0.4849
300	463.64	-260	0.064	0.642	0.8596
400	755.56	-332	0.081	1.046	1.4009
500	1214.29	-397	0.098	1.682	2.2515
600	2040	-456	0.112	2.825	3.7825
700	3966.67	-511	0.126	5.494	7.3548
800	13600	-560	0.139	18.837	25.2163
900	-15300	-606	0.151	[21.191]	-

Table 3: *The same as Table 1 but with the lenses positioned 1000 mm away from the mirror. All parameters are given in mm!*

PR2

The PR2 chamber is very similar to the PR3 chamber. However, the difficulty for the OpLev is the fact that only one vieport can be used for the incoming and out-going beam. This issue may be solved by either putting the beam collimeter inside the chamber on the side where the missing viewport should be or by using a mirror on this specific location which either reflects the beam back to the PR2 mirror or directly toward the viewport (pick-off). Also, the latter configuration may be used for focusing the beam on the PR2 mirror which reflects the light toward the viewport again.

Here, we will give the calculations for these two latter cases. If the beam is picked-off by the inner mirror, L will be ~1800 mm. If the inner mirror is focusing the beam toward the PR2 mirror, L will be similar to PR3 (~850 mm). In both cases the angle of incidence is 52°.

f	$\frac{Lf}{L-f}$	z_f	w_{0f}	w_{image}	X_2 in terms of d
100	105.88	-95	0.041	0.065	0.0927
200	225	-180	0.078	0.139	0.197
300	360	-253	0.111	0.222	0.3152
400	514.29	-314	0.141	0.318	0.4503
500	692.31	-364	0.167	0.428	0.6061
600	900	-405	0.19	0.556	0.788
700	1145.45	-438	0.21	0.708	1.0029
800	1440	-465	0.227	0.89	1.2608
900	1800	-485	0.243	1.112	1.576

Table 4: *The same as Table 1 but with the lenses positioned 1000 mm away from the mirror. All parameters are given in mm!*