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Evaluation of BS and RM noise arisen from the AC component of geomagnetism field

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1 INTRODUCTION AND MOTIVATION

In order to lock phase shift, some magnet attaches to the mirror of KAGRA interferometer. In these kind of mirror, the beam-splitter (BS) and the recycling mirror (RM) is plan to attach 4 magnets. Although these magnets is indispensable for interferometer, the displacement motion of mirror is arisen from the vibration of attached magnet since the fluctuation of geomagnetism field affects the magnetic dipole moment of a magnet and vibrate the mirror though the motion of magnet. This motion of mirror (in this article, we call this kind of noise as "magnetic noise") is critical noise source of KAGRA interferometer. In this article, we report that the explanation how the magnetic noise is arisen and the result of simulation how much the upper-limit of magnetic noise of BS and RM is. In the result of simulation, we show that the upper-limit of magnetic noise of BS and RM is fulfill KAGRA requirement except for Signal recycling mirror. However the magnetic noise of Signal recycling mirror can be archived to the requirement of KAGRA under more realistic condition.

2 FUNDAMENTAL EQUATION

In this section, we explain the process that the vibration of magnet transforms to displacement of mirror and show the formalism of transfer function.

2.1 THE FORCE APPLIED TO THE MAGNET

Here, we consider the configurations of magnet and geomagnetism field are

1. The length of magnet is $l_m[m]$,

2. The magnetic charge of magnet is $q_m[Wb]$,

3. The direction of geomagnetism field, which has magnetic field intensity $B[T]$, intersects the side of magnet at θ degree.

The picture of considering magnet and geomagnetism field is Fig. 2.1.

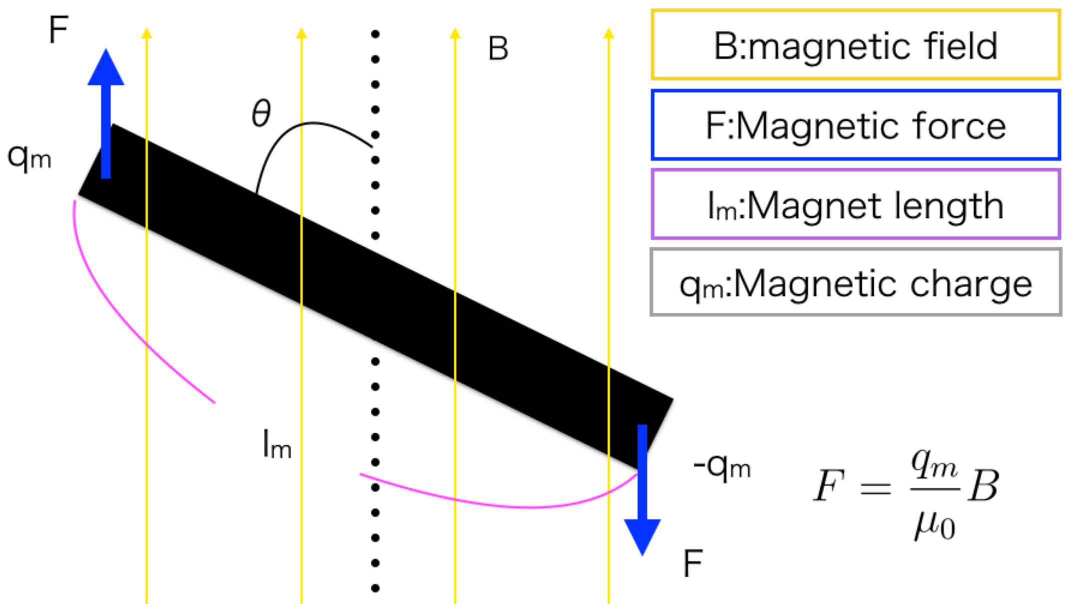

Figure 2.1: The alignment of magnet and magnetic field. μ_0 is absolute permeability of vacuum.

Since the direction of the force where both the ends of magnet generated by the geomagnetic field affects the magnetic charge of magnet are opposite, torque motion of magnet is excited around the center of magnet. The intensity of the torque N is expressed as a following equation.

$$
\begin{aligned}
N &= |\mathbf{r} \times \mathbf{F}| \\
&= \frac{l_m}{2} F \sin\theta - \frac{l_m}{2}(-F)\sin\theta \\
&= \frac{q_m}{\mu_0} l_m B \sin\theta \qquad (2.1)
\end{aligned}
$$

When the upper-limit of the noise is evaluated, all we have to consider is that the direction of geomagnetism field is orthogonal to the side of magnet, which means $\sin\theta \simeq 1$. Then we consider the torque applied to magnet as $\frac{q_m l_m}{\mu_0} B$ though this article.
Note that the translational motion of magnet is vanish since magnetic forces, which are always opposite direction, applied both ends of magnet are canceled out by each other.

In this section, we evaluate the equation of motion of rotation of mirror arisen from torque force of magnet explained to previous section. Since we should evaluate the upper-limit of magnetic noise, the statement of mirror should be

1. mirror is suspended by 4 wires,
2. 4 magnets is attached to mirror,
3. all direction of the magnetic pole of magnets are same.

Especially, the condition of the magnetic pole is very strong because in actually magnets are attached opposite pole direction each other. Note that if the configuration of magnet would like to be more realistic, you should take the intensity of torque 100 times as small as the configuration which the direction of magnetic pole are same. (In fact, the amount of realistic factor 1/100 is "rough" estimated value, not "precise" estimated value.) The picture of mirror and wire are Fig. 2.2.

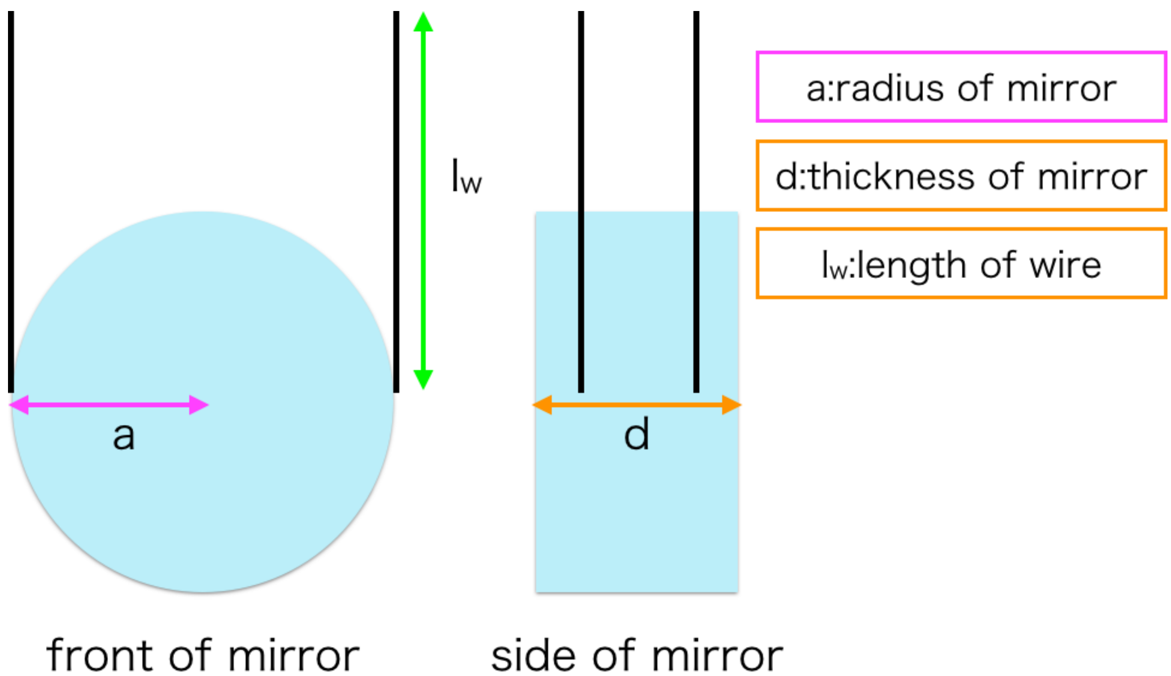

Figure 2.2: The configuration of mirror and wire.

The motion of mirror is only yaw-mode and pitch-mode since torque force applied magnet only makes magnet move in this configuration (See Sec.2.1.). Then the equation of motion is to be restricted to rotational mode (yaw-mode and pitch-mode) and expressed as a following equation;

$$I_i \frac{\partial^2 \phi_i}{\partial t^2} + \frac{\pi f_{res,i}}{Q} \frac{\partial \phi_i}{\partial t} + k_i \phi_i = N_{mag} \frac{q_m l_m}{\mu_0} B, \quad i = \text{yaw}, \text{pitch}. \tag{2.2}$$

where I_i is the moment of inertia of mirror, ϕ_i is the rotational angle of mirror from equilibrium state, $f_{res,i} \equiv \frac{1}{2\pi}\sqrt{\frac{k_i}{I_i}}$ is the resonance frequency of pitch/yaw mode of mirror, Q is the Q-value of the suspension system, k_i is the stiffness of the suspension system and N_{mag} is the number of magnets attached to mirror.
In this article, moment of inertia of mirror is always expressed as a following equation since the moment of inertia of pitch-mode and yaw-mode is same;

$$I_i = \left(\frac{a^2}{4} + \frac{l^2}{12}\right) M, \tag{2.3}$$

where a is the radius of mirror, l is the thickness of mirror and M is the mass of mirror. Q-value is expected one forth of the Q-value of wire. k_i should be calculated as the simulation of relevant suspension system.

2.3 TRANSFER FUNCTION FROM FLUCTUATION OF GEOMAGNETISM TO THE DISPLACEMENT MOTION OF MIRROR

In this section, we evaluate the displacement motion of mirror from the intensity of rotational angle evaluated to previous section. The fourier transformation of Eq.(2.2) is expressed as a following equation;

$$\begin{aligned} |\tilde{\phi}_i(f)| &= \frac{1}{4\pi^2 I_i}\frac{q_m l_m}{\mu_0} N_{mag} \frac{\left|f^2 - f_{res,i}^2\right|}{\left(f^2 - f_{res,i}^2\right)^2 + \frac{f^2 f_{res,i}^2}{4Q^2}} |\tilde{B}(f)| \quad [\mathrm{rad}/\sqrt{\mathrm{Hz}}] \\ &\equiv \frac{q_m l_m}{\mu_0} N_{mag} \tilde{H}(f)|\tilde{B}(f)|, \end{aligned} \tag{2.4}$$

where $\tilde{H}(f)$ is transfer function from the external force applied mirror to the rotational angle of mirror.
In KAGRA, since laser is planed to incident at 1mm accuracy from the center of mirror (see Fig. 2.3), the upper-limit of displacement of mirror $x_{disp,a}$ can evaluate as a following equation;

$$|\tilde{x}_{disp,a}| = 10^{-3}|\tilde{\phi}_i| \tag{2.5}$$

In the particular case of BS, since the laser incidents to BS at 45 degree, the displacement motion from rotational angle of mirror goes to $10^{-3}/\sqrt{2}$. Then the displacement of BS $x_{BS,disp,i}$ and RM $x_{RS,disp,i}$ are given by

$$|\tilde{x}_{BS,disp,i}| = \frac{10^{-3}}{\sqrt{2}}|\tilde{\phi}_{BS,i}| \quad [\mathrm{m}/\sqrt{\mathrm{Hz}}], \tag{2.6}$$

$$|\tilde{x}_{RM,disp,i}| = 10^{-3}|\tilde{\phi}_{RM,i}| \quad [\mathrm{m}/\sqrt{\mathrm{Hz}}]. \tag{2.7}$$

3 THE PARAMETER OF SUSPENSION SYSTEM

In this section, we summarize the necessary parameters which is planned to the mirror, the magnet and the suspension system of KAGRA. The planned parameters of the configuration

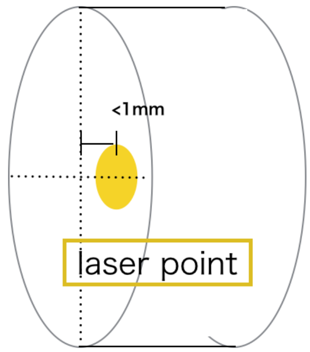

Figure 2.3: The spot where the laser incidents to the mirror

around mirror are written in these articles [1, 2, 3]. The table of parameters are shown at Table.3.1.

	BS	RM
the mass of mirror M [kg]	19.8	10.7
the radius of mirror a [m]	0.185	0.125
the thickness of mirror l [m]	0.08	0.1
the magnetic moment of magnet $\frac{q_m l_m}{\mu_0}$ [J/T]	0.074	0.074
the Q-value of suspension system	2.5×10^5	2.5×10^5
the number of magnet	4	4

Table 3.1: The parameters of suspension system.

4 SIMULATION OF MAGNETIC NOISE

In this section, we conduct the simulation of evaluating the magnetic noise of BS and RM and compare the result of simulation with KAGRA requirement. KAGRA requirement of the displacement of mirror is given by [4].

In this simulation, geomagnetism field data B is used by the direct measurement data where is taken in kamioka mine, VI room of Y-arm in 2013. The fourier transformation of this data is shown as Fig. 4.1. Note that more than 10Hz of geomagnetism field is upper-limit of itself because of the limit of measurement device and the line noise at 60Hz is EM noise from a power source.

The simulation of transfer function from external force to the rotational angle of mirror is calculated by Takanori Sekiguchi (it correspond to $\tilde{H}(f)$ of Eq.(2.4) in this article). Any other parameter which is necessary to evaluate the magnetic noise is given by Table. 3.1. The results of the simulation of magnetic noise for BS, power recycling mirror(PRM) and signal recycling mirror(SRM) with KAGRA requirement of these mirrors are plotted as Fig. 4.2 to 4.4.

In this result, the magnetic noise of BS and PRM fulfill the requirement of KAGRA. On the

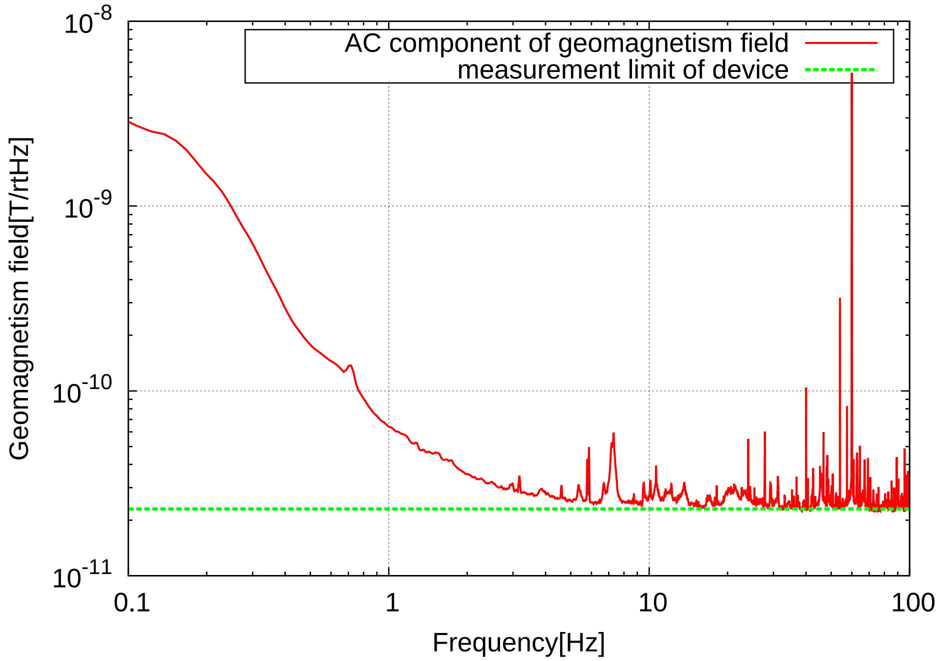

Figure 4.1: The AC component of geomagnetism field in kamioka mine.

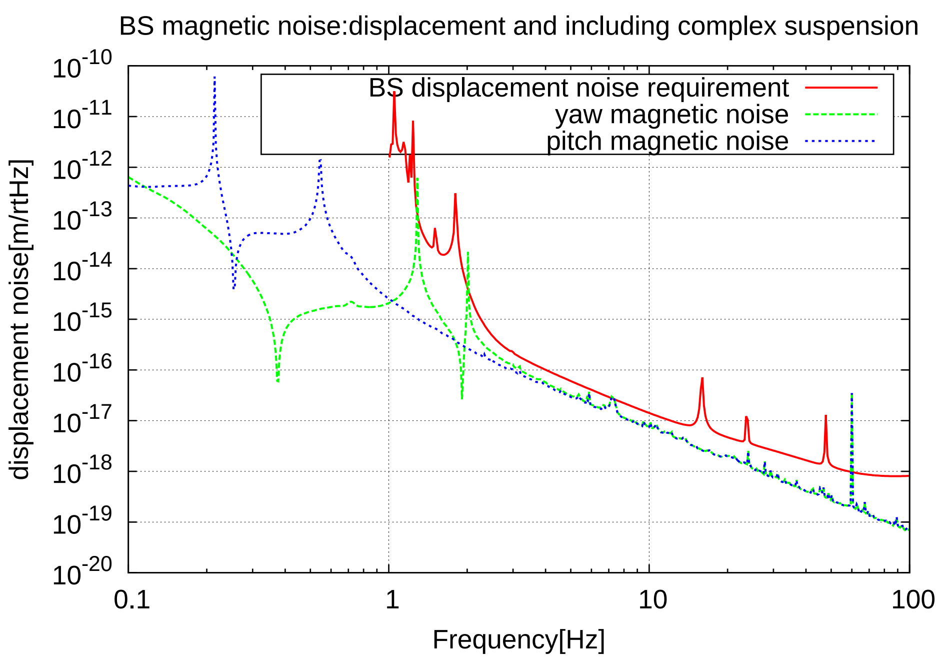

Figure 4.2: The result of magnetic noise simulation and KAGRA requirement of BS.

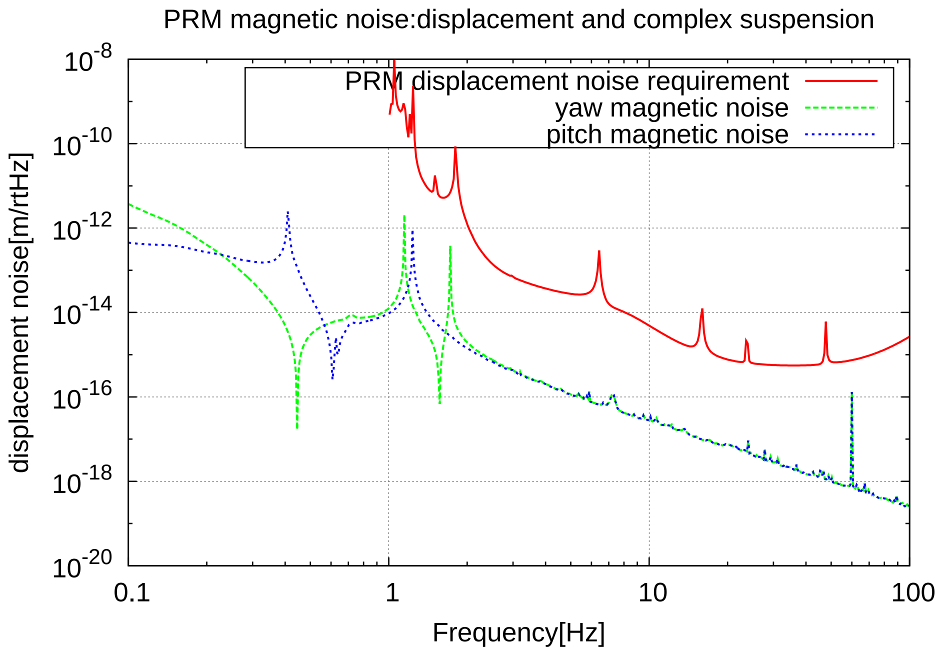

Figure 4.3: The result of magnetic noise simulation and KAGRA requirement of PRM.

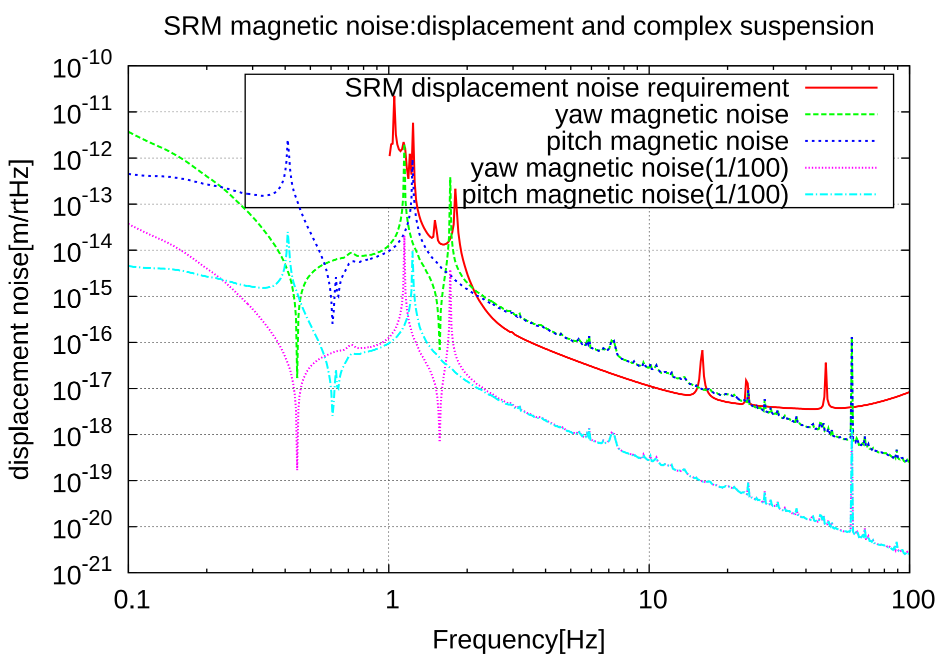

Figure 4.4: The result of magnetic noise simulation and KAGRA requirement of SRM.

other hands, the magnetic noise of SRM exceeds KAGRA requirement around 10Hz. However, the magnetic noise of SRM could be less than requirement under more realistic condition which the transfer function is 100 times as small as upper-limit of SRM.

5 CONCLUSION

We evaluate the upper-limit of magnetic noise of BS and RM. The condition of suspension system is summarized as below itemizes;

- The direction of geomagnetism field orthogonal to the side of magnet.
- 4 suspension wires and 4 magnets are attached to mirror.
- all direction of the magnetic pole of magnet is same.
- laser incidents at 1mm accuracy from the center of mirror.

In the simulation of magnetic noise, the transfer function from external force to rotational angle of mirror is calculated by Takanori Sekiguchi and geomagnetism field data is used by the measurement data at kamioka mine in 2013. The result of the simulation shows that the BS and PRM fulfill the KAGRA requirement and the SRM can archive the KAGRA requirement under more realistic condition.

References

[1] http://gwdoc.icrr.u-tokyo.ac.jp/DocDB/0005/T1100571/002/Recycler%20suspension%20structure.pdf.

[2] http://gwwiki.icrr.u-tokyo.ac.jp/JGWwiki/LCGT/subgroup/ifo/MIF/OptParam.

[3] http://gwdoc.icrr.u-tokyo.ac.jp/cgi-bin/DocDB/ShowDocument?docid=3239.

[4] https://granite.phys.s.u-tokyo.ac.jp/svn/LCGT/trunk/mif/doc/DesignDocument/data/DisplacementNoise/BRSE/DisplacementNoiseRequirement.dat.