Thermal design in vacuum

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In this report, board design of electronic circuit by taking into account thermal transfer are described.

In the air, we can assume that natural convection is a dominant heat transfer process. In this case, recommended pattern width w is given as a function of current rating I as follow.

$$w = 1 \text{ mm } / \text{Ampare} \times I \tag{1}$$

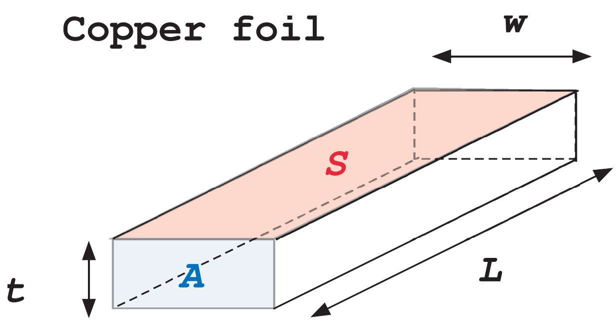

Figure 1: Copper foil electric line

1 Heating by electorical resistance

Copper has specific electric resistance of

$$\rho = 0.0172 \ \Omega \ \cdot (\mathrm{mm}^2) \ /\mathrm{m}$$
$$= 1.72 \times 10^{-8} \ \Omega \cdot \mathrm{m} \quad (2)$$

The electric resistance shown in Fig. 1 is represented as follows.

$$R = \rho \times \frac{L}{A} \quad (3)$$

Here L and A are path length and cross-section of the electric circuit line, respectively.

When a current I path through the line, a heating power of W is expected as follows.

$$W = V \cdot I = R \cdot I^2$$
$$= \frac{\rho\, L}{A}\, I^2 \ = \frac{\rho\, L}{t\, w}\, I^2 \quad (4)$$

Most of printed and flexible circuits use copper foil of 35 μm thickness.

$$t = 35 \ \mu\mathrm{m} \quad (5)$$

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[Example]
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In the case of 1 mm line width,

$$R = 0.0172 \ /0.035 \ /1 \times L = 0.49 \times L \ \ [\Omega]$$

2 Allowable temperature rise

Most of silicon transister and photo-diode has maximum operational temperature of 125 °C in their spec. sheet. Roughly speacking, temperature difference from room temperature of 25 °C is 100 °C = 100 Kelvin. When a safety factor of 2 is applied, allowable temperature rise is

$$\Delta T \leq 50 \tag{6}$$

3 Thermal resistance

Thermal resistance is defined as follows.

$$\Delta T = R_{\mathrm{T}} \cdot W \tag{7}$$

ΔT	:	temperature rise	K
R_{T}	:	thermal resistance	K /W
W	:	heating power	W

There are three ways for heat transfer. The thermal resistances of each processes are described below.

3.1 Convection

$$R_{\mathrm{conv}} = \frac{1}{h_c \cdot S} = \frac{1}{h_c\, w\, L} \tag{8}$$

Here h_c is a If convection is a dominant process, tenperature rise is expected as follows.

$$\begin{aligned} \Delta T &= R_{\mathrm{conv}} \cdot W \\ &= \frac{1}{h_c\, w\, L}\, \frac{\rho\, L}{t\, w}\, I^2 \\ &= \frac{\rho}{h_c\, t} \left(\frac{I}{w}\right)^2 \end{aligned} \tag{9}$$

$$\frac{w}{I} = \sqrt{\frac{\rho}{\Delta T\, h_c\, t}} \tag{10}$$

$$h_c = \begin{cases} 5 - 25 & \mathrm{W/m^2/K} \quad \text{(natural convection)} \\ 20 - 300 & \mathrm{W/m^2/K} \quad \text{(forced convection)} \end{cases} \tag{11}$$

Typical h_c of 10 is used for rough estimation.

$$\frac{w}{I} = \sqrt{\frac{1.72 \cdot 10^{-8}}{50 \times 10 \times 35 \cdot 10^{-6}}} \sim 10^{-3} \tag{12}$$

$$\therefore \quad w = \ 1 \text{ mm /Ampare } \times I \tag{13}$$

3.2 Conduction

$$R_{\text{cond}} = \frac{L}{\lambda \cdot A} \tag{14}$$

$$\lambda = \begin{cases} 400 \quad \text{W/m/K} \quad & \text{for copper} \\ 240 \quad \text{W/m/K} \quad & \text{for aluminum} \end{cases} \tag{15}$$

Here L and A are path-length and cross-section of the coductor, respectively.

In the case of pattern thickness of 35 μm, width of 1 mm and its length of L,

$$R_{\text{cond}} = \frac{L}{400 \times \text{35e-6} \times \text{1e-3}} \;=\; 7 \cdot 10^4 \times L \tag{16}$$

3.3 Radiation

熱平衡温度の計算

$$W = \sigma\, \varepsilon\, F\, (T_1^4 - T_2^4)\; S$$

$$\begin{aligned} \sigma &: \ \text{Stefan-Boltzmann constant} \\ \varepsilon &: \ \text{Emissivity} \\ F &: \ \text{View (form) factor} \end{aligned}$$

$$\frac{\rho\, L}{t\, w}\, I^2 = \sigma\, \varepsilon\, F\, (T_1^4 - T_2^4)\; w\, L$$

$$\frac{w}{I} = \sqrt{\frac{1}{\varepsilon F}\, \frac{\rho}{t\, \sigma\, (T_1^4 - T_2^4)}} \qquad (17)$$

In radiative heat transfer, a view factor, $F_{A \to B}$, is the proportion of the radiation which leaves surface A that strikes surface B

$$\begin{aligned} \sigma &= 5.67 \times 10^{-8}\ \ \text{W}\ /\text{m}^2\ /\text{K}^4 \\ T_2 &= 25^\circ\text{C} = 298.16\ \text{K}\ \simeq 300\ \text{K} \\ T_1 &= 300 + 50 = 350\ \text{K} \end{aligned}$$

$$\begin{aligned} \frac{w}{I} &= \sqrt{\frac{1}{\varepsilon F} \times \frac{1.72 \cdot 10^{-8}}{35 \cdot 10^{-6} \times 5.67 \cdot 10^{-8} \times (350^4 - 300^4)}} \\ &= \sqrt{\frac{1}{\varepsilon F} \times 1.25 \times 10^{-6}} \\ &\geq 1.12 \times 10^{-3} = 1.18\ \text{mm} \end{aligned} \qquad (18)$$

$$\begin{aligned} \varepsilon &: \ \text{emissivity of the copper foil} \\ F &: \ \text{view (form) factor} \end{aligned}$$

4 Case study: in air

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_{\text{conv}}} + \frac{1}{R_{\text{cond}}} + \frac{1}{R_{\text{rad}}} \tag{19}$$

Copper foil of 35 μm in thickness, of 1 mm in width and L in langth has the total thermal resistance

$$\frac{1}{R_{\text{total}}} = \frac{L}{100} + \frac{1}{7 \cdot 10^4 \times L} + \frac{L}{1.3 \cdot 10^4} \tag{20}$$

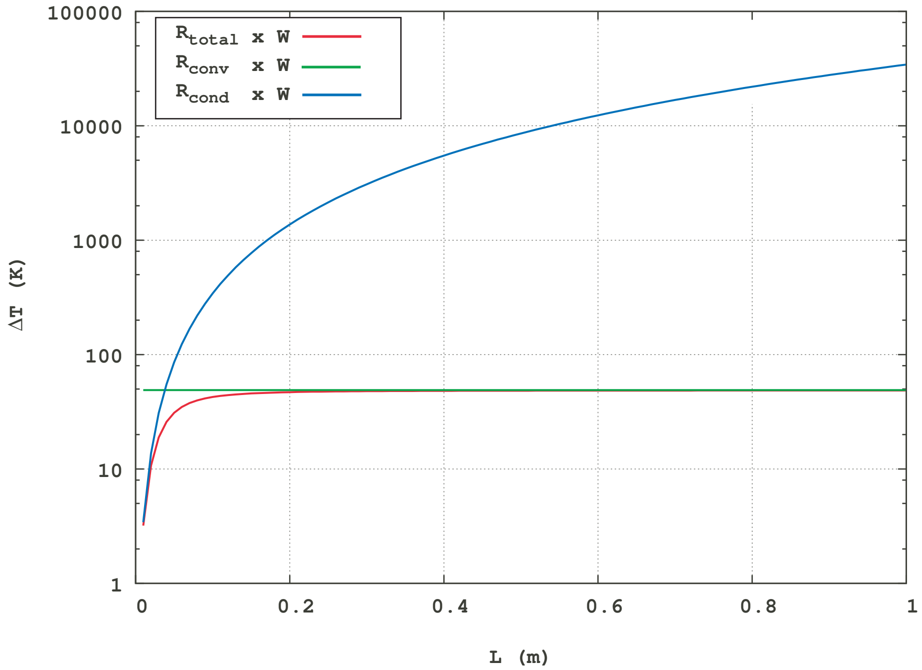

Figure 2: