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OSEM Coil/Magnet/Flag Calculation

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JGW-DCC

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of the KAGRA collaboration.

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Table of Contents

1 Introduction

1.1 Purpose and Scope

Gives the calculation of the position of the magnet in the field of the OSEM coil to give maximum force/current and minimum cross-coupling from position of the coil to force.

1.2 References

LIGO-T1000164: Calculation and measurement of the OSEM actuator sweet spot position

1.3 Version history

1/15/2015: Pre-rev-v1 draft. Based on LIGO-T1000164, but with updates for KAGRA.

2/2/2015: -v1. Calculations for optic/RM and IM/IRM OSEM-magnet combos.

1/21/2016: -v2. New combos for IM flag with double magnet and new small BS magnet.

2/16/2016: -v3. Minor fix to double-magnet calculation.

2/25/2016: -v4. Fix to small BS magnet diameter. Add SR calculation.

2 Theory

The "sweet spot" for the magnet is the position relative to the coil for which the force is maximum and the variation in the force for small excursions of the coil is zero. For a single turn coil and a point dipole magnet, the sweet spot is one radius from the center. For a coil with many turns and/or a finite-sized magnet, it is necessary to integrate over the volume of each.

The theory for the force on a current line element in a magnetic field is derived in the Mathematica notebook MagDipole.nb accompanying this document in the DCC. Note that this notebook and the extracts from it below had to be updated from the version included with LIGO-T1000164-v3 to allow for changes introduced in Mathematica 9. Formerly, the Mathematica vector analysis functions were in a separate package, `Calculus`VectorAnalysis`, and assumed a default set of Cartesian coordinates

```
Coordinates[]
```

```
{Xx, Yy, Zz}
```

From Mathematica 9, the functions were added to the Mathematica kernel and the calling convention for `Grad[]`, `Div[]` and `Curl[]` was changed to require the coordinates to be supplied explicitly.

Briefly, if the magnitude and coordinates of a current element within the coil are

```
sourcecurrent = {j1x, j1y, j1z};
sourcepos = {dx, dy, dz};
```

and the coordinates of an arbitrary test point are

```
coordinates = {Xx, Yy, Zz};
```

then the distance between them is

```
sourcefieldvec = coordinates - sourcepos
rsf = Sqrt[DotProduct[sourcefieldvec,sourcefieldvec]]
```

and the magnetic vector potential from the line element is

```
currentA = mu0/(4 Pi) sourcecurrent/rsf
```

$$\left\{\frac{\text{j1x mu0}}{4\pi\sqrt{(-\text{dx}+\text{Xx})^2+(-\text{dy}+\text{Yy})^2+(-\text{dz}+\text{Zz})^2}},\ \frac{\text{j1y mu0}}{4\pi\sqrt{(-\text{dx}+\text{Xx})^2+(-\text{dy}+\text{Yy})^2+(-\text{dz}+\text{Zz})^2}},\ \frac{\text{j1z mu0}}{4\pi\sqrt{(-\text{dx}+\text{Xx})^2+(-\text{dy}+\text{Yy})^2+(-\text{dz}+\text{Zz})^2}}\right\}$$

giving a field of

```
currentB = mu0/(4 Pi) Curl[sourcecurrent/rsf, coordinates]
```

$$\left\{\frac{\text{mu0}\left(\frac{\text{j1z}\,(\text{dy}-\text{Yy})}{\left((\text{dx}-\text{Xx})^2+(\text{dy}-\text{Yy})^2+(\text{dz}-\text{Zz})^2\right)^{3/2}}-\frac{\text{j1y}\,(\text{dz}-\text{Zz})}{\left((\text{dx}-\text{Xx})^2+(\text{dy}-\text{Yy})^2+(\text{dz}-\text{Zz})^2\right)^{3/2}}\right)}{4\pi},\right.$$

$$\frac{\text{mu0}\left(-\frac{\text{j1z}\,(\text{dx}-\text{Xx})}{\left((\text{dx}-\text{Xx})^2+(\text{dy}-\text{Yy})^2+(\text{dz}-\text{Zz})^2\right)^{3/2}}+\frac{\text{j1x}\,(\text{dz}-\text{Zz})}{\left((\text{dx}-\text{Xx})^2+(\text{dy}-\text{Yy})^2+(\text{dz}-\text{Zz})^2\right)^{3/2}}\right)}{4\pi},$$

$$\left.\frac{\text{mu0}\left(\frac{\text{j1y}\,(\text{dx}-\text{Xx})}{\left((\text{dx}-\text{Xx})^2+(\text{dy}-\text{Yy})^2+(\text{dz}-\text{Zz})^2\right)^{3/2}}-\frac{\text{j1x}\,(\text{dy}-\text{Yy})}{\left((\text{dx}-\text{Xx})^2+(\text{dy}-\text{Yy})^2+(\text{dz}-\text{Zz})^2\right)^{3/2}}\right)}{4\pi}\right\}$$

The field gradient is

```
currentgradB = Grad[currentB, coordinates]
```

$$\left\{\left\{\frac{\mathrm{mu0}\left(\frac{3\,\mathrm{j1z}\,(\mathrm{dx}-\mathrm{Xx})\,(\mathrm{dy}-\mathrm{Yy})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}-\frac{3\,\mathrm{j1y}\,(\mathrm{dx}-\mathrm{Xx})\,(\mathrm{dz}-\mathrm{Zz})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}\right)}{4\,\pi},\right.\right.$$

$$\frac{1}{4\,\pi}\mathrm{mu0}\left(\frac{3\,\mathrm{j1z}\,(\mathrm{dy}-\mathrm{Yy})^2}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}-\frac{\mathrm{j1z}}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{3/2}}-\frac{3\,\mathrm{j1y}\,(\mathrm{dy}-\mathrm{Yy})\,(\mathrm{dz}-\mathrm{Zz})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}\right),$$

$$\left.\frac{1}{4\,\pi}\mathrm{mu0}\left(\frac{\mathrm{j1y}}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{3/2}}+\frac{3\,\mathrm{j1z}\,(\mathrm{dy}-\mathrm{Yy})\,(\mathrm{dz}-\mathrm{Zz})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}-\frac{3\,\mathrm{j1y}\,(\mathrm{dz}-\mathrm{Zz})^2}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}\right)\right\},$$

$$\left\{\frac{1}{4\,\pi}\mathrm{mu0}\left(-\frac{3\,\mathrm{j1z}\,(\mathrm{dx}-\mathrm{Xx})^2}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}+\frac{\mathrm{j1z}}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{3/2}}+\frac{3\,\mathrm{j1x}\,(\mathrm{dx}-\mathrm{Xx})\,(\mathrm{dz}-\mathrm{Zz})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}\right),\right.$$

$$\frac{\mathrm{mu0}\left(-\frac{3\,\mathrm{j1z}\,(\mathrm{dx}-\mathrm{Xx})\,(\mathrm{dy}-\mathrm{Yy})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}+\frac{3\,\mathrm{j1x}\,(\mathrm{dy}-\mathrm{Yy})\,(\mathrm{dz}-\mathrm{Zz})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}\right)}{4\,\pi},$$

$$\left.\frac{1}{4\,\pi}\mathrm{mu0}\left(-\frac{\mathrm{j1x}}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{3/2}}-\frac{3\,\mathrm{j1z}\,(\mathrm{dx}-\mathrm{Xx})\,(\mathrm{dz}-\mathrm{Zz})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}+\frac{3\,\mathrm{j1x}\,(\mathrm{dz}-\mathrm{Zz})^2}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}\right)\right\},$$

$$\left\{\frac{1}{4\,\pi}\mathrm{mu0}\left(\frac{3\,\mathrm{j1y}\,(\mathrm{dx}-\mathrm{Xx})^2}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}-\right.\right.$$

The potential of a dipole element `{m2x,m2y,m2z}` in the field is

```
currentdipolepot = currentB.{m2x,m2y,m2z}
```

$$\frac{m2z\ mu0\left(\frac{j1y\ (dx-Xx)}{\left((dx-Xx)^2+(dy-Yy)^2+(dz-Zz)^2\right)^{3/2}}-\frac{j1x\ (dy-Yy)}{\left((dx-Xx)^2+(dy-Yy)^2+(dz-Zz)^2\right)^{3/2}}\right)}{4\pi}+$$

$$\frac{m2y\ mu0\left(-\frac{j1z\ (dx-Xx)}{\left((dx-Xx)^2+(dy-Yy)^2+(dz-Zz)^2\right)^{3/2}}+\frac{j1x\ (dz-Zz)}{\left((dx-Xx)^2+(dy-Yy)^2+(dz-Zz)^2\right)^{3/2}}\right)}{4\pi}+$$

$$\frac{m2x\ mu0\left(\frac{j1z\ (dy-Yy)}{\left((dx-Xx)^2+(dy-Yy)^2+(dz-Zz)^2\right)^{3/2}}-\frac{j1y\ (dz-Zz)}{\left((dx-Xx)^2+(dy-Yy)^2+(dz-Zz)^2\right)^{3/2}}\right)}{4\pi}$$

and the force on it is

```
currentdipoleforce = -Grad[currentB.{m2x,m2y,m2z}, coordinates]
```

$$
\begin{aligned}
\Bigg\{ & -\frac{1}{4\pi}\,\mathrm{m2z}\,\mathrm{mu0}\left(\frac{3\,\mathrm{j1y}\,(\mathrm{dx}-\mathrm{Xx})^2}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}-\frac{3\,\mathrm{j1x}\,(\mathrm{dx}-\mathrm{Xx})\,(\mathrm{dy}-\mathrm{Yy})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}-\frac{\mathrm{j1y}}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{3/2}}\right)-\\
& \frac{1}{4\pi}\,\mathrm{m2y}\,\mathrm{mu0}\left(-\frac{3\,\mathrm{j1z}\,(\mathrm{dx}-\mathrm{Xx})^2}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}+\frac{\mathrm{j1z}}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{3/2}}+\frac{3\,\mathrm{j1x}\,(\mathrm{dx}-\mathrm{Xx})\,(\mathrm{dz}-\mathrm{Zz})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}\right)-\\
& \frac{\mathrm{m2x}\,\mathrm{mu0}\left(\frac{3\,\mathrm{j1z}\,(\mathrm{dx}-\mathrm{Xx})\,(\mathrm{dy}-\mathrm{Yy})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}-\frac{3\,\mathrm{j1y}\,(\mathrm{dx}-\mathrm{Xx})\,(\mathrm{dz}-\mathrm{Zz})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}\right)}{4\pi},\\
& -\frac{1}{4\pi}\,\mathrm{m2z}\,\mathrm{mu0}\left(\frac{3\,\mathrm{j1y}\,(\mathrm{dx}-\mathrm{Xx})\,(\mathrm{dy}-\mathrm{Yy})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}-\frac{3\,\mathrm{j1x}\,(\mathrm{dy}-\mathrm{Yy})^2}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}+\frac{\mathrm{j1x}}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{3/2}}\right)-\\
& \frac{\mathrm{m2y}\,\mathrm{mu0}\left(-\frac{3\,\mathrm{j1z}\,(\mathrm{dx}-\mathrm{Xx})\,(\mathrm{dy}-\mathrm{Yy})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}+\frac{3\,\mathrm{j1x}\,(\mathrm{dy}-\mathrm{Yy})\,(\mathrm{dz}-\mathrm{Zz})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}\right)}{4\pi}-\\
& \frac{1}{4\pi}\,\mathrm{m2x}\,\mathrm{mu0}\left(\frac{3\,\mathrm{j1z}\,(\mathrm{dy}-\mathrm{Yy})^2}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}-\frac{\mathrm{j1z}}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{3/2}}-\frac{3\,\mathrm{j1y}\,(\mathrm{dy}-\mathrm{Yy})\,(\mathrm{dz}-\mathrm{Zz})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}\right),\\
& -\frac{\mathrm{m2z}\,\mathrm{mu0}\left(\frac{3\,\mathrm{j1y}\,(\mathrm{dx}-\mathrm{Xx})\,(\mathrm{dz}-\mathrm{Zz})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}-\frac{3\,\mathrm{j1x}\,(\mathrm{dy}-\mathrm{Yy})\,(\mathrm{dz}-\mathrm{Zz})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}\right)}{4\pi}-\\
& \frac{1}{4\pi}\,\mathrm{m2y}\,\mathrm{mu0}\left(-\frac{\mathrm{j1x}}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{3/2}}-\frac{3\,\mathrm{j1z}\,(\mathrm{dx}-\mathrm{Xx})\,(\mathrm{dz}-\mathrm{Zz})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}+\frac{3\,\mathrm{j1x}\,(\mathrm{dz}-\mathrm{Zz})^2}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}\right)-\\
& \frac{1}{4\pi}\,\mathrm{m2x}\,\mathrm{mu0}\left(\frac{\mathrm{j1y}}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{3/2}}+\frac{3\,\mathrm{j1z}\,(\mathrm{dy}-\mathrm{Yy})\,(\mathrm{dz}-\mathrm{Zz})}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}-\frac{3\,\mathrm{j1y}\,(\mathrm{dz}-\mathrm{Zz})^2}{\left((\mathrm{dx}-\mathrm{Xx})^2+(\mathrm{dy}-\mathrm{Yy})^2+(\mathrm{dz}-\mathrm{Zz})^2\right)^{5/2}}\right)\Bigg\}
\end{aligned}
$$

Because of cylindrical symmetry it is convenient to transform to cylindrical coordinates `{r1, theta1, z1}` about the centre of the coil and `{r2, theta2, z2}` about the centre of the magnet:

```
fdr1dr2dtheta1dtheta2dz1dz2 = (
   -r1*r2*currentdipoleforce
   /. dx -> r1*Cos[theta1]
   /. dy -> r1*Sin[theta1]
   /. dz -> z1
   /. j1x -> coilsigma*Sin[theta1]
   /. j1y -> -coilsigma*Cos[theta1]
   /. j1z -> 0
   /. Xx -> r2*Cos[theta2]
   /.  Yy -> r2*Sin[theta2]
   /. Zz -> z2
   /. m2x -> 0
   /. m2y -> 0
   /. m2z -> mz
);
```

where `coilsigma` is the current density per unit area in the coil and `mz` is the magnetic moment per unit volume in the magnet.

Effectively, the above integrand is integrated over all six variables as follows:

```
force[z_] := Integrate[
  fdr1dr2dtheta1dtheta2dz1dz2,
  {z1, -coillen/2, coillen/2},
  {z2, z-l/2, z+l/2},
  {theta1, 0, 2Pi},
  {theta2, 0, 2Pi},
  {r1, coilrad1, coilrad2},
  {r2, 0, a}
 ]
```

where `coillen`, `coilrad1` and `coilrad2` are the coil length and inner and outer radii, `l` and `a` are the magnet length and radius, and `z` is the distance from the centre of the coil.

In practice, of the 6 integrations required, only `z1` and `z2` can be done analytically, or at least *could* in older versions of Mathematica. Newer versions of Mathematica seem to have gotten dumber but fortunately, because the results took a long time to compute from scratch, they were were archived and so are still available. See SweetSpot.nb for the expressions, which are too long to reproduce here.

The integrals over `theta1` and `theta2` can be combined by applying the transformation **`theta1 - theta2 -> deltatheta`**, and multiplying by `2*Pi`. The three remaining integrals, `deltatheta`, `r1` and `r2` can then be done numerically in a few seconds.

The coupling of small displacements of the coil to force on the magnet can be calculated from the second derivative of the force. In –v1 of this document, as well as versions up through –v3 of the predecessor document, LIGO-T1000164, this was reported as a value for `coupling` in the tables, but the values were complete nonsense due to two compounding errors:

- The first (rather than second) derivative had been calculated.
- The Mathematica numerical derivative function `ND[]` turns out to be very unreliable for some functions including the force as calculated above, and the value was not zero (as

expected for a first derivative at a maximum), but non-zero and by coincidence in the ball-park of what the second derivative should have been.

After trying a bunch of different approaches to get a reliable second derivative value, the method settled on for this version uses the Mathematica `FunctionInterpolation[]` function to create a fitted approximation to the force function calculated numerically as above, followed by `Series[]` to extract a power series, `Normal[]` to convert it to a regular expression and `Coefficient[]` to extract the square term:

```
deriv2[KPR] = -2*
  Coefficient[Normal[Series[fzi[KPR][z], {z, zmax[KPR], 2}]], z^2]
```

The final calculation method was checked by plotting the second derivative approximation together with the full curve as in Figure 1.

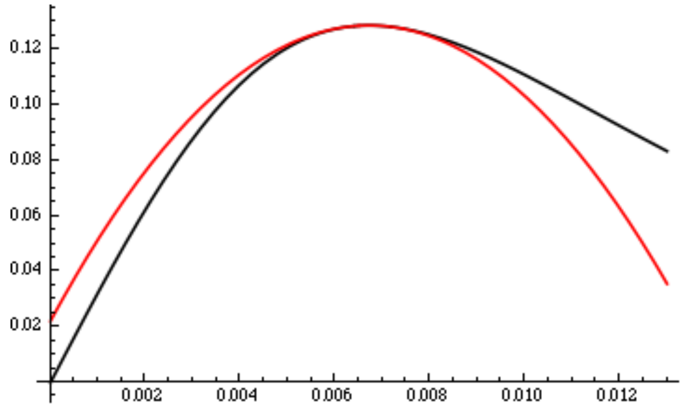


Figure 1: Force as a function of displacement (for original design of optic flag and magnet) in black against an approximation using second derivative (red)A successful

3 Parameters/Results

The parameters of the OSEM coil are given in Promec drawings

`10212-Gr.12-body BOSEM intermediate.pdf`

and

`10203-Gr.12-Osem intermediate mass.pdf`

(Akutsu-san has redesigned many of the details of the OSEM body relative to these drawings, but the coil is unchanged.) The coil outer radius can be calculated from the inner radius (9 mm) plus the wire diameter (0.36 mm) and the number of layers (27). In fact the 600 turns do not quite fill the channel allowed for the coil.

The original optic flag design for PRx was detailed in `10304-Gr.13-Varia details.pdf` and used a 3 mm long, 6 mm diameter magnet.

The original IM flag design was detailed in 10207-Gr.12-Varia details.pdf and used a 10 mm long, 10 mm diameter magnet.

Fabian adjusted both of these designs to use the sweet-spot positions calculated in –v1 of these documents, and these were been used for the iKAGRA PR3 suspension.

More recently it has been decided to

- Make all the magnets Ni-plated SmCo to reduce Barkhausen noise.
- Reduce all the optic magnet sizes.

- Add extra magnets with opposite polarity to the IM flags so that the net magnetic moment of each flag is zero, and the coupling to external fields is reduced.
- Make the IM flags self-assembling by gluing ferromagnetic disks into the locating recesses for the magnets (instead of gluing the magnets in directly), and allowing the magnets to stick magnetically to the discs.

New magnet parameters (per Yuta Michimura):

PRs TM: SmCo 6 mm diameter, 3 mm thick

PRs IM: SmCo 10 mm diameter, 10 mm long

SRs TM: SmCo 2 mm diameter, 5 mm long

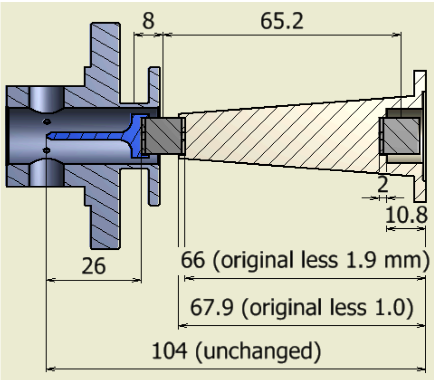
SRs IM: SmCo 10 mm diameter, 10 mm long

BS TM: SmCo 2 mm diameter, 3 mm long

BS IM: SmCo 10 mm diameter, 10 mm long

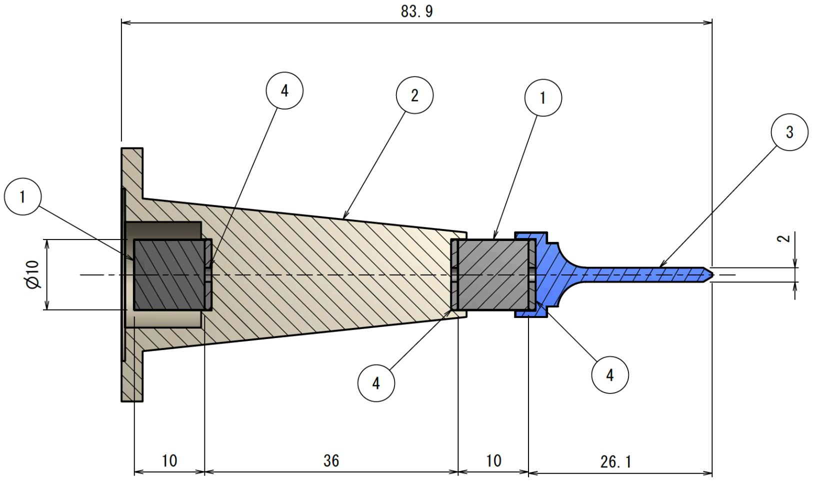
(Note that in –v3 of this document, the magnet size used for the BS optic flags was completely wrong, giving a strength high by a factor of ≈3.)

Because of the ease of breaking the long IM flags, a revised design was prepared for the BS using ferromagnetic disks that could be glued into the flag tip and base parts and be attracted by the magnet, so that the entire flag assembly would be held together magnetically.



To approximate this using the formalism of the magnetic force calculation, a dipole moment corresponding to the actual volume of the magnet material is assumed to be uniformly distributed over the volume of the magnet plus disks, which are each 1 mm thick.

A very similar version with a slightly reduced overall length of 83.9 mm will be used for the PR and SR IM:



Although the magnetic material is now SmCo rather than NdFeB, the difference in magnetic moment per unit volume is very small, and the calculation uses the same value that was measured for NdFeB by Mark Barton and used for all magnet calculations in LIGO. (See LIGO-T1000164.)

The parameters and results for these combinations are given in Table 1.

Table 1: Parameters and results.

Parameter	3xØ6 magnet for PR	5xØ2 magnet for SR	3xØ2 magnet for BS	Opposed 10xØ10 magnets for PR/SR, disk version	Opposed 10xØ10 magnets for BS, disk version	Description
`l (length)`	3 mm	5 mm	3 mm	10+1+1 mm	10+1+1 mm	length of magnet
`a (radius)`	3 mm	1 mm	1 mm	5 mm	5 mm	radius of magnet
`coillen`	8 mm	8 mm	8 mm	8 mm	8 mm	length of coil
`coilrad1`	9 mm	9 mm	9 mm	9 mm	9 mm	inner radius of coil
`coilrad2`	18.72 mm	18.72 mm	18.72 mm	18.72 mm	18.72 mm	outer radius of coil (inner radius plus 27 layers @ 0.36 mm)
`coilturns`	600	600	600	600	600	number of turns
`mz (SmCo; same as for NdFeB)`	8.78 10^5 A/m; 1.10 T	8.78 10^5 A/m; 1.10 T	8.78 10^5 A/m; 1.10 T	8.78 10^5 A/m; 1.10 T	8.78 10^5 A/m; 1.10 T	magnetic moment/volume
`dipole`	0.0745 A.m	0.0138 A.m	0.08828 A.m	0.690 A.m	0.690 A.m	net magnetic moment
`coilsigma`	7.72 10^6 A/m^2	7.72 10^6 A/m^2	7.72 10^6 A/m^2	7.72 10^6 A/m^2	7.72 10^6 A/m^2	coil current density
`fmax` (theory)	0.129 N/A	0.0225 N/A	0.0138 N/A	1.26 N/A	1.25 N/A	maximum force (theory)
`zmax` (theory)	6.72 mm	7.15 mm	6.97 mm	7.99 mm	8.01 mm	sweet spot (theory)
`z95minus/plus`	5.2/8.5 mm	5.5/9.0 mm	5.4/8.8 mm	6.2/10.0 mm	6.2/10.2 mm	95% force positions
`pairoffset`	-	-	-	65.2 mm	46.0 mm	magnet distance (between centers)
`deriv2` (theory)	4724 N/A/m^2	739 N/A/m^2	473 N/A/m^2	4724 N/A/m^2	4724 N/A/m^2	displacement-force cross-coupling

4 Conclusion

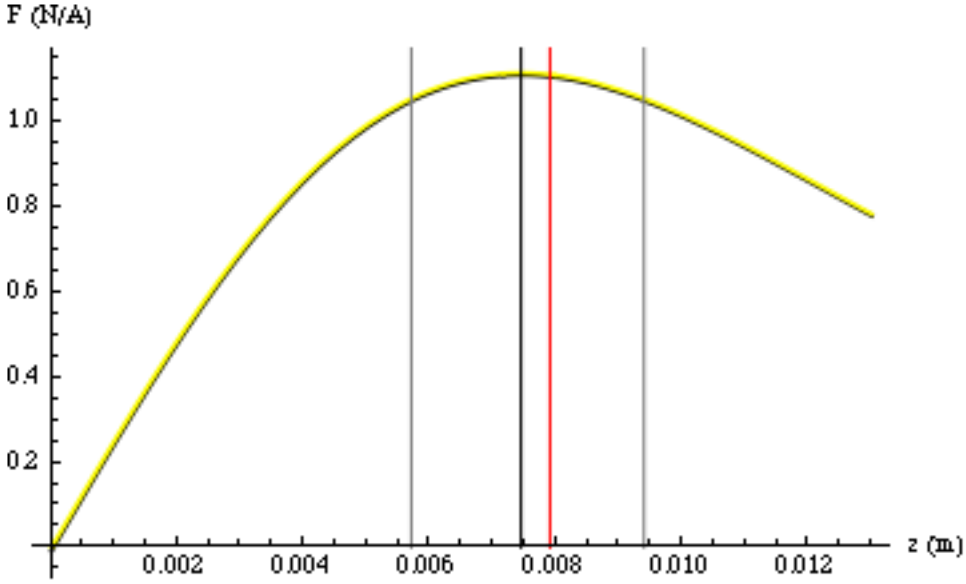


Figure 2: Strength as a function of distance between coil center and magnet center for IM flags with single (yellow) and double (black) magnets but no disks, position of maximum force in black and the original Promec design in red.

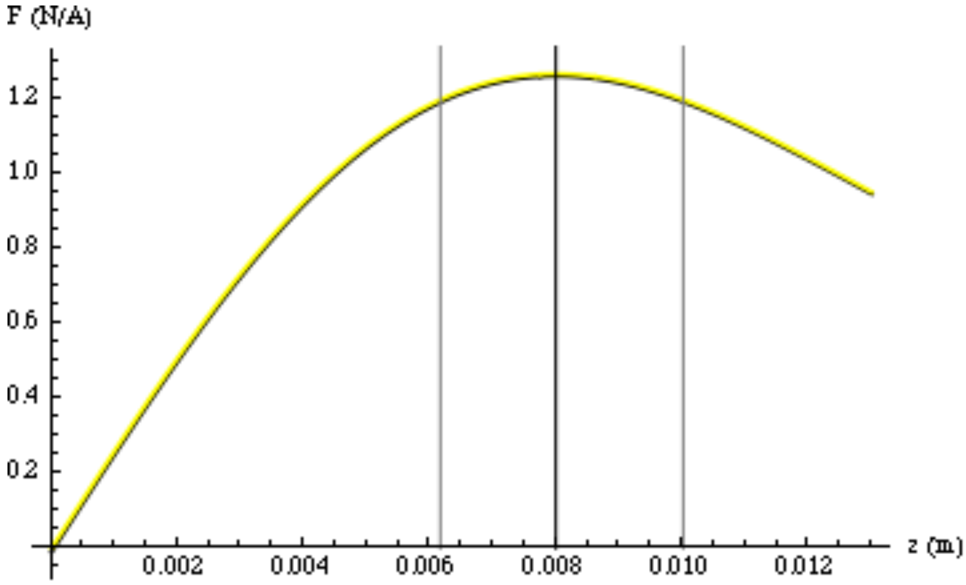


Figure 3: Strength as a function of distance between coil center and magnet center for IM flags with single (yellow) and double (black) magnets with disks. Note that although the disk near the coil is 1 mm thick it only produces an offset of 0.55 of the optimum center position.

All versions of the design have the sweet spot well within the zone where the force is at least 95% of the maximum.

As expected, adding a second magnet to the BS-IM flags makes very little difference to the sweet spot.