1 ITRF94 coordinate system

Reference site: `http://vldb.gsi.go.jp/sokuchi/surveycalc/main.html`

Ellipsoid	GRS80
Semi-major axis	6378137m
Flattening	1/298.257222101
Semi-minor axis	$b_{GRS80} = a_{GRS80}(1 - f_{GRS80})$

Height: $H_e = H_o + H_g$ where

H_e: ellipsoid height
H_g: geoid height = height of geoid surface from GRS80 surface.
H_o: orthometric height = height from geoid surface

The Geospatial Information Authority of Japan gives H_g from (B, L) at `http://vldb.gsi.go.jp/sokuchi/surveycalc/main.html`.

B and L are the latitude and the longitude in the ITRF94 geodesic coordinate system. The 3-D international terrestrial reference system (X, Y, Z) of the position specified by the ITRF94 geodesic coordinate (B, L) are computed by

$$X = \left(\frac{a_{GRS80}^2}{\sqrt{a_{GRS80}^2 \cos^2 B + b_{GRS80}^2 \sin^2 B}} + H_e \right) \cos B \cos L, \qquad (1)$$

$$Y = \left(\frac{a_{GRS80}^2}{\sqrt{a_{GRS80}^2 \cos^2 B + b_{GRS80}^2 \sin^2 B}} + H_e \right) \cos B \sin L, \qquad (2)$$

$$Z = \left(\frac{b_{GRS80}^2}{\sqrt{a_{GRS80}^2 \cos^2 B + b_{GRS80}^2 \sin^2 B}} + H_e \right) \sin B \qquad (3)$$

The Geospatial Information Authority of Japan gives (X, Y, Z) from (B, L) at `http://vldb.gsi.go.jp/sokuchi/surveycalc/main.html`.

LALDetectors.h requires the azimuth angule ζ_{GRS80} and the altitude angle $\mathcal{A}_{GRS80}$ of the arms. The LALDetector.h document says " The c LALFrDetector structure stores the directions along the two arms of an interferometer in an altitude/azimuth representation with respect to the local tangent plane to the reference ellipsoid, known as the local horizontal. The altitude $\mathcal{A}$ is the angle the direction vector makes with the horizontal, $\mathcal{A} > 0$ meaning above horizontal, $\mathcal{A} < 0$ below. The azimuth angle ζ is found by projecting the direction onto the local horizontal plane, then measuring the angle clockwise from North to this projected direction.".

Define the following vectors. $\vec{u}$ is the unit vector from the BS to the X-End. $\vec{v}$ is the unit vector from the BS to the Y-End. $\vec{n}$ is the normal unit vector to the local horizontal at the BS. $\vec{m}$ is the unit vector on the local horizontal at the BS and directed to the local North. $\vec{l}$ is the unit vector on the local horizontal at the BS and directed to the local East ($\vec{l} = \vec{m} \times \vec{n}$).

Then the azimuth angule ζ_{GRS80} and the altitude angle $\mathcal{A}_{GRS80}$ are defined by

$$\vec{n} = (\cos L \cos B, \sin L \cos B, \sin B), \tag{4}$$
$$\vec{m} = (-\cos L \sin B, -\sin L \sin B, \cos B), \tag{5}$$
$$\vec{l} = (\sin L, -\cos L, 0), \tag{6}$$
$$\sin\left(\mathcal{A}_{GRS80,X-arm}\right) = \vec{n} \cdot \vec{u}, \tag{7}$$
$$\tan \zeta_{GRS80,X-arm} = \frac{\vec{l} \cdot \vec{u}}{\vec{m} \cdot \vec{u}}, \tag{8}$$
$$\sin\left(\mathcal{A}_{GRS80,Y-arm}\right) = \vec{n} \cdot \vec{v}, \tag{9}$$
$$\tan \zeta_{GRS80,Y-arm} = \frac{\vec{l} \cdot \vec{v}}{\vec{m} \cdot \vec{v}} \tag{10}$$

For KAGRA, it turns out that $\zeta_{GRS80,X-arm} > 0$ (X-arm is directed to the local North East) and $\zeta_{GRS80,Y-arm} < 0$ (Y-arm is directed to the local North West). Then it was checked that $\zeta_{GRS80,X-arm} - \zeta_{GRS80,Y-arm} - \pi/2 = 4 \times 10^{-6}$ radians.

The locations of the KAGRA mirrors are given by Prof. Yoshio Saito (the KAGRA project manager) as the document JGW-G140105-v1 (2014) and shown in the Tables 1, 2, and 3.

Table 1: Beam Splitter.

B [ddmmss]	L [dddmmss]	H_e [m]	H_g [m]	H_o [m]
362442.69722	1371821.44171	414.181	41.0464	373.135
B [radians]	L [radians]	X [m]	Y [m]	Z [m]
0.6355068497	2.396441015	-3777336.024	3484898.411	3765313.697

Table 2: X-End.

B [ddmmss]	L [dddmmss]	H_e [m]	H_g [m]	H_o [m]
362531.18475	1372007.07060	424.407	41.1788	383.228
B [radians]	L [radians]	X [m]	Y [m]	Z [m]
0.6357419239	2.396953119	-3778473.700	3482367.772	3766522.541

Table 3: Y-End.

B [ddmmss]	L [dddmmss]	H_e [m]	H_g [m]	H_o [m]
362607.96387	1371721.48451	403.934	40.8740	363.06
B [radians]	L [radians]	X [m]	Y [m]	Z [m]
0.6359202341	2.396150335	-3775170.073	3484932.092	3767422.582

Table 4: X-arm: $\vec{U}$ = (X-end) - (Beam Splitter)

U_x [m]	U_y [m]	U_z [m]	$\lvert\vec{U}\rvert$ [m]
-1137.676	-2530.639	1208.844	3026.507
$\vec{u}_x$	$\vec{u}_y$	$\vec{u}_z$	$\lvert\vec{u}\rvert$
-0.3759040	-0.8361583	0.3994189	1.0000000
$\mathcal{A}_{GRS80}$ [radians]	ζ_{GRS80} [radians]	–	–
0.0031414	1.054113	–	–

Table 5: Y-arm: $\vec{V}$ = (Y-end) - (Beam Splitter)

V_x [m]	V_y [m]	V_z [m]	$\lvert\vec{V}\rvert$ [m]
2165.951	33.681	2108.885	3023.222
$\vec{v}_x$	$\vec{v}_y$	$\vec{v}_z$	$\lvert\vec{v}\rvert$
0.7164378	0.01114076	0.6975620	1.0000000
$\mathcal{A}_{GRS80}$ [radians]	ζ_{GRS80} [radians]	–	–
-0.0036270	-0.5166798	–	–