

光-マグノン-マイクロ波-超伝導量子ビットのリンク あるいは量子ピタゴラススイッチ

東大先端研
宇佐見康二

Quantum optics with collective excitations in solid

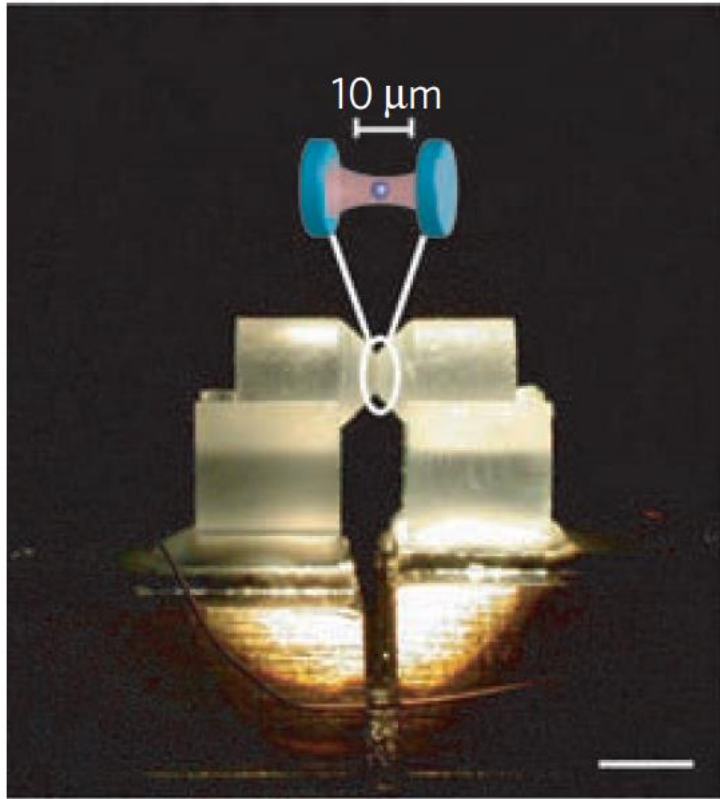
Koji Usami

RCAST, the University of Tokyo

Wonderful world of quantum optics

Cavity QED

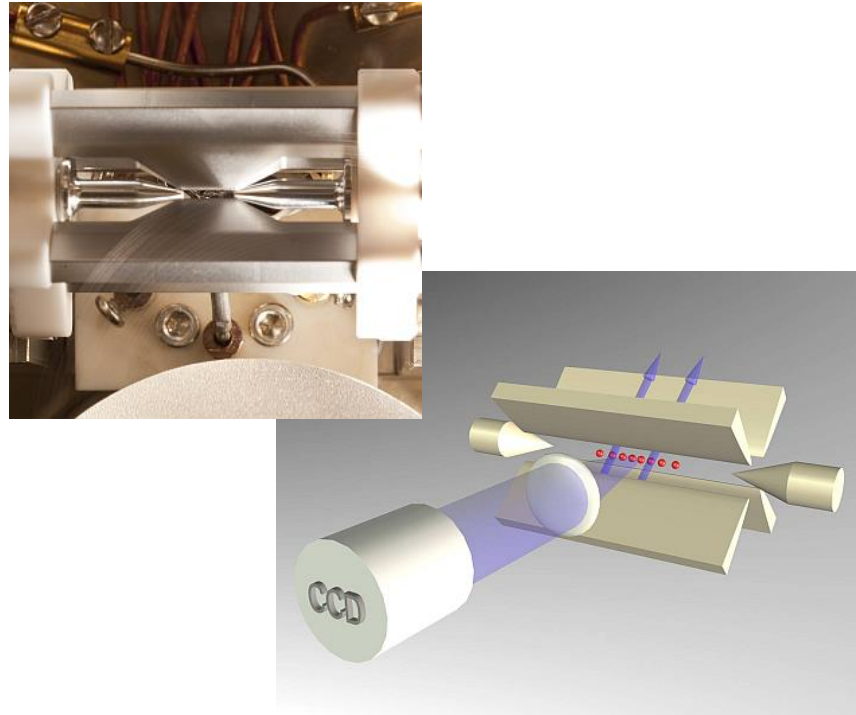
- Atom
- Photon



H.J. Kimble,
Nature **453**, 1023 (2008).

Ion Trap

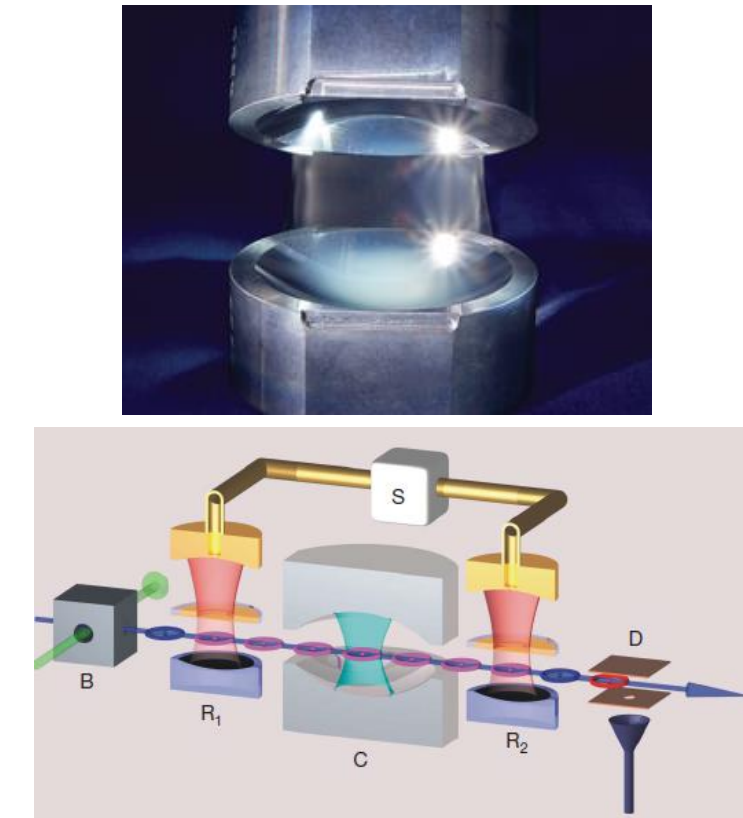
- Ion
- Phonon



F. Schmidt-Kaler, *et al.*,
Nature **422**, 408 (2003).

Cavity QED with Rydberg atom

- Rydberg atom
- Microwave photon

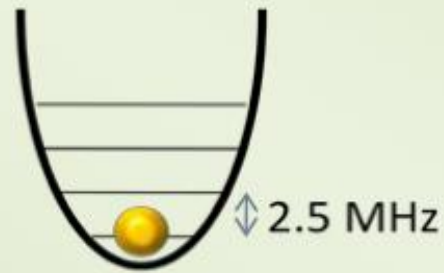


S. Gleyzes, *et al.*,
Nature **446**, 297 (2007)

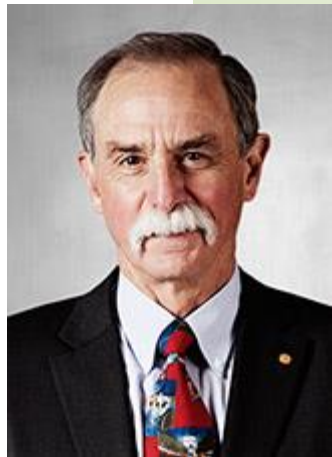
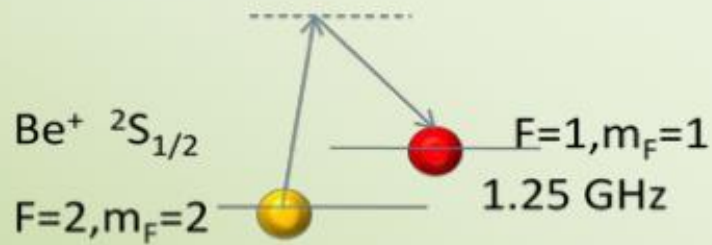
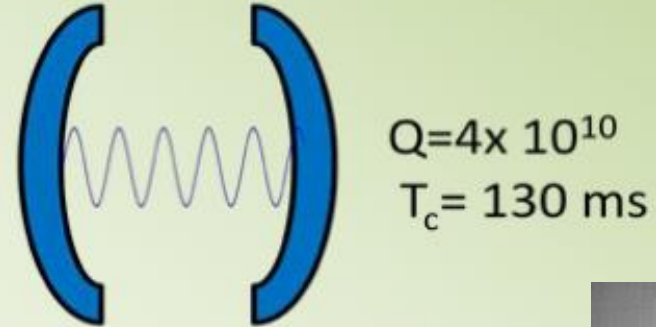
Nobel Prize in Physics 2012

Control of individual quantum systems

Ion in a trap



Photon in a cavity

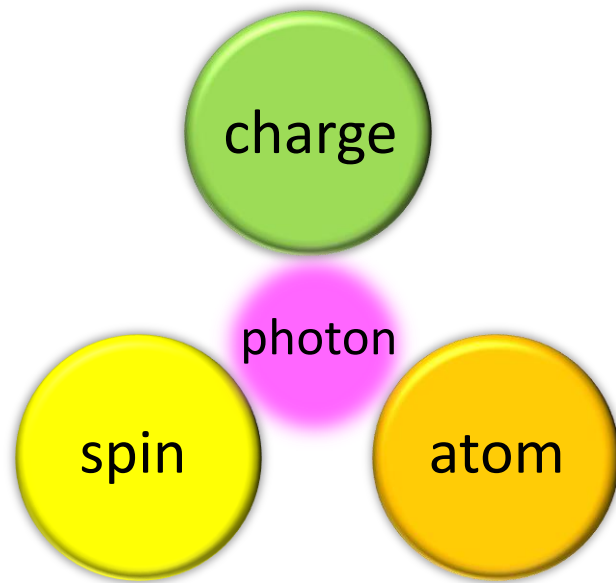


David J. Wineland

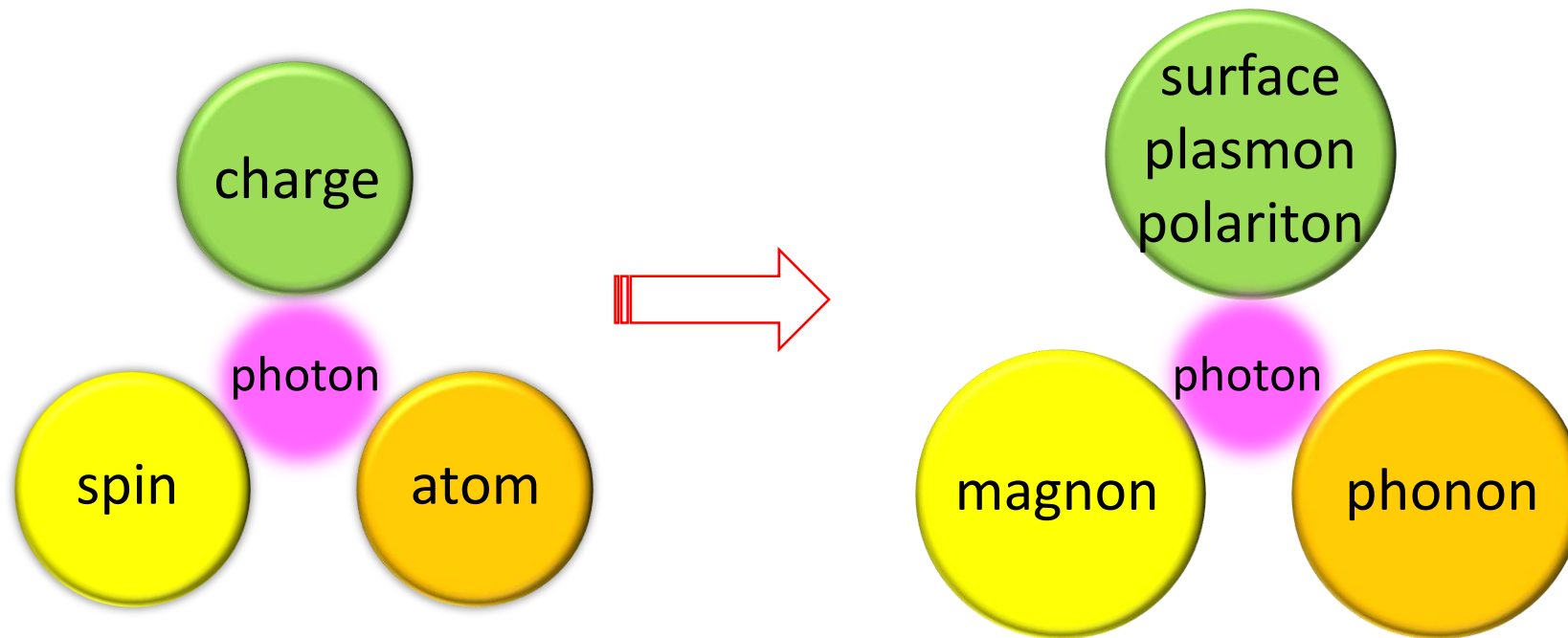


Serge Haroche

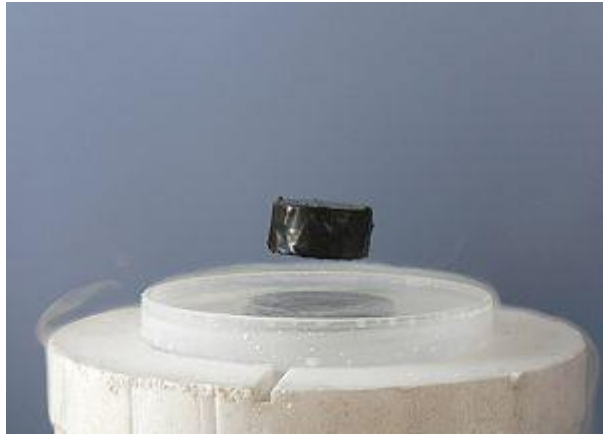
Individual quantum systems



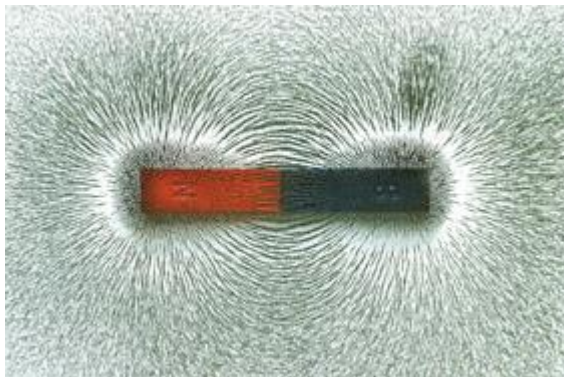
Collective excitations



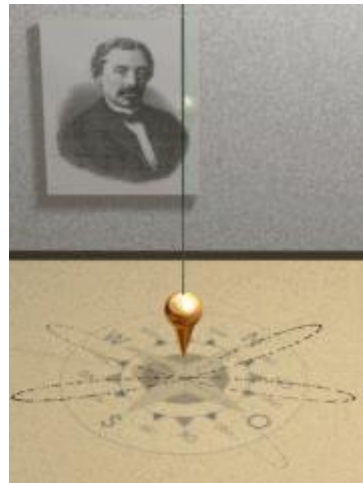
Macroscopic quantum systems



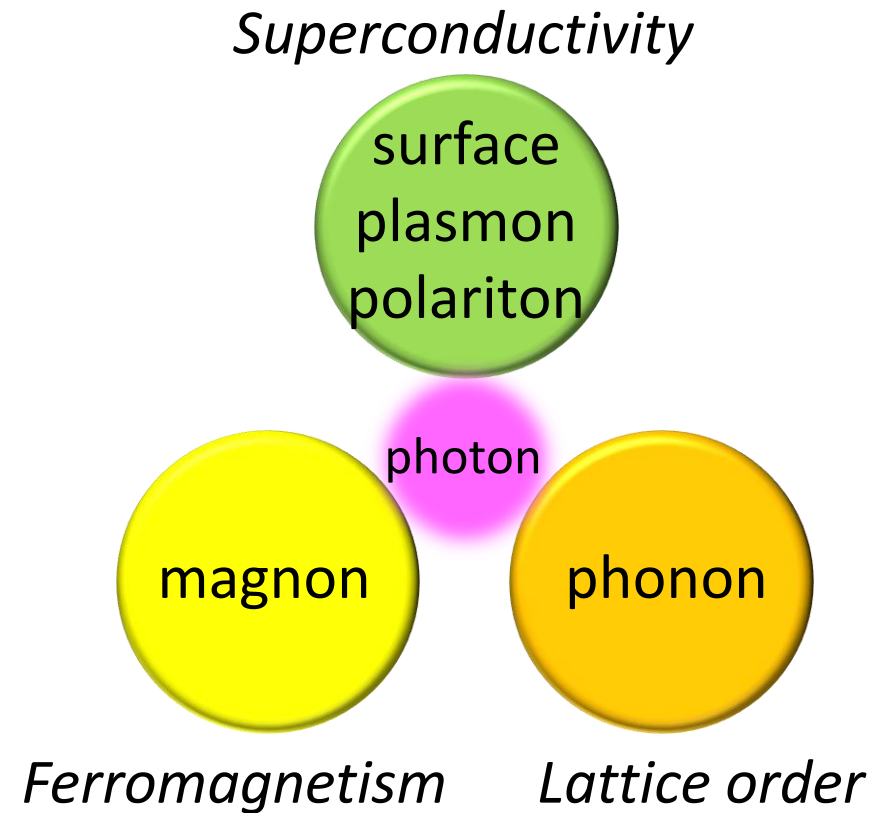
<http://ja.wikipedia.org/>



<http://kids.gakken.co.jp>

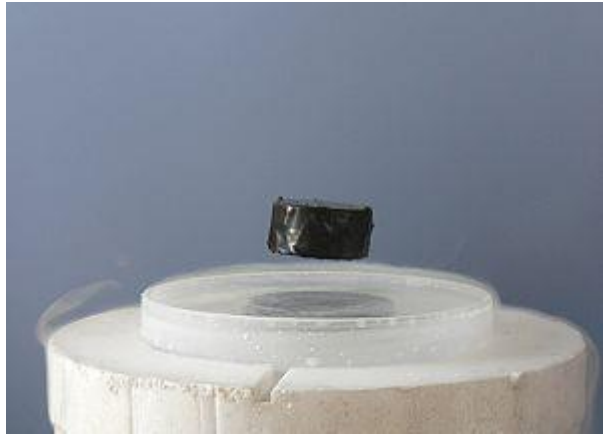


<http://en.wikipedia.org/>

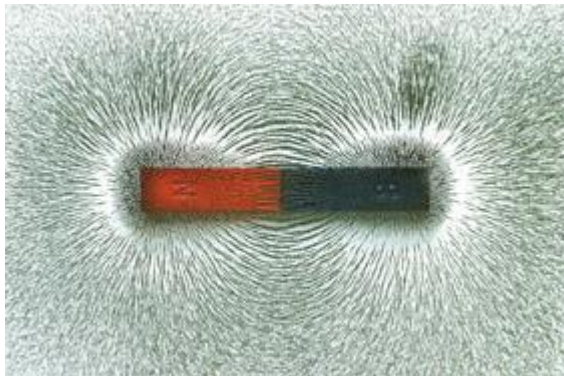


Collective excitations

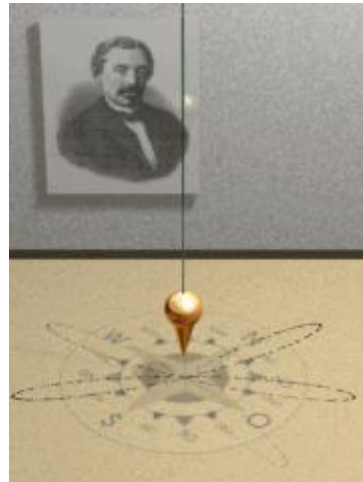
Spontaneous breaking of symmetry : Nambu-Goldstone bosons



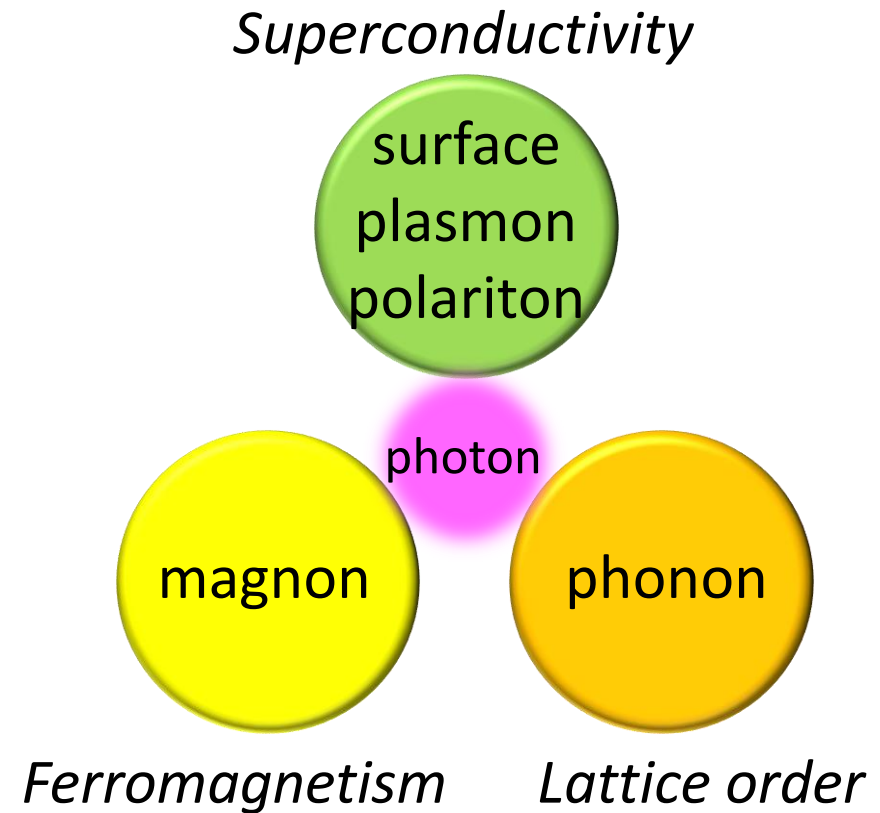
<http://ja.wikipedia.org/>



<http://kids.gakken.co.jp>



<http://en.wikipedia.org/>

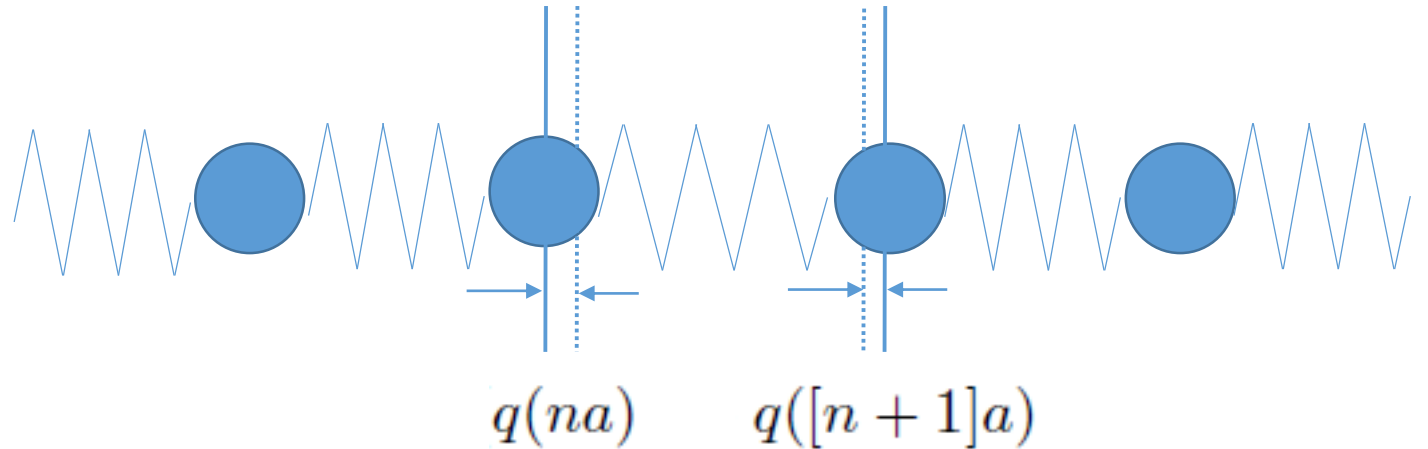


Phonons and nano-mechanics

Atomic chain

Potential:

$$V = \frac{1}{2} \kappa \sum_{n=1}^{N_a} (q(na) - q([n+1]a))^2$$



Equation of motion:

$$\begin{aligned} m\ddot{q}(na) &= -\frac{\partial V}{\partial q(na)} \\ &= -\kappa (2q(na) - q([n-1]a) - q([n+1]a)) \end{aligned}$$

Coupled equations...

Continuum limit ($a \rightarrow 0$)

Coupled equation of motion:

$$\begin{aligned} m\ddot{q}(na) &= -\frac{\partial V}{\partial q(na)} \\ &= -\kappa (2q(na) - q([n-1]a) - q([n+1]a)) \end{aligned}$$

$$\begin{aligned} &= -\left((q(x_n) - q(x_n - a)) - (q(x_n + a) - q(x_n)) \right) \\ &= -\left(\frac{\partial q(x_n + a)}{\partial x} - \frac{\partial q(x_n)}{\partial x} \right) a \\ &= -\left(\frac{\partial^2 q(x_n + a)}{\partial x^2} \right) a^2 \end{aligned}$$

(1+1)-D boson field equation:

$$m\ddot{q}(x) = \kappa \left(\frac{\partial^2 q(x)}{\partial x^2} \right) a^2 \implies \left(\frac{1}{v_s^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} \right) q(x, t) = 0$$

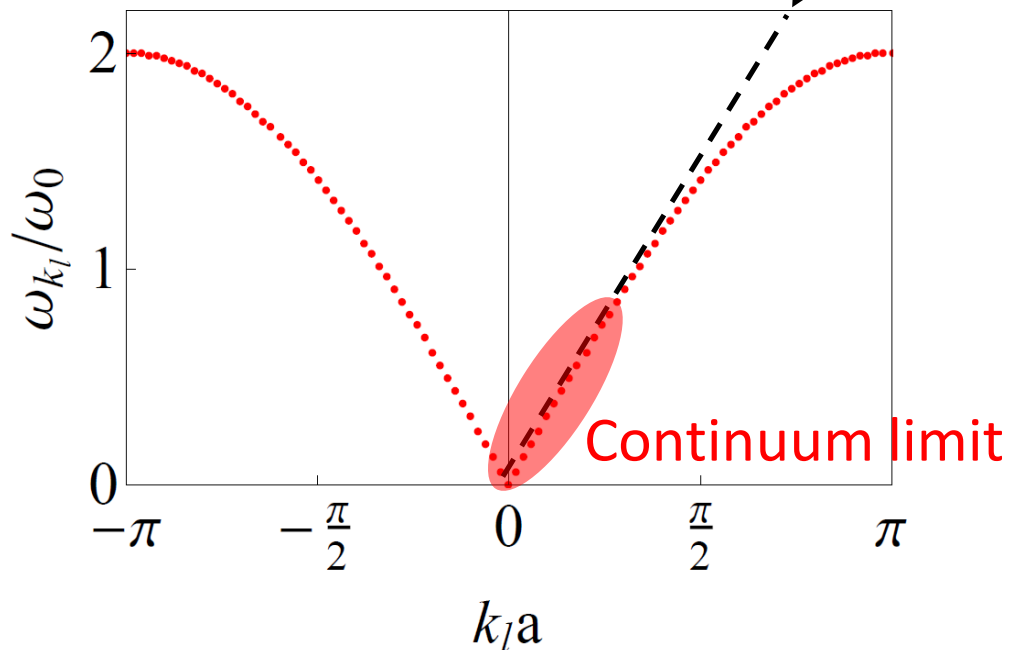
Continuum limit ($a \rightarrow 0$)

(1+1)-D boson field equation:

$$m\ddot{q}(x) = \kappa \left(\frac{\partial^2 q(x)}{\partial x^2} \right) a^2 \quad \Rightarrow \quad \left(\frac{1}{v_s^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} \right) q(x, t) = 0$$

Dispersion relation in continuum limit:

$$\omega_k = v_s k$$



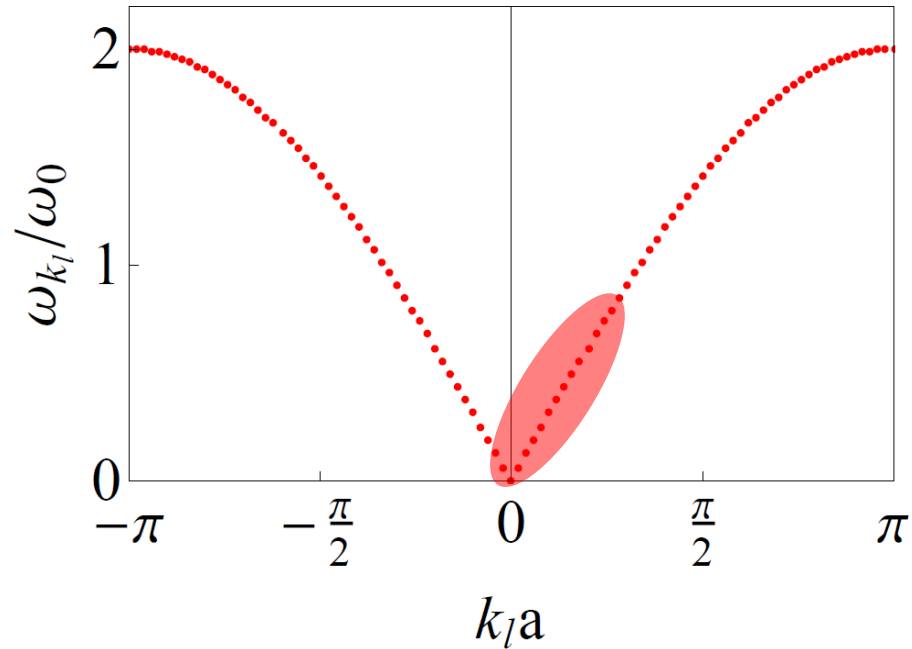
Sound velocity:

$$v_s = \sqrt{\frac{\kappa a^2}{m}} = \sqrt{\frac{\kappa a}{\left(\frac{m}{a}\right)}} = \sqrt{\frac{c_{11}}{\rho}}$$

Elastic constant

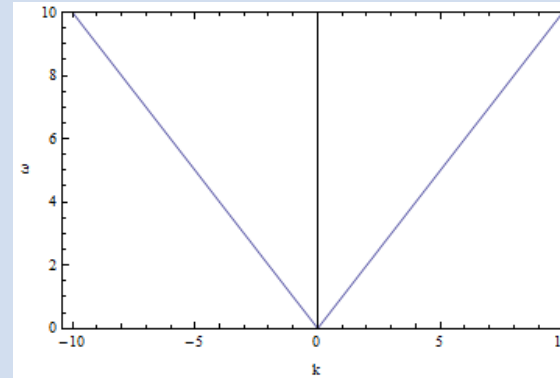
Mass density

Boundary conditions make the discrete modes



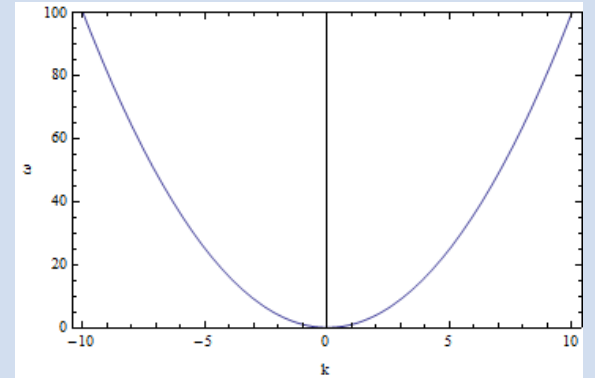
Type A

$\omega \propto k$ (linear dispersion)

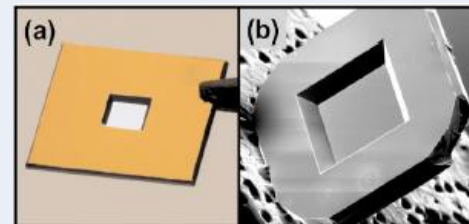


Type B

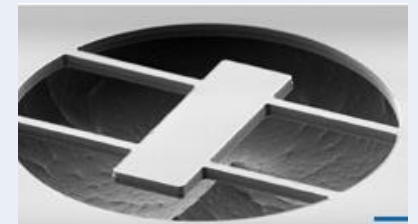
$\omega \propto k^2$ (quadratic dispersion)



Doubly-clamped cantilevers

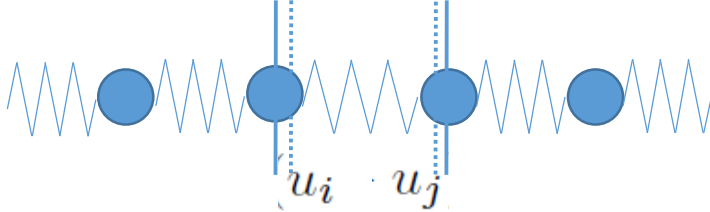
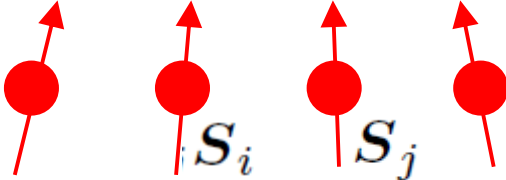


Free-free cantilevers

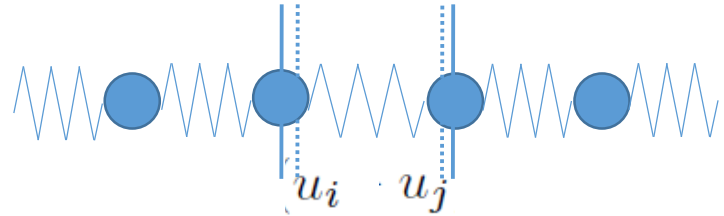
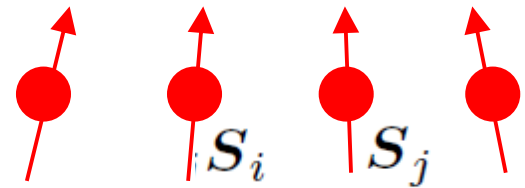


Ferromagnets and magnons

Interacting particles

	Atomistic viewpoint (microscopic)
Mechanics (Phonons)	 <p>Covalent/molecular/ionic bonding</p>
Magnets (Magnon)	 <p>Exchange interaction</p>

Interacting particles

	Atomistic viewpoint (microscopic)	
Mechanics (Phonons)	$U_{\text{el}} = \frac{1}{2} \sum_{i,j} K_{ij} (u_i - u_j)^2$	 <p>Covalent/molecular/ionic bonding</p>
Magnets (Magnon)	$U_{\text{ex}} = - \sum_{i,j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$	 <p>Exchange interaction</p>

Continuum limit ($k \rightarrow 0$ limit)

	Atomistic viewpoint (microscopic)	Continuum field viewpoint (macroscopic)
Mechanics (Phonons)	$\mathcal{U}_{\text{el}} = \frac{1}{2} \sum_{i,j} K_{ij} (u_i - u_j)^2$	$\mathcal{U}_{\text{el}} = C_{\alpha\mu\beta\nu} \frac{\partial u_\alpha}{\partial x_\mu} \frac{\partial u_\beta}{\partial x_\nu}$
Magnets (Magnon)	$\mathcal{U}_{\text{ex}} = - \sum_{i,j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$	$\mathcal{U}_{\text{ex}} = C_{\mu\nu} \frac{\partial M_\alpha}{\partial x_\mu} \frac{\partial M_\alpha}{\partial x_\nu}$

Continuum limit ($k \rightarrow 0$ limit)

	Atomistic viewpoint (microscopic)	Continuum feild viewpoint (macroscopic)
Mechanics (Phonons)	$\mathcal{U}_{\text{el}} = \frac{1}{2} \sum_{i,j} K_{ij} (u_i - u_j)^2$	$\mathcal{U}_{\text{el}} = C_{\alpha\mu\beta\nu} \frac{\partial u_\alpha}{\partial x_\mu} \frac{\partial u_\beta}{\partial x_\nu}$
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Elastic rigidity (elastic constant)

Magnetic rigidity

that is, Harmonic potential

	Atomistic viewpoint (microscopic)	Continuum feild viewpoint (macroscopic)
Mechanics (Phonons)	$\mathcal{U}_{el} = \frac{1}{2} \sum_{i,j} K_{ij} (u_i - u_j)^2$	$\mathcal{U}_{el} = C_{\alpha\mu\beta\nu} \frac{\partial u_\alpha}{\partial x_\mu} \frac{\partial u_\beta}{\partial x_\nu}$
Magnets (Magnon)	$\mathcal{U}_{ex} = - \sum_{i,j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$	$\mathcal{U}_{ex} = C_{\mu\nu} \frac{\partial M_\alpha}{\partial x_\mu} \frac{\partial M_\alpha}{\partial x_\nu}$

1D lumped system
(spring)

Rigidity

$$\mathcal{H}(X) = \frac{1}{2} k x^2$$

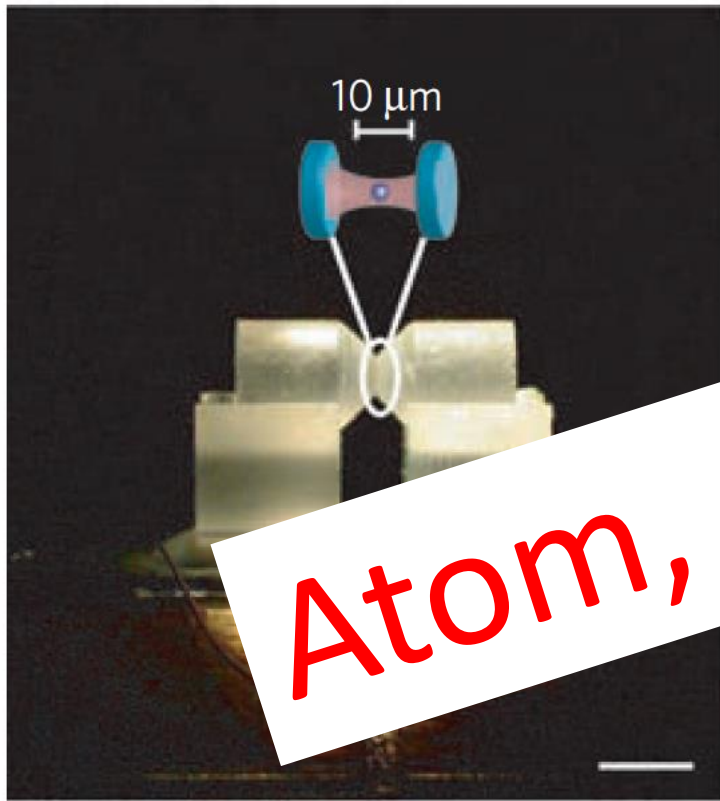
$$\mathcal{H}(X) = \cancel{\mathcal{H}(X_0)} + \cancel{\frac{\partial \mathcal{H}(X_0)}{\partial x} x} + \frac{1}{2} \underbrace{\frac{\partial^2 \mathcal{H}(X_0)}{\partial x^2}}_k x^2$$

neglect equilibrium

Wonderful world of quantum optics

Cavity QED

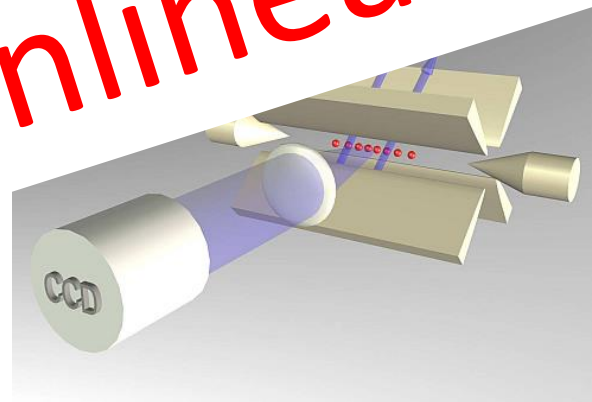
- Atom
- Photon



H.J. Kimble,
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Ion Trap

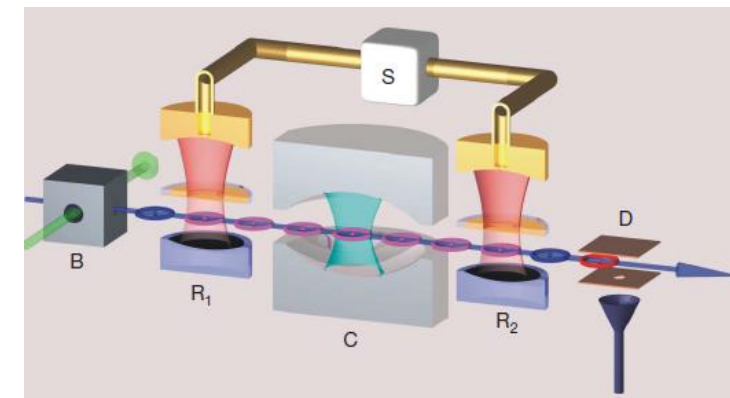
- Ion
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F. Schmidt-Kaler, *et al.*,
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Cavity QED with Rydberg atom

- Rydberg atom
- Microwave photon

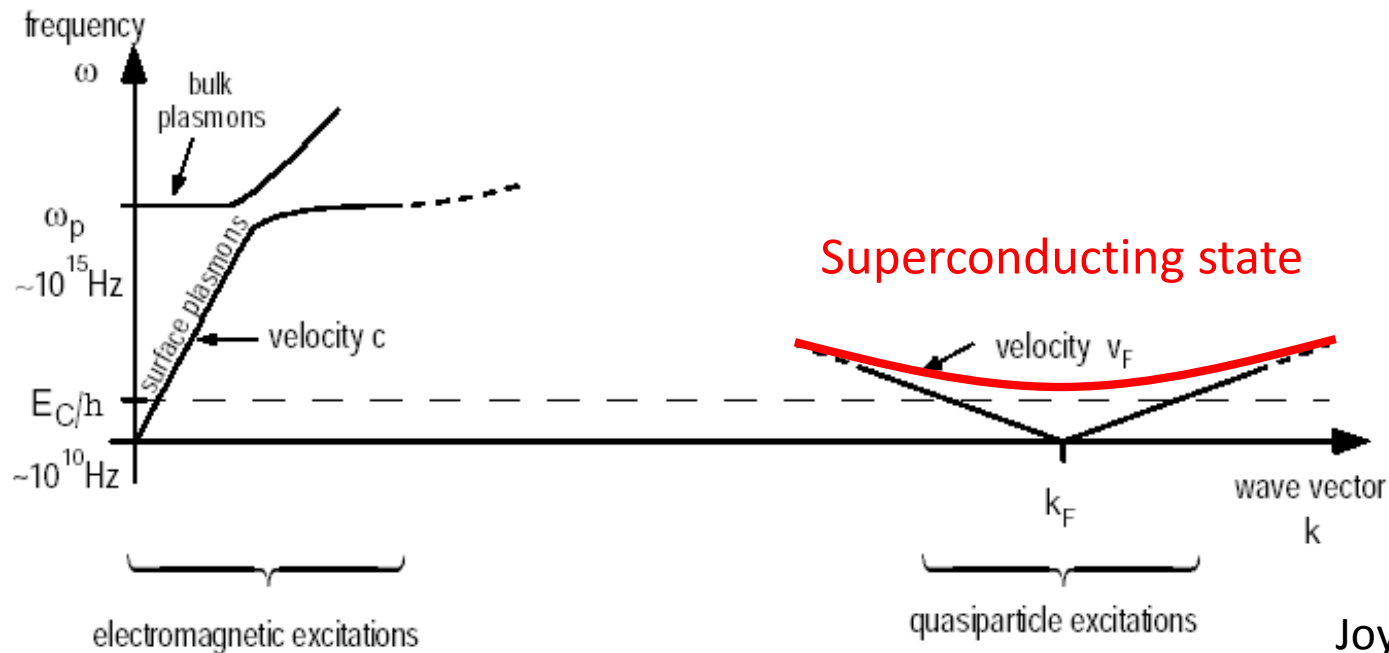
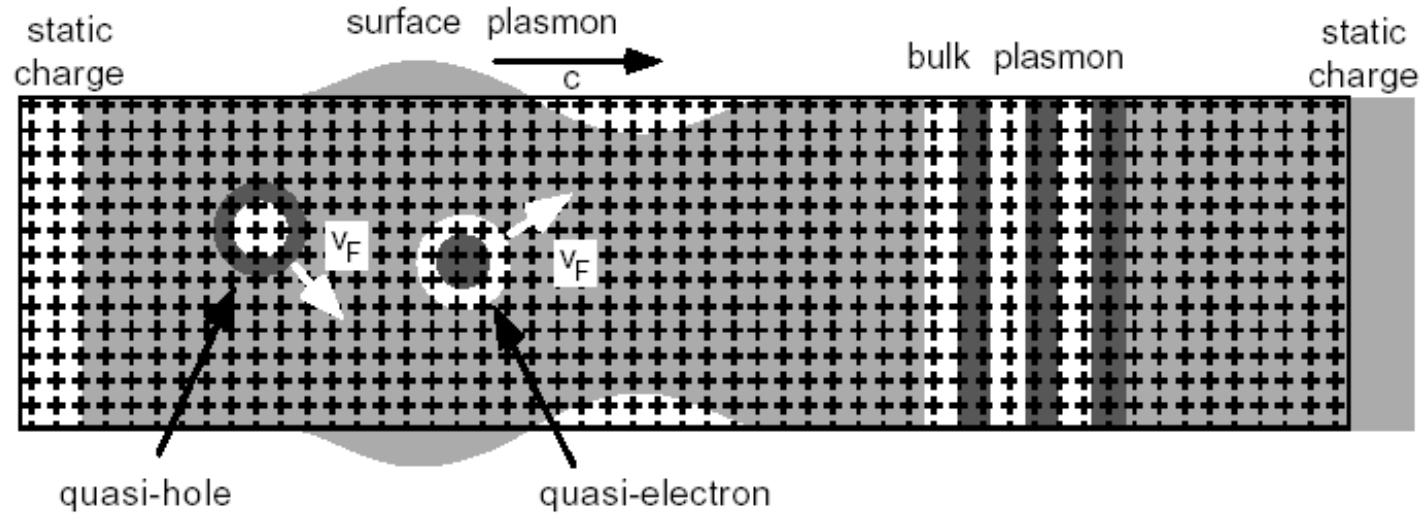


S. Gleyzes, *et al.*,
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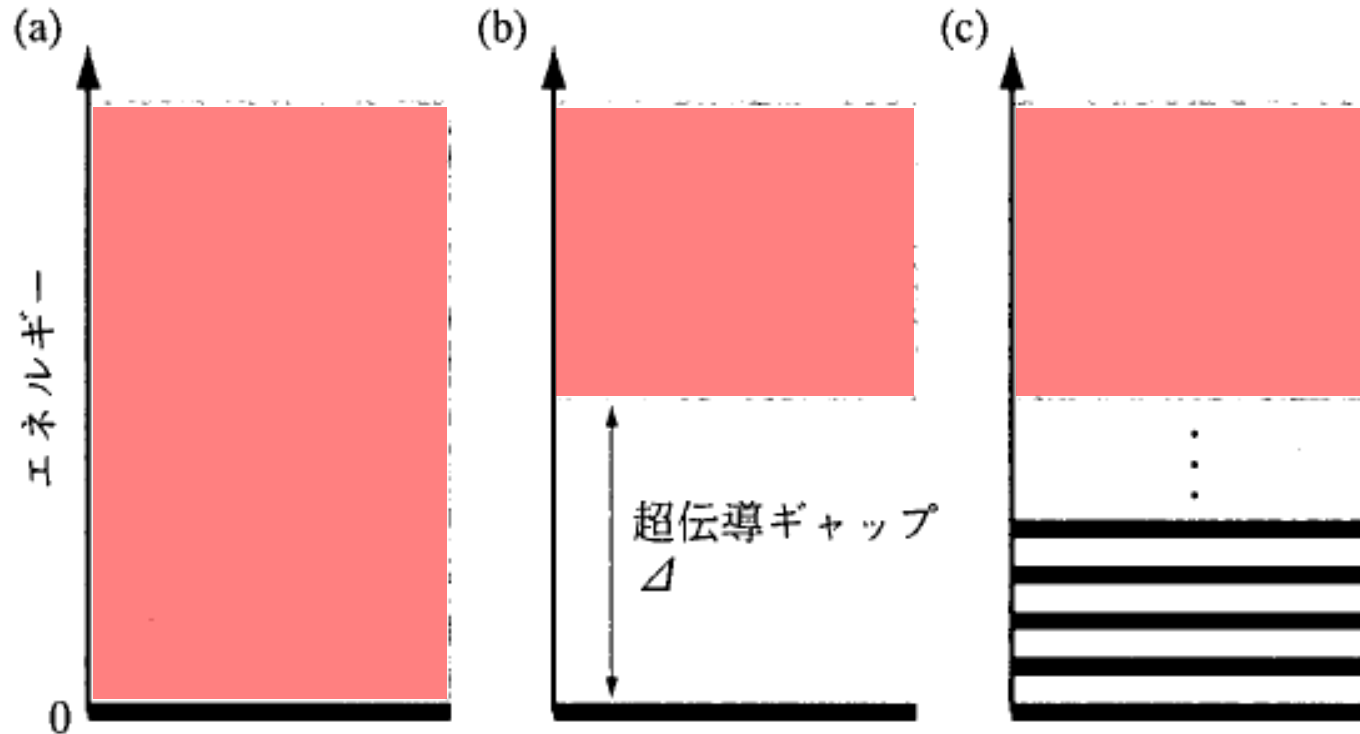
Atom, a nonlinear element!

Superconductivity and Josephson atoms

Elementary excitation in metals



Superconducting LC circuit



- Rydberg atom
- Microwave photon

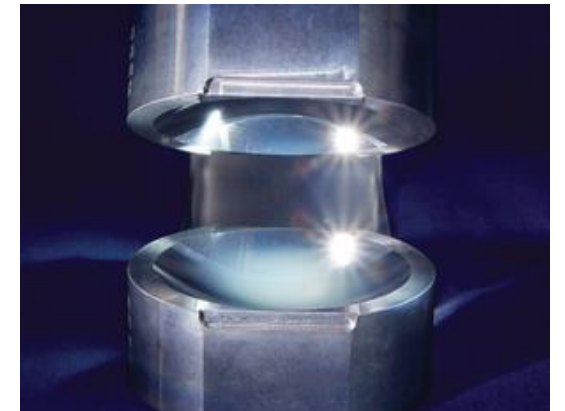


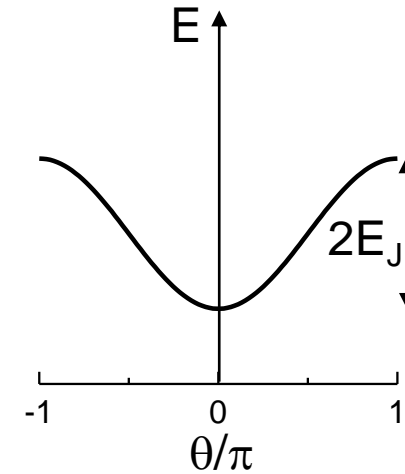
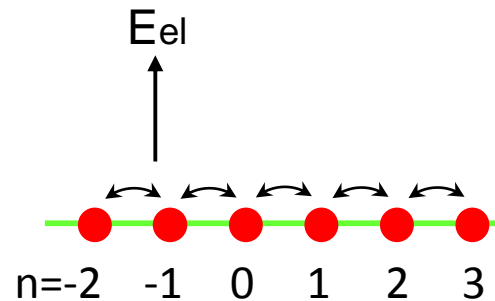
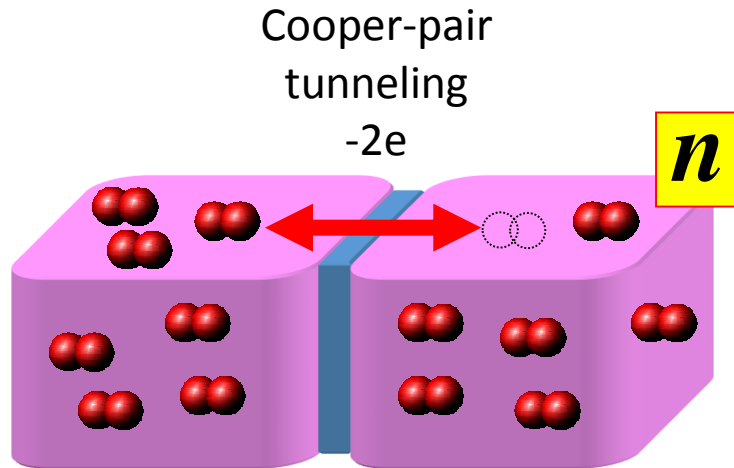
図1 模式的なエネルギー準位図. 最下部の黒線が基底エネルギー準位, 薄赤色部分は連続的な準位分布を示す. (a) フェルミ電子系, (b) 超伝導状態, (c) 超伝導共振器,

Josephson effect (lossless nonlinearity)



B. D. Josephson 1962

number $n \Leftrightarrow$ phase difference θ $[n, \theta] = -i$



$$H = -\frac{E_J}{2} \sum_n \{ |n\rangle \langle n+1| + |n+1\rangle \langle n| \} = - \int_0^{2\pi} d\theta E_J \cos \theta |\theta\rangle \langle \theta|$$

Tight-binding model in 1d lattice \Rightarrow Bloch band

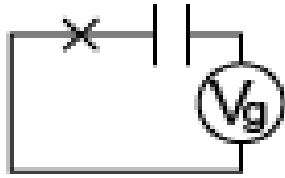
$$|\theta\rangle = \sum_n e^{in\theta} |n\rangle$$

Superconducting qubits

– artificial atoms in electrical circuit

small ← E_J/E_C → large

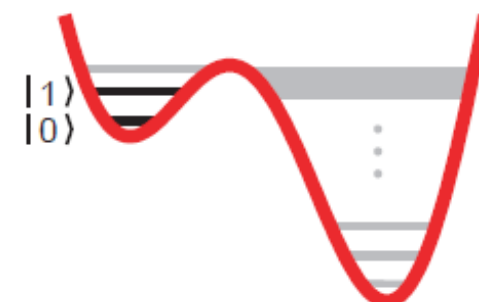
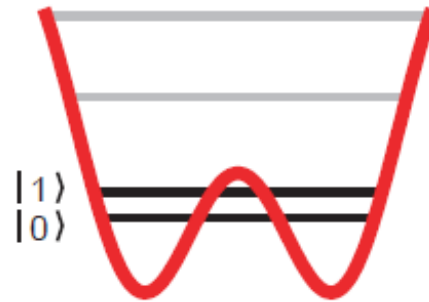
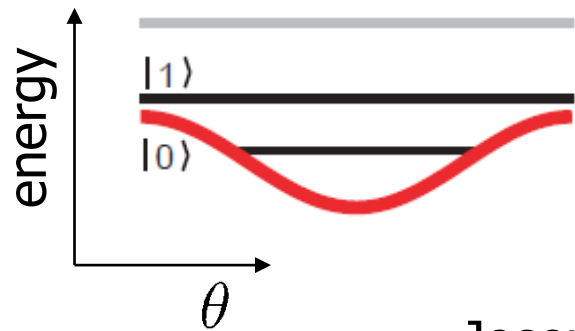
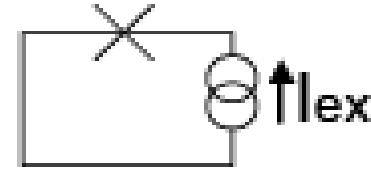
Charge qubit



Flux qubit



Phase qubit



Josephson energy E_J = confinement potential
 charging energy E_C = kinetic energy

typical qubit energy $E_{01} \sim 10 \text{ GHz} \sim 0.5 \text{ K}$

typical experimental temperature $T \sim 0.01 \text{ K}$

Superconducting qubits

– artificial atoms in electrical circuit

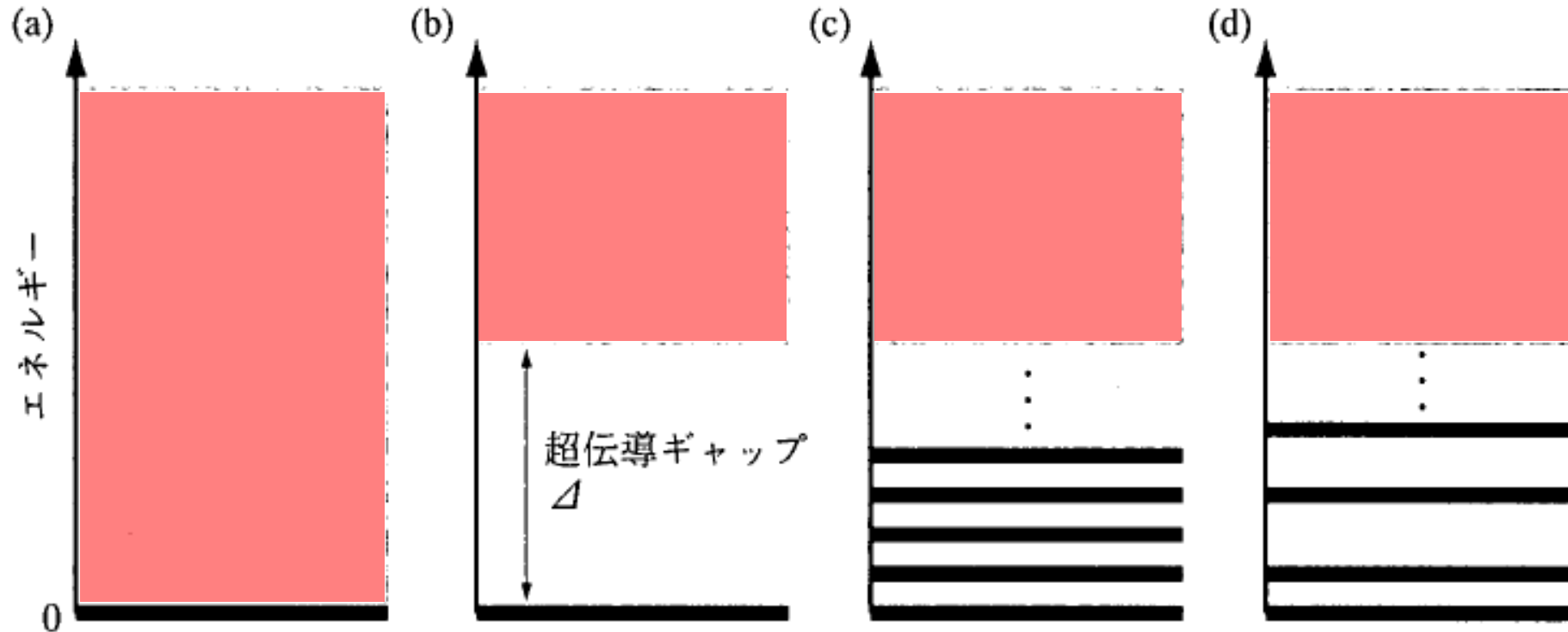
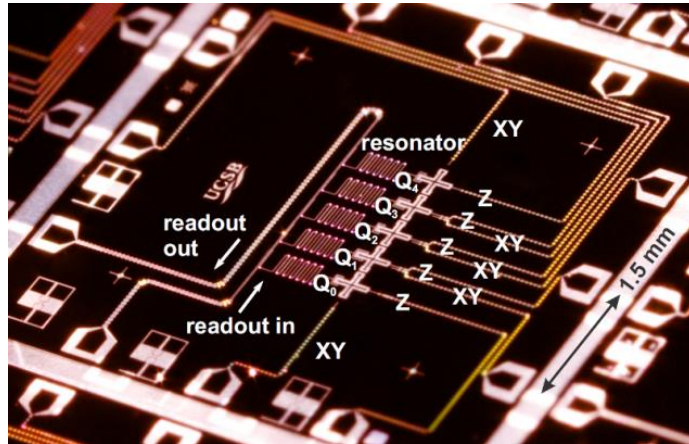


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Advances of superconducting qubits

- Artificial quantum system in electrical circuits
- Strong coupling with microwave
- Design flexibility, integrability
- Improved coherence time

● 2D qubits

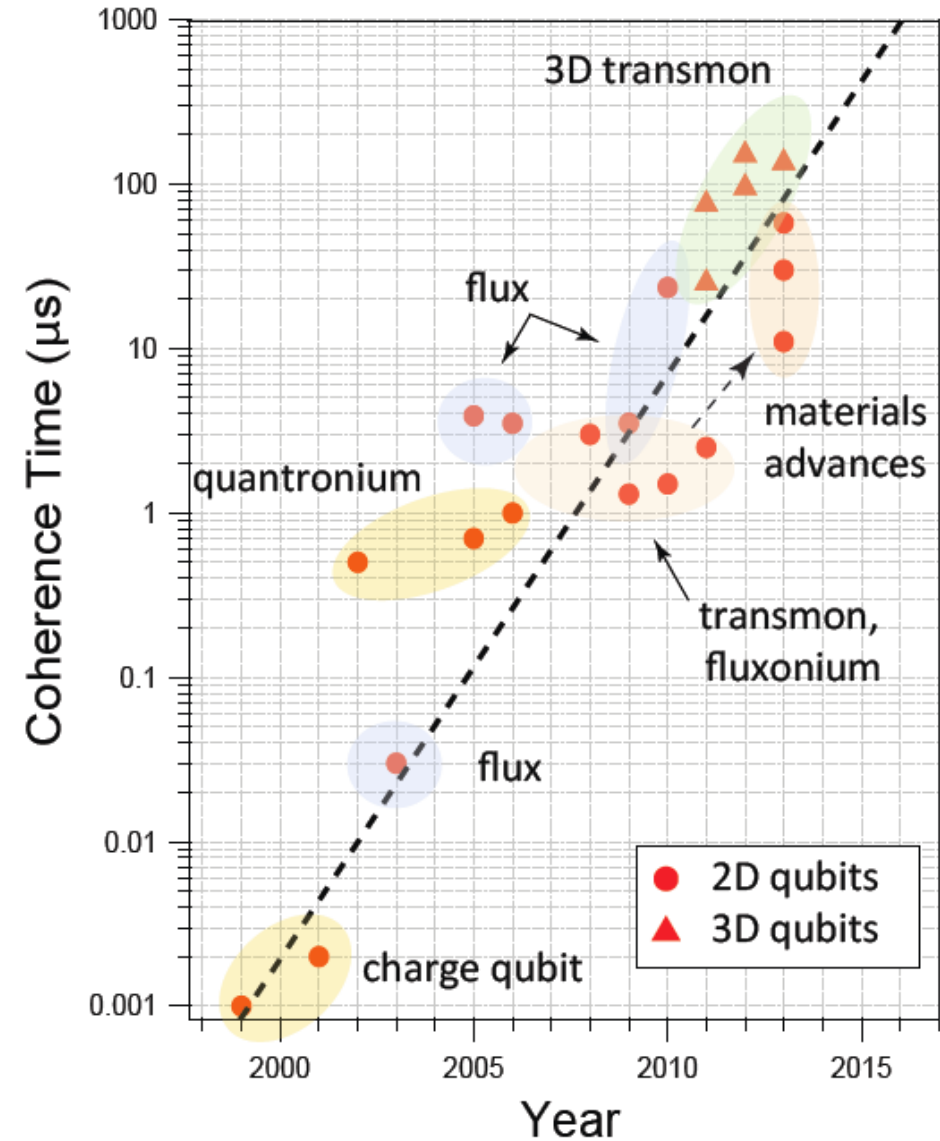


R. Barends et al. Nature (2014) UCSB

▲ 3D qubits



Courtesy of S. Kono



Courtesy of W. Oliver and P. Welander (MIT-LL)

Quantum transducers

Linking superconducting qubits

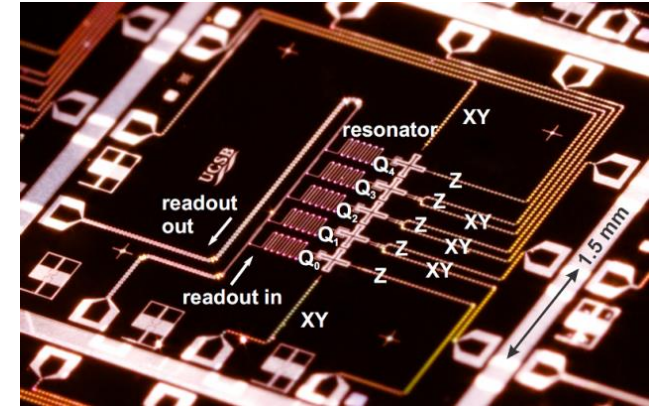
Superconducting Qubit

Dilution fridge (10mK)



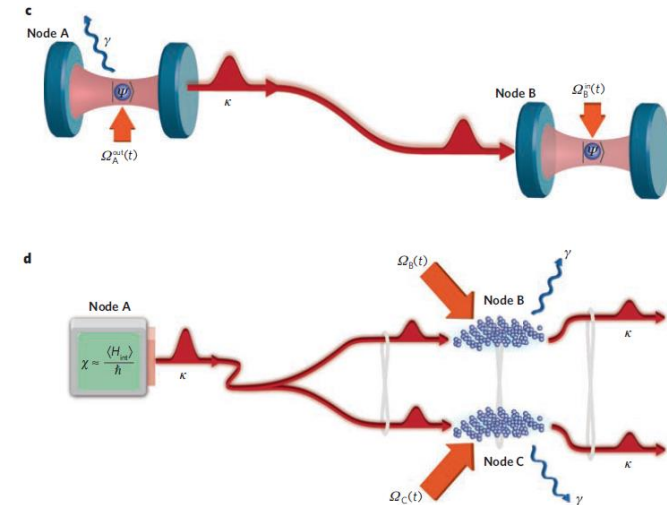
Optical photons
(~200THz)

Quantum circuits



R. Barends et al. Nature (2014).

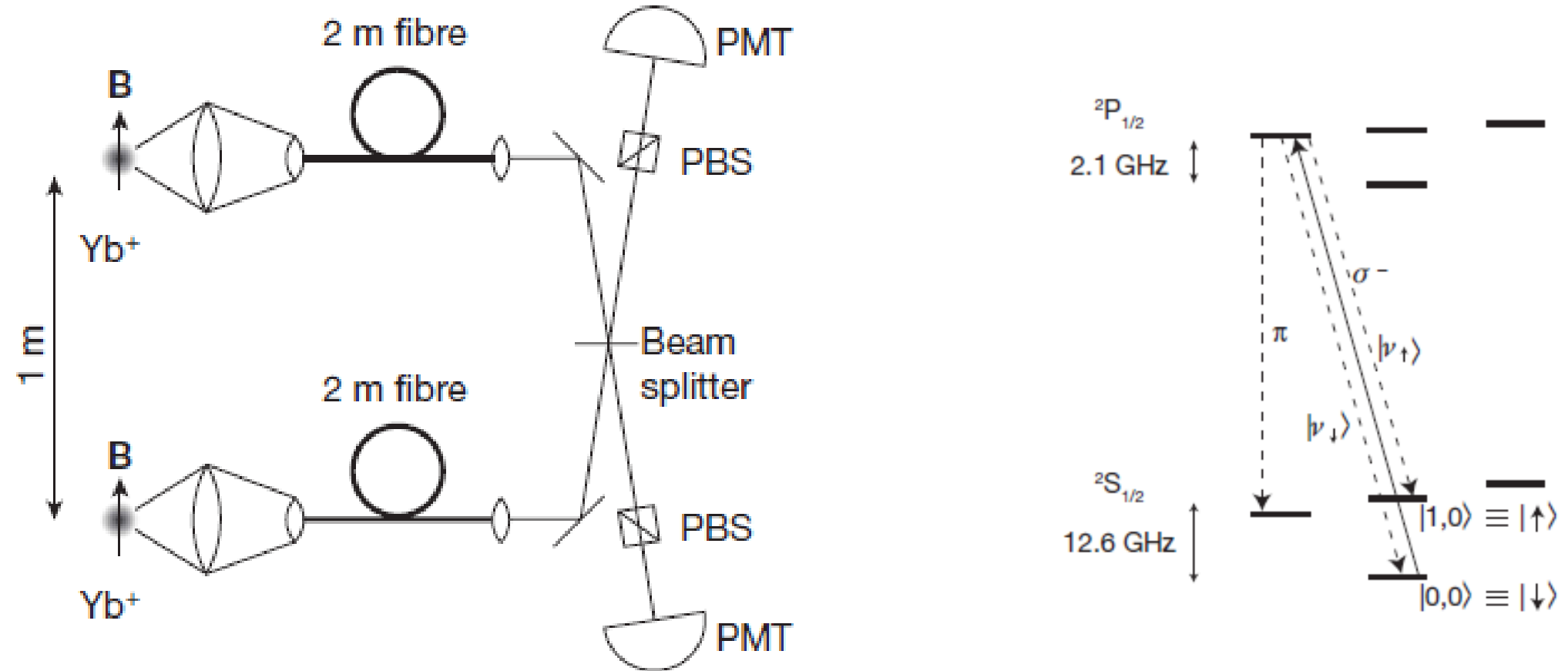
Quantum optics



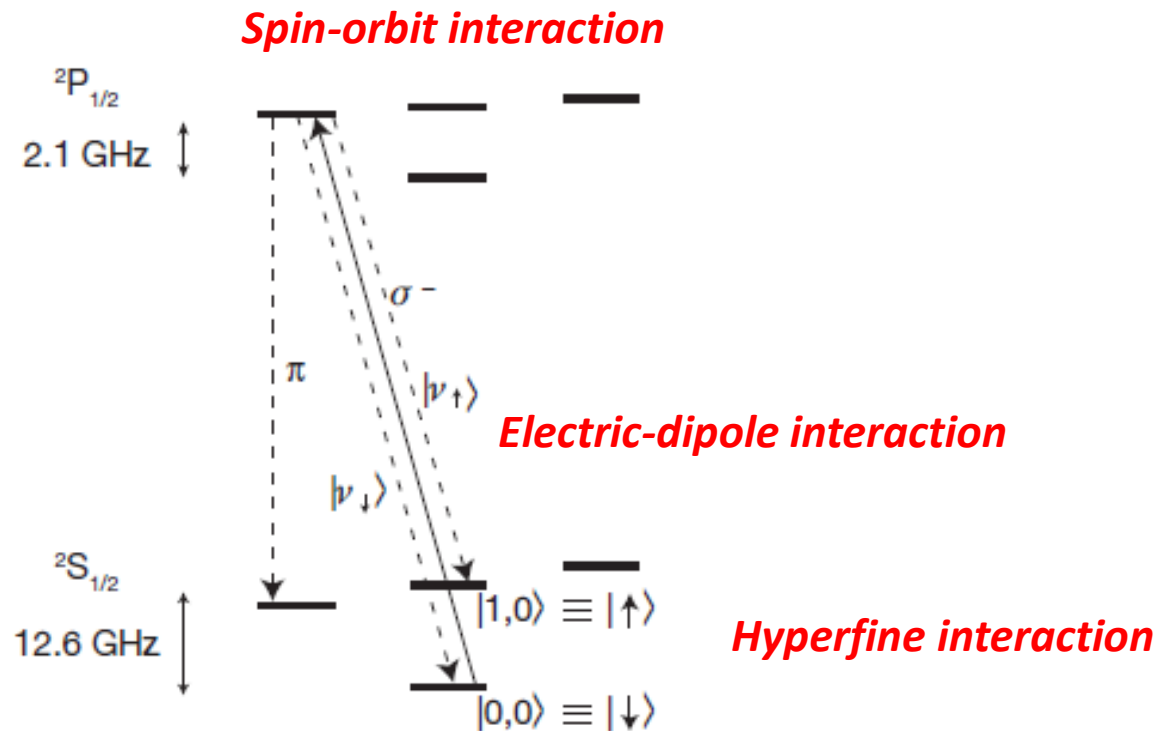
H.J. Kimble, Nature **453**, 1023 (2008).

Linking single-atom qubits

b

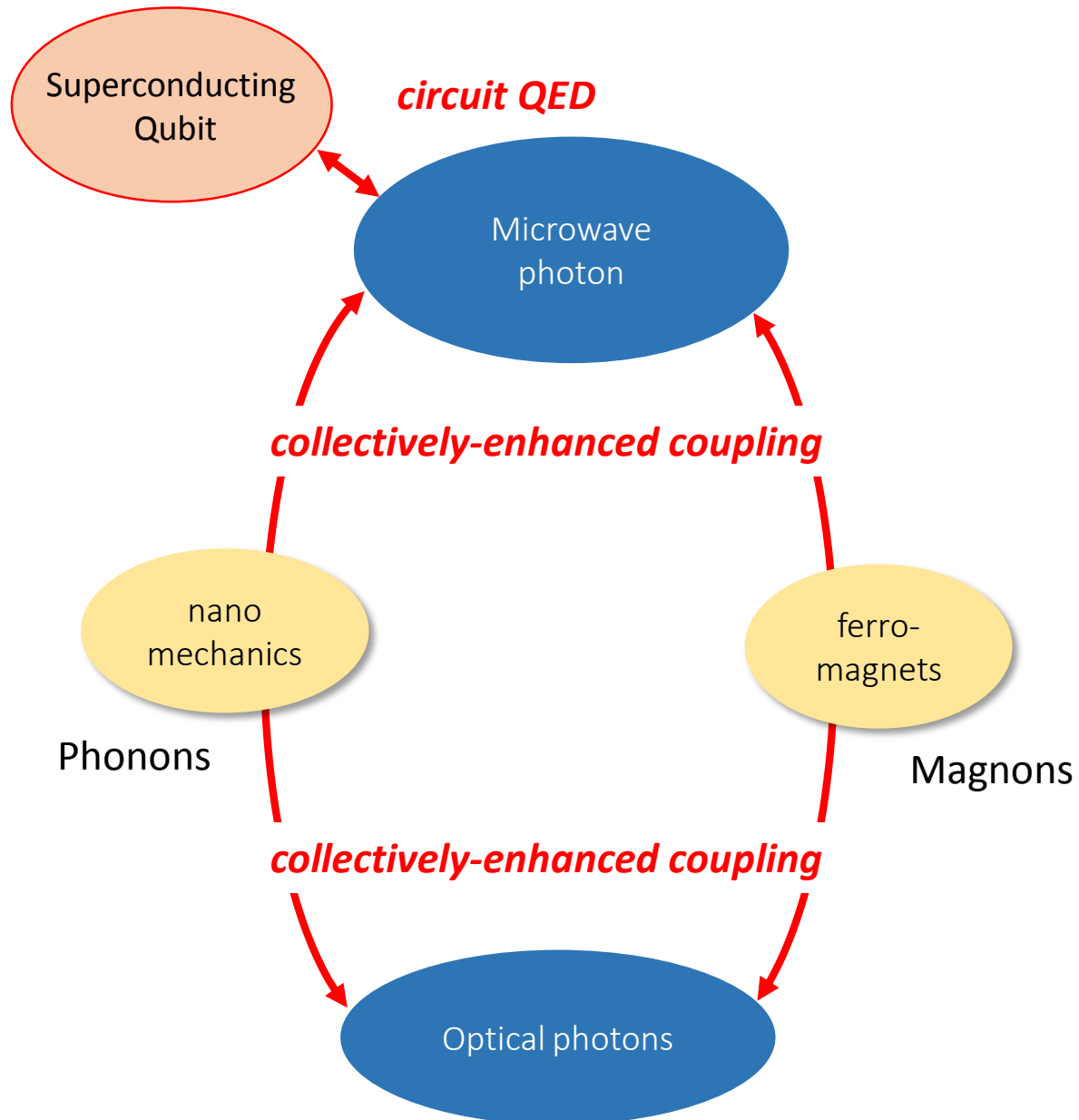


Quantum transducer for linking single-atom qubits



- Nuclear spin (n)
Hyperfine interaction
- Electron spin (s)
Spin-orbit coupling
- Orbital (l)
Electric-dipole interaction
- light

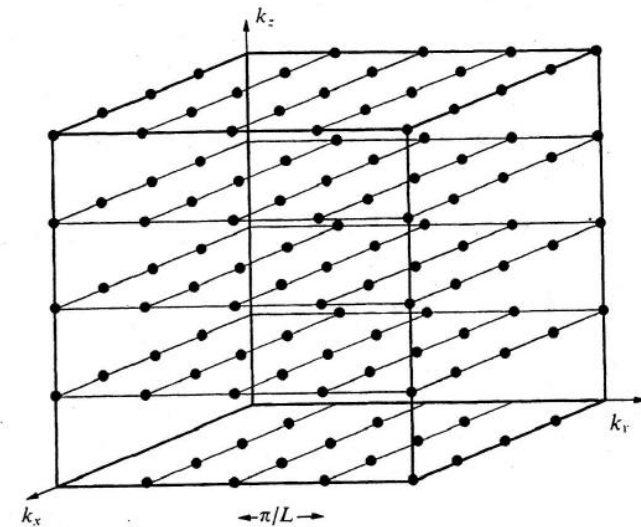
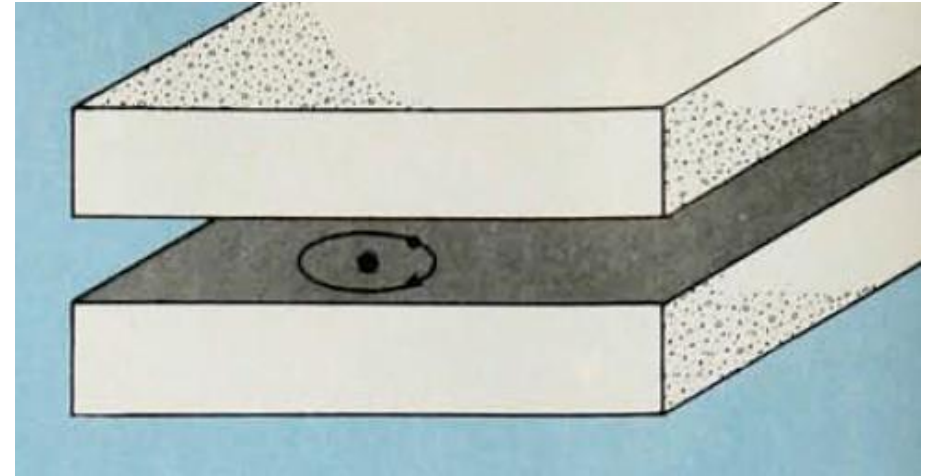
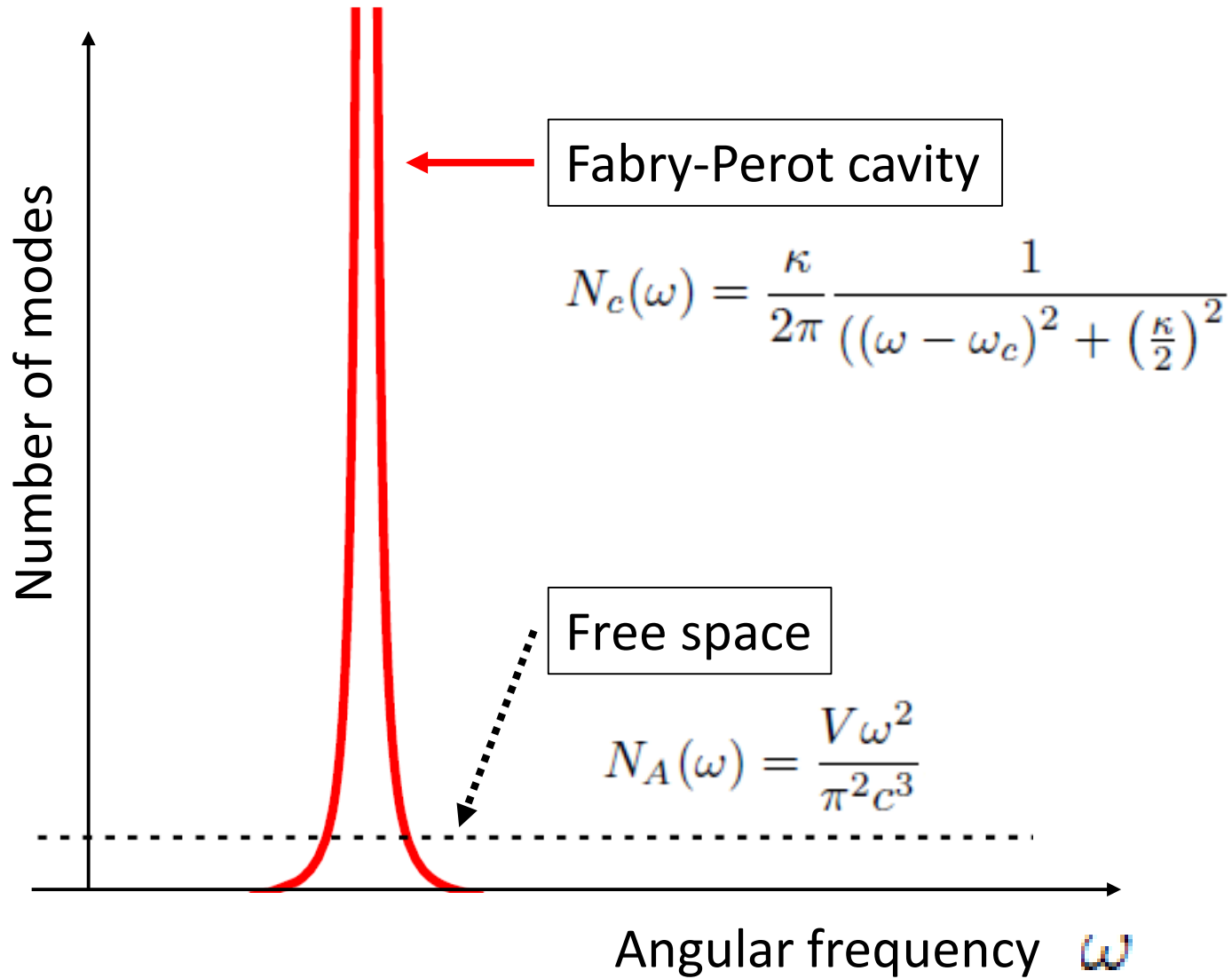
Quantum transducer architecture



- Superconducting qubit
circuit QED
- Microwave
collectively-enhanced coupling
- Magnon/phonon
collectively-enhanced coupling
- light

Circuit QED

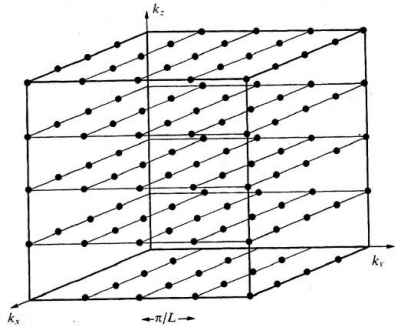
Purcell enhancement



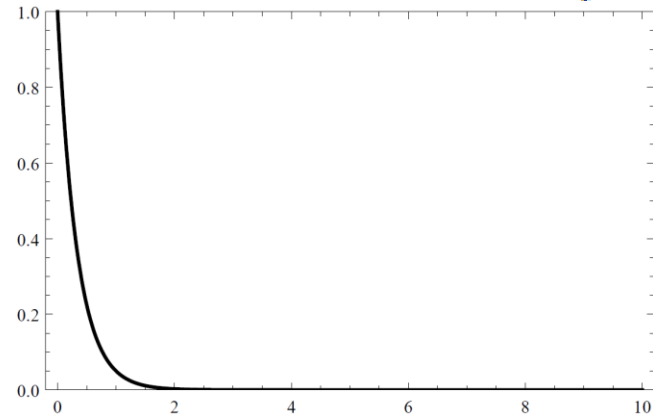
Jaynes-Cummings Hamiltonian

Einstein Hamiltonian

$$H_I = -i\hbar \sum_j \lambda_j \left(\hat{\sigma}_+ \hat{a}_j e^{i(\omega_A - \omega_j)t} - \hat{\sigma}_- \hat{a}_j^\dagger e^{-i(\omega_A - \omega_j)t} \right)$$



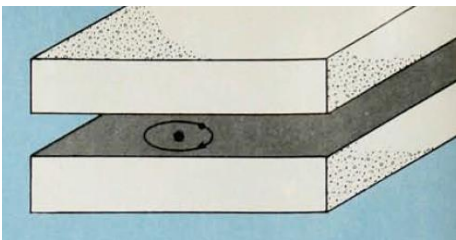
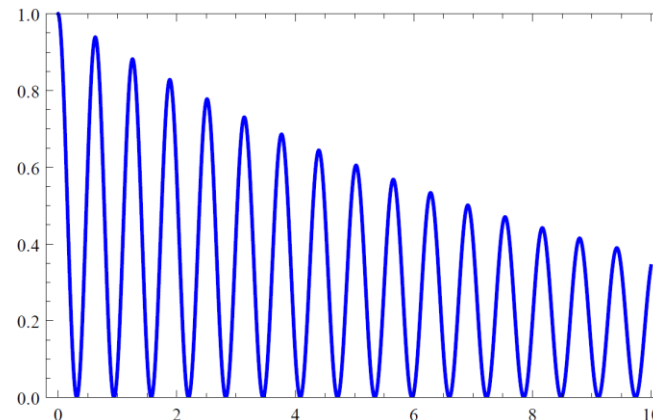
Fermi's golden rule



Jaynes-Cummings Hamiltonian

$$H_{JC} = -i\hbar g \left(\hat{\sigma}_+ \hat{a} - \hat{\sigma}_- \hat{a}^\dagger \right)$$

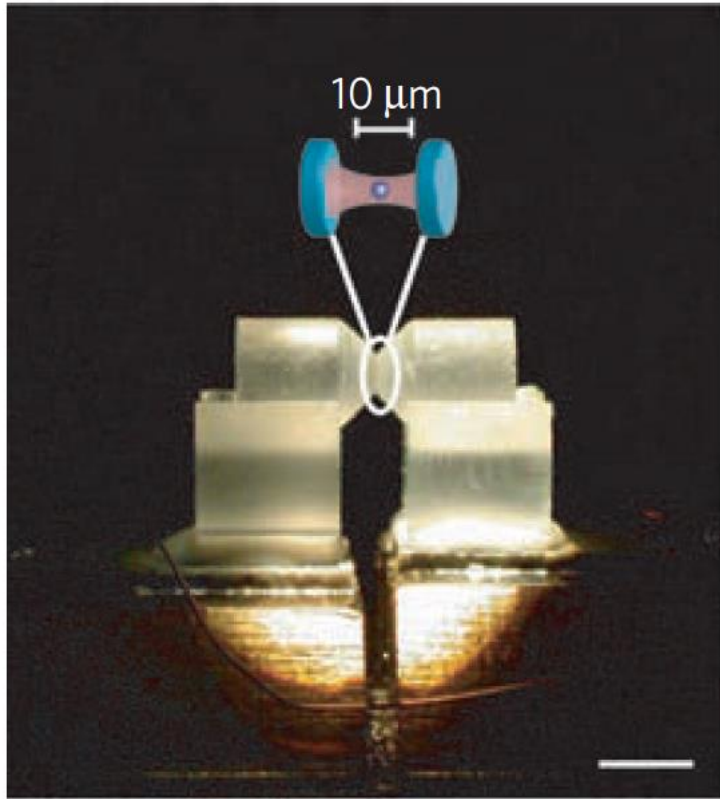
Rabi oscillation



QED systems

Cavity QED

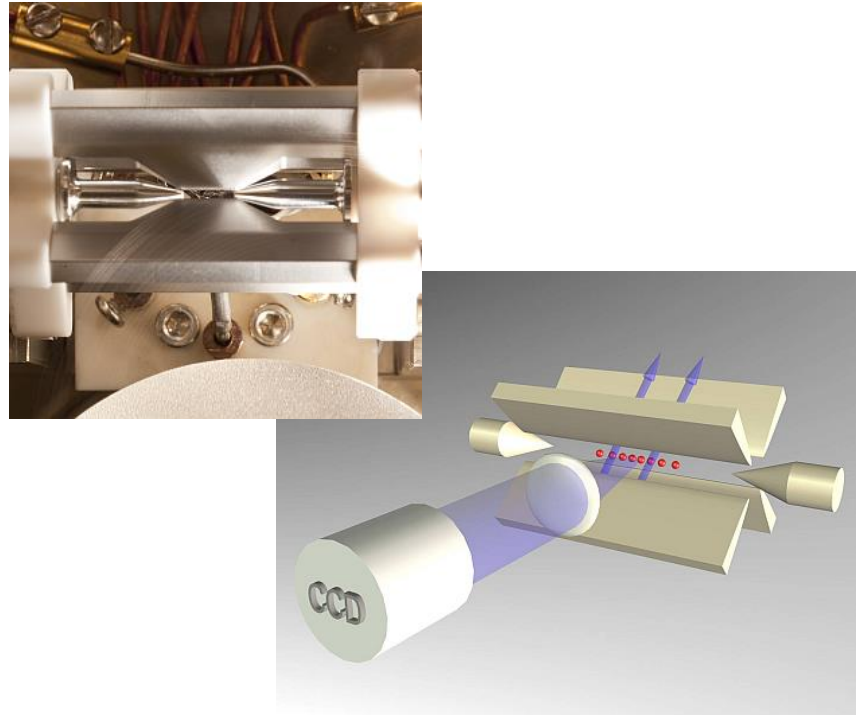
- Atom
- Photon



H.J. Kimble,
Nature **453**, 1023 (2008).

Ion Trap

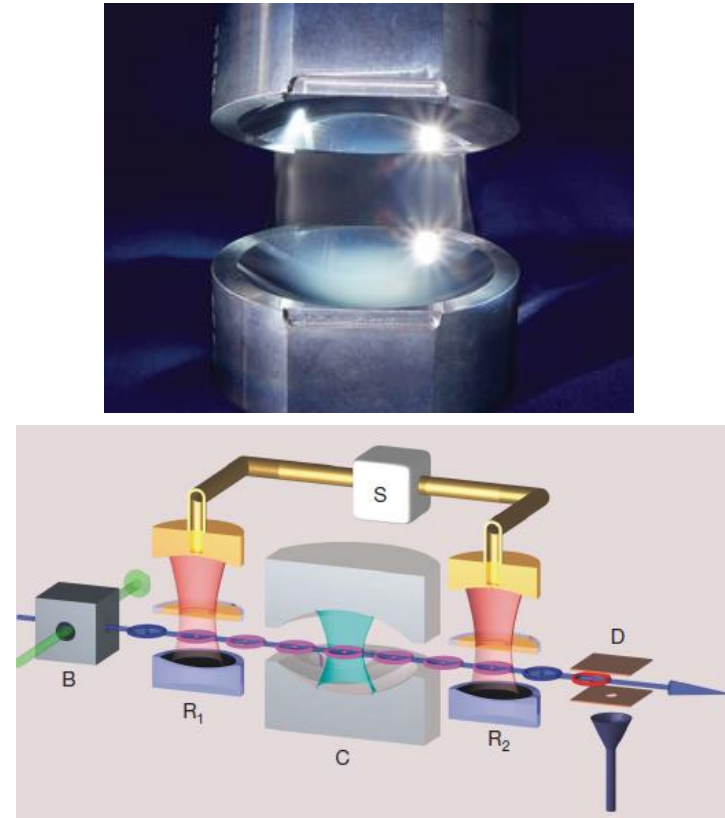
- Ion
- Phonon



F. Schmidt-Kaler, *et al.*,
Nature **422**, 408 (2003).

Cavity QED with Rydberg atom

- Rydberg atom
- Microwave photon

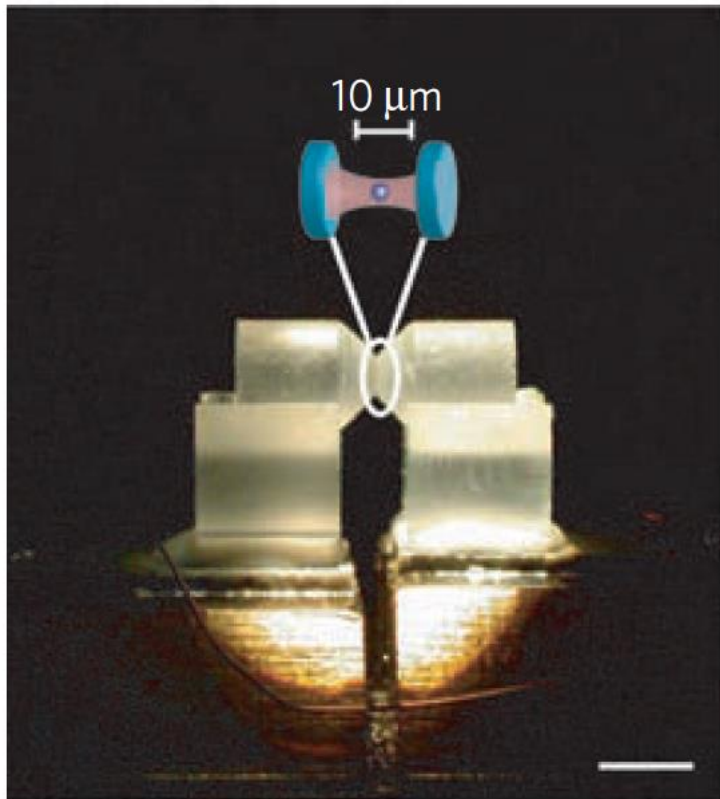


S. Gleyzes, *et al.*,
Nature **446**, 297 (2007)

Variants of cavity QED systems

Cavity QED

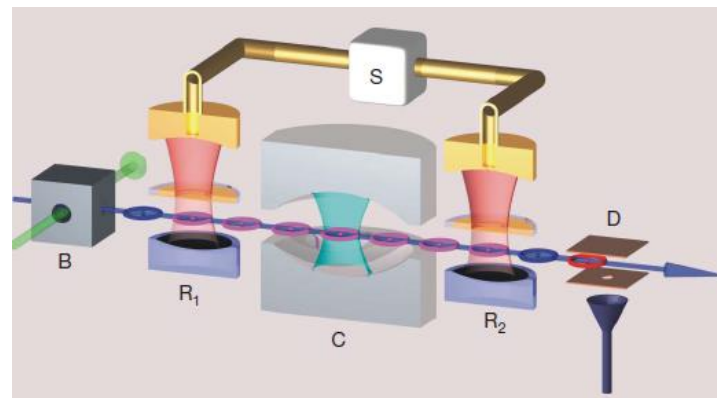
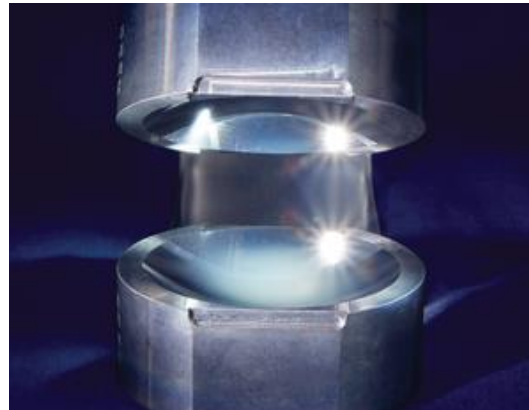
- Atom
- Photon



H.J. Kimble,
Nature **453**, 1023 (2008).

Cavity QED with Rydberg atom

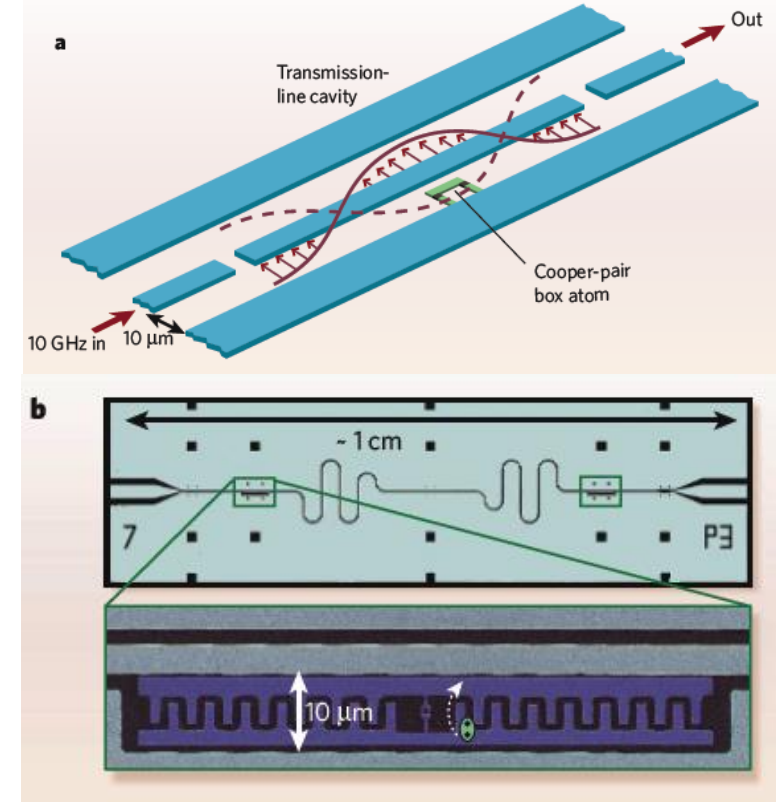
- Rydberg atom
- Microwave photon



S. Gleyzes, *et al.*,
Nature **446**, 297 (2007)

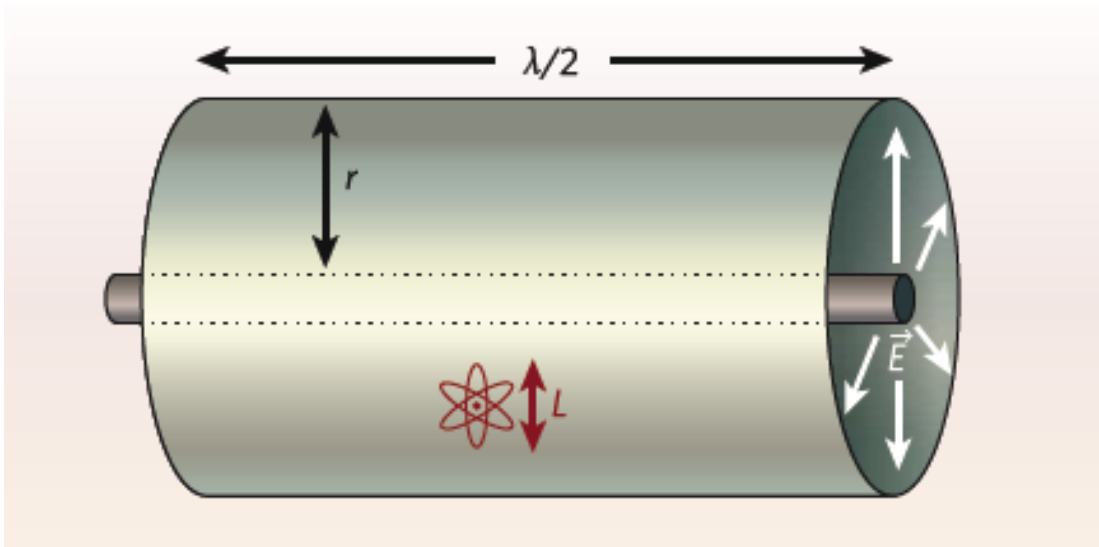
Superconducting Circuit

- Artificial atom
- Microwave photon



R. J. Schoelkopf and S. M. Girvin,
Nature **451**, 664 (2008).

Circuit QED



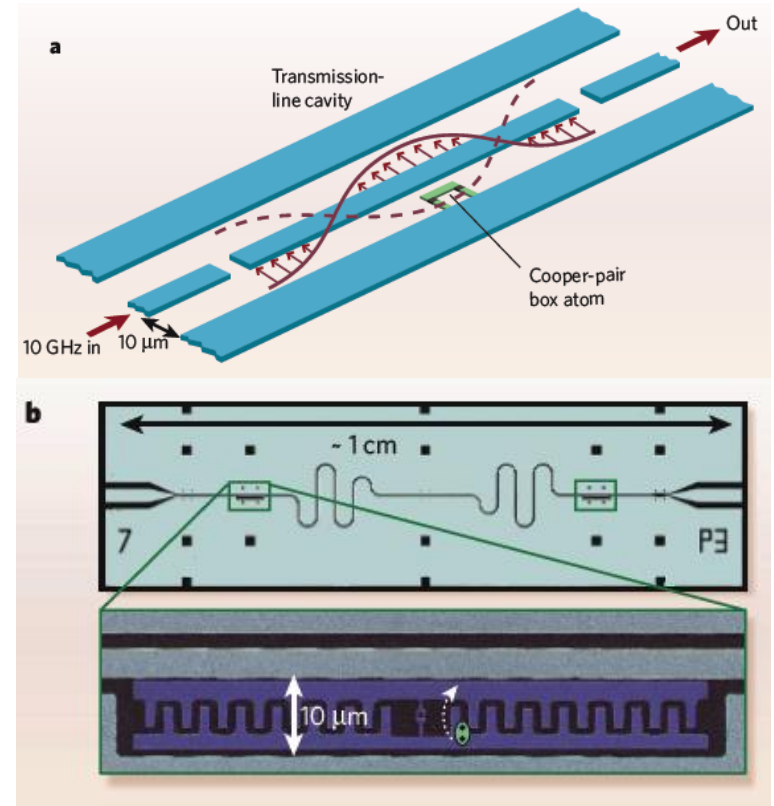
Fundamental limit of the coupling strength:

$$g_e = \frac{ea_0}{\hbar} \sqrt{\frac{\hbar\omega}{2\epsilon_0 V}} = ea_0 \sqrt{\frac{\omega}{2\hbar\epsilon_0 V}} \Rightarrow g_c = \sqrt{\alpha\omega}$$

for cavity volume of $V \sim \lambda a_0^2$

Superconducting Circuit

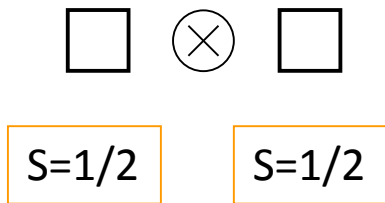
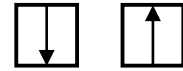
- Artificial atom
- Microwave photon



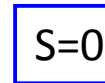
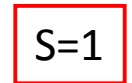
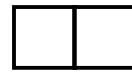
Collectively-enhanced coupling

Angular Momentum addition

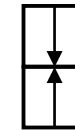
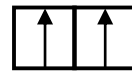
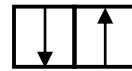
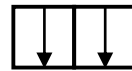
Young's Tableau



=



=



3 state

Triplet
(totally symmetric)

1 state

Singlet
(totally anti-symmetric)

Observation of Superradiant and Subradiant Spontaneous Emission of Two Trapped Ions

R. G. DeVoe and R. G. Brewer

*IBM Research Division, Almaden Research Center, 650 Harry Road,
San Jose, California 95120-6099*

(Received 19 October 1995)

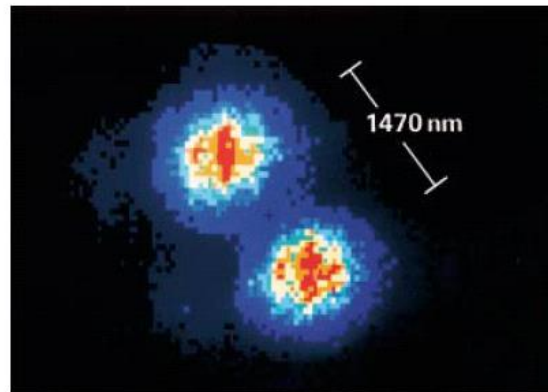
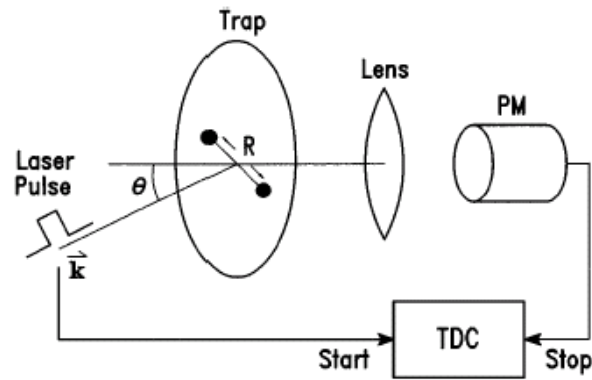
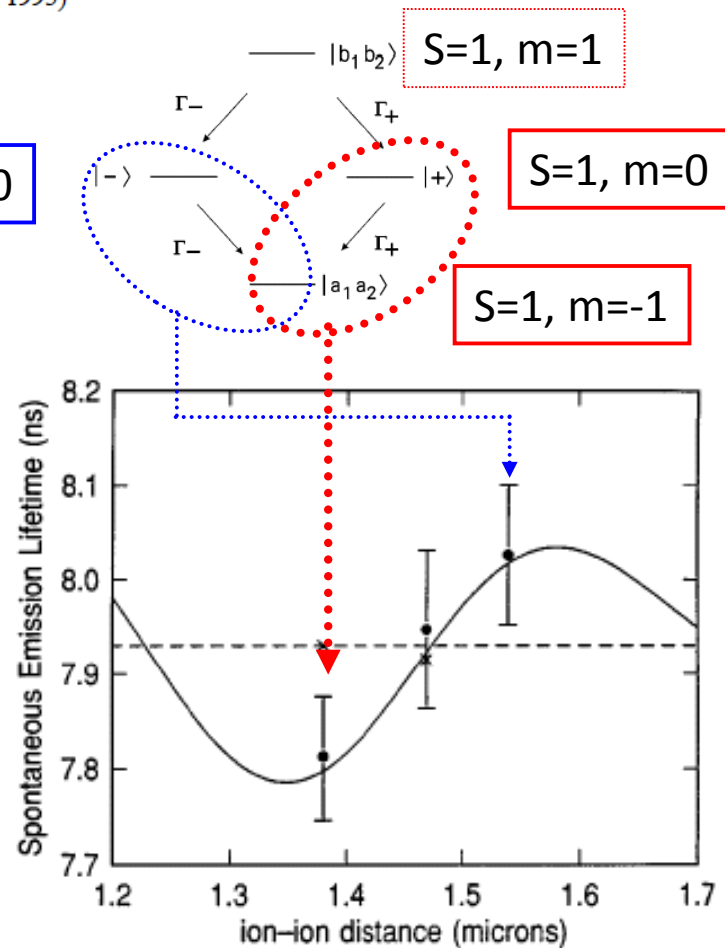


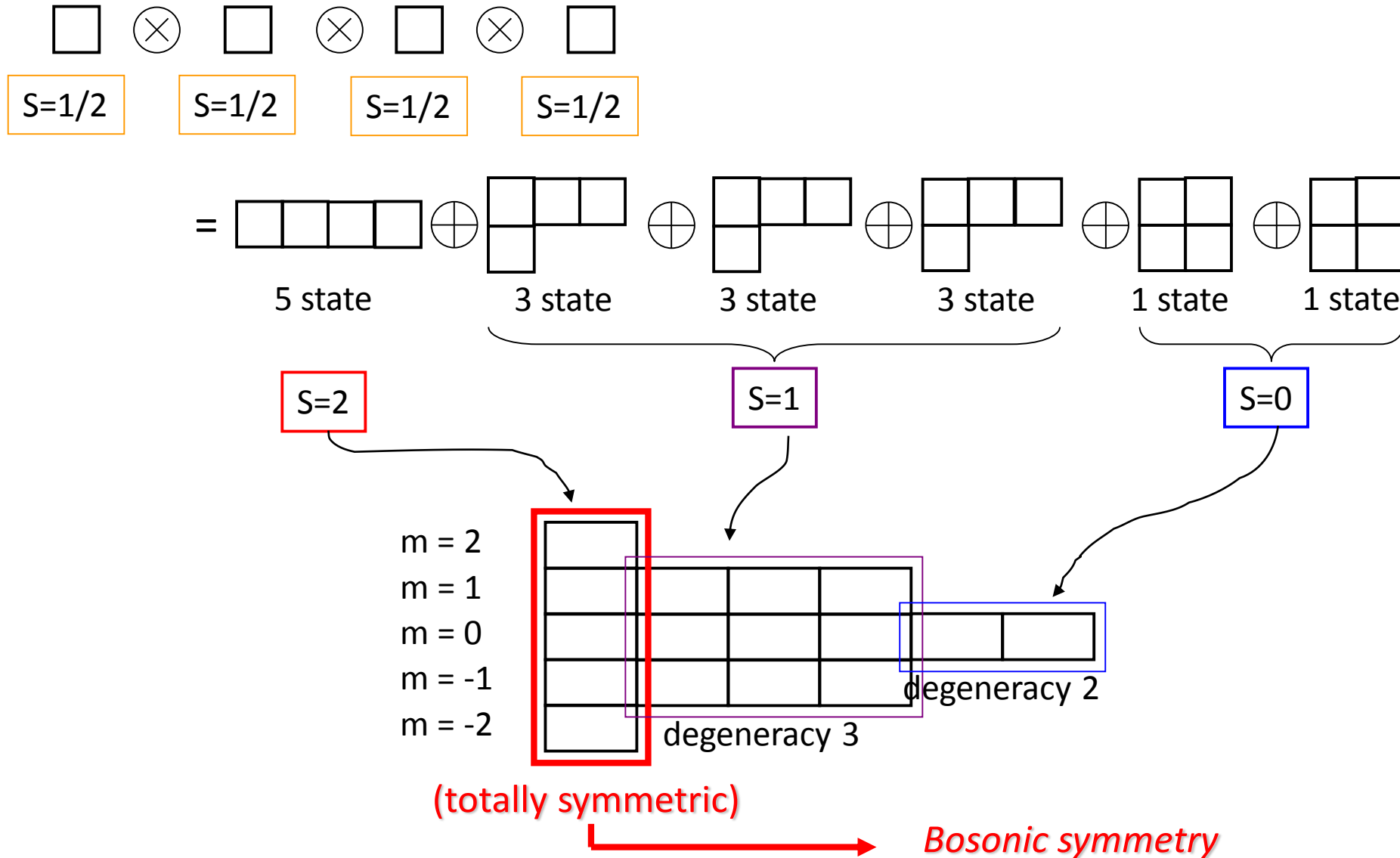
FIG. 4. (color) Diffraction-limited image of a two-ion crystal with $R = 1470$ nm. This determines the orientation of the interatomic vector \vec{R} enabling a no-free-parameter fit.

$S=0, m=0$

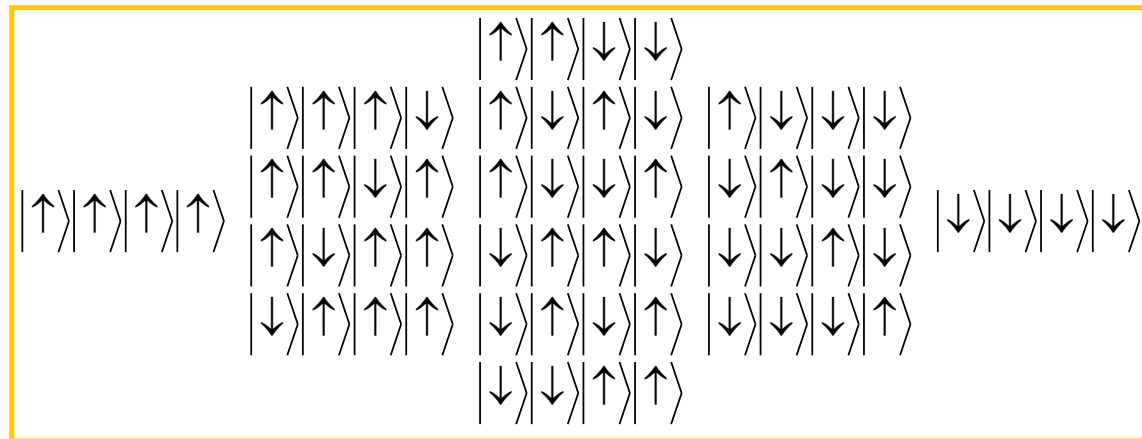


Permutation Symmetry

- in case of 4 spin-1/2 particles -



Second Quantization of Spin



$$|S = 2, m = 2\rangle = |\uparrow\rangle|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle \leftarrow |4\rangle_{\uparrow}|0\rangle_{\downarrow} \text{ (CSS)}$$

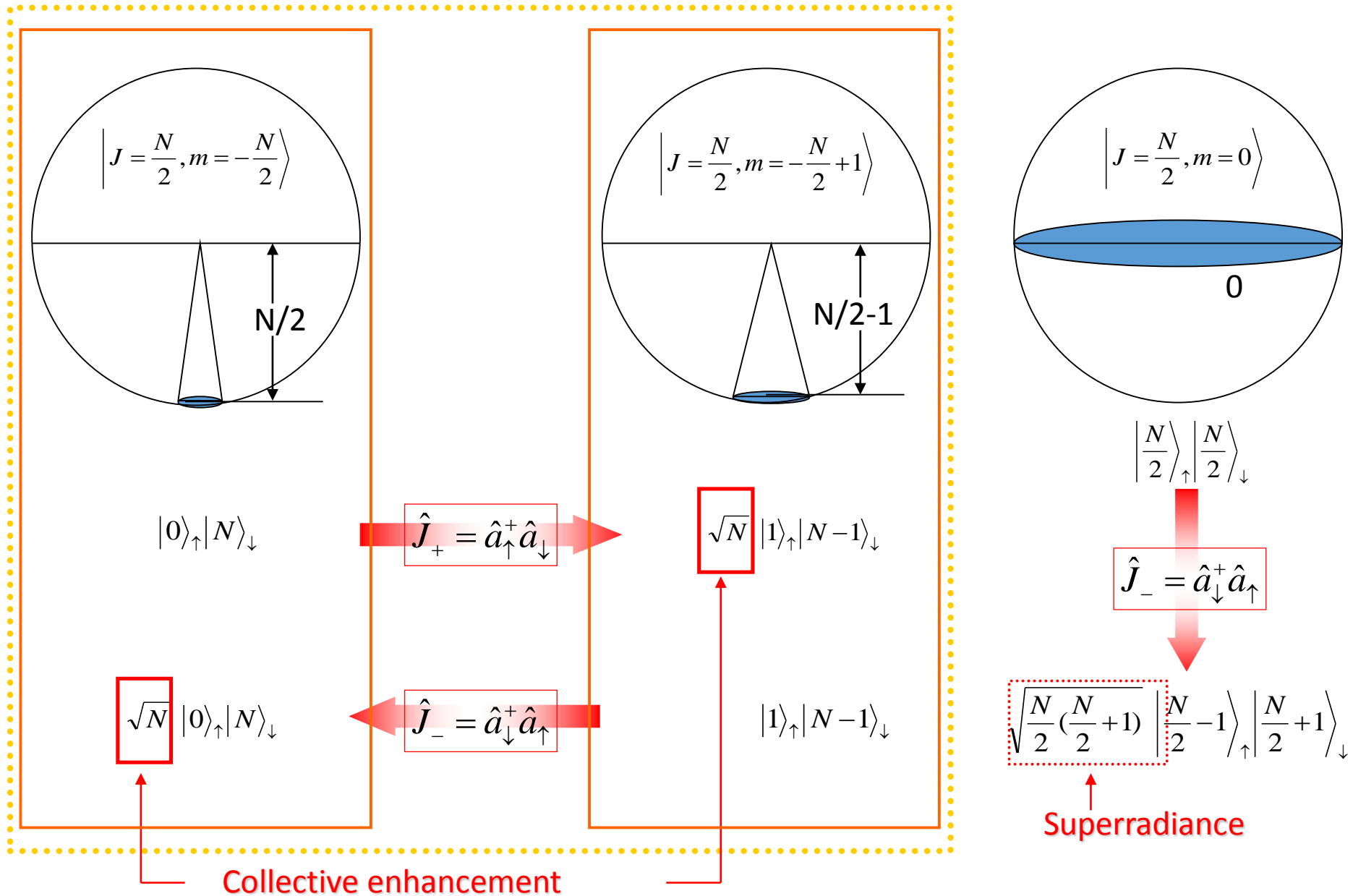
$$|S = 2, m = 1\rangle = \frac{1}{2} (|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle|\downarrow\rangle + |\uparrow\rangle|\uparrow\rangle|\downarrow\rangle|\uparrow\rangle + |\uparrow\rangle|\downarrow\rangle|\uparrow\rangle|\uparrow\rangle + |\downarrow\rangle|\uparrow\rangle|\uparrow\rangle|\uparrow\rangle) \leftarrow |3\rangle_{\uparrow}|1\rangle_{\downarrow} \text{ (Dicke, W)}$$

$$|S = 2, m = 0\rangle = \frac{1}{\sqrt{6}} (|\uparrow\rangle|\uparrow\rangle|\downarrow\rangle|\downarrow\rangle + |\uparrow\rangle|\downarrow\rangle|\uparrow\rangle|\downarrow\rangle + |\uparrow\rangle|\downarrow\rangle|\downarrow\rangle|\uparrow\rangle + |\downarrow\rangle|\uparrow\rangle|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle|\downarrow\rangle|\uparrow\rangle + |\downarrow\rangle|\downarrow\rangle|\uparrow\rangle|\uparrow\rangle)$$

$$|S = 2, m = -1\rangle = \frac{1}{2} (|\uparrow\rangle|\downarrow\rangle|\downarrow\rangle|\downarrow\rangle + |\downarrow\rangle|\uparrow\rangle|\downarrow\rangle|\downarrow\rangle + |\downarrow\rangle|\downarrow\rangle|\uparrow\rangle|\downarrow\rangle + |\downarrow\rangle|\downarrow\rangle|\downarrow\rangle|\uparrow\rangle) \leftarrow |2\rangle_{\uparrow}|2\rangle_{\downarrow} \text{ (Super-radiant)}$$

$$|S = 2, m = -2\rangle = |\downarrow\rangle|\downarrow\rangle|\downarrow\rangle|\downarrow\rangle \leftarrow |0\rangle_{\uparrow}|4\rangle_{\downarrow} \text{ (CSS)} \quad |1\rangle_{\uparrow}|3\rangle_{\downarrow} \text{ (Dicke, W)}$$

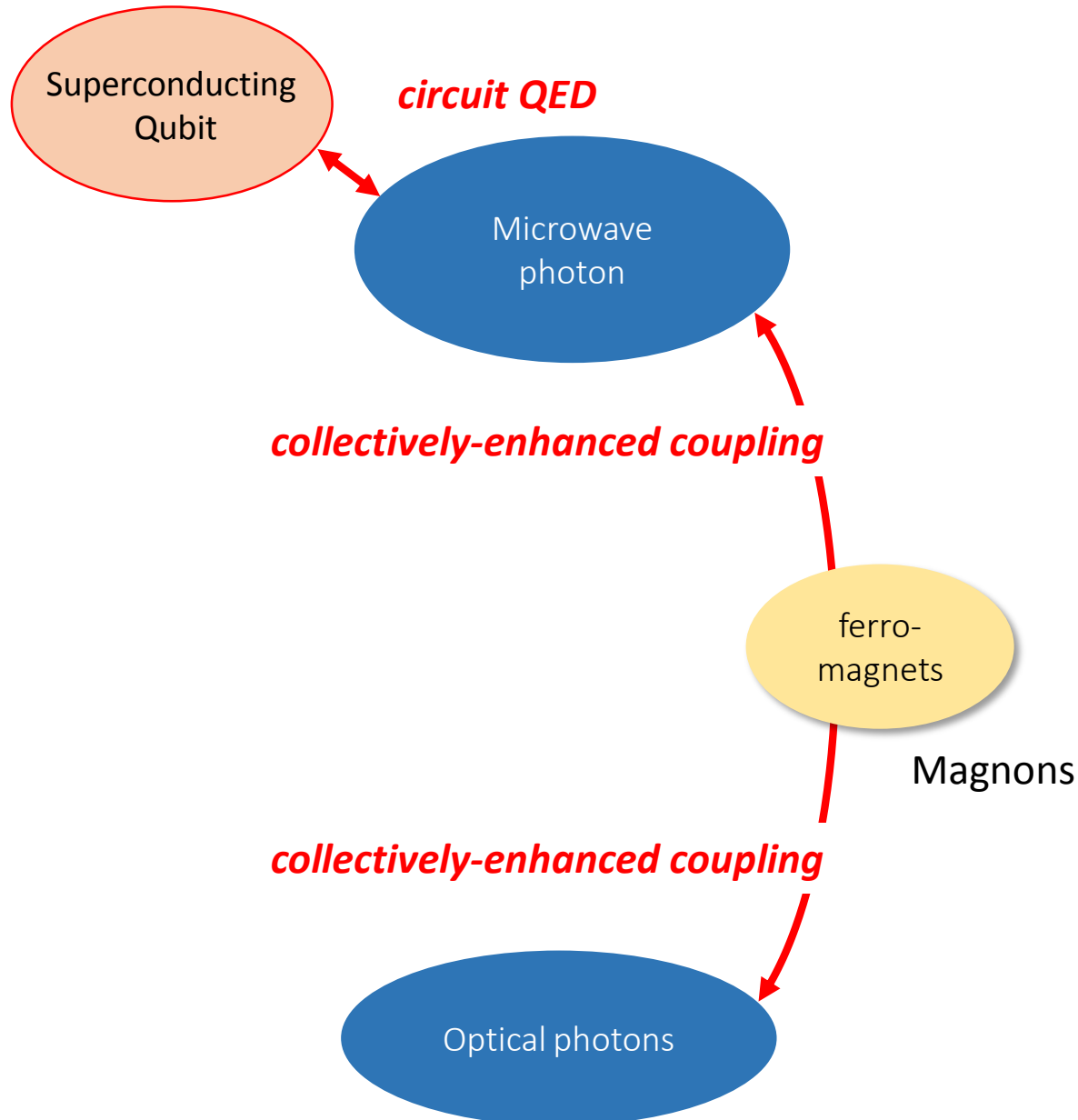
Collective enhancement



Cavity QED and collective enhancement

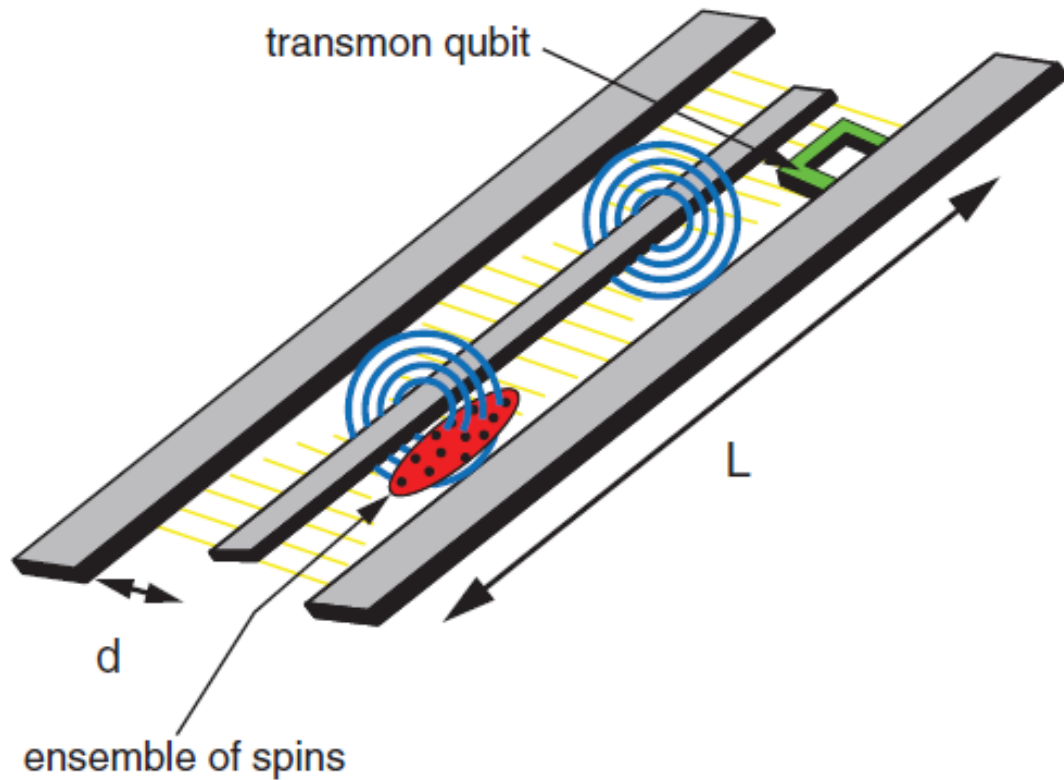
	Cavity QED	Collective enhancement
Coherent coupling rate	$g_e = d \sqrt{\frac{\omega}{2\hbar\epsilon_0 V}}$ <p>Rydberg atom (pointing to d) Micro cavity (pointing to V)</p>	$g_E = g_e \sqrt{N}$
Single-atom decay rate	$\gamma_A = \frac{2\pi}{3} \frac{V\omega^3}{\pi^2 c^3} g_e^2$	$\gamma_A = \frac{2\pi}{3} \frac{V\omega^2}{\pi^2 c^3} g_e^2$
Cavity decay rate	κ_c	$\kappa_L = \frac{c}{L}$
Cooperativity (SNR)	$C_s = \frac{4g_e^2}{\gamma_A \kappa_c}$	$C_e = \frac{4g_E^2}{\gamma_A \kappa_L} = \frac{4N g_e^2}{\gamma_A \kappa_L}$

Magnon transducer architecture



- Superconducting qubit
circuit QED
- Microwave
collectively-enhanced coupling
- Magnon
collectively-enhanced coupling
- light

Paramagnetic spin ensemble quantum transducers



- **Circuit QED**

$$g_e = ea_0 \sqrt{\frac{\omega}{2\hbar\epsilon_0 V}}$$

- **Collective enhancement**

$$g_m = g_0 \sqrt{N}$$

where single-spin coupling strength:

$$g_0 = \frac{2\mu_B}{\hbar} \sqrt{\frac{\mu_0 \hbar \omega}{2V}} = \frac{2\mu_B}{c} \sqrt{\frac{\omega}{2\hbar\epsilon_0 V}} = \alpha g_e$$

Experiments

Superconducting qubit- magnon coupling

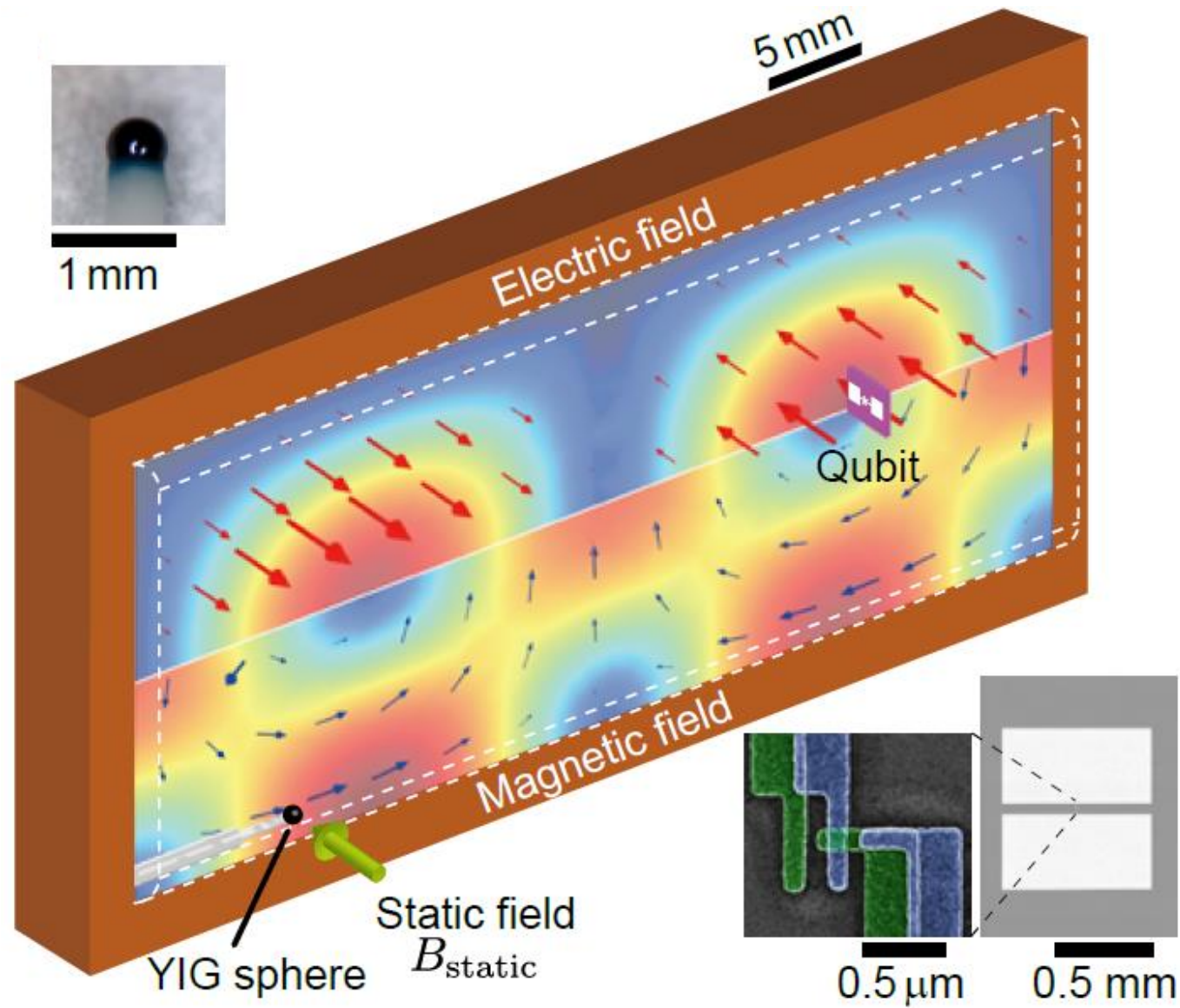


**SUPERCONDUCTING
QUBIT(TRANSMON)**



**FERROMAGNET
CRYSTAL**

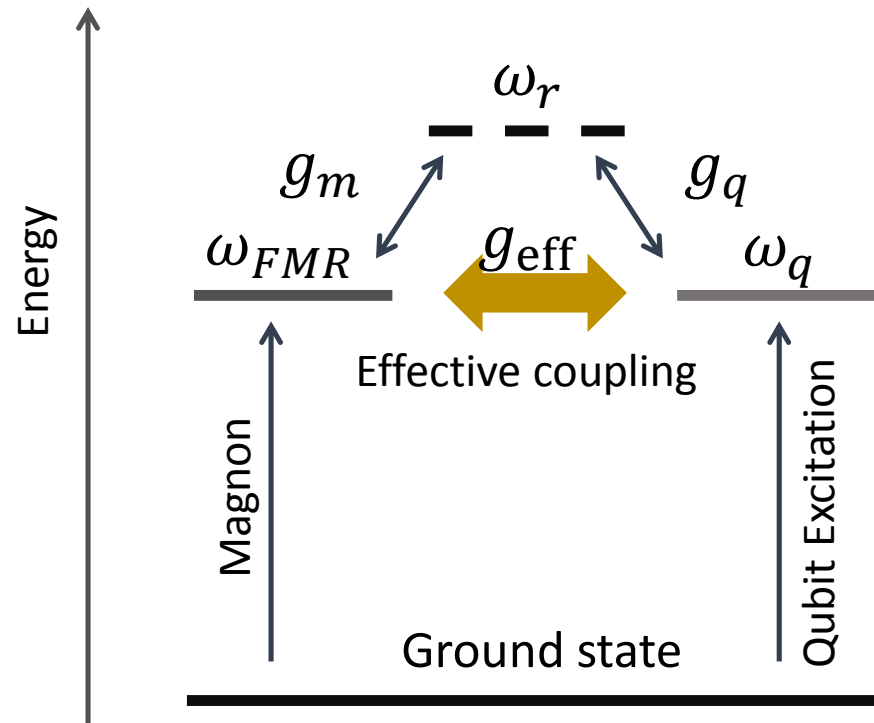
Coupling superconducting qubit with magnon



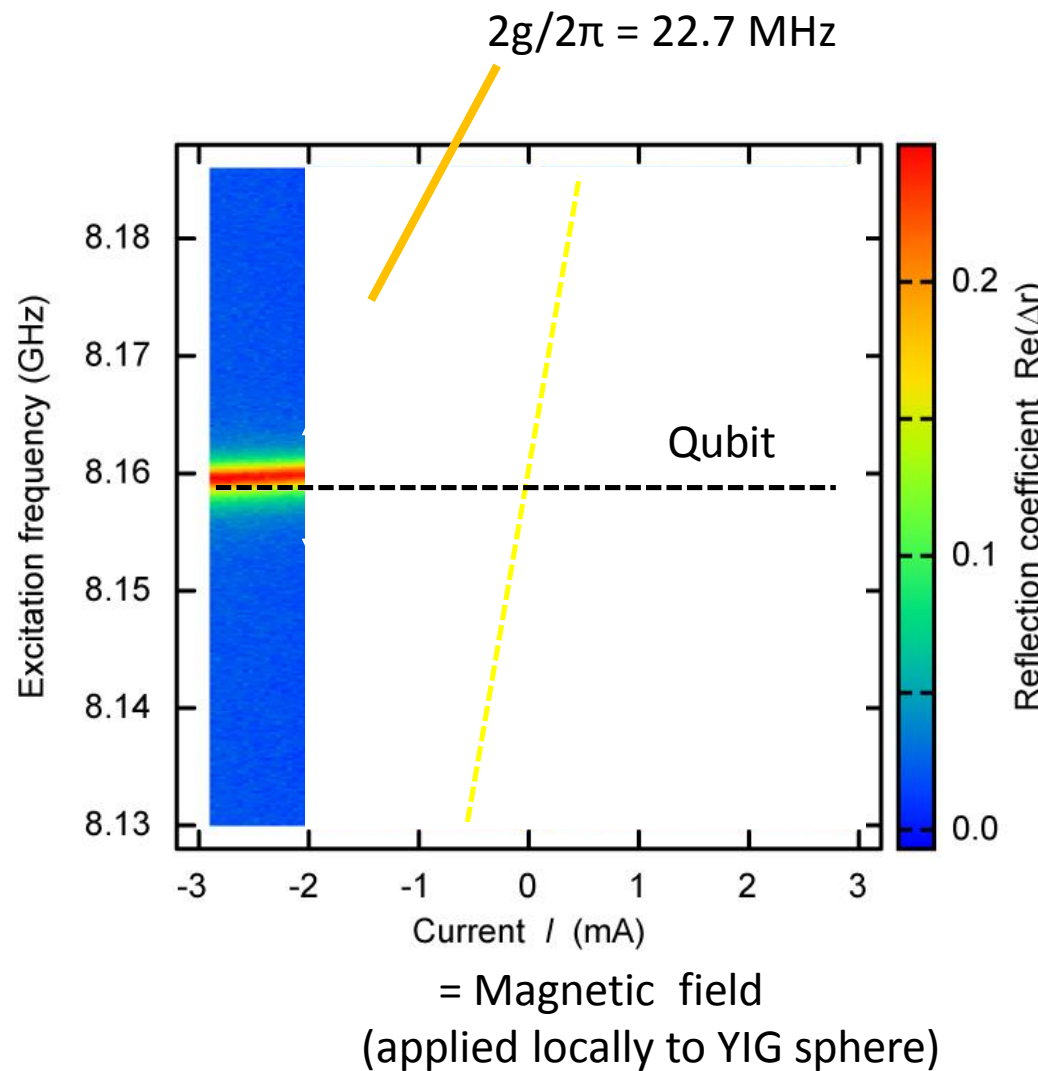
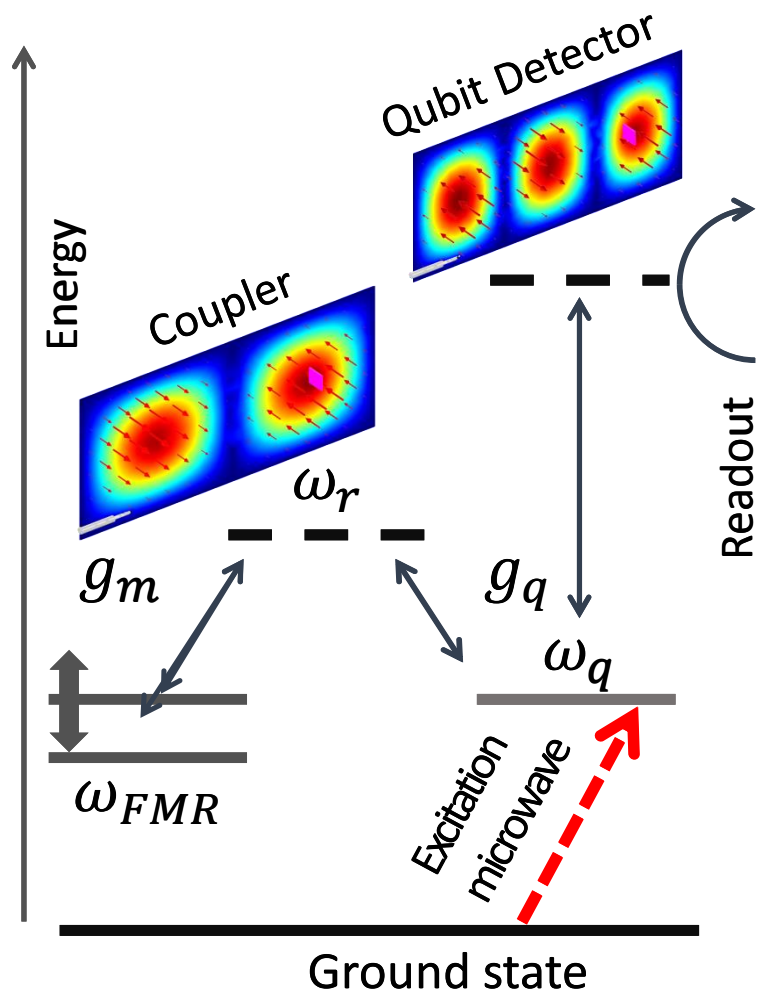
Magnon – cavity - qubit hybrid system

- Magnon-qubit coupling mediated by cavity

$$\hat{\mathcal{H}}/\hbar \sim \frac{g_m g_q}{\omega_r - \omega_{\text{qubit}}} (\hat{a}_m^\dagger \hat{\sigma}^- + \hat{a}_m \hat{\sigma}^+)$$

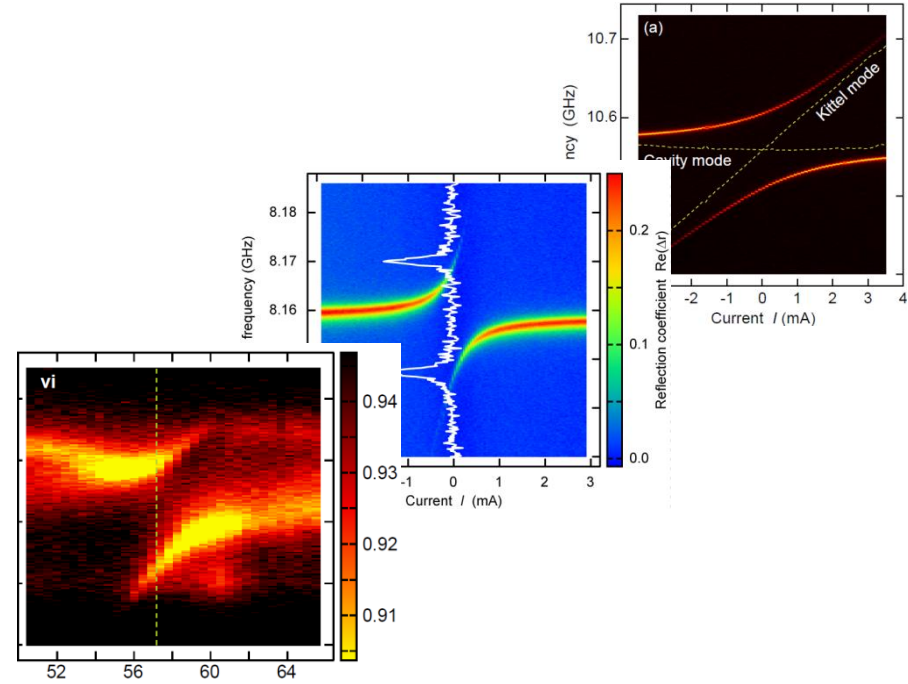
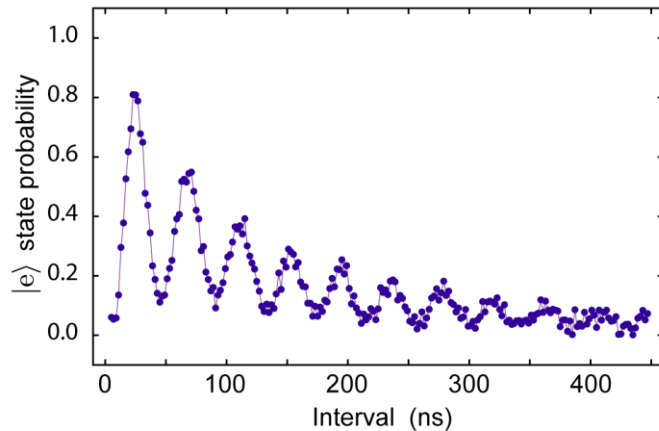


Magnon Vacuum Rabi Splitting



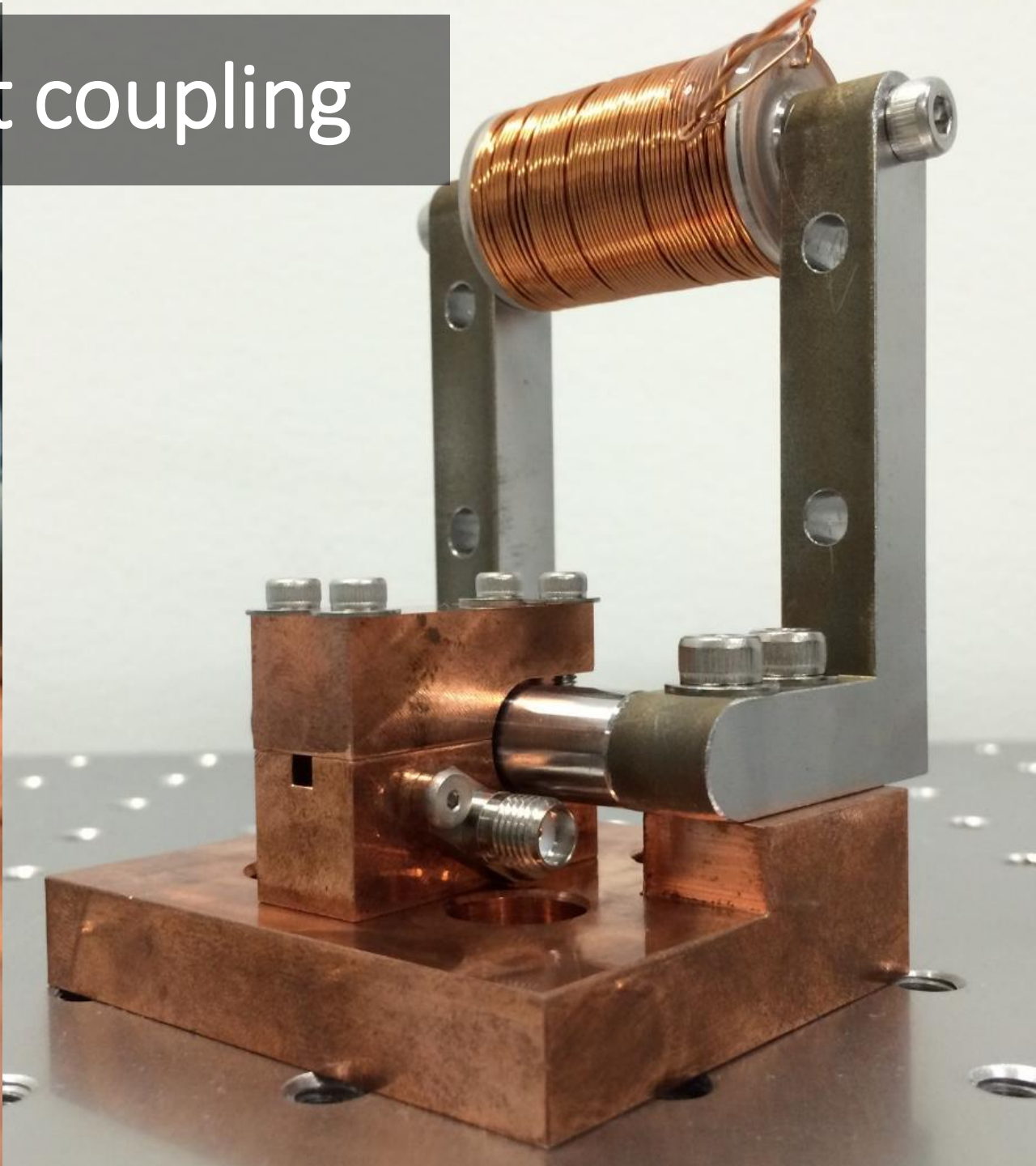
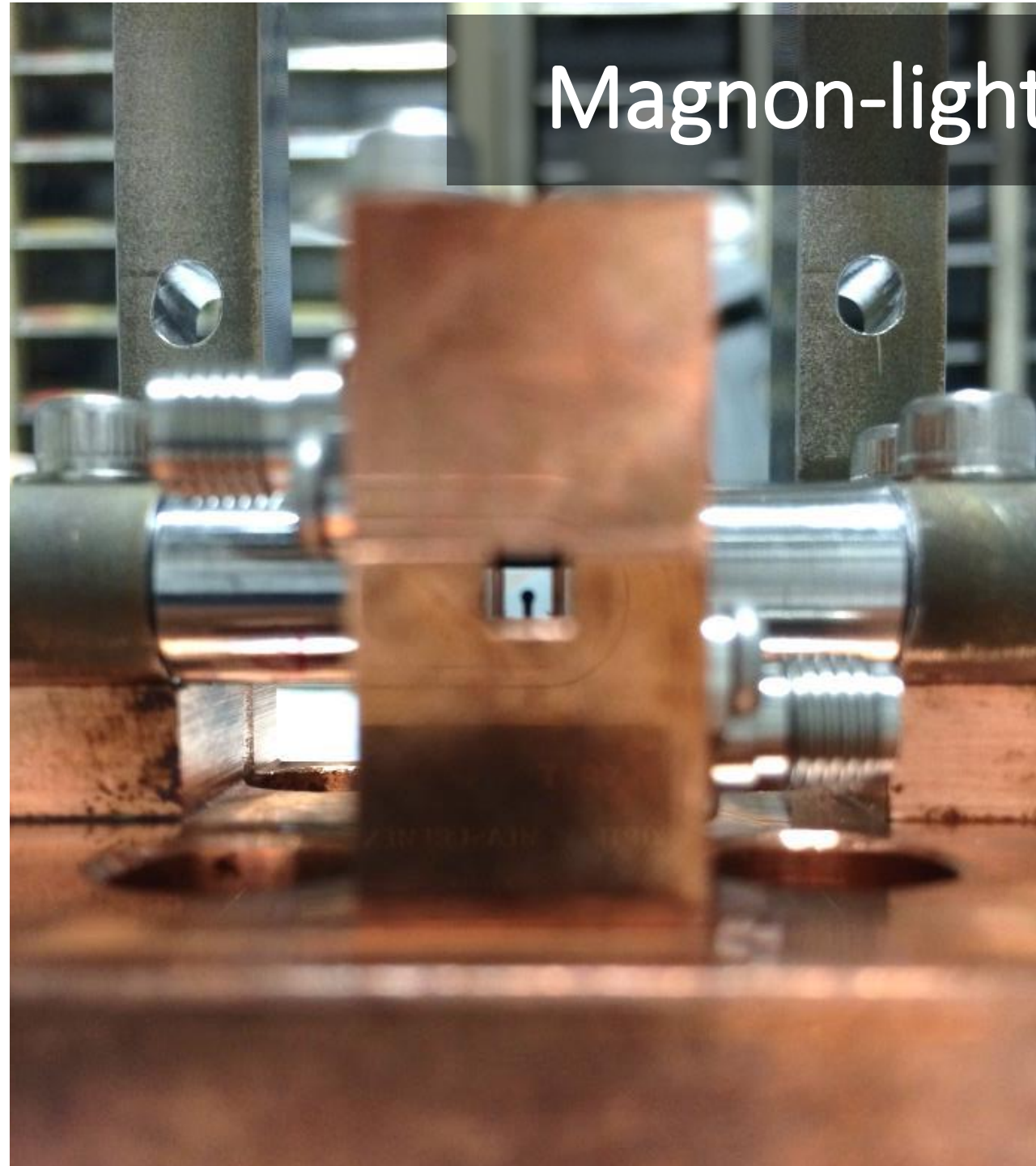
SUMMARY 1

- Strong magnon-photon coupling
- Strong magnon-qubit coupling
- Tunable coupling by parametric driving



- Time-domain control & readout magnons
- Non-classical magnon control
- Magnon-state tomography
- Coupling to light

Magnon-light coupling



Collective enhancements

Microwave
photon

ferro-
magnets

Optical photons

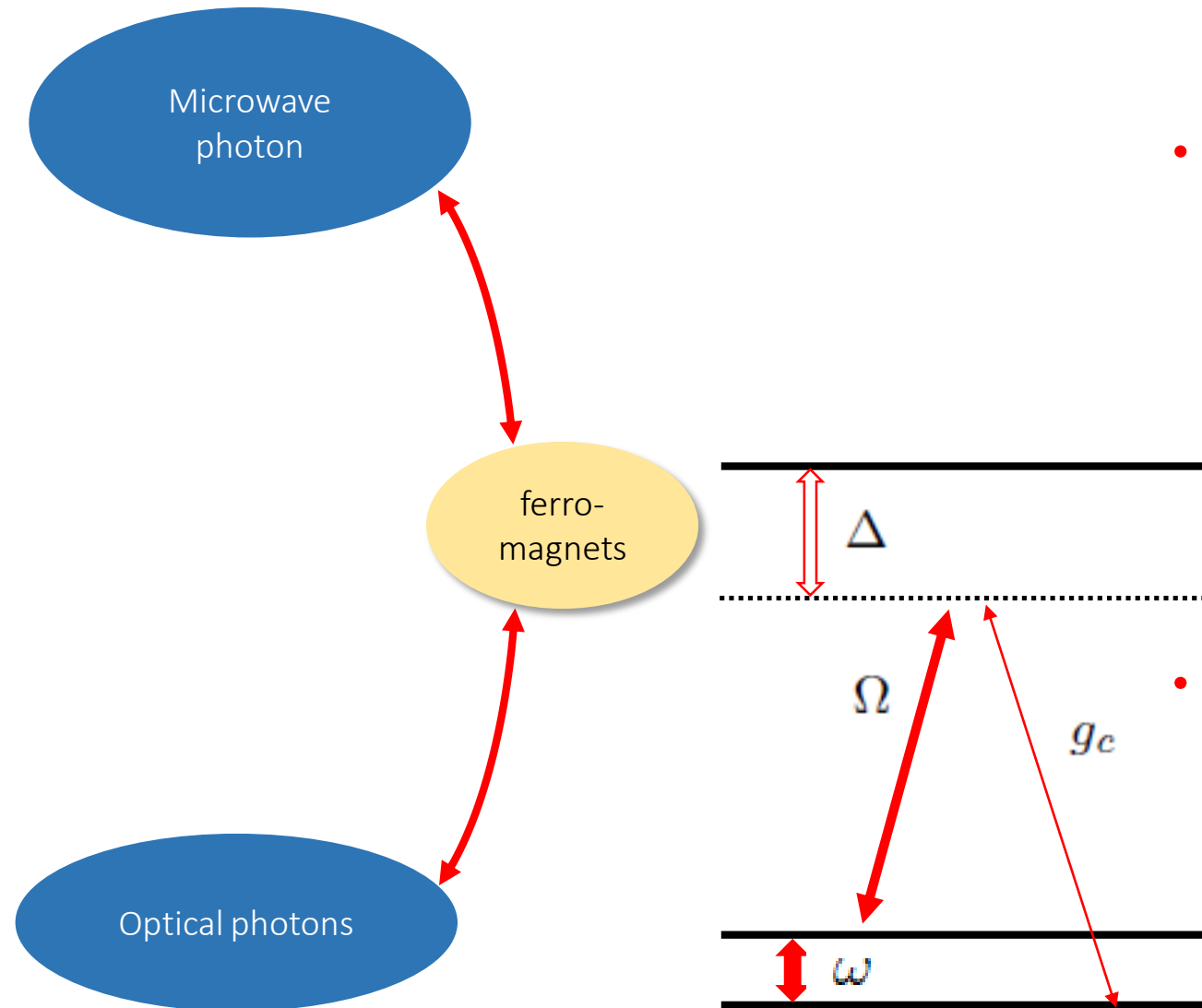
- **(Resonant) collective enhancement**

$$g_m = g_0 \sqrt{N}$$

where single-spin coupling strength:

$$g_0 = \frac{2\mu_B}{\hbar} \sqrt{\frac{\mu_0 \hbar \omega}{2V}} = \frac{2\mu_B}{c} \sqrt{\frac{\omega}{2\hbar \epsilon_0 V}} = \alpha g_e$$

Collective enhancements



- (Resonant) collective enhancement

$$g_m = g_0 \sqrt{N}$$

where single-spin coupling strength:

$$g_0 = \frac{2\mu_B}{\hbar} \sqrt{\frac{\mu_0 \hbar \omega}{2V}} = \frac{2\mu_B}{c} \sqrt{\frac{\omega}{2\hbar \epsilon_0 V}} = \alpha g_e$$

- Raman collective enhancement (DLCZ)

$$\zeta = N \frac{|\Omega g_e|^2}{\Delta^2 \kappa}$$

Light-magnon coupling scheme

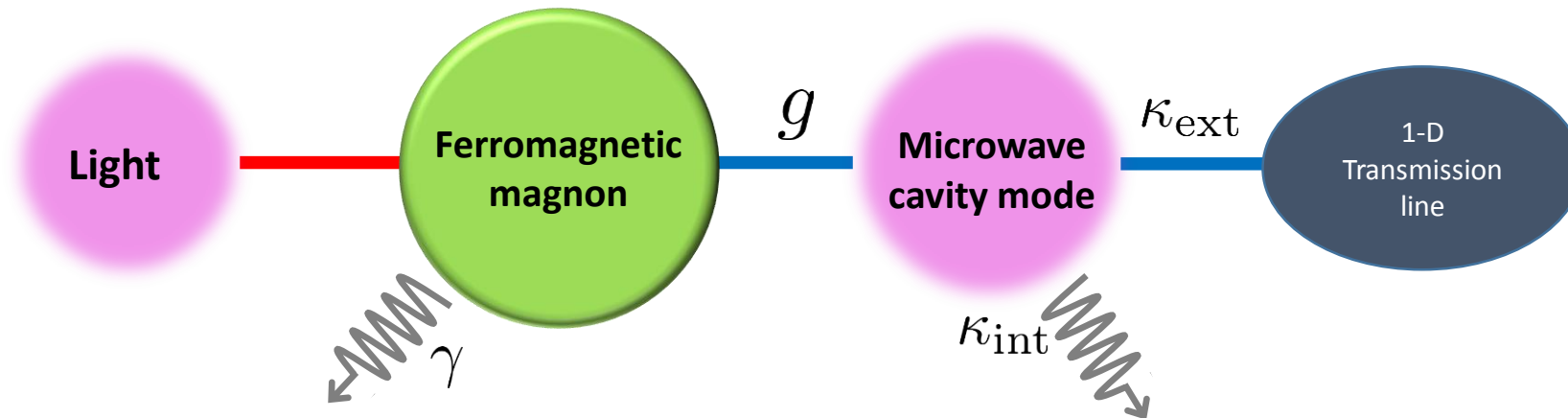
Faraday interaction:
$$\hat{H}_I = \int_0^\tau \hbar G \hat{j}_x(t') \hat{s}_x(t') A c dt'$$

Collective spin operator

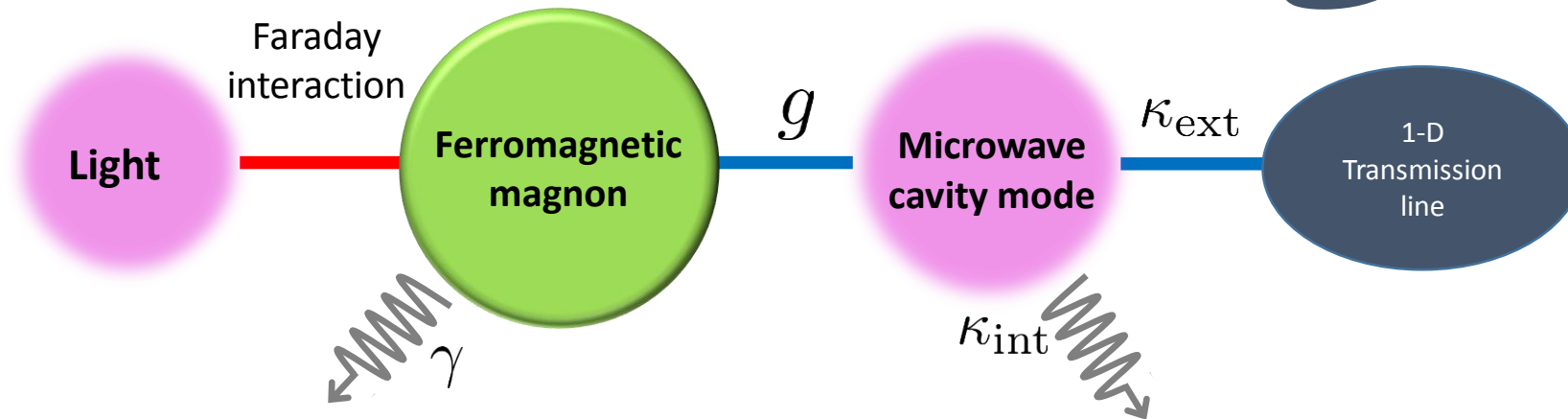
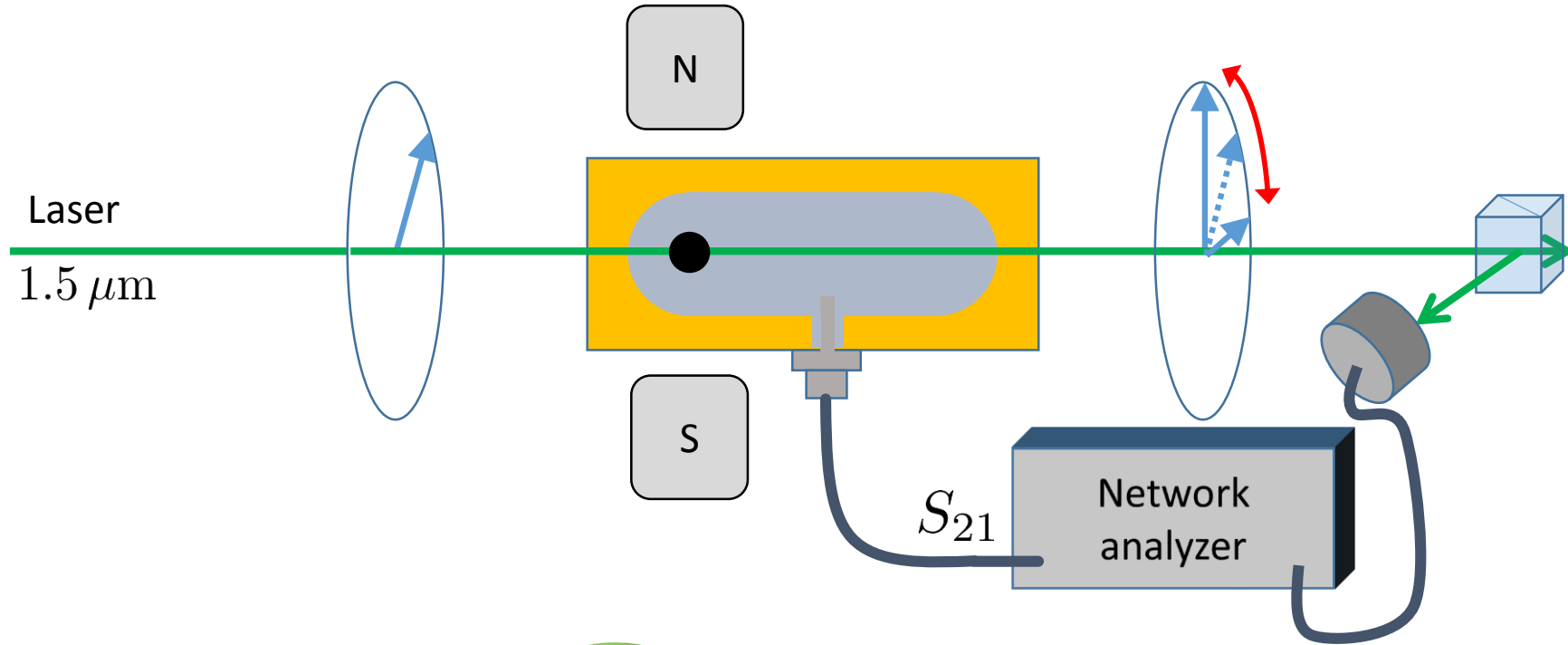
Stokes operator

Evolution of Stokes operator:

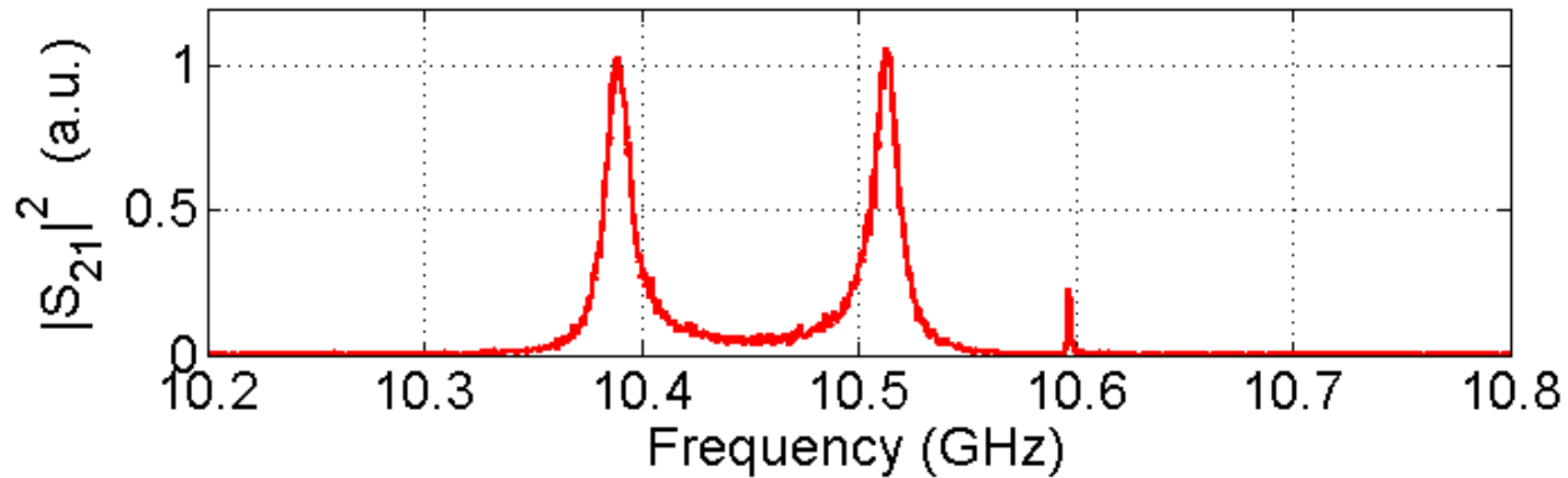
$$\frac{\partial}{\partial t} \hat{s}_\alpha(t) = \frac{1}{i\hbar} [\hat{s}_\alpha(t), \hat{H}_I] \quad (\text{Faraday rotation})$$



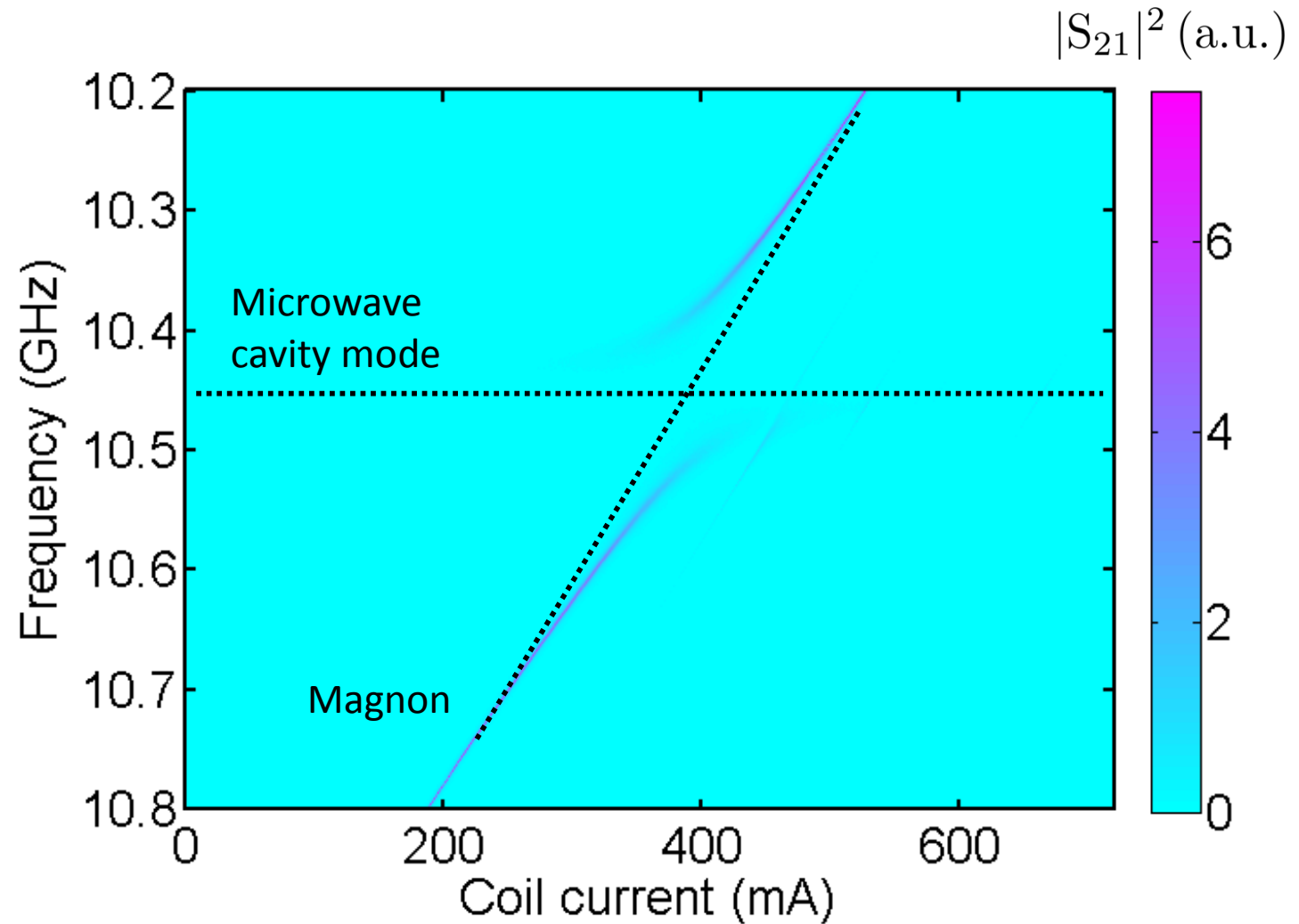
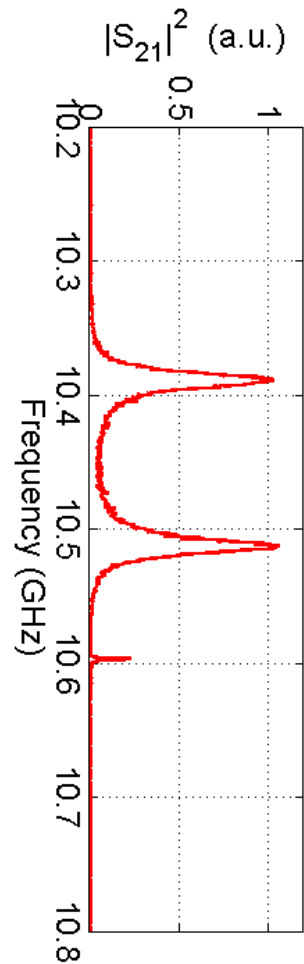
Light-magnon coupling scheme



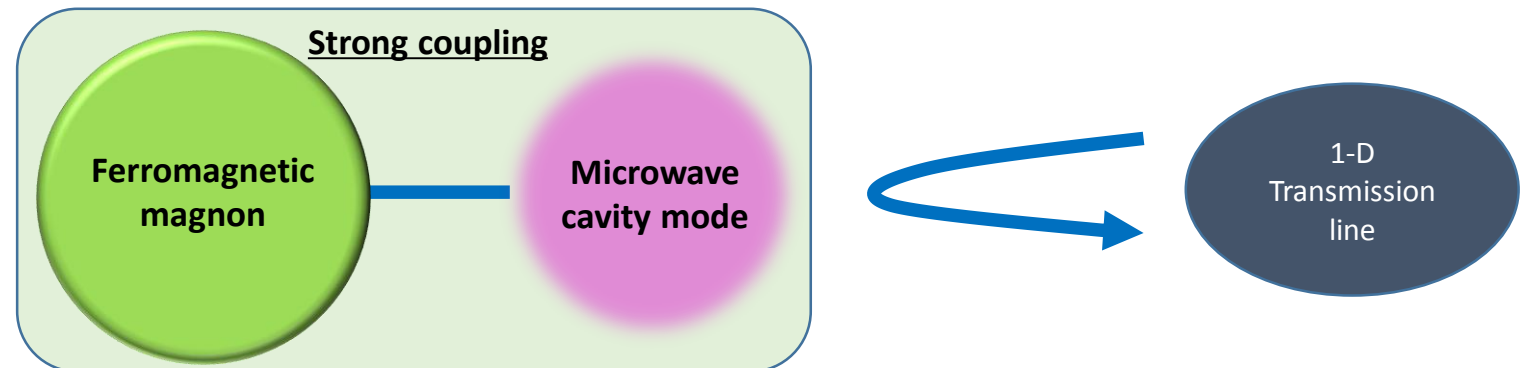
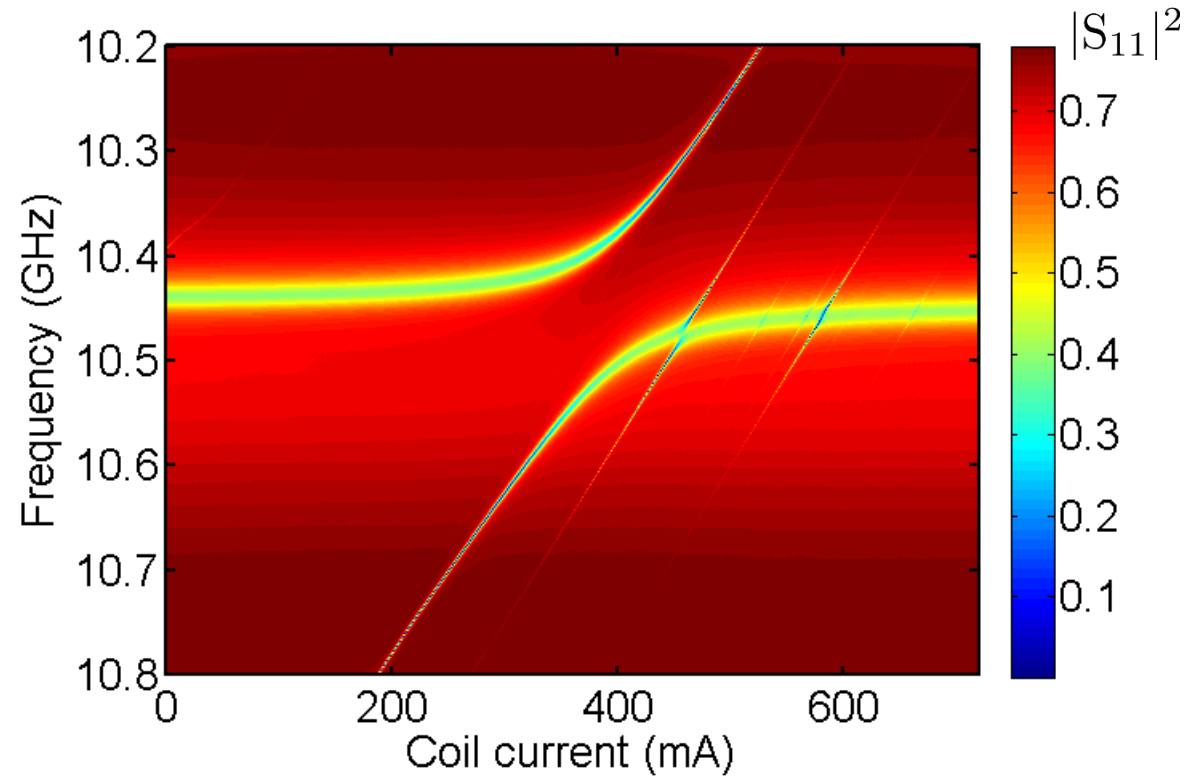
Sideband appear at normal mode frequencies



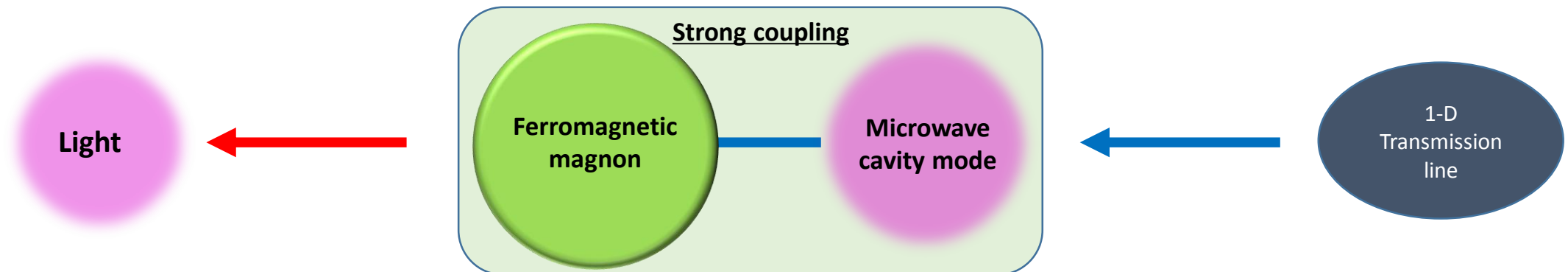
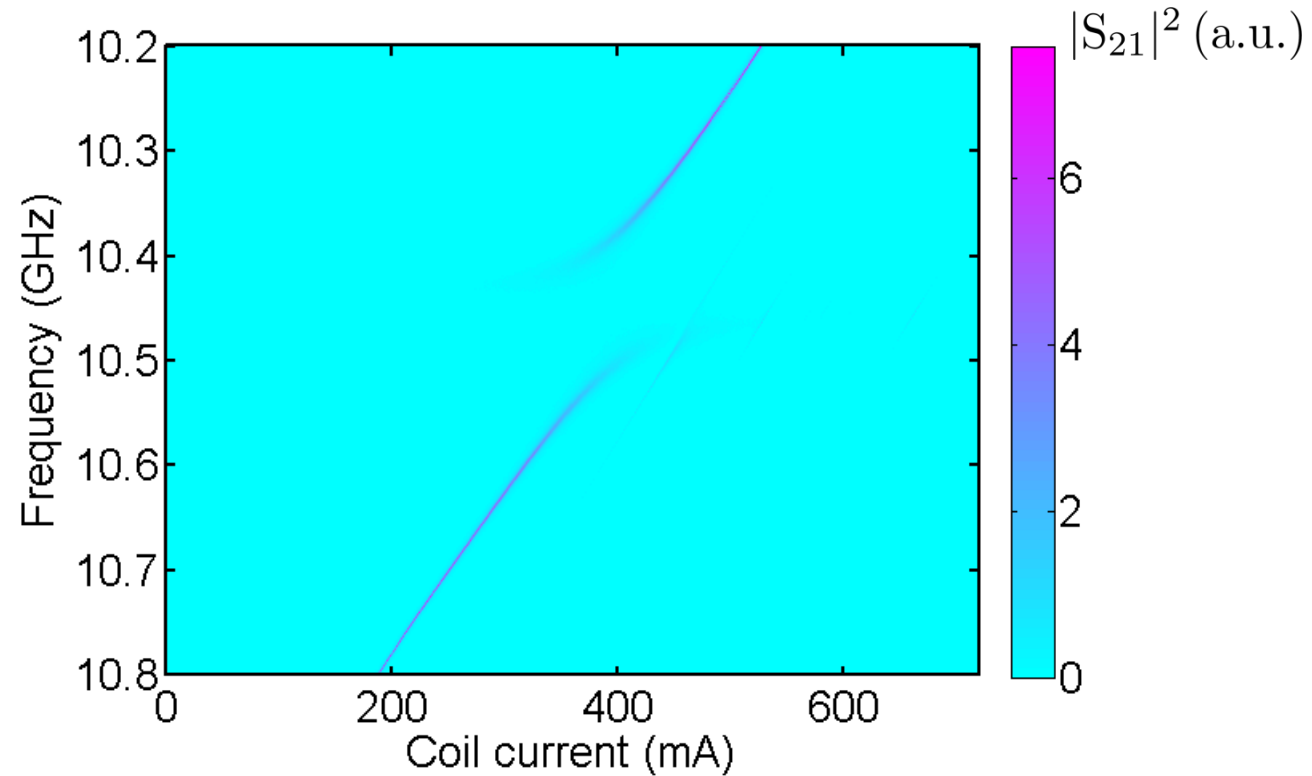
Magnon-cavity coupling probed by light



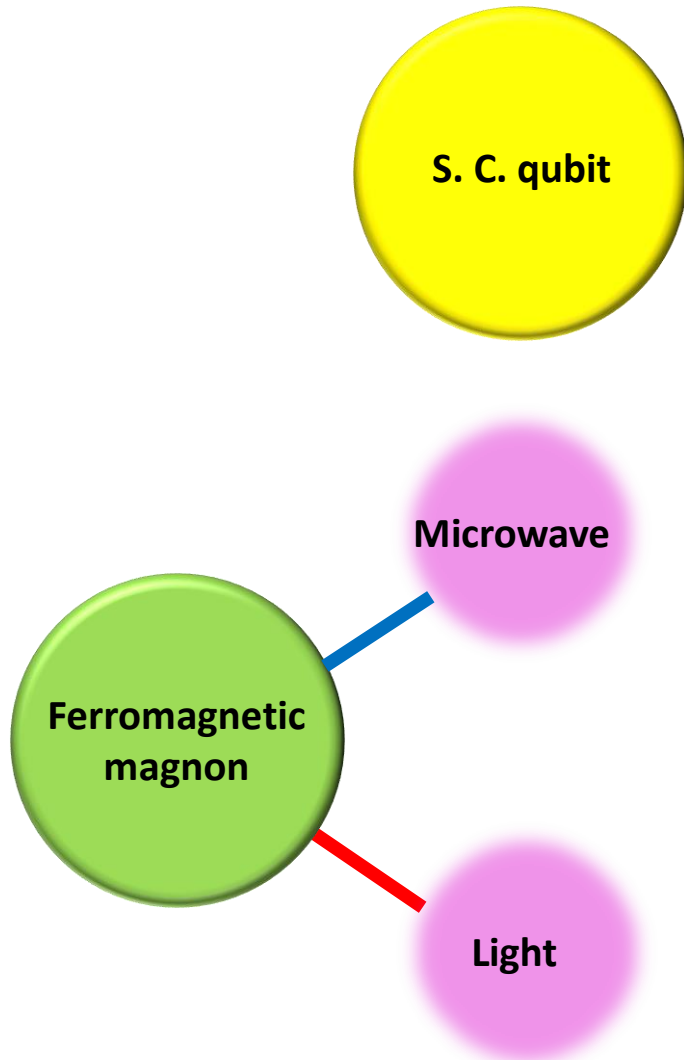
Microwave-light transducer



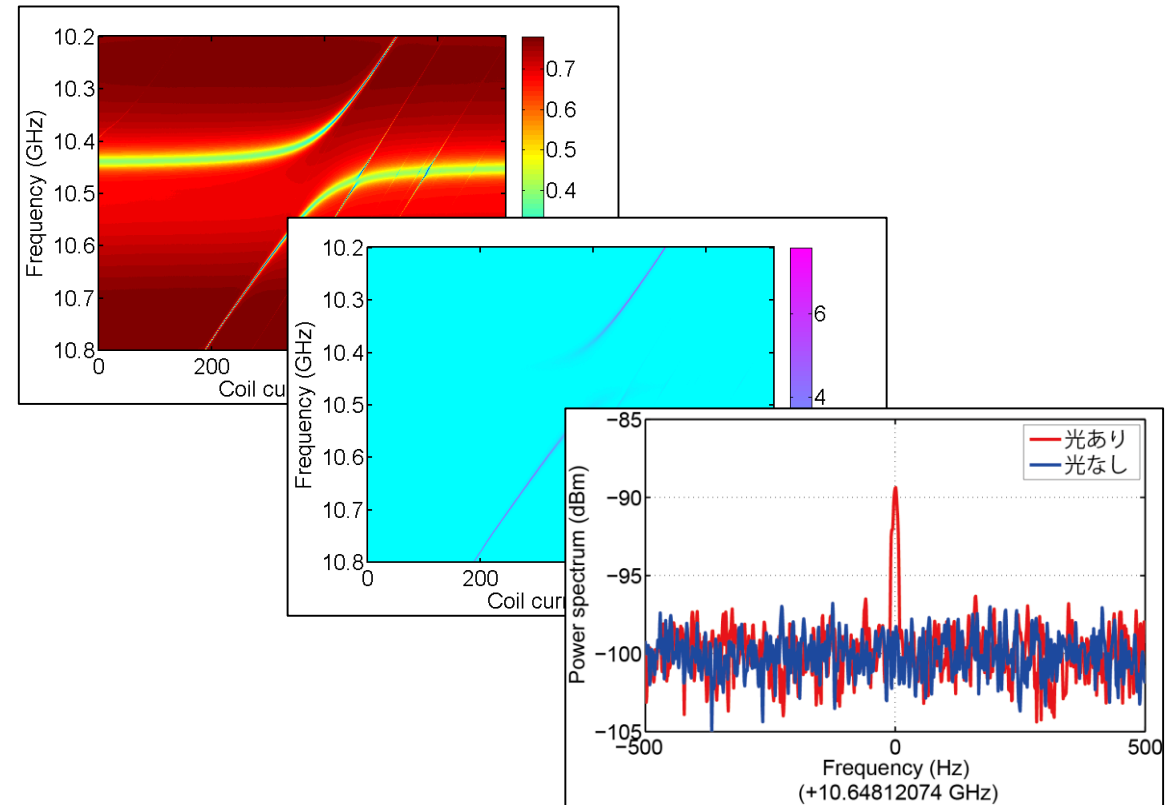
Microwave-light transducer



SUMMARY 2



- ◆ Realization of strong coupling magnon and M.W.
- ◆ Observation of the coupling system using light
- ◆ Realization of coherent m.w. generation using light



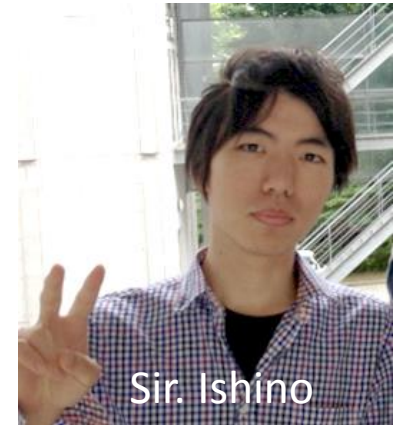
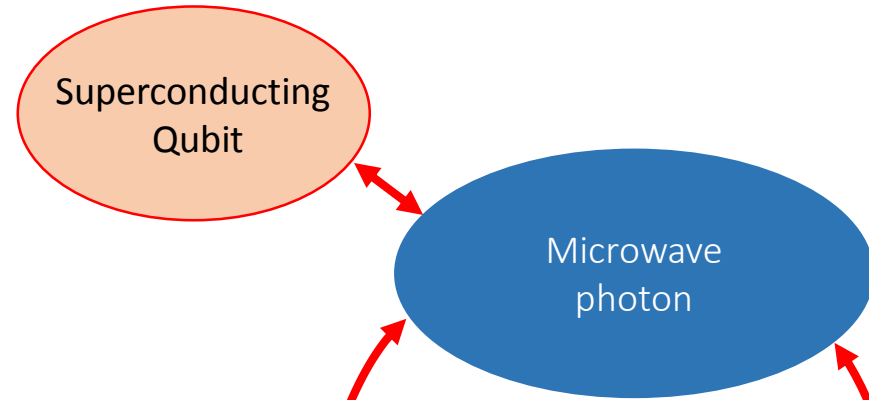
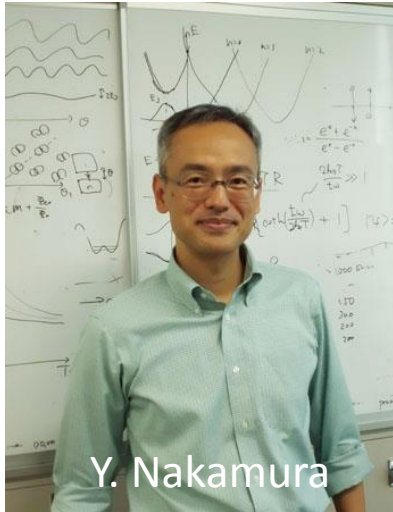
Why microwave?

- Small dissipation (superconductivity)
- Strong nonlinearity (Josephson effect)
- Strong confinement (ultra-small mode volume)

Why collective excitations?

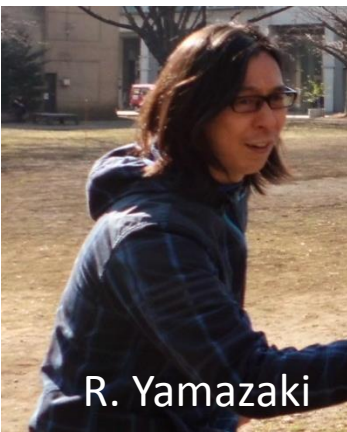
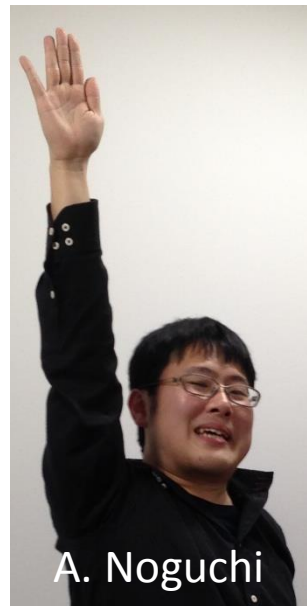
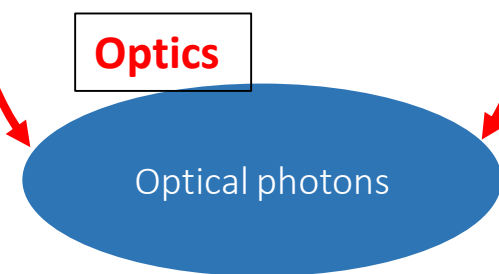
- Large dimensions
- Easy mode-matching
- Collectively-enhanced coupling

Overview of Research activities



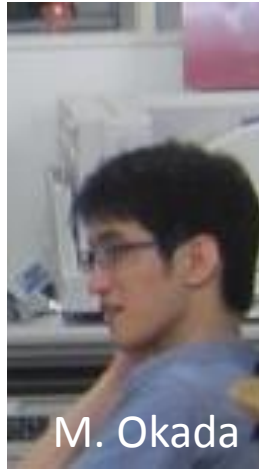
Phonons

Magnons



Overview of Research activities

Optomechanics-NMR



Waseda
Iwase lab

Kyoto
Takeda lab

Cavity cooling of magnon



UEC
Hakuta lab

Tokyo
Nomura lab



Thank you!

