Length-Sensing OpLevs for KAGRA

Simon Zeidler

Basics

Length-Sensing Optical Levers are needed in order to measure the shift of mirrors along the optical path of the incident main-laser beam with time. The main idea behind this is to maintain a low-frequency control of the mirror's motions.

The principle for such a measurement is to use a second laser (or, as in case of KAGRA, a collimated monochromatic light source) besides the main-laser beam, and to direct it onto the central area of the mirror. The motions that a suspended mirror may have (it may yaw and swing) will then change the position of the reflected beam. Thus, the issue is to disentangle the two possible movements from the measured change of the position.

The most easiest way is to put a lens in the reflected beam and to use a small beam splitter together with two position-sensing detectors (PSD) to measure the swing along the main-laser beam and the yaw independently.

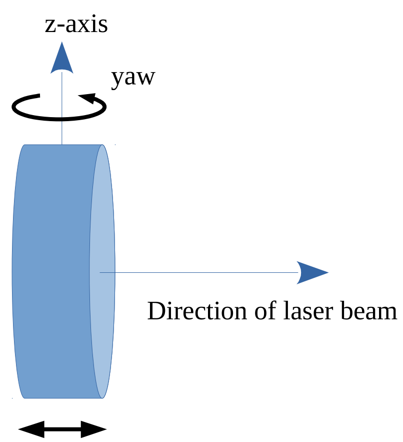

Figure 1: *Diagram of the basic concepts according to the movements of a mirror having a suspension like the mirrors used in KAGRA.*

The calculations for the disentanglement can easily be done by using the ABCD-matrix model. Thereby, it should be noted that a shift *d* along the main-beam's axis corresponds to a displacement (or shift) of the reflected beam of

$$X_1 = 2d \cdot \sin(\alpha)$$

where α is the incident angle on the mirror. In contrast, an angular displacement δ of the mirror (yaw, in our case) would lead also to an angular displacement θ :

$$\theta = 2\delta$$

In Figure 2, these basic relations are displayed graphically to give a better impression.

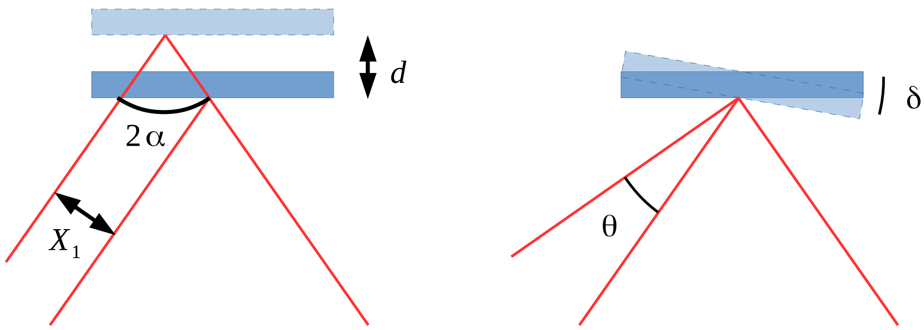

Figure 2: *View from top of the mirror in the cases of a horizontal shift (left) and a yaw around the z-axis (right). The paths of the OpLev-beam are given in red.*

It should be noted that, strictly speaking, only the second case (yaw) is related to an optical lever. However, in order to keep things simple, I will use “optical lever” also for the first case.

If we put a lens in the path of the reflected beam, there will be a displacement measurable also in the image and the focal plane of the lens which are due to an actual shift of these planes along the optical axis. But while a horizontal shift of the mirror will not change the focal plane of the lens, an angular displacement will do. Conversely, an angular displacement will not change the localization of the image plane but a horizontal shift of the mirror will do.

Using the ABCD-matrix model, we can write

$$\begin{pmatrix} X_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 1 & D \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -f^{-1} & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} X_1 \\ \theta_1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 - D/f & D + L(1 - D/f) \\ -f^{-1} & 1 - f^{-1} \end{pmatrix} \cdot \begin{pmatrix} X_1 \\ \theta_1 \end{pmatrix}$$

where *D* and *L* are the distance from the sensor plane to the lens and from the lens to the mirror, respectively (see also Figure 3).

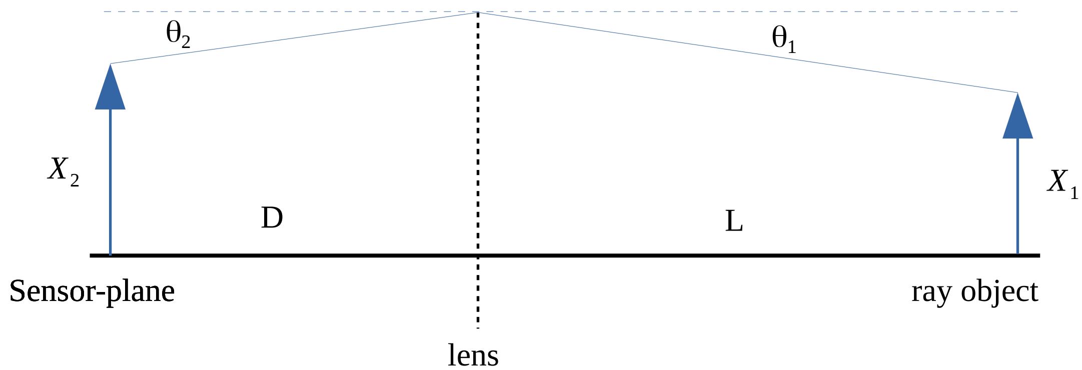

Figure 3: *Simplified diagram to visualize the basic parameters used in the calculation.*

The displacement visible on a PSD is thus

$$X_2 = (1 - D/f)X_1 + (D + L(1 - D/f))\theta_1$$

Obviously, when the sensor plane (the PSD) lies on the focal plane of the lens ($D = f$), then X_2 is sensitive only to a yaw of the mirror (θ_1):

$$X_2 = f\,\theta_1$$

When the sensor plane lies on the image plane of the mirror ($D = Lf/(L - f)$), then X_2 is sensitive only to the displacement X_1 :

$$X_2 = \frac{-f}{L - f} \cdot X_1$$

In reality, however, a PSD can never be put 100%ly in the correct position to match with the focal or the image plane. Thus, there will be always a misplacement δD in the positioning of the PSD. If we put this misplacement in the equations above, we will get a more realistic expression for X_2 in the image and the focal plane:

$$X_2 = \left(\frac{-f}{L - f} - \frac{\delta D}{f}\right) \cdot X_1 + \delta D\left(1 - \frac{L}{f}\right) \cdot \theta_1 \qquad \rightarrow \text{image plane}$$

$$X_2 = \delta D \cdot X_1 + (f + \delta D(1 + L)) \cdot \theta_1 \qquad \rightarrow \text{focal plane}$$

It is possible to calculate the misplacement, of course, when all other parameters are known. However, the main issue is that these other parameters are hard to specify without significant errors in the real conditions of KAGRA.

Beam Profile

The beam profile of the used collimeter has been measured in the laboratory and is given in Figure 4.

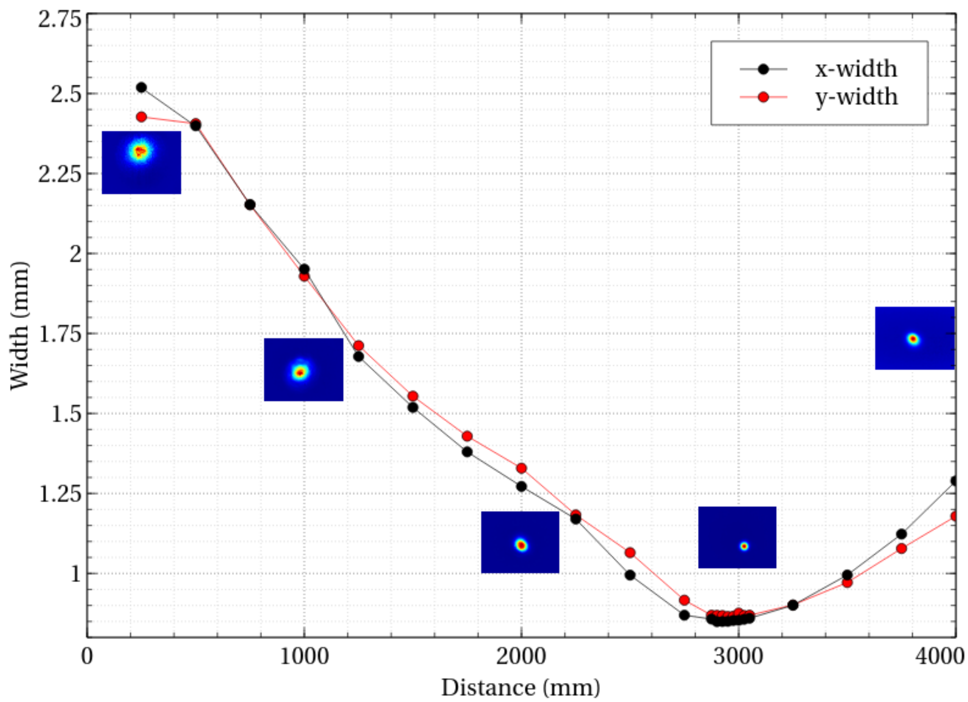

Figure 4: *Development of the beam profile of the used collimeter in two dimensions. The small pictures are photographs of the actual profile. The width means the actual diameter of the beam.*

By putting a lens in the beam, the profile will change according to the focal length of the lens and its position. If we assume the beam as being approximately Gaussian, the changed profile can be calculated by using the beam-profile parameter q:

$$q = z + i z_R$$

with z being the distance to the waist of the beam and $z_R = \pi w_0^2 / \lambda$ where w_0 is the half diameter of the beam's waist.

By taking the ABCD-matrix model of a lens (see above), the new q-parameter q_f becomes

$$q_f = \frac{q}{1 - \frac{q}{f}}$$

which, after some calculation, is turning to

$$q_f = \frac{z - \frac{z^2}{f} + \frac{z_R}{f}}{\left(1 - \frac{z}{f}\right)^2 + \frac{z_R{}^2}{f^2}} + i \frac{z_R}{\left(1 - \frac{z}{f}\right)^2 + \frac{z_R{}^2}{f^2}}$$

In this matter, *z* appears to be the position of the lens in terms of its distance to the beam's waist.

Beam Splitter

Configuration of the Length-Sensing OpLevs for the beam splitter in KAGRA

In theory, the angle of incidence of the collimated beam onto the BS (beam splitter) mirror is 33° and the distance from either viewport to the mirror is 988.8 mm (viewports and mirror should be symmetrically aligned). **The overall distance from mirror to the lens (*L*) is (right now) difficult to specify. But, it should be around 1500 mm!**

With these basic parameters we can estimate the profile of the beam behind the lens (considering $\delta D = 0$). According to Figure 4, the waist of the collimated beam is at around 3000 mm away from the collimeter, so that *z* can be approximated with -500 mm. λ , the wavelength, is ca. 630 nm, and w_0 is 0.43 mm (see Figure 4):

f	$\frac{Lf}{L-f}$	z_f	w_{0f}	w_{image}	X_2 in terms of *d*
100	107.14	-95	0.039	0.073	0.0778
200	230.77	-179	0.074	0.158	0.1676
300	375	-252	0.106	0.257	0.2723
400	545.45	-313	0.133	0.373	0.3961
500	750	-365	0.158	0.514	0.5446
600	1000	-408	0.18	0.685	0.7262
700	1312.5	-443	0.2	0.899	0.9531
800	1714.29	-472	0.216	1.174	1.2449
900	2250	-496	0.231	1.541	1.6339

Table 1: *List of all important parameters and their development when using different lenses positioned 1500 mm away from the mirror. All parameters are given in mm!*

For the actual measurements, the most important parameters are the width of the beam in the image plane and the sensitivity (last two columns in Table 1). Basically, as *L* would decrease, both values will increase for all *f*. The minimum value *L* could have is thus ~1000 mm and the respective parameters become:

f	$\frac{Lf}{L-f}$	z_f	w_{0f}	w_{image}	X_2 in terms of d
100	111.1	-95	0.039	0.091	0.121
200	250	-179	0.074	0.205	0.2723
300	428.57	-252	0.106	0.352	0.4668
400	666.67	-313	0.133	0.547	0.7262
500	1000	-365	0.158	0.821	1.0893
600	1500	-408	0.18	1.232	1.6339
700	2333.33	-443	0.2	1.916	2.5416
800	4000	-472	0.216	3.284	4.3571
900	9000	-496	0.231	7.39	9.8035

Table 2: *The same as Table 1 but with the lenses positioned 1000 mm away from the mirror. All parameters are given in mm!*