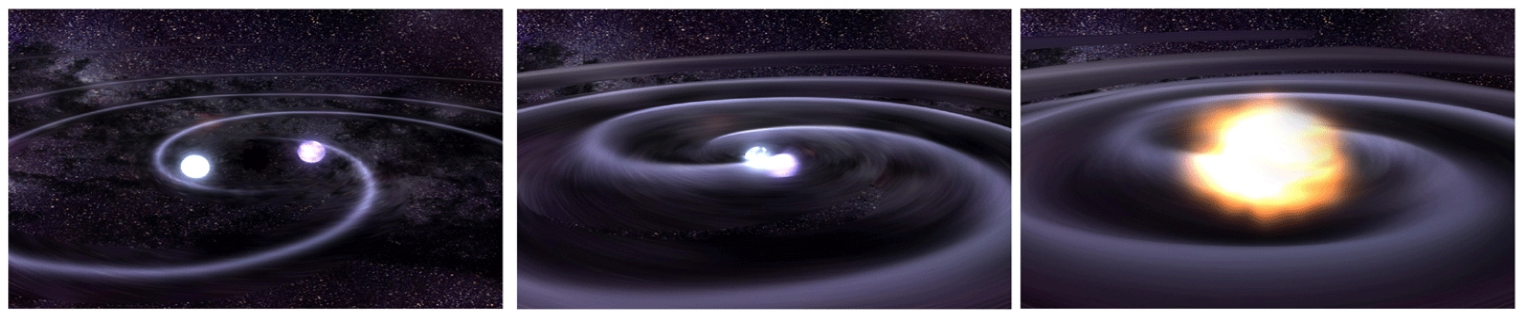


MCMC PE for “eccentric” CBC inspirals

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In collaboration with Marc Favata, K.G. Arun, Blake Moore

Motivation



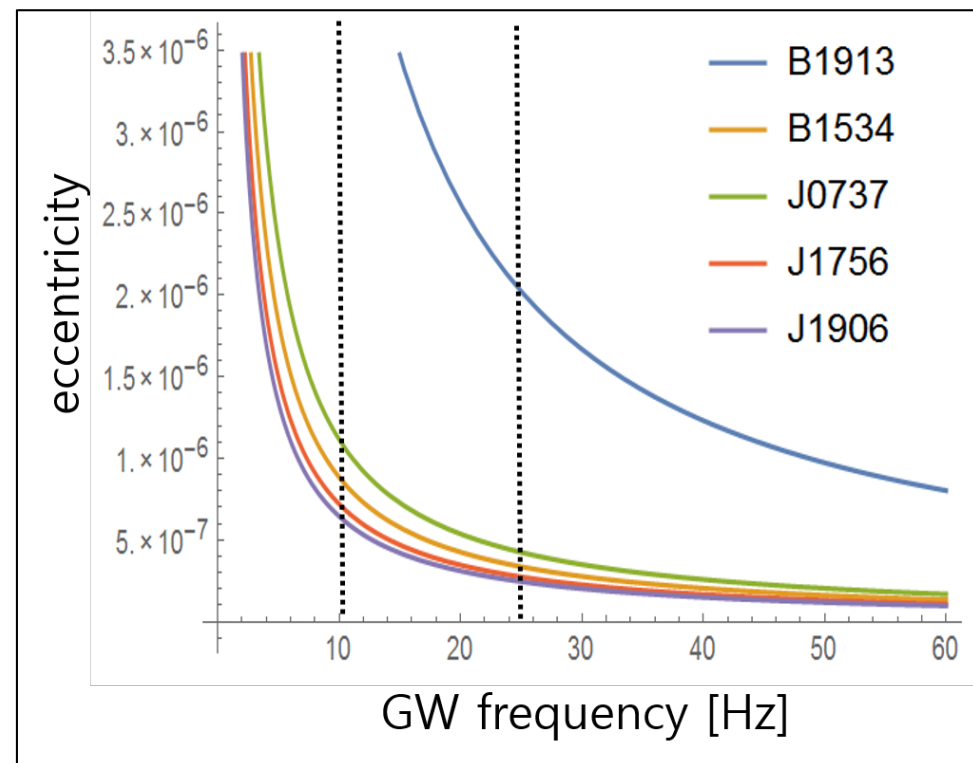
- Eccentric CBC inspirals are most relevant to populations formed by stellar dynamics and those formed with strong supernova kicks
- GW search/PE use “circular” templates. Eccentricity changes the phase of inspiral GW waveform from a circular case. This affects PE accuracy of intrinsic parameters such as masses (Favata 2013)
- 2nd and 3rd G detectors can be capable of observing “eccentric” CBC inspirals. Low frequency band ($f_{\text{gw}} = 10 - 30$ Hz) is important.
- This work: **MCMC PE for GW151226-like BBH inspirals assuming different eccentricities ($e=0.0001 - 0.2$)**

CBC inspirals and eccentricity

Circular templates are ok
for CBCs like known NS-NS mergers.
 $e < 10^{-5}$ at 10 Hz or higher
(e.g. Hulse-Taylor binary)

Some exotic CBCs may be “eccentric”
 $O(0.1\%) - O(1\%)$ of disk population
(Kowalska, Bulik, et al. 2011)

We examine **systematic error in eta**
(systematic mass ratio) due to “eccentricity”

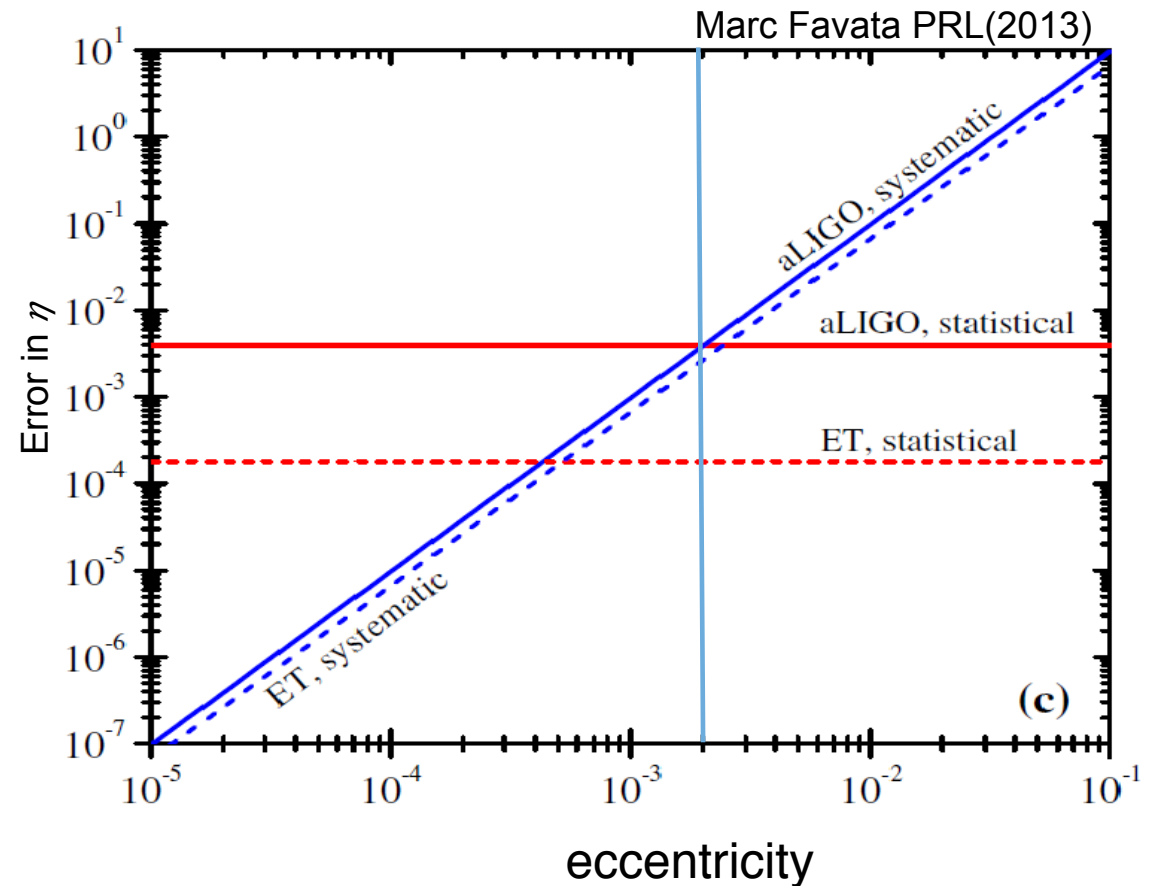


Systematic bias in the symmetric mass ratio due to neglecting eccentricity [Fisher Matrix study]

η (η) : symmetric mass ratio

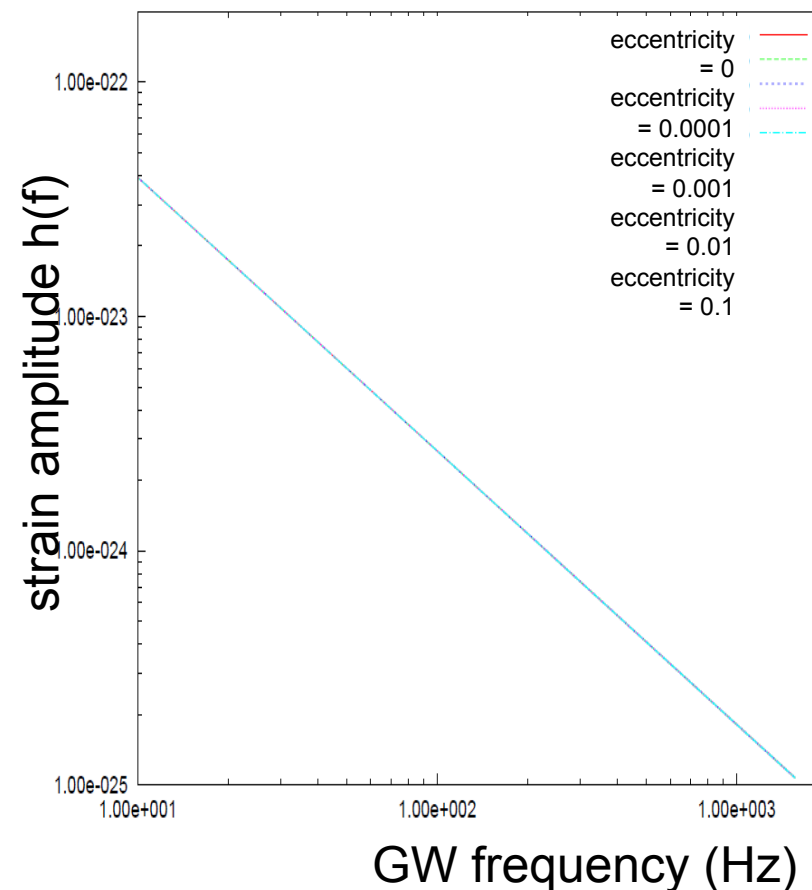
$$\eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

systematic error becomes larger than statistical error in η measurement for CBCs with $e > 0.002$ at 10Hz (aLIGO) with $e > \sim 0.0004$ (ET)



“eccentric” GW waveform (inspiral)

- Waveform model: TaylorF2 (restricted, non-spinning) [Favata et al. in prep]
- Phase term = 3.5pN SPA
+ 3.0pN secular phase corrections for eccentricity
- We implemented **eccentricity corrections** to **TaylorF2** (in LALSuite/LALSimulation). No spin, No amplitude corrections.



phase corrections

TaylorF2 phase = SPA + phaseEcc + etc.

$$\begin{aligned} \psi = & 2f\pi t_0 - 2\phi_0 + \psi_0 + \frac{3}{128} \frac{1}{\eta v^5} \left\{ 1 + v^2 \left(\frac{3715}{756} + \frac{55}{9} \eta \right) - 16\pi v^3 + v^4 \left(\frac{15293365}{508032} + \frac{27145}{504} \eta + \frac{3085}{72} \eta^2 \right) \right. \\ & + v^5 \left(\frac{38645}{756} \pi + \frac{38645}{756} \pi \ln[v^3] + \eta \left(-\frac{65\pi}{9} - \frac{65}{9} \pi \ln[v^3] \right) \right) + v^6 \left(\frac{11583231236531}{4694215680} - \frac{6848}{21} \gamma_e - \frac{640}{3} \pi^2 \right. \\ & \left. \left. + \left(-\frac{15737765635}{3048192} + \frac{2255}{12} \pi^2 \right) \eta + \frac{76055}{1728} \eta^2 - \frac{127825}{1296} \eta^3 - \frac{3424}{21} \ln[16v^2] \right) \right\} \end{aligned}$$

phaseEcc depends on e_0 , v_0 and eta

e_0 : eccentricity@ f_0

$$v_0 = (\pi M f_0)^{1/3}$$

$$M = m_1 + m_2$$

f_0 is set to be 10Hz

$$\begin{aligned} & - \frac{2355}{1462} e_0^2 \left(\frac{v_0}{v} \right)^{19/3} \left[1 + v^2 \left(\frac{299076223}{81976608} + \frac{18766963}{2927736} \eta \right) + v_0^2 \left(\frac{2833}{1008} - \frac{197}{36} \eta \right) + -\frac{2819123}{282600} \pi v^3 + \frac{377}{72} \pi v_0^3 \right. \\ & + v^4 \left(\frac{16237683263}{3330429696} + \frac{24133060753}{971375328} \eta + \frac{1562608261}{69383952} \eta^2 \right) + v^2 v_0^2 \left(\frac{847282939759}{82632420864} - \frac{718901219}{368894736} \eta - \frac{3697091711}{105398496} \eta^2 \right) \\ & + v_0^4 \left(-\frac{1193251}{3048192} - \frac{66317}{9072} \eta + \frac{18155}{1296} \eta^2 \right) + v^5 \left(-\frac{2831492681}{118395270} \pi - \frac{11552066831}{270617760} \pi \eta \right) + v^3 v_0^2 \left(-\frac{7986575459}{284860800} \pi + \frac{555367231}{10173600} \pi \eta \right) \\ & + v^2 v_0^3 \left(\frac{112751736071}{5902315776} \pi + \frac{7075145051}{210796992} \pi \eta \right) + v_0^5 \left(\frac{764881}{90720} \pi - \frac{949457}{22680} \pi \eta \right) + v^6 \left(-\frac{43603153867072577087}{132658535116800000} + \frac{536803271}{19782000} \gamma_e \right. \\ & + \frac{15722503703}{325555200} \pi^2 + \left(\frac{299172861614477}{689135247360} - \frac{15075413}{1446912} \pi^2 \right) \eta + \frac{3455209264991}{41019955200} \eta^2 + \frac{50612671711}{878999040} \eta^3 + \frac{3843505163}{59346000} \ln[2] \\ & - \frac{1121397129}{17584000} \ln[3] + \frac{536803271}{39564000} \ln[16v^2] \left. \right) + v^4 v_0^2 \left(\frac{46001356684079}{3357073133568} + \frac{253471410141755}{5874877983744} \eta - \frac{1693852244423}{23313007872} \eta^2 \right. \\ & - \frac{307833827417}{2497822272} \eta^3 \left. \right) - \frac{1062809371}{20347200} \pi^2 v^3 v_0^3 + v^2 v_0^4 \left(-\frac{356873002170973}{249880440692736} - \frac{260399751935005}{8924301453312} \eta + \frac{150484695827}{35413894656} \eta^2 \right. \\ & + \frac{340714213265}{3794345856} \eta^3 \left. \right) + v_0^6 \left(\frac{26531900578691}{168991764480} - \frac{3317}{126} \gamma_e + \frac{122833}{10368} \pi^2 + \left(\frac{9155185261}{548674560} - \frac{3977}{1152} \pi^2 \right) \eta - \frac{5732473}{1306368} \eta^2 \right. \\ & \left. \left. - \frac{3090307}{139968} \eta^3 + \frac{87419}{1890} \ln[2] - \frac{26001}{560} \ln[3] - \frac{3317}{252} \ln[16v_0^2] \right) \right\} \end{aligned}$$

PE for GW151226-like BBH [preliminary]

Software injection simulation

GW151226 mass parameters

$m_1 = 15.6 M_{\text{sun}}$, $m_2 = 8.2 M_{\text{sun}}$, **$\eta = 0.2258$**

Assign eccentricity @ 10 Hz, overlap calculation $f_{\text{gw}} = [25 \text{ Hz, ISCO}]$

To examine PE accuracy,
we consider a single detector (**aLIGO**) ;
set SNR ~ 20 (distance = 500 Mpc)

Systematic error in “eta” due to eccentricity

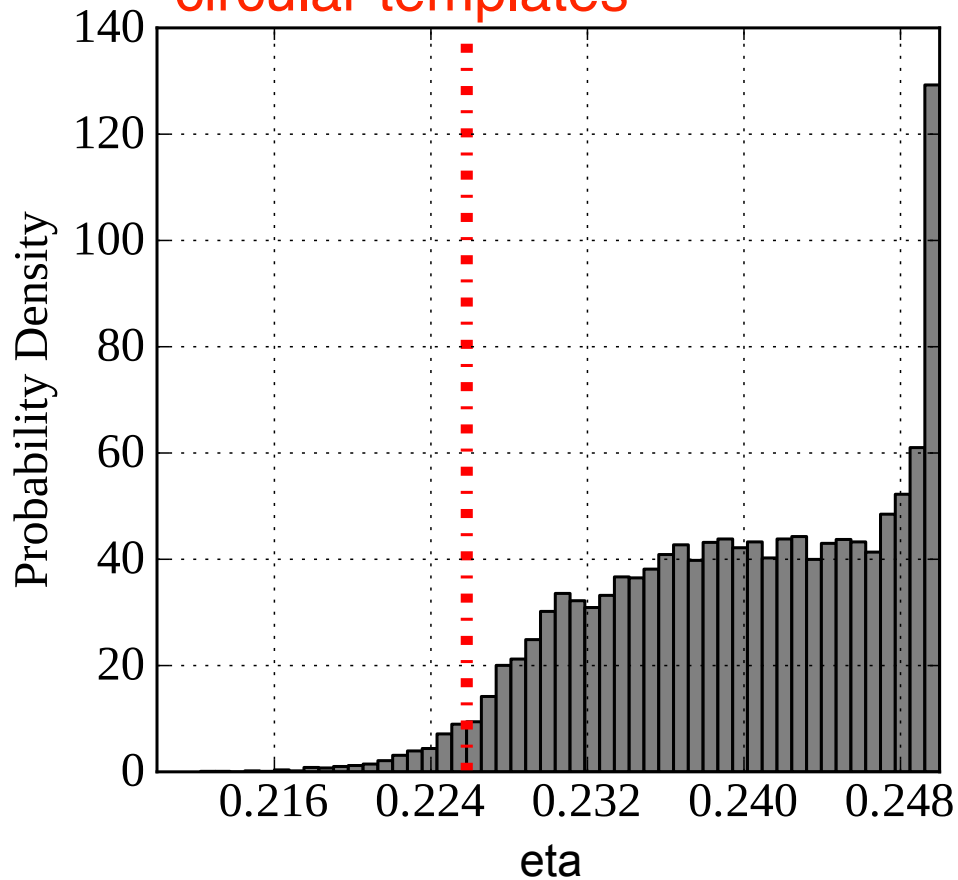
“Eccentric” templates are necessary for $e > \sim 0.2$ (@10Hz)

Injection:
 $e=0.2$,
 $\eta=0.2258$

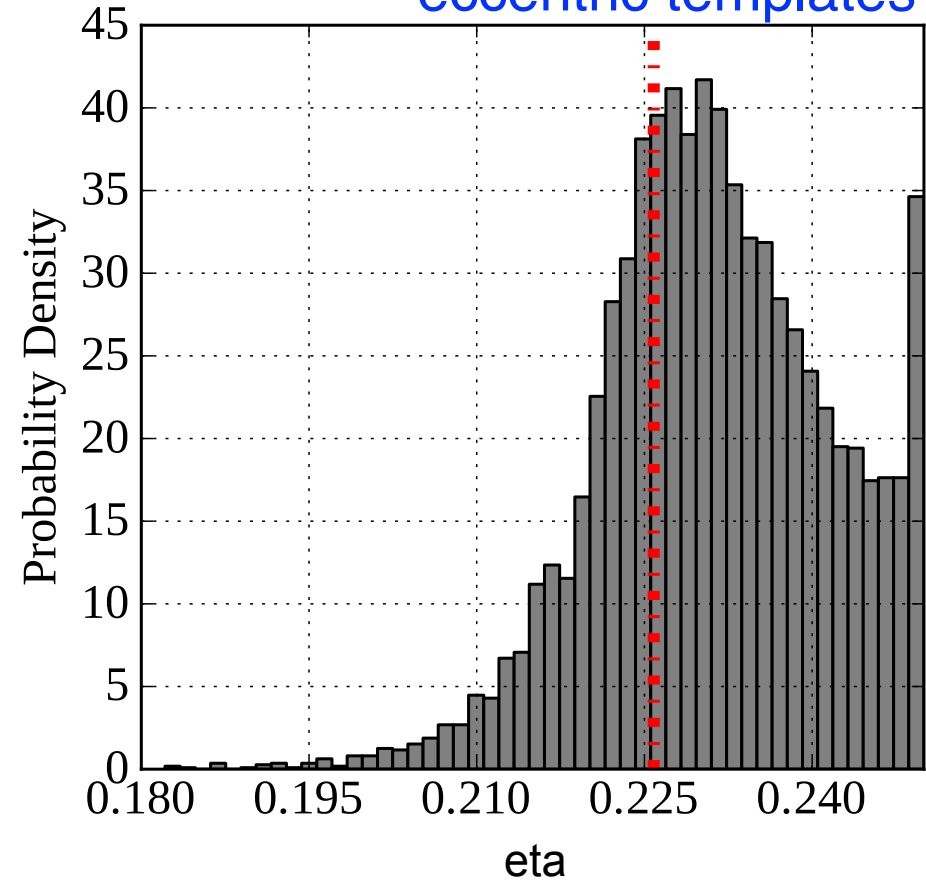
Detector:
aLIGO (single IF)
 $f_{\text{low}}=25\text{Hz}$

prior for e :
uniform
 $e=[0.0, 1.0]$

eccentric injection
circular templates



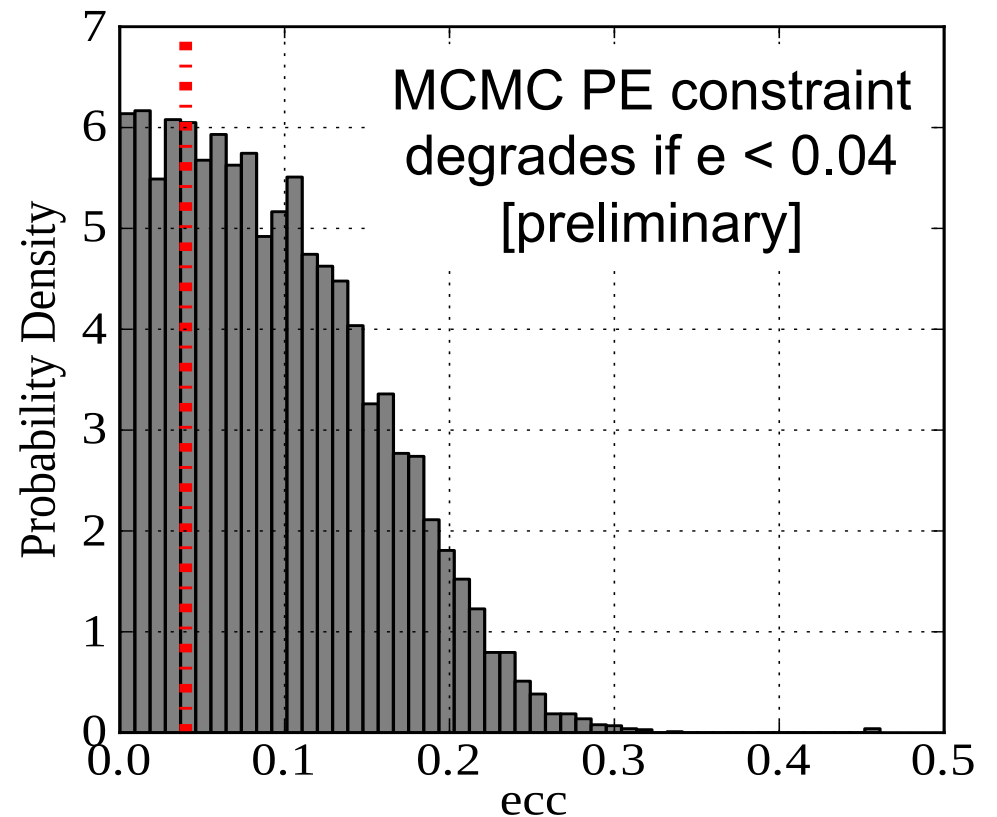
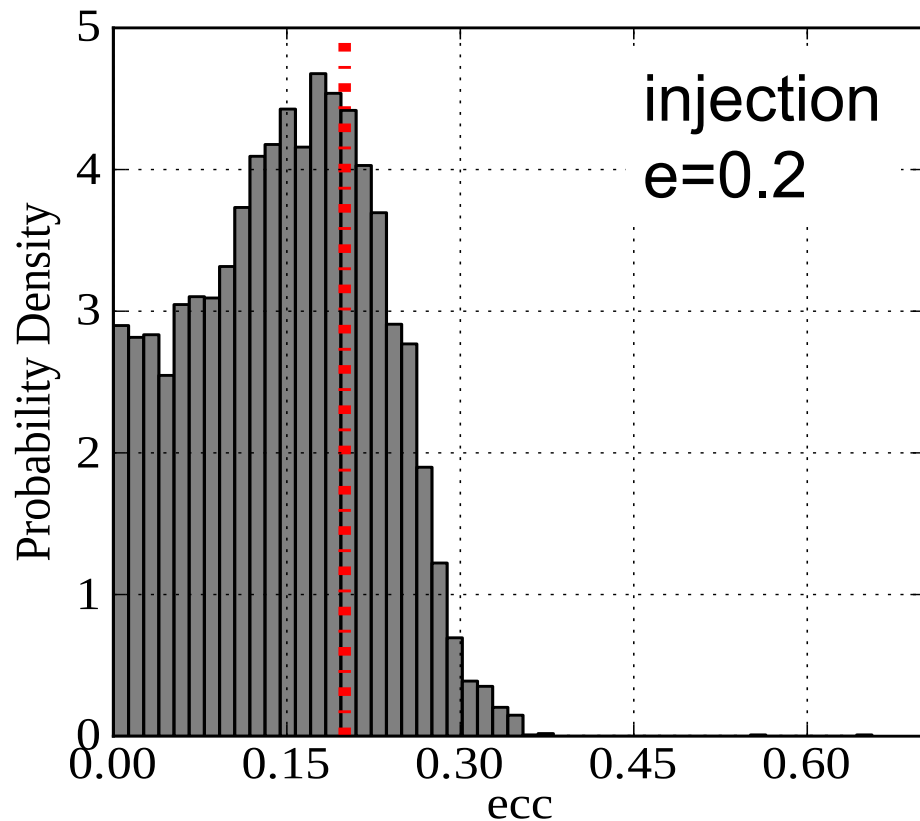
eccentric injection
eccentric templates



Can we constrain eccentricity from CBC inspirals?

With aLIGO (25Hz low cut-off frequency), $e \sim O(0.01)$ can be constrained. Search/PE for small e inspirals are expected to be computationally expensive

[eccentric injection vs eccentric templates]



Summary

- We implemented up to 3.0 pN phase corrections due to eccentricity for TaylorF2 in LALSuite (local branch).
LALSimulation, LALInference_mcmc can utilize this waveform model
- We examine PE accuracy for symmetric mass ratio and eccentricity for GW151226-like BBH inspiral assuming different eccentricities ($0.0001 < e < 0.2$ @ 10Hz)
- With aLIGO, inspirals with $e \sim O(0.04-0.1)$ can be interesting for PE purposes ($f_{\text{low}}=25$ Hz) **preliminary**
- MCMC PE results are roughly consistent with the Fisher matrix prediction **preliminary**

Future work

- We will extend MCMC PE for “eccentric” CBCs varying masses and low cut-off $f_{\text{gw}} = [10, 30]$ Hz (computationally expensive, though)
- Eccentric taylorF2 inspiral waveform will be included in KAGALI

CBC PE for KAGALI

- Additioning pN corrections for amplitude, eccentricity (TaylorF2)
- CBC PE for software AND hardware injection
- KGWG DAS members: Hyung Won Lee, Jeongcho Kim,
Chunglee Kim

Yeong-Bok Bae (new postdoc at KASI) will join from 2016.09