Report on the Calculation of the Magnetic Field of a "SolBlack" Coated Wide-Angle Baffle

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1 Introduction

In this report, the calculation of magnetic fields caused by the Solblack coating of a wide-angle baffle (WAB), which are installed in front of the ITMX and ITMY mirrors inside the respective cryostats (Aso, 2014) is presented. A review on the design of the WABs can be found here, and a report on the suspension characterization can be found here.

The baffles are coated with an absorber having a smoothly fractured surface to increase the spatial distribution of scattering light while decreasing the total amount of it. The most efficient one for our purpose (from the viewpoint of the price) proofed to be Solblack [GENESIA (2013); Akutsu et al. (2016)]. However, there is a possibility that Solblack can be magnetized [Tokoku (2014)] as it contains some amount of nickel. Thus, its influence on other magnets, especially the OSEM-actuators, might affect the sensitivity of the KAGRA interferometer. To make sure that the influence is low enough, I have made near-field simulations on the magnetic field of a Solblack coated WAB and will present the results of these calculations here. It should be noted that the presented results refer only to extreme case scenarios to make sure that in the reality the disturbances will be much lower and thus safer.

2 Magnetic Fields

For the calculation of the influence of magnetic fields, I will give a short review of the theory that has been used.

In the concept presented here, I assume the Solblack coating consists of many small and independent magnetic dipoles. This has the advantage that a near-field problem for the overall baffle structure converges down to a far-field problem of many small dipoles, where an analytical solution is possible. The far-field solution of the magnetic field of a small dipole is given by

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi r^2} \cdot \frac{3\vec{r}(\vec{\mu} \cdot \vec{r}) - \vec{\mu} r^2}{r^3}. \tag{1}$$

$\vec{\mu}$ is the magnetic moment of the dipole and μ_0 is the magnetic susceptibility of the vacuum. The force on a magnetic probe (in our case the OSEM-actuator on the mirror surface) is given by

$$\vec{F} = m\ddot{\vec{r}} = \nabla \left(\vec{\mu}_{\mathrm{OSEM}} \cdot \vec{B}(\vec{r}) \right). \tag{2}$$

$\vec{\mu}_{\mathrm{OSEM}}$ is the magnetic dipole moment of the OSEM actuator. For this expression also an analytical solution exists (as can be proofed by a careful analysis). We can write

$$\vec{F} = \frac{3\mu_0}{4\pi r^5} \cdot \left[(\vec{\mu} \cdot \vec{r})\vec{\mu}_{\mathrm{OSEM}} + (\vec{\mu}_{\mathrm{OSEM}} \cdot \vec{r})\vec{\mu} + (\vec{\mu} \cdot \vec{\mu}_{\mathrm{OSEM}})\vec{r} - \frac{5 \cdot (\vec{\mu} \cdot \vec{r})(\vec{\mu}_{\mathrm{OSEM}} \cdot \vec{r})\vec{r}}{r^2} \right]. \tag{3}$$

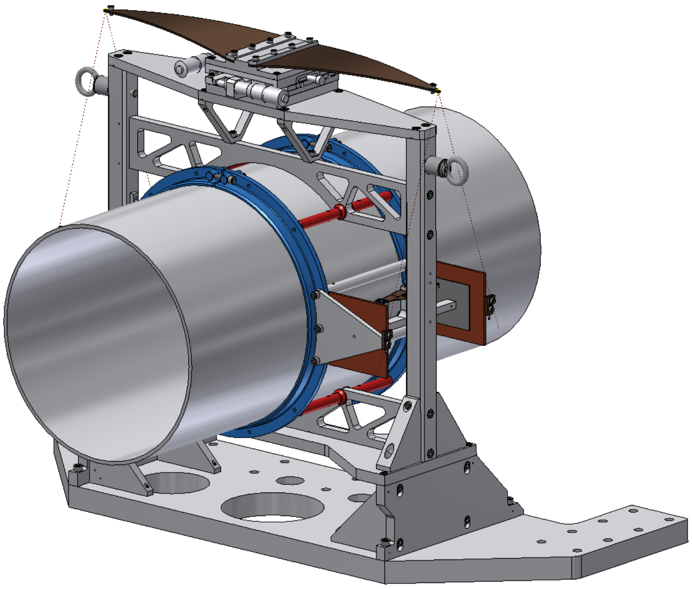

Figure 1: Design of the wide-angle baffles that are used in KAGRA (credit: Yoshiyuki Obuchi). The baffle itself is suspended as a single pendulum with vertical springs.

To derive the change of the force when there are small changes of $\vec{r}$ due to seismic vibrations, the (constant) force in the mean position $\vec{r}_0$ needs to be subtracted:

$$\Delta\vec{F}(\vec{r}) = \vec{F}(\vec{r}) - \vec{F}(\vec{r}_0) \tag{4}$$

In the following calculations, these formulae have been taken to derive the upper limit of the magnetic influence of a vibrating WAB onto the OSEMs.
In order to make the calculations comparable to the real situation in KAGRA, I have taken real-time records of the chamber-motion at the side to simulate $\Delta\vec{F}(t)$. In addition to that (since the baffles are suspended), I have taken respective records of the baffle-motion done in the clean-room at Mitaka to compare the chamber-motion results with those of suspensions. After getting $\Delta\vec{F}(t)$, I calculated the force's power-spectral density from which the pure motion of mirror and thus the influence on h could be anticipated.
Note: there are no measurements of the real-time motion done with the installed baffles!

Once the movement of the actuator, and thus the mirror, is known, we can calculate its influence on the gravitational-wave strain noise and compare the results with the goal sensitivity of KAGRA. However, it should be noted that in the actual design, there will be four actuators aligned anti-symmetrically so that the resulting magnetic moment of the whole system will be zero. In the calculations which are presented here, I will assume an extreme case of just having one actuator that responds to the solblack magnetic field. The strain noise effect due to the movements of the mirror because of a vibrating force on the magnet can be described to be

$$h_{WAB} = \frac{\Delta\vec{F}(\omega)}{L \cdot m\omega^2}, \tag{5}$$

where L is the length of the arms of the interferometer (for KAGRA: 3000 m).

3 Baffle Model

The particular shape of the wide-angle baffle structure installed in KAGRA can be seen from Fig. 1. However, in order to simplify the issue, I removed any thickness and so the cylinder in

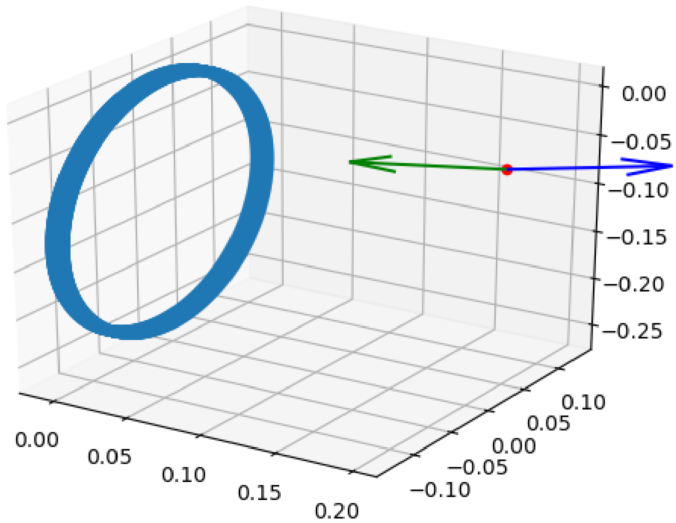

Figure 2: The WAB (blue dots) and an OSEM magnet (red dot) as it is defined in the model. From the WAB, only the first cm is shown for the purpose of a better visibility. The green arrow symbolizes the direction of the attracting force on the OSEM magnet while the blue arrow is the direction of $\vec{B}$ at the OSEM. All dimensions are given in [m]

the model (representing the magnetized baffle part) consists only of one layer of dipoles. The baffle has ~ 12.9 cm radius and is ~ 57 cm long (see Fig. 3 for a basic concept).
An important parameter is the estimation of μ of the magnetic dipoles from the coating. Fortunately, there are already measurements on the magnetic behavior of a solblack coating by Tokoku (2014) on which the calculations presented here are based. From the given values of the magnetization of a sheet of solblack (maximum $6{\cdot}10^4\,\mathrm{Am}^{-1}$), $|\mu|$ for a cubic-shaped dipole element of 1 mm length can be estimated[1] to be $6{\cdot}10^{-5}\,\mathrm{Am}^2$. The model of the WAB consists of circles made of dipoles placed to form a cylinder. It was assumed that there is a small space between each dipole of ~ 0.1 mm to the next dipole in one circle, and a space of 1 mm between each circle. In total I worked with $4.2{\cdot}10^5$ dipoles for the WAB model. As the wide-angle baffle is placed as close as possible to the mirror, its minimal distance to the actuators is not bigger than 20 cm. The magnetic moment of the actuator, μ_{OSEM}, should have an absolute value $\gtrsim 0.0022\,\mathrm{Am}^2$ (Fiori et al., 2014) and has the same direction of $\vec{\mu}$ for all the dipoles (which is the extreme case).

Since the WABs are suspended, the basic outcome of the investigations so far are not applied directly to the realistic case and form an even more stringent result which may be called a "super-extreme" scenario. Due to the characterizations of the WAB's suspensions (see JGW document 10303), we can define a movement transfer-function in frequency-space that can give us a more realistic worst-case scenario (see also Fig. 3)). As the basic transfer function would apply only for the WAB, in principle one would need to do the model calculations twice, with and without suspension. However, given the fact that we are dealing mainly with small displacements only, we can assume that our whole system is linear which let us use the transfer-function directly on

[1] This assumption is reasonable in so far as in this model the extreme case of a magnetization in one direction for all dipoles is considered and therefore their sizes can be set to be a bit rougher.

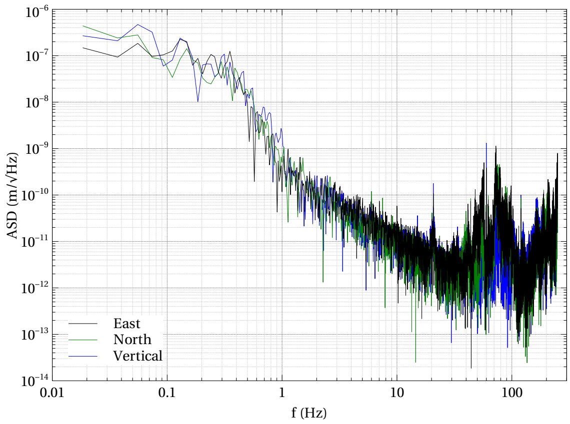

Figure 3: Spectra of the seismic noise in the Kamioka mine calculated from a time series of seismic vibration measured at the CLIO side in August 2011. The data show the amplitude spectral density of the noise in three directions: toward east, north, and vertical.

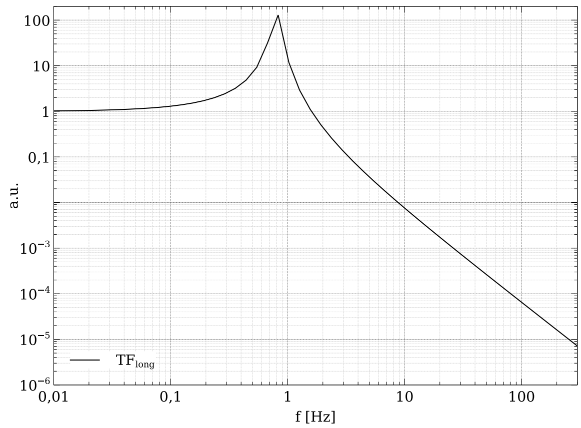

Figure 4: Transfer function of the longitudinal degree of freedom for the WABs. The resonance is at 0.8Hz with a damping-rate of 0.08. Among all resonances, the longitudinal one has the highest impact in terms of OSEM actuation.

the OSEM movement in frequency-space. For the "super-extreme" case, the baffle is mounted directly with the ground, meaning that its seismic vibration will be the same as in the Kamioka mine. For the more realistic one, the transfer-function in frequency-space is multiplied directly with the strain-noise implied by the magnetic coupling of the seismic vibrations to the OSEM. For the ground-noise of the Kamioka mine, I have used the data set from Yusuke Sakakibara, taken in 2011 at CLIO in three directions: two represent the arms of KAGRA (toward east and north) and one represents the noise in vertical direction. The amplitude spectral density of the seismic vibration can be seen in Fig. 3.

4 Results and Discussion

The results of the calculations can be seen in Fig. 4. In the graph, the strain-noise power-spectral density from the modeled baffle is shown for the two cases of being suspended and non-suspended, respectively, in comparison with KAGRAs goal sensitivity. The first observation and conclusion that can be made is that the influence of a vibrating WAB on the gravitational wave strain is for all frequencies below KAGRA's goal sensitivity. Even for the assumed "super extreme" case of having a non-suspended baffle. It should be again emphasized that also the suspended baffle would be an extreme case in our model since we are using equally aligned dipole moments in the solblack coating as well as taking into account the influence of only one (isolated) OSEM actuator that is combined with the mirror (neglecting the antisymmetric alignment of all four OSEM actuators). In its maximal approach to the sensitivity curve of KAGRA at around 6 Hz, the created strain-noise of the suspended baffle is approximately one order (for the "super-extreme" case) and ca. 3 orders (for the suspended case) of magnitude smaller. Even the narrow peaks at around 50 to 80 Hz seem to be of a minor issue, especially for the suspended baffle.

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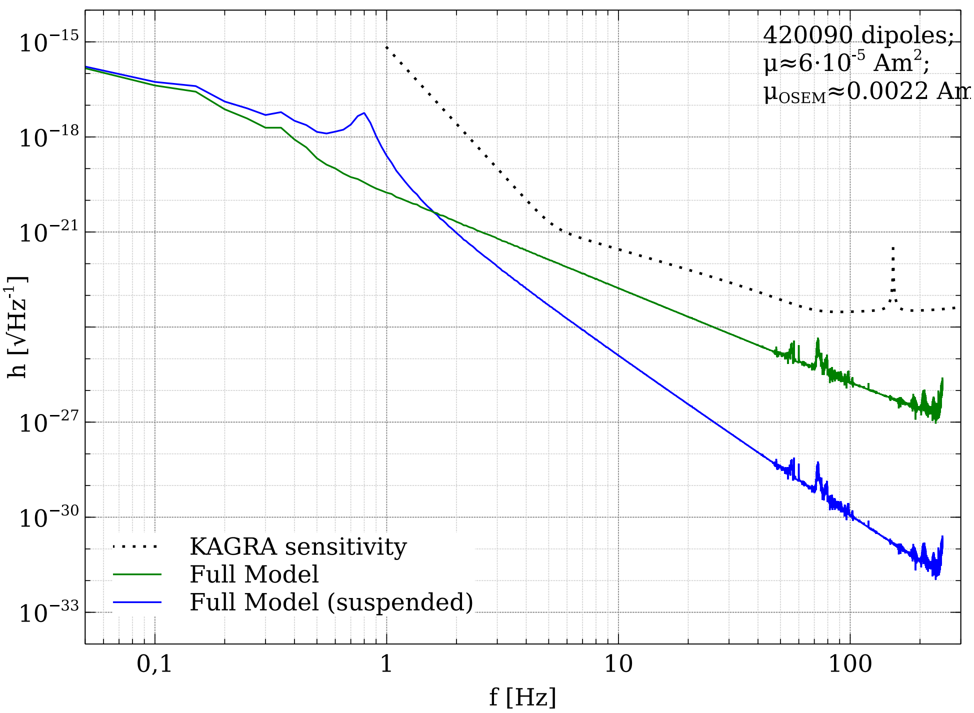

Figure 5: Calculated strain-noise for the suspended and the non-suspended wide-angle baffle (extreme and "super-extreme" case) in comparison with the goal sensitivity of KAGRA.

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