Report on the Calculation of the Magnetic Field of a SolBlack Coated Wide-Angle Baffle

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1 Introduction

In this report, the calculation of magnetic fields caused by the solblack coating of a wide-angle baffle, which is going to be installed in front of the ITMX and ITMY mirrors inside the respective cryostats (Aso, 2014) is presented. The basic conceptional design of these baffles is based on the work by Tomotada Akutsu (see Fig. 1) and shows basically a cylindrical body with a conical form toward the face that is closest to the mirror. These baffle are going to be coated with a light-absorbing envelope having a smooth surface to suppress scattering of stray-light that may hit the baffle. There are several candidates for such a coating. However, the most efficient one for our purpose (from the viewpoint of the price) is still solblack (GENESIA, 2013; Akutsu, 2014) as its abilities to suppress stray-light are still quite sufficient for us. Unfortunately, it turned out that solblack is a magnetic material Tokoku (2014) as it contains nickel. Thus, its influence on other magnets, especially the OSEM-actuators, might affect the measurements. To make sure that the influence is low enough for KAGRAs sensitivity, I have made near-field simulations on the magnetic field of a solblack coated wide-angle baffle and will present the results of these calculations here. It should be noted that the presented results refer only to extreme case scenarios to make sure that in the reality the disturbances will be much lower and thus safer.

2 Magnetic Fields

For the calculation of the influence of magnetic fields, I will give a short review of the theory that I have used.
In my concept, I assume the solblack coating consists of many small and independent magnetic dipoles. The far-field solution of the magnetic field of a small dipole is given by

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi r^2} \cdot \frac{3\vec{r}(\vec{\mu} \cdot \vec{r}) - \vec{\mu} r^2}{r^3}. \quad (1)$$

$\vec{\mu}$ is the magnetic moment of the dipole and μ_0 is the magnetic susceptibility of the vacuum. The force on a magnetic probe (in our case the OSEM-actuator on the mirror surface) is given by

$$\vec{F} = m\ddot{\vec{r}} = \nabla \left(\vec{\mu}_{OSEM} \cdot \vec{B}(\vec{r}) \right). \quad (2)$$

$\vec{\mu}_{OSEM}$ is the magnetic dipole moment of the OSEM actuator. As the movement of the wide-angle baffle shall be small compared to the size of the dipoles (movement in nm scale

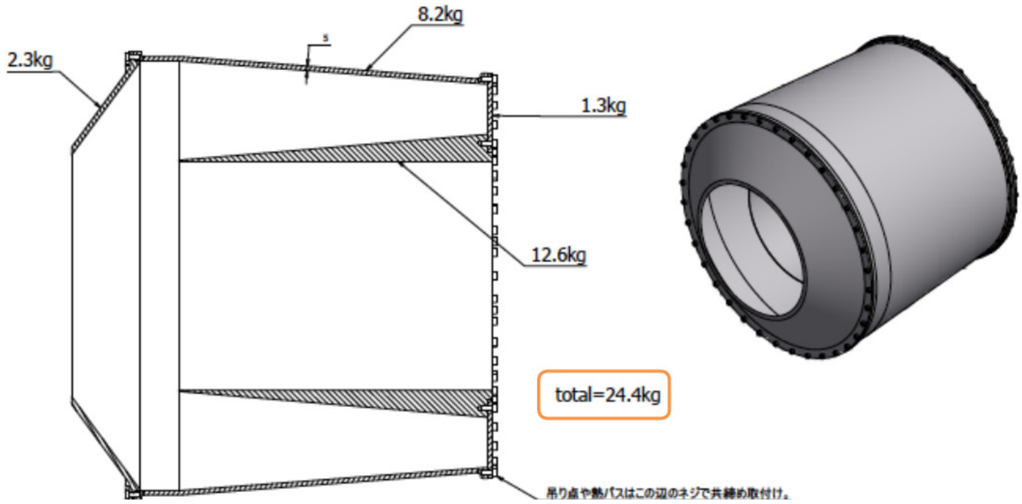

Figure 1: *Conceptional design of the wide-angle baffles that are going to be used in KAGRA (credit: Tomotada Akutsu).*

while size of dipole in mm scale), we can simplify[1] the problem with a Taylor approximation:

$$\vec{F}(\vec{r}) \approx \vec{F}(\vec{r}(0)) + \sum_{j=1}^{3} \frac{\partial \vec{F}(\vec{r})}{\partial r_j}|_{\vec{r}=\vec{r}(0)} \Delta r_j. \tag{3}$$

I am using the view that $\vec{r} = \vec{r}_{OSEM} - \vec{r}_0$ where $\vec{r}_0$ is the position of the coating-dipole and $\vec{r}_{OSEM}$ the position of the OSEM-actuator. $\vec{r}(0)$ marks the relative position $\vec{r}$ at the starting point. Together with Eq. (2), Eq. (3) becomes

$$F_i(\vec{r}) = \sum_{k=1}^{3} \mu_{OSEM,k} \frac{\partial B_k(\vec{r})}{\partial r_i}|_{\vec{r}=\vec{r}(0)} + \sum_{j,k=1}^{3} \mu_{OSEM,k} \frac{\partial^2 B_k(\vec{r})}{\partial r_i \partial r_j}|_{\vec{r}=\vec{r}(0)} \Delta r_j. \tag{4}$$

As any "DC" signal (or force) will be balanced by the OSEM-actuator, in the resulting (noise producing) force only the last term on the right-hand side of Eq. (4) remains and may be summarized as

$$F_{res,i} = \sum_{j,k=1}^{3} \mu_{OSEM,k} \frac{\partial^2 B_k(\vec{r})}{\partial r_i \partial r_j}|_{\vec{r}=\vec{r}(0)} \Delta r_j = \sum_{j=1}^{3} K_{i,j} \Delta r_j. \tag{5}$$

From this equation, we can already calculate the transferred noise of a shaking baffle to the mirror and, thus, determine the spectral density of its movements when we imagine that $\Delta\vec{r}$ is here the movement of the respective dipole $\Delta\vec{r}_0$ while $F_{res,i}$ represents the force-response on the OSEM-actuator. In the calculations, we have to sum over all dipoles of the baffle on the right-hand side of Eq. (5) to get the resulting force.
Once the movement of the actuator, and thus the mirror, is known, we can calculate its influence on the gravitational-wave strain noise and compare the results with the goal sensitivity of KAGRA. However, it should be noted that in the actual design, there will be four actuators aligned anti-symmetrically so that the resulting magnetic moment of the whole system will be zero. In the calculations which are presented here, I will assume an extreme case of just

[1] Simplification means here to save calculation time for the computer

having one actuator that responds to the solblack magnetic field. The strain noise effect due to the movements of the mirror because of a vibrating force on the magnet can be described to be

$$h_{WAB} = 2\frac{\vec{F}_{res}(\omega)}{L \cdot m\omega^2}, \tag{6}$$

where L is the length of the arms of the interferometer (for KAGRA: 3000 m). The reason for the factor of 2 is that because of the reflection of the light on the mirror, the light path is actually doubled.

It is possible to further simplify the problem given in Eq. (5) if (and this is actually the case as can be seen from Fig. 3) the noise in all three spatial dimensions has the same (or almost the same) power spectrum. Then, this problem can be simplified to become a linear differential equation in frequency and we can define a frequency dependent response function to calculate the force-noise-spectrum out of the amplitude-noise-spectrum from the dipoles (Saulson, 1994). Taking the assumption that $\Delta r_{0,i}(\omega) = a_j \cdot e^{i\omega t}$ and $\Delta r_{OSEM,j}(\omega) = b_j \cdot e^{i\omega t}$, we can write

$$m\omega^2 \Delta r_{OSEM,i} = \Delta r_{0,i} \sum_{j=1}^{3} |K_{i,j}|, \tag{7}$$

where $\Delta\vec{r}_{OSEM}$ is the vector of the local movement of the OSEM-actuator due to the noise of the magnetic dipoles of the solblack coating. The reason for taking the absolute value of the $K_{i,j}$ is that we actually taken the Fourier spectrum of the right-hand side of Eq. (5). The frequency response function, or the relation between input noise and output noise spectrum, is thus

$$G_i(\omega) = \frac{\Delta r_{OSEM,i}}{\Delta r_{0,i}} = \frac{\sum_{j=1}^{3} |K_{i,j}|}{m\omega^2}. \tag{8}$$

For having more than one dipole, in above equation one would have to sum first all $K_{i,j}$ for all dipoles before taking the absolute value.

3 Baffle Model

The particular shape of the wide-angle baffle that is going to be installed in KAGRA can be seen from Fig. 1. However, in order to simplify the issue, I assumed the baffle to be a simple cylinder of 40cm diameter and 40cm length (in first order) to which a doughnut-shaped plate of 10 cm broadness is attached on the side facing the mirror (in second order). I did not consider the inner structure of the baffle as I do think that it has no important influence on the outcome of the calculations. More important is the estimation of μ of the dipoles of the coating. Fortunately, there are already measurements on the magnetic behavior of a solblack coating by Tokoku (2014). From the given values of the magnetization of a sheet of solblack (maximum $6{\cdot}10^4$ Am^{-1}), $|\mu|$ for a cubic-shaped dipole element of 1 mm length can be estimated[2] to be $6{\cdot}10^{-5}$ Am^2. For the model itself, it was assumed that there is a small space between each dipole of ~ 0.1 mm to the next dipole. In total I worked with $4{\cdot}10^5$ dipoles for the cylinder and $7.4{\cdot}10^4$ dipoles for the doughnut-shaped element facing the mirror. As the wide-angle baffle is placed as close as possible to the mirror, the distance of the mirror-side of the baffle to the actuators is not bigger than 20 cm. The magnetic moment of the actuator, μ_{OSEM}, should have an absolute value $\gtrsim$0.0022 Am^2 (Fiori et al., 2014) and has the same direction of $\vec{\mu}$ of all the dipoles (extreme case). In Fig. 3, a comparison between the two approaches (cylinder and cylinder + doughnut plate) is shown. Basically, there is only a small difference between the first and second order approximation by a factor of ~ 1.42. However, in the calculations I used only the combined model of cylinder and doughnut plate only.

[2]This assumption is reasonable in so far as in this model the extreme case of a magnetization in one direction for all dipoles is considered for which the size of the dipoles may be set to be a bit rough.

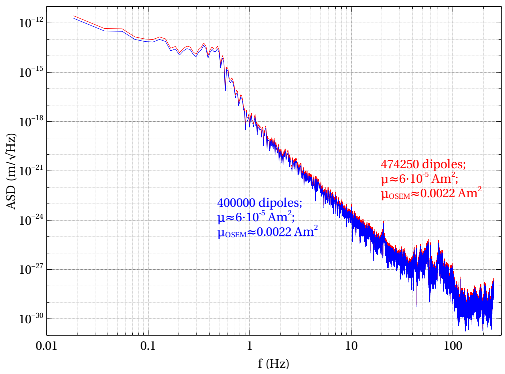

Figure 2: *Comparison of the amplitude spectral density of the vibration of the mirror caused by the model of only one cylinder (400000 dipoles) and the combined model of cylinder and doughnut plate (474250 dipoles), both suspended. Shown are the spectra for the noise in east direction.*

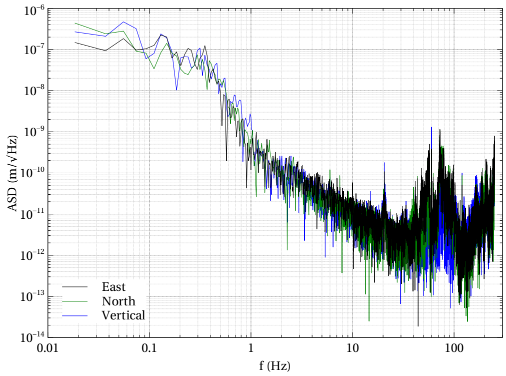

Figure 3: *Spectra of the seismic noise in the Kamioka mine calculated from a time series of seismic vibration measured at the CLIO side in August 2011. The data show the amplitude spectral density of the noise in three directions: toward east, north, and vertical.*

Another approach has been made for the suspension of the baffle which is still under discussion and not decided yet. Therefore, I treat here two (extreme but possible) cases: first, the baffle is mounted directly with the ground, meaning that its seismic vibration will be the same as in the Kamioka mine. Second, the baffle has a very simple suspension as a pendulum with a wire having a length of 1 m. In this case, the transfer function is assumed to follow a Lorentzian with an eigenfrequency of ~ 0.5 Hz and a maximal amplitude of ~ 63. In order to get the spectrum from the suspended baffle, I just multiplied the transfer function with the amplitude spectral density of the seismic noise to get $\Delta r_0(\omega)$ For the seismic noise of the Kamioka mine, I have used the data set from Yusuke Sakakibara, taken in 2011 at CLIO in three directions: two represent the directions of the arms of KAGRA (toward east and north) and one represents the noise in vertical direction. The amplitude spectral density of the seismic vibration can be seen in Fig. 3.

4 Results and Discussion

The concrete results of the calculations can be inspected in Fig. 4. In the graph, the strain-noise of one baffle which is, without loss of generality, placed in the east-arm of KAGRA is shown for the two cases of being suspended and non-suspended, respectively, in comparison with KAGRAs goal sensitivity. The first observation and conclusion that can be made is at the same time the most important for the applications of KAGRA: the influence of a vibrating wide-angle baffle on the gravitational wave strain is negligible. Even for the assumed extreme cases of having equally aligned dipole moments in the solblack coating as well as taking into account the influence of only one (isolated) OSEM actuator that is combined with the mirror (neglecting the antisymmetric alignment of all four OSEM actuators). In its maximal approach to the sensitivity curve of KAGRA, the created strain-noise of the suspended baffle is still four to five orders of magnitude (at around 2 Hz) smaller. Even for the non-suspended baffle, the strain-noise of the baffle approaches the sensitivity to half an order of magnitude (in the peaks at around 50 to 80 Hz). That might be already critical but since we made so many extreme assumptions, this can be seen as an absolute maximum of

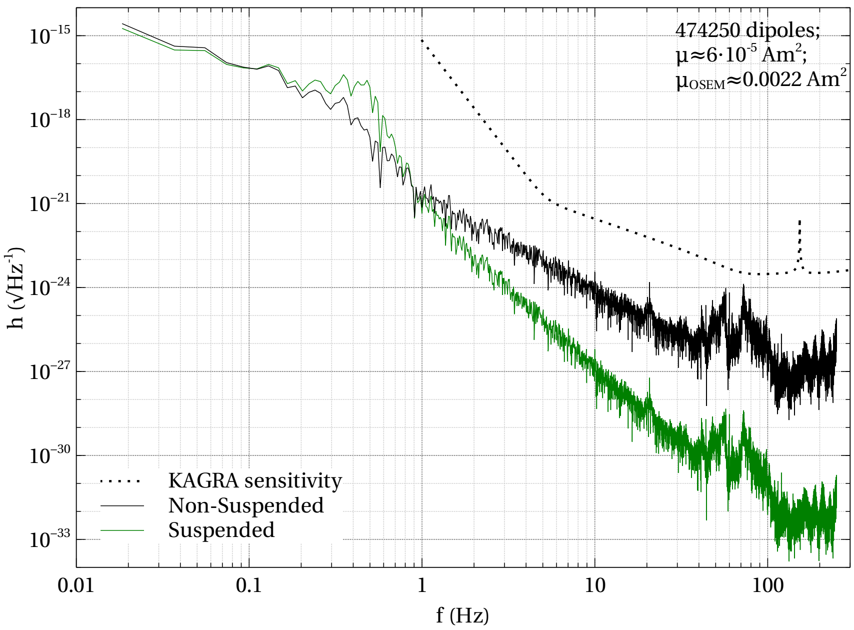

Figure 4: *Calculated strain-noise for the suspended and the non-suspended wide-angle baffle (extreme case) in comparison with the goal sensitivity of KAGRA.*

the strain-noise that can be expected.

It should be mentioned also that in the real baffle-mirror system the distance between the baffle and the OSEMs will not bigger than 20 cm. Indeed, we might deal with a distance of 60 − 70 cm which will further decrease the expecting influence of the magnetic field.

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