Investigation of crackling noise in the vibration isolation system of KAGRA gravitational wave detector(2)

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Introduction

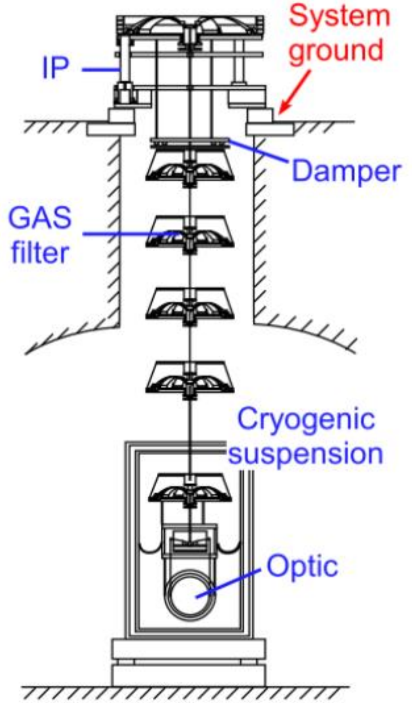
- The GAS (geometric anti-spring) filter is the main vertical isolator of KAGRA main optics.
- Crackling noise is a potential noise excited by the variation of the stress of the blades of the GAS filter due to the seismic motion.
- How to measure it?—Use a Michelson interferometer and a common-mode driving.
- Comparing the crackling noise in KAGRA and our lab: Scaling law.

KAGRA project

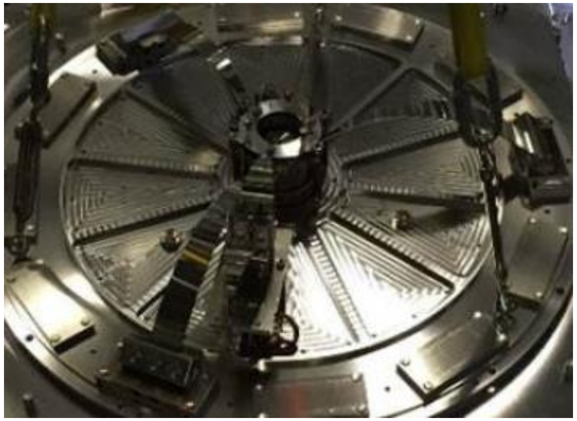
- The second generation gravitational wave detector in Japan.
- Underground site.
- Cryogenic operation.
- To operate as a cryogenic Michelson in 2018.

VIS of KAGRA

- Main optics of KAGRA are isolated from seismic noise by the seismic attenuation systems (SAS), including type-A, type B, type-C.
- For type-A SAS, six stages of GAS filters isolate vertical vibration. One stage of an inverted pendulum and seven stages of pendulums isolate horizontal vibration.



Type-A SAS of KAGRA,
T. Sekiguchi Dthesis



- GAS filter is a kind of blade spring. The blades are radially compressed to result in an anti-spring effect to lower the resonant frequency.
- Crackling noise may arise in the highly-compressd blades.

What is crackling noise

- A ubiquitous phenomenon existing in many driven systems (intermittent noise of earthquake, Barkhausen noise, crumpled paper).
- Arising when the response in a driven system behaves discrete and impulsive.
- Main features:

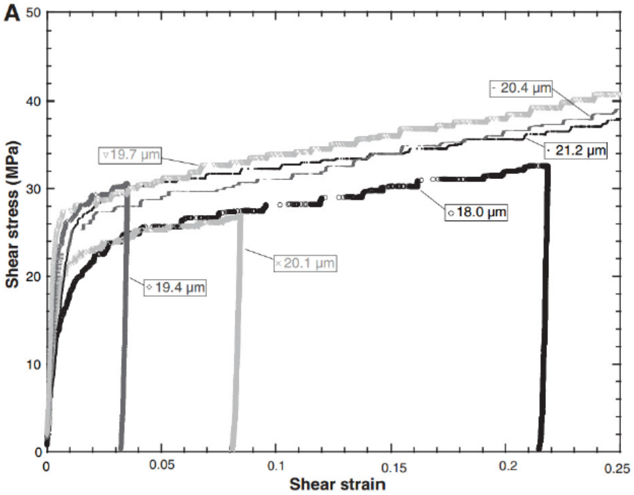
  Universal in many different systems. → Universality

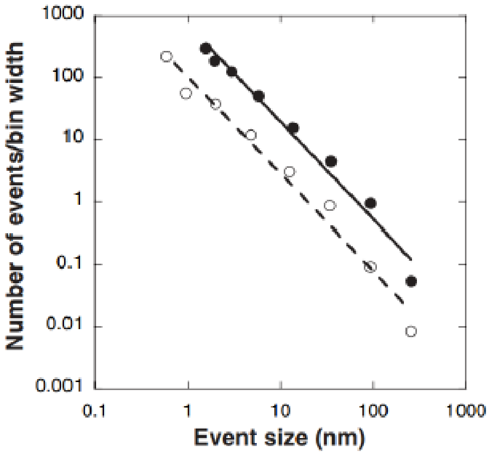
  Power-law → Scale-free

  Wide-size distribution

- Observed in the stress-strain curve of small-scale crystals

D.M Dimiduk et.al, Science, 2006



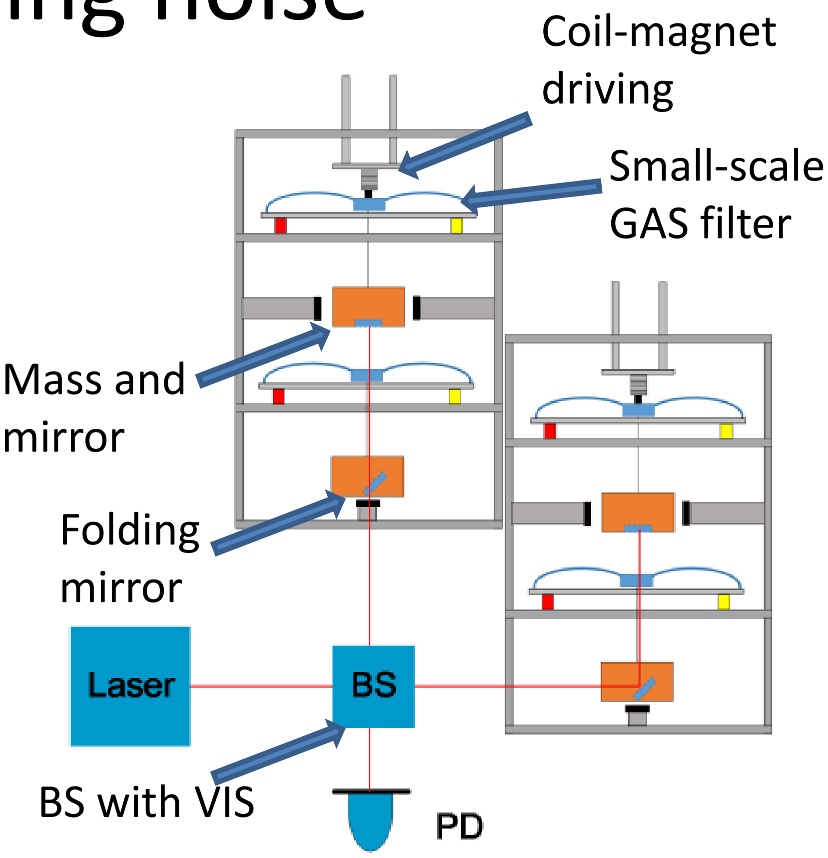


Crackling noise in the GAS filter

- Crackling noise refers to the rapidly burstlike motion of the microstructures of the blades of the GAS filter.
- Crackling noise is driven by the low-frequency seismic motion.
- Crackling noise can induce random noise in the observation frequency band of KAGRA.
- Especially, the floor has a tilt of 1/300 for the water drainage system in KAGRA, so that at least 0.3% vertical motion of the mirror will couple into KAGRA's readout.
- Rare previous research. Requiring experimental investigation and theoretical interpretation.

How to measure crackling noise

- A Michelson interferometer with two balanced arm.
- Small-scale GAS filter.
- Common-mode driving to excite crackling.
- The differential mode motion of the two suspended end mass from crackling events is recorded by the Michelson interferometer.



Scaling problem

- Scaling problem: how can we estimate the crackling noise in the KAGRA GAS filter according to the measurement of the lab-scale GAS filter?
- Crackling noise depends on

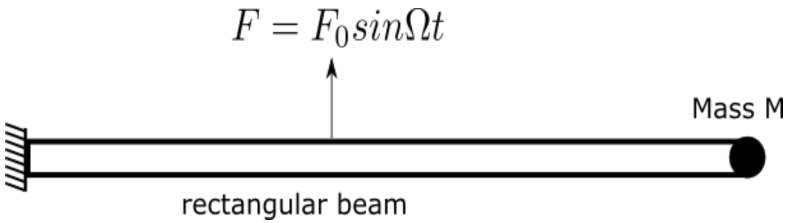
  Local crackling event rate, which can be estimated from

  - ☐Simulation/calculation
  - ☐Microscopical experiment

  Transmission from local crackling events to the mass, which can be calculated by

  - ☐Continuum transverse vibration model
  - ☐Finite element analysis (FEA) method
- The scaling problem can be simplified to a vibration transmission problem if the local crackling rate can be approximately the same in different-scale blades.

Transverse vibration model



Analytical solution of the transverse vibration of a rectangular beam when (given by Rao et al.*)

1. one end is fixed with a mass M attached at the other end
2. a driving force $F = F_0 \sin\Omega t$ at $x = \xi$

is

$$w(x,t) = F_0 \sum_{i=1}^{\infty} \frac{W_i(x)W_i(\xi)}{\omega_i^2 - \Omega^2}(\sin\Omega t - \frac{\Omega}{\omega_i}\sin\omega_i t)$$

$w(x,t)$: displacement at x, t

$W_i(x)$: mode shape

ω_i: resonant frequency of the nth mode

*Rao, Singiresu S. *Vibration of continuous systems*. John Wiley & Sons, 2007.

Transverse vibration model

The mode shape $W_n(x)$ is given by

$$W_n(x) = C_{2n}\left[(\cos\beta_n x - \cosh\beta_n x) - \frac{\cos\beta_n l + \cosh\beta_n l}{\sin\beta_n l + \sinh\beta_n l}(\sin\beta_n x - \sinh\beta_n x)\right]$$

β_n: nth natural frequency
l: beam length
C_{2n}:a constant

β_n is given by

$$1 + \frac{1}{\cos\beta l + \cosh\beta l} - R\beta l(\tan\beta l - \tanh\beta l) = 0$$

$$R = \frac{M}{\rho A l}$$

M: attached mass
ρ: density of beam
A: area of cross section of the beam about the direction of gravity

C_{2n} can be calculated from the normalization function.

$$\int_0^l \rho A(x) W_i^2(x)\,dx + W_i^2(l)M = 1, \qquad i = 1, 2, \cdots$$

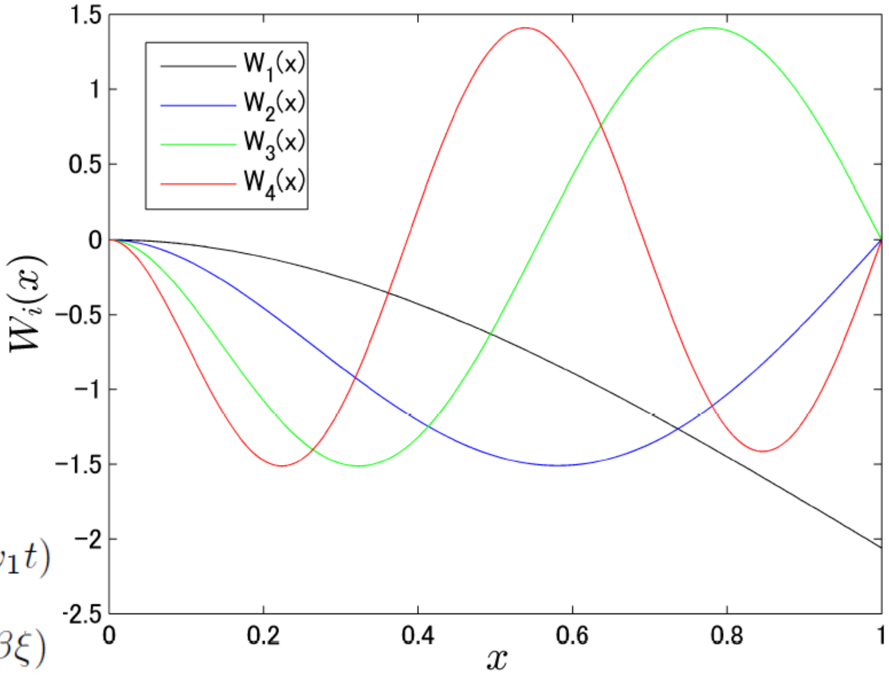
Transverse vibration model

- Higher order modes ($W_i(x)$ $i > 1$)can be neglected.
- After normalization and some approximation, we can get the response at tip as

$$w(l,t) = \frac{F_0}{M} H(\beta\xi) \frac{1}{\omega_1^2 - \Omega^2} \left(\sin \Omega t - \frac{\Omega}{\omega_1} \sin \omega_1 t\right)$$

$$H(\beta\xi) = \frac{(\cos\beta\xi - \cosh\beta\xi) - \frac{\cos\beta l + \cosh\beta l}{\sin\beta l + \sinh\beta l}(\sin\beta\xi - \sinh\beta\xi)}{(\cos\beta l - \cosh\beta l) - \frac{\cos\beta l + \cosh\beta l}{\sin\beta l + \sinh\beta l}(\sin\beta l - \sinh\beta l)}$$

The initial four vibration modes $W_i(x)$



Discussion

- As we only care about the response at Ω, the response can be revised as

$$w(l,t) = \frac{F_0}{M} H(\beta\xi) \frac{1}{\omega_1^2 - \Omega^2} \sin \Omega t$$

- Segment the full-scale blade (blade A) and lab-scale blade (blade B) into n equal slices, respectively.

- Take the crackling event rate in the slice of A and the corresponding slice of B is equivalent.
- RMS force from crackling events should be proportional to the square root of volume V.
- $H(\beta\xi)$ can be taken as equal for the corresponding slices in our case.

Discussion

- Only consider the response at $\Omega \gg \omega_1$. The derived scaling law is denoted as

$$\frac{w_B(\omega)}{w_A(\omega)} = \sqrt{\frac{V_B}{V_A}\frac{M_A}{M_B}}$$

$V_{A,B}$: Volume of blade A and blade B

- In our case, the volume of KAGRA blade is about 100 times the lab-scale blade. The attached mass of KAGRA blade is about 150 times the lab-scale blade. This gives the ration of crackling noise of KAGRA blade to lab-scale blade is

$$\frac{w_{lab}}{w_{KAGRA}} \sim 15$$

Conclusion

- Crackling noise refers to the rapidly burstlike motion of the microstructures of the blade of the GAS filter.
- Crackling noise is a potential noise for KAGRA gravitational wave detector.
- A Michelson interferometer using small-scale GAS filter was designed to investigate the crackling noise.
- A rough estimation of the relationship between crackling noise in the full-scale blades and that in the lab-scale blade was given.

Thanks very much for your attention