Gravitational Casimir Effect

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Quach, PRL 114, 081104 (2015)

東京大学
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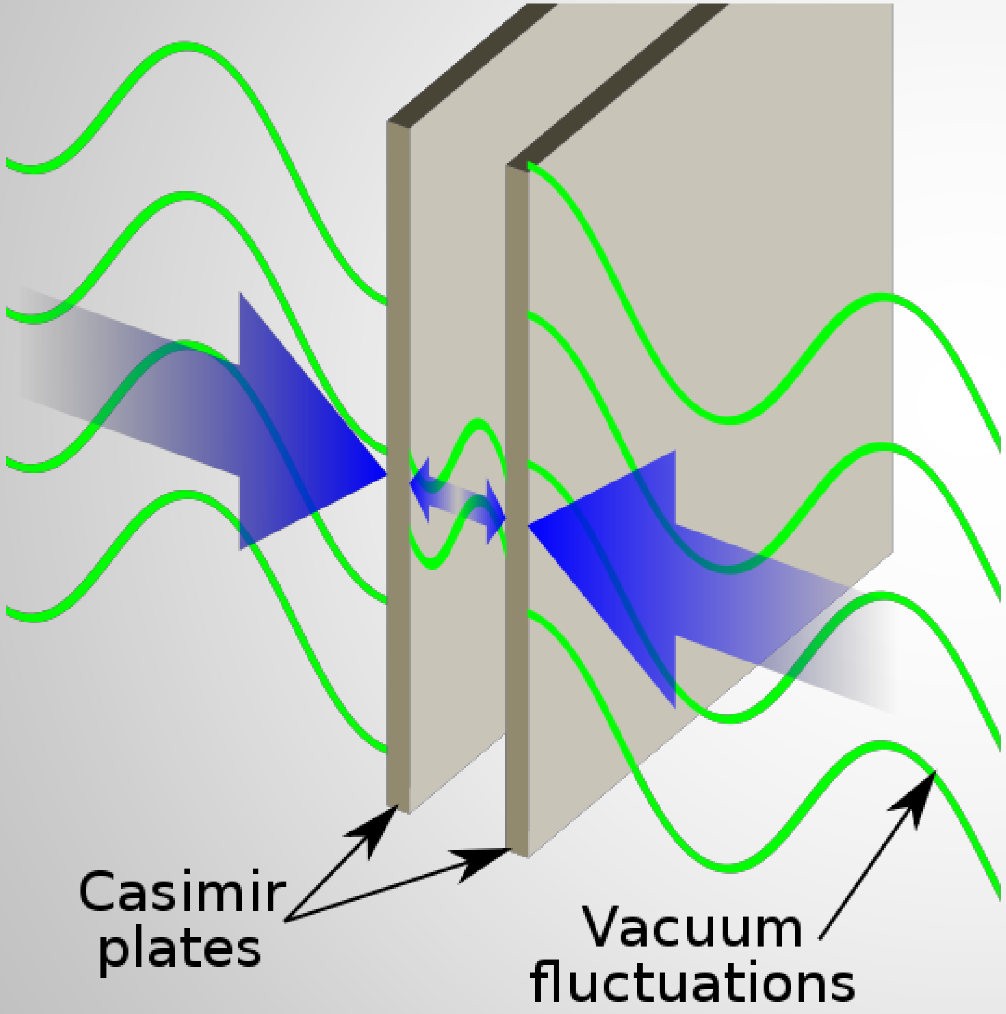
Casimir Effect




Hendrik Casimir (1948)

$$E_n = \hbar\omega(n + \frac{1}{2})$$

$$E_0 = \frac{\hbar\omega}{2}$$

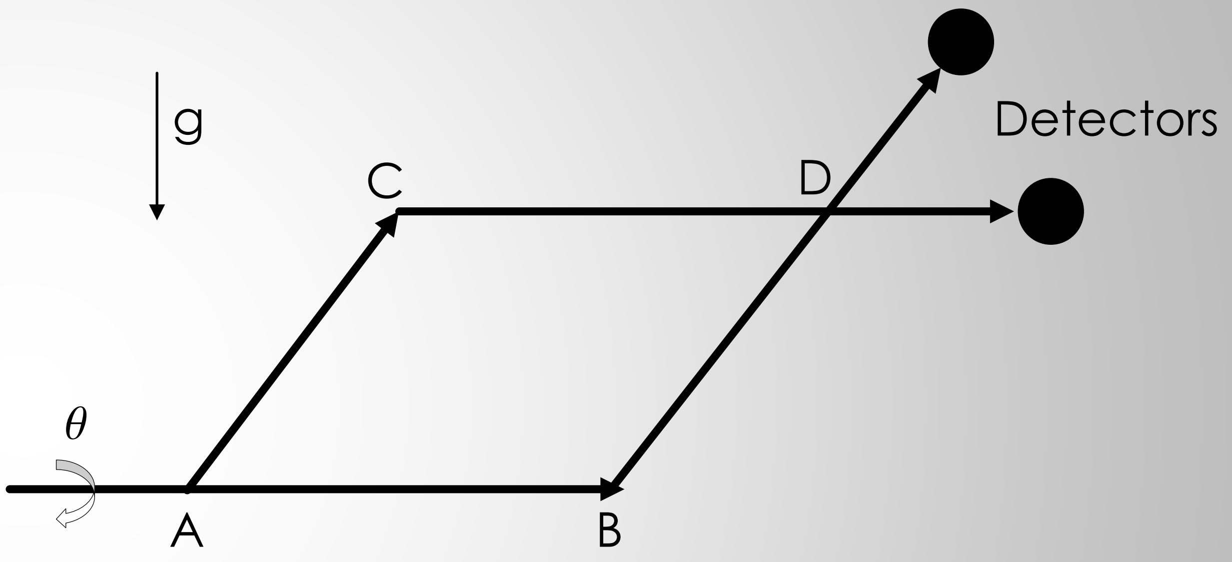
$$P_I(a) = -\frac{\pi^2}{240}\frac{\hbar c}{a^4}$$

Outline

- Motivation
- Gravitational Casimir Effect
- Ordinary Materials
- Superconductors

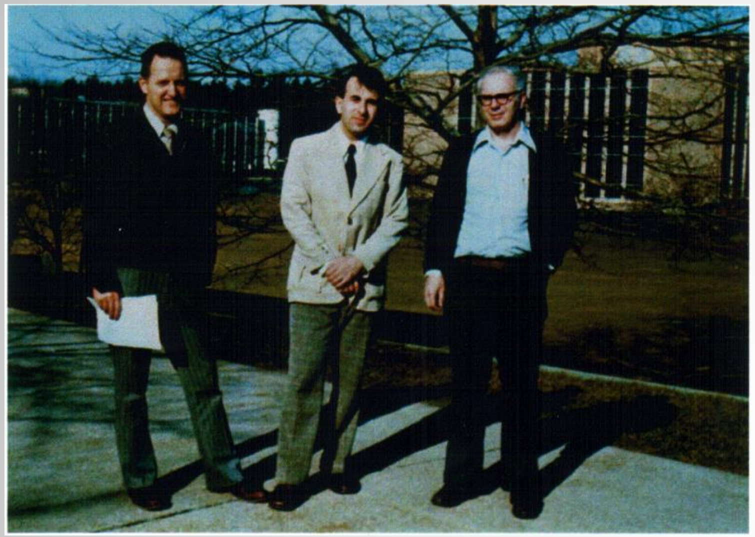
QM & GR Interface

Hawking Radiation

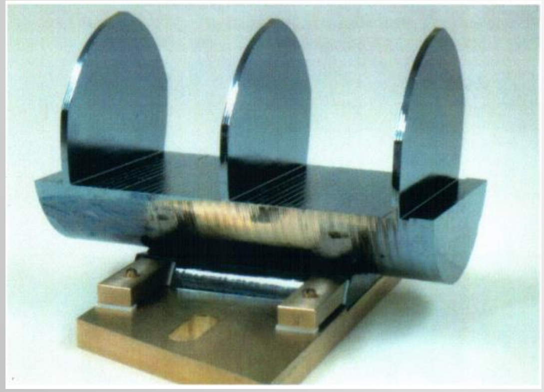


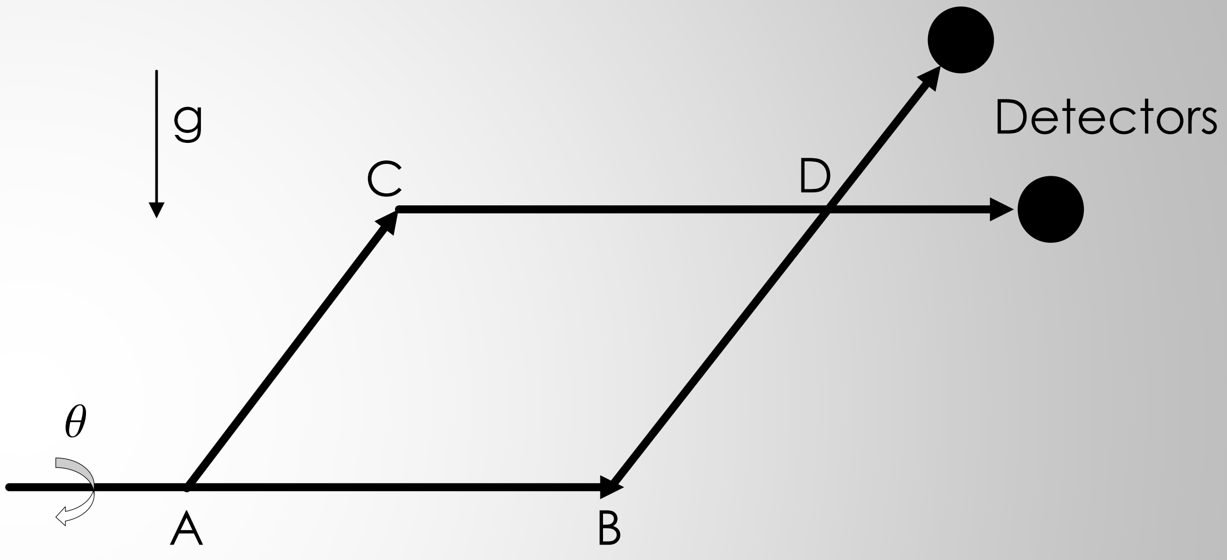
COW Experiment

QM & GR Interface



A. Overhauser, R. Colella, S. Werner





COW Experiment

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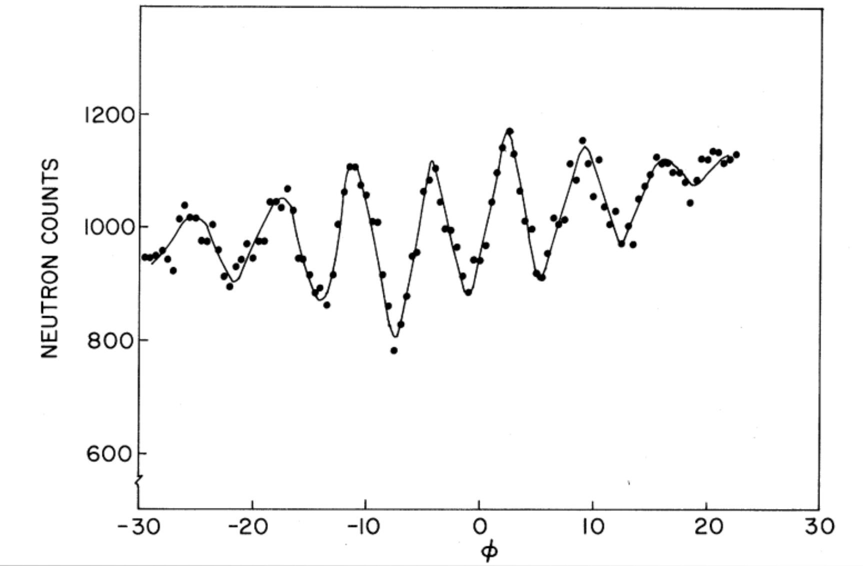
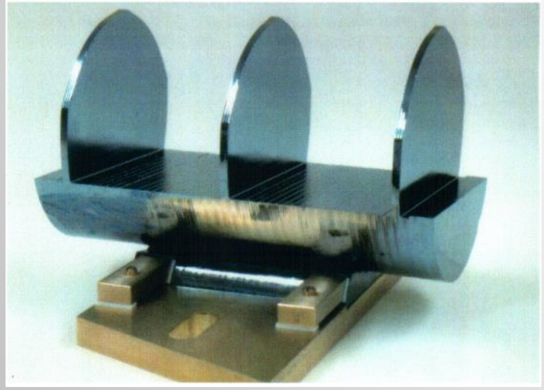
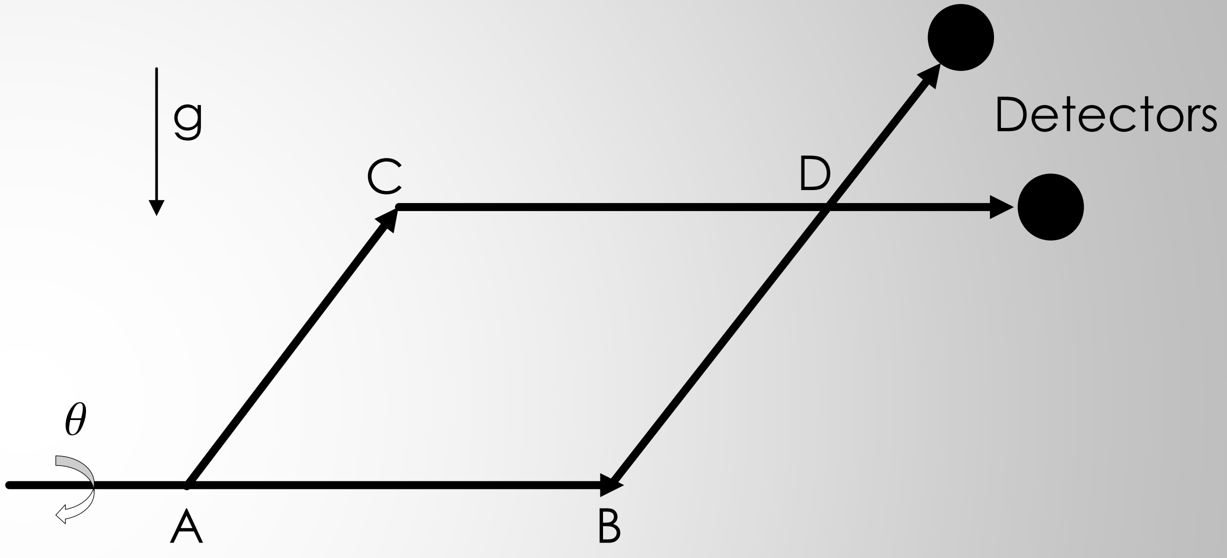


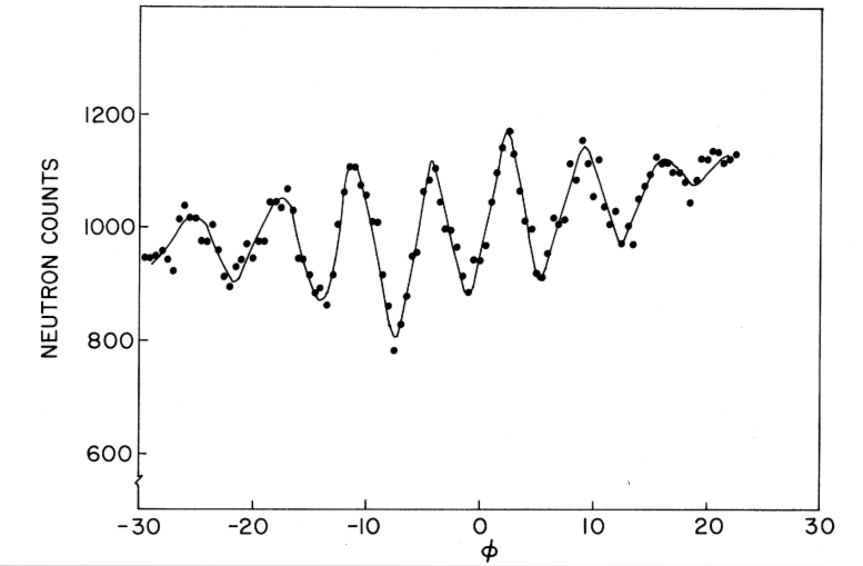
Colella, Overhauser, Werner, PRL 1975



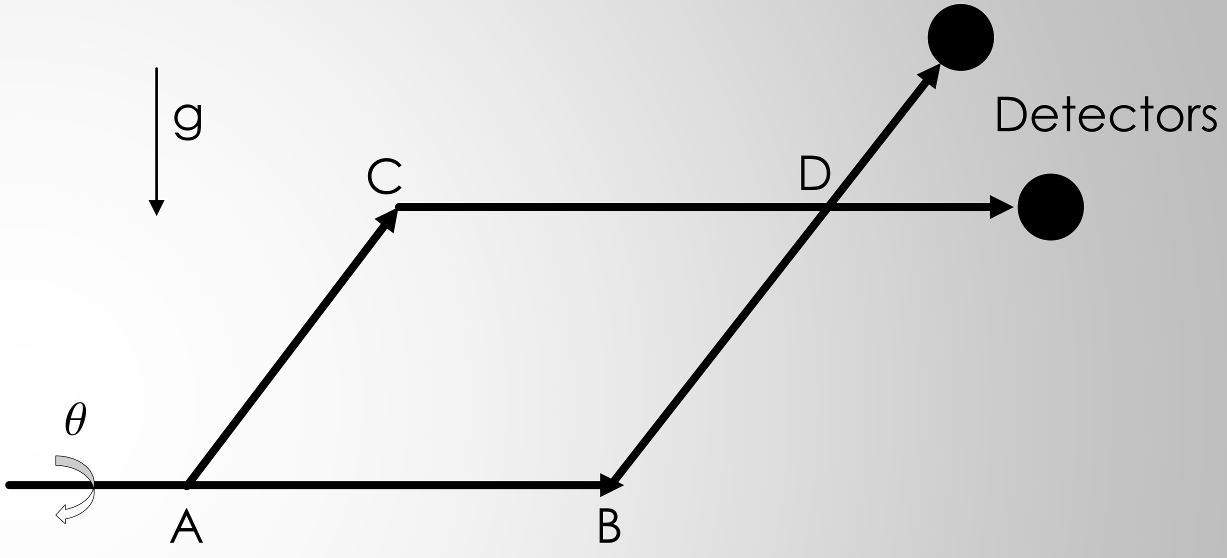


COW Experiment

QM & GR Interface



Colella, Overhauser, Werner, PRL 1975



$$H = \frac{\boldsymbol{p}^2}{2m_i} + m_g \boldsymbol{g}\boldsymbol{r} - \boldsymbol{\omega}\boldsymbol{L}$$

COW Experiment

$$\Delta\beta_g \approx \frac{m_g}{m_i} \boldsymbol{g}\boldsymbol{K}TT'$$

Lammerzahl, Gen. Rel. Grav.1996

WEP violation in QM

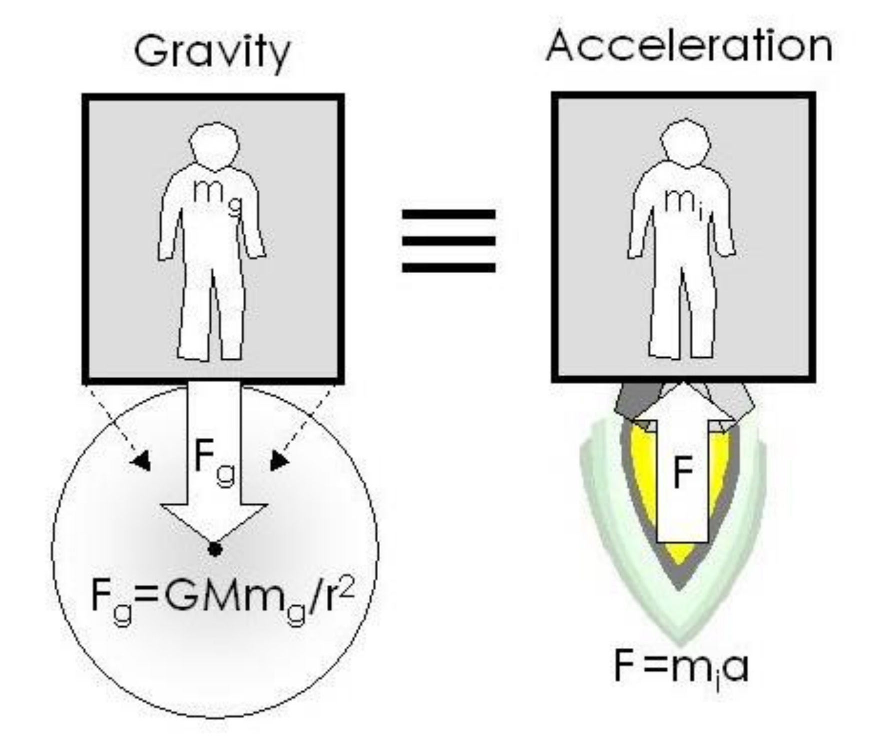
Example: Dirac Equation

$$(i\hbar\gamma^{\alpha}D_{\alpha} - mc)\psi = 0$$

$$D_{\alpha} = h_{\alpha}^{i}D_{i}, \qquad D_{i} = \partial_{i} + \frac{i}{4}\sigma_{\alpha\beta}\omega_{i}^{\alpha\beta}$$

$$ds^2 = V^2(dx^0)^2 - W^2(d\boldsymbol{x}\cdot d\boldsymbol{x})$$

Obukhov, PRL, 2001



There is no experiment that can be done, in a small confined space, which can detect the difference between a uniform gravitational field and an equivalent uniform acceleration.

WEP violation in QM

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Obukhov, PRL, 2001

(i) Gravitational field

$$\boldsymbol{g} = -GM\frac{\boldsymbol{r}}{r^3}$$

$$V = \left(1 - \frac{GM}{2c^2 r}\right)\left(1 + \frac{GM}{2c^2 r}\right)^{-1}, \qquad W = \left(1 + \frac{GM}{2c^2 r}\right)^{2}$$

$$H_a = \beta mc^2 + \beta\frac{p^2}{2m} - \beta m\boldsymbol{g}\cdot\boldsymbol{x} - \frac{\hbar}{2c}\boldsymbol{\Sigma}\cdot\boldsymbol{g} - \frac{\hbar}{mc^2}\beta\boldsymbol{\Sigma}\cdot(\boldsymbol{g}\times\boldsymbol{p})$$

(ii) Accelerated frame

$$V = 1 + \frac{\boldsymbol{a}\cdot\boldsymbol{x}}{c^2}, \qquad W = 1$$

$$H_a = \beta mc^2 + \beta\frac{p^2}{2m} + \beta m\boldsymbol{a}\cdot\boldsymbol{x} + \frac{\hbar}{2c}\boldsymbol{\Sigma}\cdot\boldsymbol{a} + \frac{\hbar}{2mc^2}\beta\boldsymbol{\Sigma}\cdot(\boldsymbol{a}\times\boldsymbol{p})$$

(Foldy–Wouthuysen transformation)

$$\frac{\hbar g}{c} = 2\times 10^{-23}\,\text{eV}$$

WEP violation in QM

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(gravitational spin-orbit coupling)

$$H_a = \beta mc^2 + \beta\frac{p^2}{2m} - \beta m\boldsymbol{g}\cdot\boldsymbol{x} - \frac{\hbar}{2c}\boldsymbol{\Sigma}\cdot\boldsymbol{g} - \frac{\hbar}{mc^2}\beta\boldsymbol{\Sigma}\cdot(\boldsymbol{g}\times\boldsymbol{p})$$

(ii) Accelerated frame

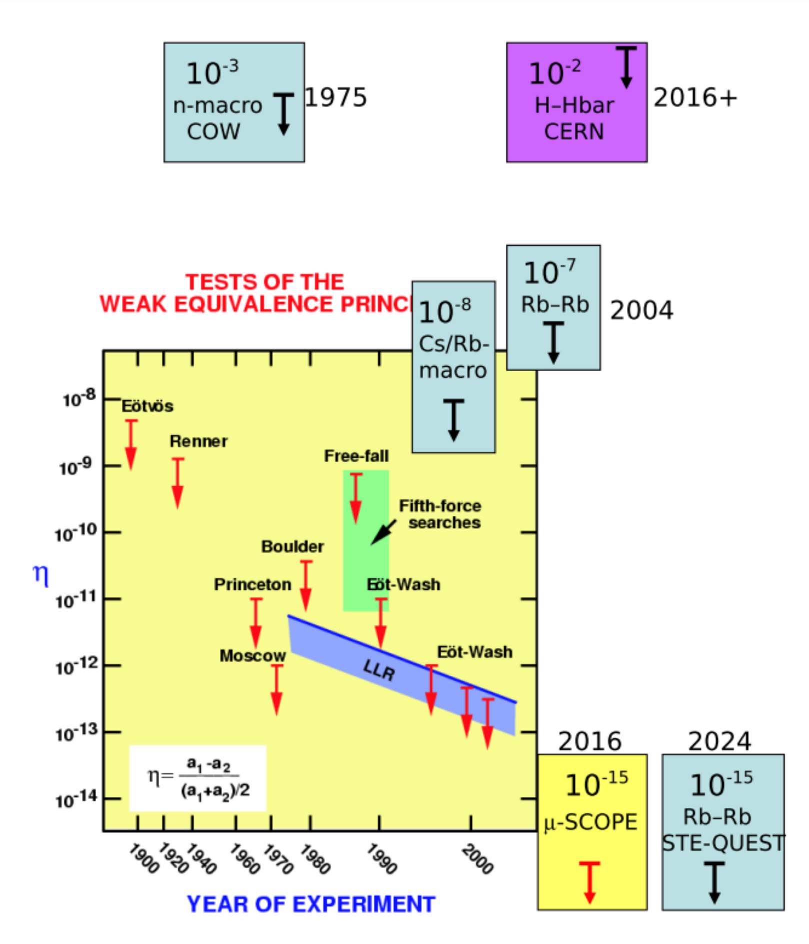
$$V = 1 + \frac{\boldsymbol{a}\cdot\boldsymbol{x}}{c^2}, \qquad W = 1$$

(inertial spin-orbit coupling)

$$H_a = \beta mc^2 + \beta\frac{p^2}{2m} + \beta m\boldsymbol{a}\cdot\boldsymbol{x} + \frac{\hbar}{2c}\boldsymbol{\Sigma}\cdot\boldsymbol{a} + \frac{\hbar}{2mc^2}\beta\boldsymbol{\Sigma}\cdot(\boldsymbol{a}\times\boldsymbol{p})$$

$$\frac{\hbar g}{c} = 2\times10^{-23}\,\text{eV}$$

WEP violation in QM



Altschul, et al., Advances in Space Research (2015)

(i) Accelerated frame

$$V = 1 + \frac{\boldsymbol{a}\cdot\boldsymbol{x}}{c^2}\ , \qquad W = 1$$

$$H_a = \beta mc^2 + \beta m\boldsymbol{a}\cdot\boldsymbol{x} + \beta\frac{p^2}{2m} + \frac{\hbar}{2c}\boldsymbol{\Sigma}\cdot\boldsymbol{a} + \frac{\hbar}{2mc^2}\beta\boldsymbol{\Sigma}\cdot(\boldsymbol{a}\times\boldsymbol{p})$$

(ii) Gravitational field

$$\boldsymbol{g} = -GM\frac{\boldsymbol{r}}{r^3}$$

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$$H_a = \beta mc^2 - \beta m\boldsymbol{g}\cdot\boldsymbol{x} + \beta\frac{p^2}{2m} - \frac{\hbar}{2c}\boldsymbol{\Sigma}\cdot\boldsymbol{g} - \frac{\hbar}{mc^2}\beta\boldsymbol{\Sigma}\cdot(\boldsymbol{g}\times\boldsymbol{p})$$

Eötvös ratio: $\eta = 2\frac{a_A - a_B}{a_A + a_B}$

$$\frac{\hbar g}{c} = 2\times 10^{-23}\text{eV}$$

Enhanced gravitational interaction with quantum systems

B. S. DeWitt, “Superconductors and Gravitational Drag”, Phys. Rev. Lett. 16 , 1092 (1966).

G. Papini, “London moment of rotating superconductors and lense-thirring fields of general relativity”, Nuovo Cimento B 45 , 66 (1966).

G. Papini, “Particle Wave Functions in Weak Gravitational Fields”, Nuovo Cimento B, 52, 136–41 (1967).

G. Papini, “Detection of inertial effects with superconducting interferometers”, Phys. Lett. A 24, 32–3 (1967).

G. Papini, “Superconducting and Normal Metals as Detectors of Gravitational Waves”, Lettere Al Nuovo Cimento 4, 22 1027 (1970).

J. Anandan, “Gravitational and Inertial Effects in Quantum Fluids”, Phys. Rev. Lett. 47 , 463 (1981).

Gravitational Casimir Effect

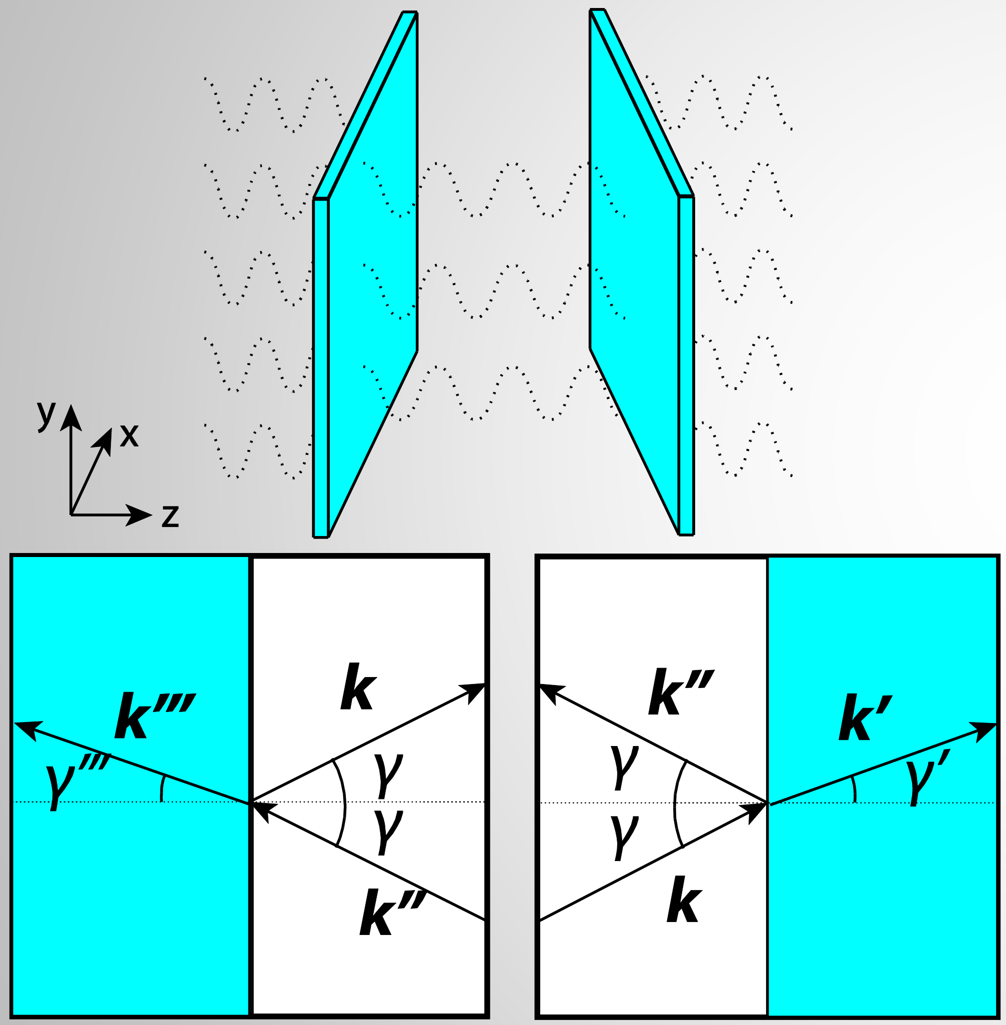
Gravitoelectromagnetism (GEM)

$$\nabla \cdot \boldsymbol{E} = \kappa \boldsymbol{\rho}^{(E)}$$
$$\nabla \cdot \boldsymbol{B} = \kappa \boldsymbol{\rho}^{(M)}$$
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} - \kappa \boldsymbol{J}^{(M)}$$
$$\nabla \times \boldsymbol{B} = \frac{\partial \boldsymbol{E}}{\partial t} + \kappa \boldsymbol{J}^{(E)}$$

$$\kappa = \frac{8\pi G}{c^4}$$
$$E_{ij} = C_{0i0j}$$
$$B_{ij} = \star\, C_{0i0j}$$

$$J_{\mu\nu\rho} = -T_{\rho[\mu,\nu]} + \frac{1}{3}\eta_{\rho[\mu}T_{,\nu]}$$
$$\rho_i^{(E)} = -J_{i00}$$
$$\rho_i^{(M)} = -\star\, J_{i00}$$
$$J_{ij}^{(E)} = J_{i0j}$$
$$J_{ij}^{(M)} = \star\, J_{i0j}$$

Gravitational Casimir Effect



$$E_0 = \frac{\hbar}{4\pi}\int_0^\infty k_\parallel dk_\parallel \sum_n (\omega_n^+ + \omega_n^\times)\sigma$$

$$\Delta\left[\left(1+\frac{\kappa\chi}{2}\right)\boldsymbol{E}^{TT}\right] = 0$$

$$\Delta\boldsymbol{B}^{TT} = 0$$ *(smoothness principle)*

$$\boldsymbol{E}^{TT} = \begin{bmatrix} \alpha(1-\frac{\sin^2\gamma}{2}) & \beta\cos\gamma \\ \beta\cos\gamma & -\alpha(1-\frac{\sin^2\gamma}{2}) \end{bmatrix} e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$$

$$\boldsymbol{B}^{TT} = \begin{bmatrix} -\beta(1-\frac{\sin^2\gamma}{2}) & \alpha\cos\gamma \\ \alpha\cos\gamma & \beta(1-\frac{\sin^2\gamma}{2}) \end{bmatrix} e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}$$

Gravitational Casimir Effect

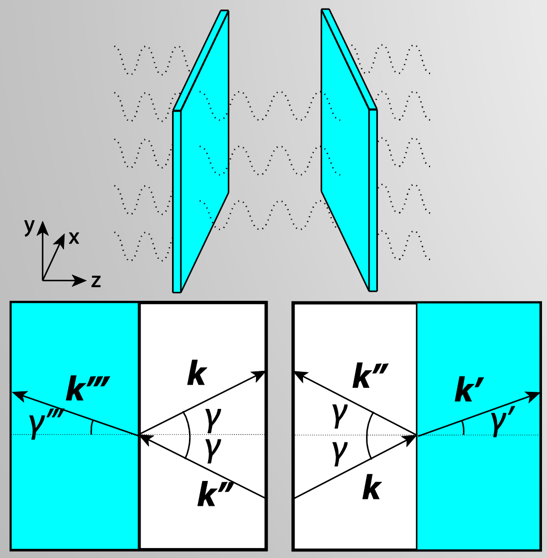
$$\alpha'\left(1+\frac{\kappa\chi}{2}\right)\left(1-\frac{S'}{2}\right)e^{-q'a/2} = \alpha\left(1-\frac{S^2}{2}\right)e^{-qa/2} + \alpha''\left(1-\frac{S^2}{2}\right)e^{qa/2}$$

$$S = \sin\gamma$$
$$C = \cos\gamma$$

$$\alpha'''\left(1+\frac{\kappa\chi}{2}\right)\left(1-\frac{S'}{2}\right)e^{-q'a/2} = \alpha\left(1-\frac{S^2}{2}\right)e^{qa/2} + \alpha''\left(1-\frac{S^2}{2}\right)e^{-qa/2}$$

$$\alpha' C' e^{-\frac{q'a}{2}} = \alpha C e^{-\frac{qa}{2}} - \alpha'' C e^{qa/2}$$

$$-\alpha''' C' e^{-\frac{q'a}{2}} = \alpha C e^{-\frac{qa}{2}} - \alpha'' C e^{-qa/2}$$



$$q^2 = -k_z^2 = k_\parallel^2 - \frac{\omega^2}{c^2}$$

$$q'^2 = -k_z'^2 = k_\parallel^2 - (1+\kappa\chi)\frac{\omega^2}{c^2}$$

Gravitational Casimir Effect

$$\alpha'\left(1+\frac{\kappa\chi}{2}\right)\left(1-\frac{S'}{2}\right)e^{-q'a/2}=\alpha\left(1-\frac{S^2}{2}\right)e^{-qa/2}+\alpha''\left(1-\frac{S^2}{2}\right)e^{qa/2}$$

$$S=\sin\gamma$$
$$C=\cos\gamma$$

$$\alpha'''\left(1+\frac{\kappa\chi}{2}\right)\left(1-\frac{S'}{2}\right)e^{-q'a/2}=\alpha\left(1-\frac{S^2}{2}\right)e^{qa/2}+\alpha''\left(1-\frac{S^2}{2}\right)e^{-qa/2}$$

$$\alpha'C'e^{-\frac{q'a}{2}}=\alpha Ce^{-\frac{qa}{2}}-\alpha''Ce^{qa/2}$$

$$-\alpha'''C'e^{-\frac{q'a}{2}}=\alpha Ce^{-\frac{qa}{2}}-\alpha''Ce^{-qa/2}$$

$$\Delta^{+}=e^{-aq'}\{\left[C'(S^2-2)-\left(1+\frac{\kappa\chi}{2}\right)C(S'^2-2)\right]^2e^{-aq}-\left[C'(S^2-2)+\left(1+\frac{\kappa\chi}{2}\right)C(S'^2-2)\right]^2e^{aq}=0$$

Gravitational Casimir Effect

$$\sum_n \omega_n(q) = \frac{1}{2\pi i}\left[\int_{i\infty}^{-i\infty} \omega(q)\, d\ln\Delta(\mathrm{q}) + \int_{C^+} \omega(q) d\ln\Delta(\mathrm{q})\right]$$

$$E_0 = \frac{\hbar}{4\pi}\int_0^\infty k_\parallel dk_\parallel \sum_n (\omega_n^+ + \omega_n^\times)\sigma$$

$$E(a) = \frac{E_0}{\sigma} - \lim_{a\to\infty}\frac{E_0}{a}$$

$$E(a) = \frac{\hbar}{4\pi^2}\int_0^\infty k_\parallel dk_\parallel \int_0^\infty [\ln(1 - r_+^2 e^{-2qa}) + \ln(1 - r_\times^2 e^{-2qa})]d\xi$$

$$r_+ = \frac{C'(S^2-2) - \left(1+\frac{\kappa\chi}{2}\right)C(S'^2-2)}{C'(S^2-2) + \left(1+\frac{\kappa\chi}{2}\right)C(S'^2-2)},\quad r_\times = \frac{\left(1+\frac{\kappa\chi}{2}\right)C'(S^2-2) - C(S'^2-2)}{\left(1+\frac{\kappa\chi}{2}\right)C'(S^2-2) + C(S'^2-2)}$$

Gravitational Casimir Effect

$$E(a) = \frac{\hbar}{4\pi^2}\int_0^\infty k_\parallel dk_\parallel \int_0^\infty [\ln(1 - r_+^2 e^{-2qa}) + \ln(1 - r_\times^2 e^{-2qa})] d\xi$$

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$$\alpha'\left(1+\frac{\kappa\chi}{2}\right)\left(1-\frac{S'}{2}\right)e^{-q'a/2} = \alpha\left(1-\frac{S^2}{2}\right)e^{-qa/2} + \alpha''\left(1-\frac{S^2}{2}\right)e^{qa/2}$$

$$\alpha' C' e^{-\frac{q'a}{2}} = \alpha C e^{-\frac{qa}{2}} - \alpha'' C e^{qa/2}$$

$$\frac{\alpha''}{\alpha} = \frac{-C'(S^2-2) + \left(1+\frac{\kappa\chi}{2}\right)C(S'^2-2)}{C'(S^2-2) + \left(1+\frac{\kappa\chi}{2}\right)C(S'^2-2)}e^{qa}$$

Gravitational Casimir Effect

$$E(a) = \frac{\hbar}{4\pi^2}\int_0^\infty k_\parallel dk_\parallel \int_0^\infty [\ln(1 - r_+^2 e^{-2qa}) + \ln(1 - r_\times^2 e^{-2qa})]d\xi$$

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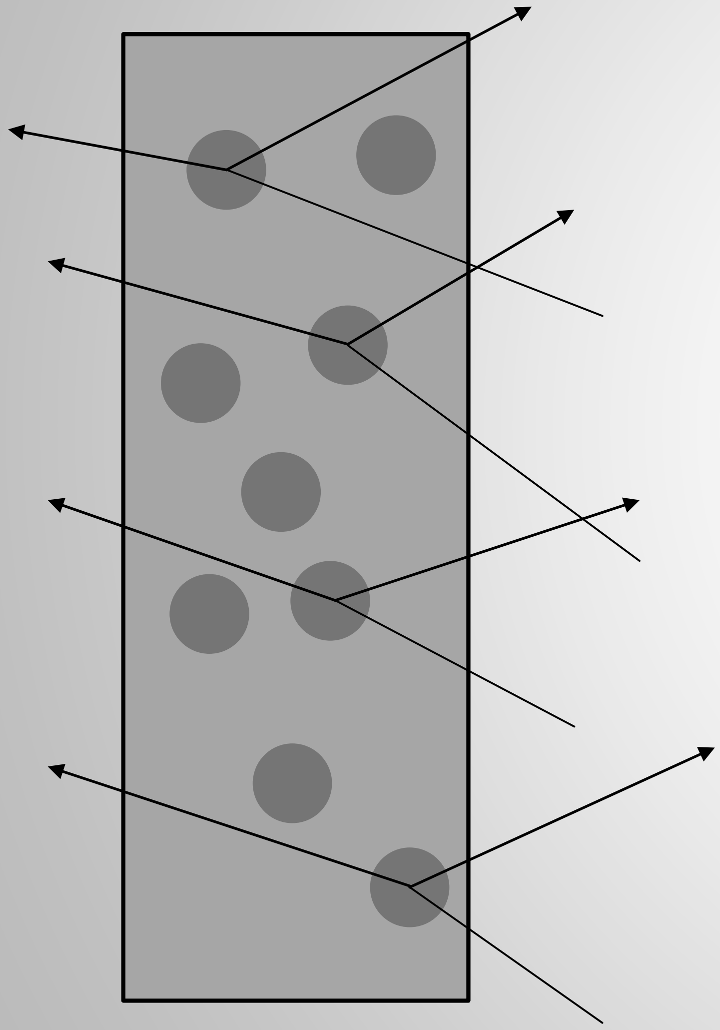
$$\left|\frac{\alpha''}{\alpha}\right| = \left|\frac{-C'(S^2-2) + \left(1+\frac{\kappa\chi}{2}\right)C(S'^2-2)}{C'(S^2-2) + \left(1+\frac{\kappa\chi}{2}\right)C(S'^2-2)} e^{qa}\right| = |r_+|$$

$$\left|\frac{\beta''}{\beta}\right| = \left|\frac{-\left(1+\frac{\kappa\chi}{2}\right)C'(S^2-2) + C(S'^2-2)}{\left(1+\frac{\kappa\chi}{2}\right)C'(S^2-2) + C(S'^2-2)} e^{qa}\right| = |\mathrm{r}_\times|$$

Ordinary material

$$G = 6.67 \times 10^{-11} \mathrm{m}^3 \mathrm{kg}^{-1} \mathrm{s}^{-2}$$



$$n = 1 + \frac{2\pi G \rho}{\omega^2}$$

$$\chi = \frac{1 - n(\omega)^2}{\kappa c^2}$$

Peters, PRD, 1974

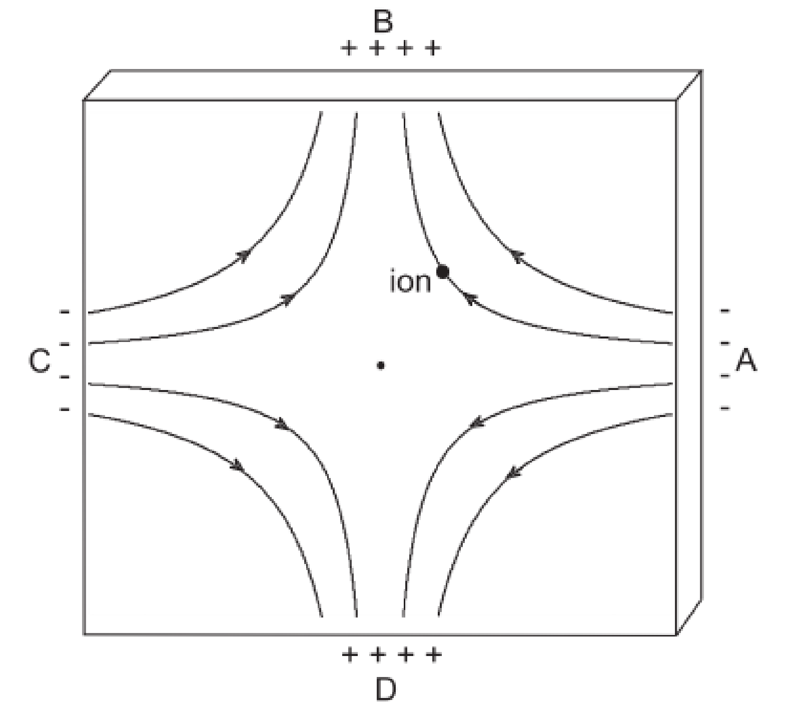
$$P(a) = -\frac{\partial E(a)}{\partial a}$$

$$O[\rho] = 10^4 \mathrm{kg\ m}^{-3}$$

$$P(10^{-6}) \approx 10^{-21} \mathrm{nPa}$$

$$O[\rho] = O[\frac{c^2}{G}] \approx 10^{27} \mathrm{kg/m}^3$$

Superconductor



Heisenberg-Coulomb effect

Minter, Wegter-McNelly, Chiao, Physica E, 2010

$$H = \frac{(\boldsymbol{p} - q\boldsymbol{A} - m\boldsymbol{h})^2}{2m} + V$$

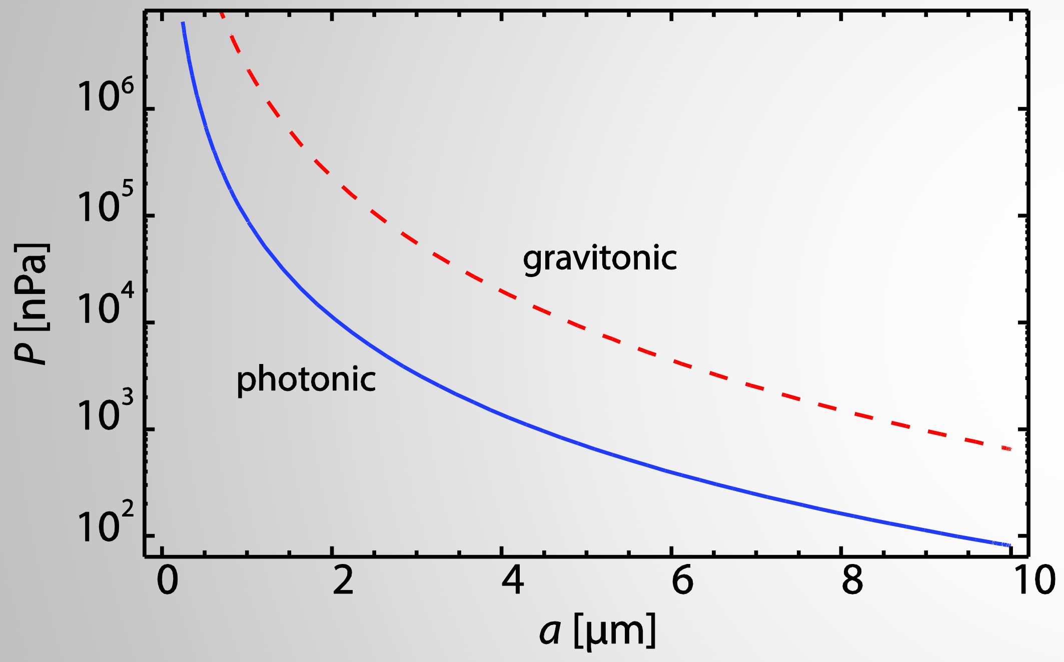
$$\boldsymbol{j}_G = \frac{1}{m}\mathrm{Re}\left[\psi^*\left(\frac{\hbar}{i}\nabla - q\boldsymbol{A} - m\boldsymbol{h}\right)\psi\right]$$
$$= \frac{1}{m}(-q\boldsymbol{A} - m\boldsymbol{h})\psi^*\psi$$

$$\boldsymbol{j}_G = \sigma_G \boldsymbol{E}_G$$

$$r_G = \left(1 + \frac{2\delta^2}{cd}\xi\right)^{-1}$$

$$r_E = \left(1 + \frac{2\lambda\delta^2}{cd^2}\xi\right)^{-1}$$

Superconductor



$d = 2$ nm $\quad \delta = 37$ nm $\quad \lambda = 83$ nm

$$H = \frac{(\boldsymbol{p} - q\boldsymbol{A} - m\boldsymbol{h})^2}{2m} + V$$

$$\boldsymbol{j}_G = \frac{1}{m}\mathrm{Re}\left[\psi^*\left(\frac{\hbar}{i}\nabla - q\boldsymbol{A} - m\boldsymbol{h}\right)\psi\right]$$
$$= \frac{1}{m}(-q\boldsymbol{A} - m\boldsymbol{h})\psi^*\psi$$

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