

Backscattering TMS

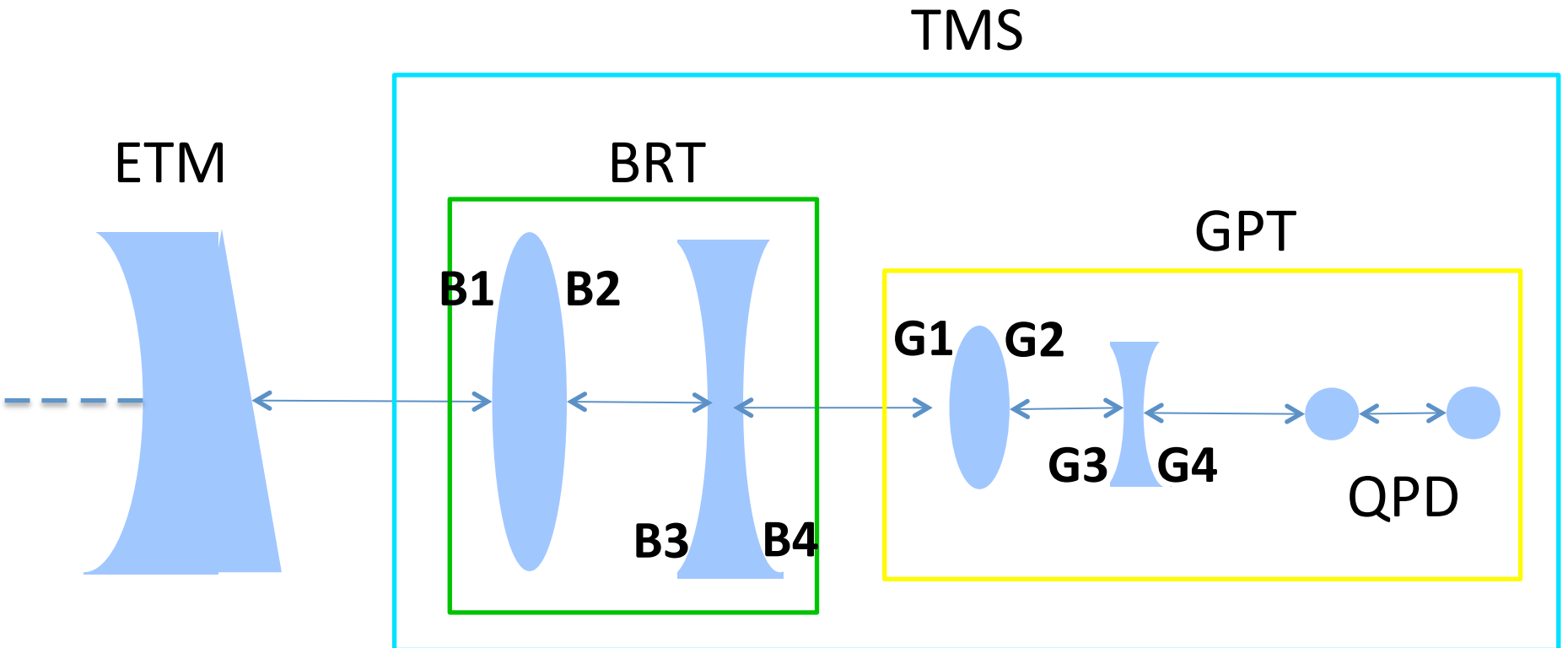
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What I did

I computed the backscattering noise on the each surface of BRT and GPT lenses.

- Simple estimation
- Including radiation pressure
- up-conversion
- up-conversion using the relative motion between ETM and TMS elements

Transmission Monitor System



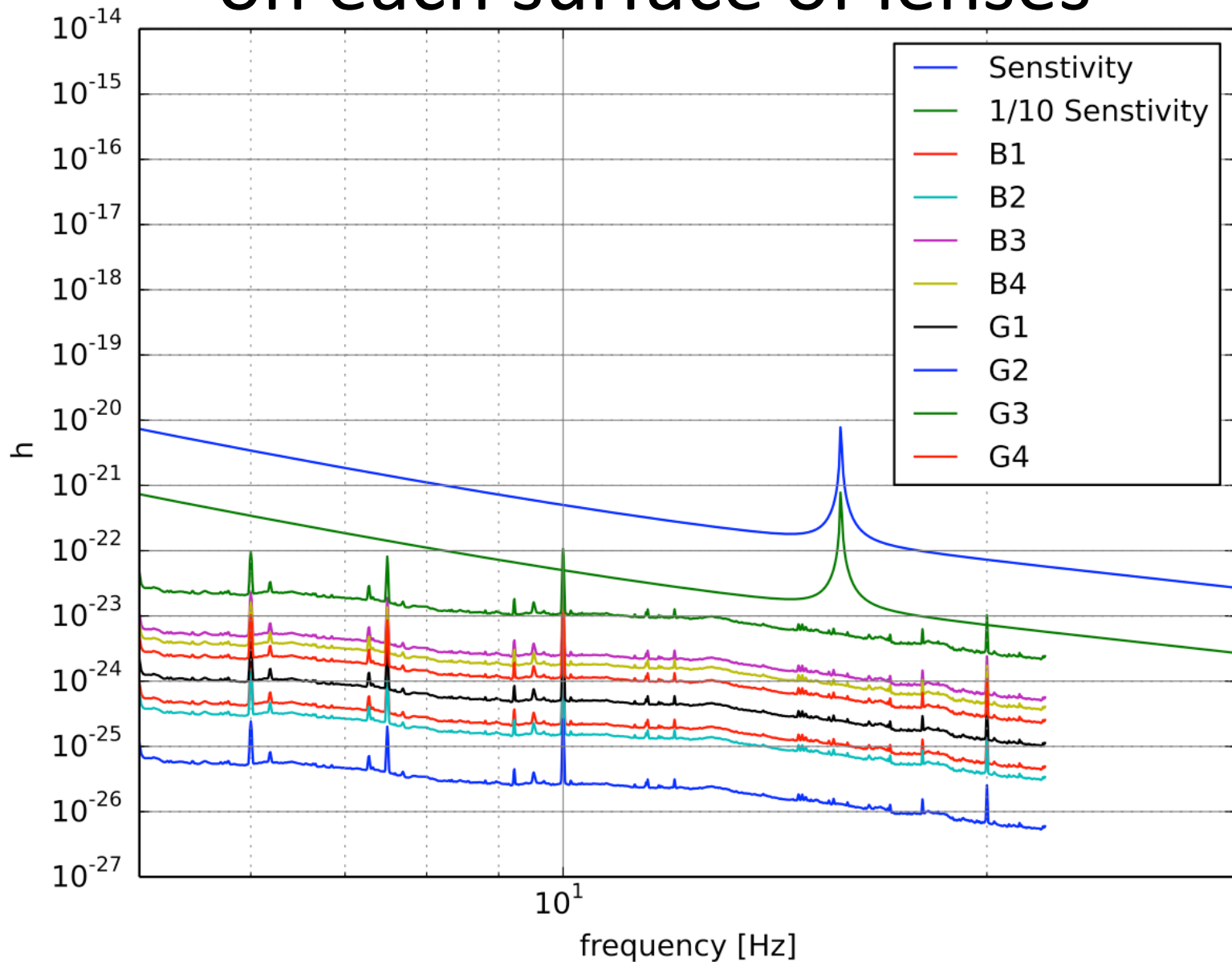
Simple estimation

$$\underline{\Phi(t) \ll 1}$$

$$h = \text{sqrt}(f_{sc}) * T/L * \delta x$$

$$f_{sc} = |\text{overlap integral}|^2 * R_{AR}$$

Simple estimation on each surface of lenses



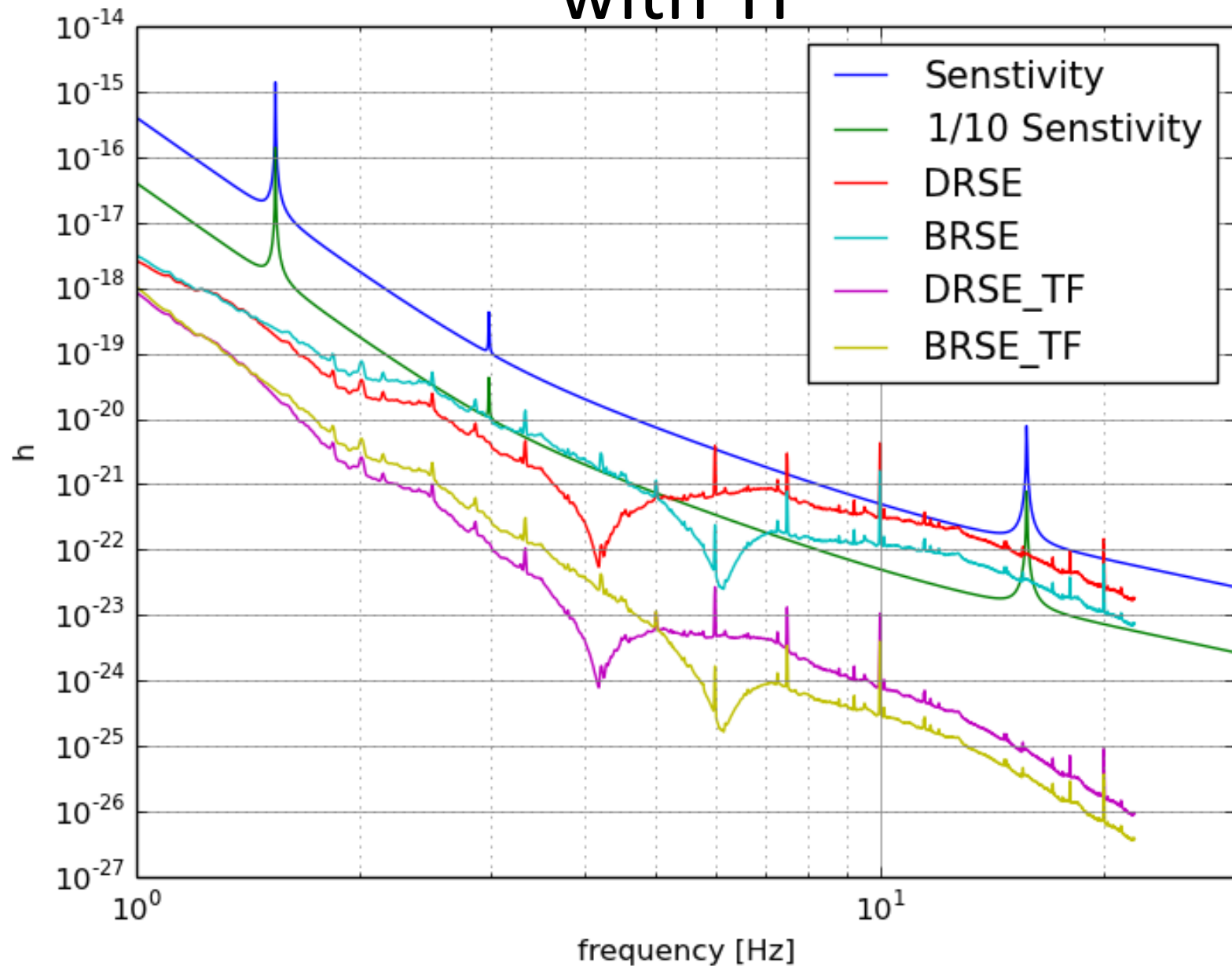
Including radiation pressure

- $h = G \cdot \sqrt{f_{sc} \cdot T \cdot P_{cav} / P_{in}} / L \cdot 4\pi / \lambda \cdot \delta x$
(G is given by Aso-san)

- Transfer Function (Simple pendulum)

$$TF = 1 / (1 - \omega^2 / \omega_0^2 + i\omega / \omega_0 \cdot 1/Q)$$

Including radiation pressure with TF



up-conversion

$$E_{sc} * e^{i\Omega t} [\cos(\phi(t)) + \underline{i \sin(\phi(t))}]$$

$$\underline{\phi(t) \ll 1}$$

$$h = G * \sqrt{f_{sc} * T * P_{cav} / P_{in}} / L * \frac{4\pi / \lambda * \delta x}{P_{\phi}(\omega)}$$

$$\underline{\phi(t) \gg 1}$$

Up-conversion ; $\phi(t) \rightarrow \sin(\phi(t))$

$$P_{\phi}(\omega) \rightarrow P_{\sin\phi}(\omega) \equiv P_a(\omega)$$

autocorrelation function

$$C_{\Phi}(\tau) = \int_{-\infty}^{\infty} \frac{P_{\Phi}(\omega) e^{i\omega\tau} d\omega}{\text{既知}}$$

既知
already know

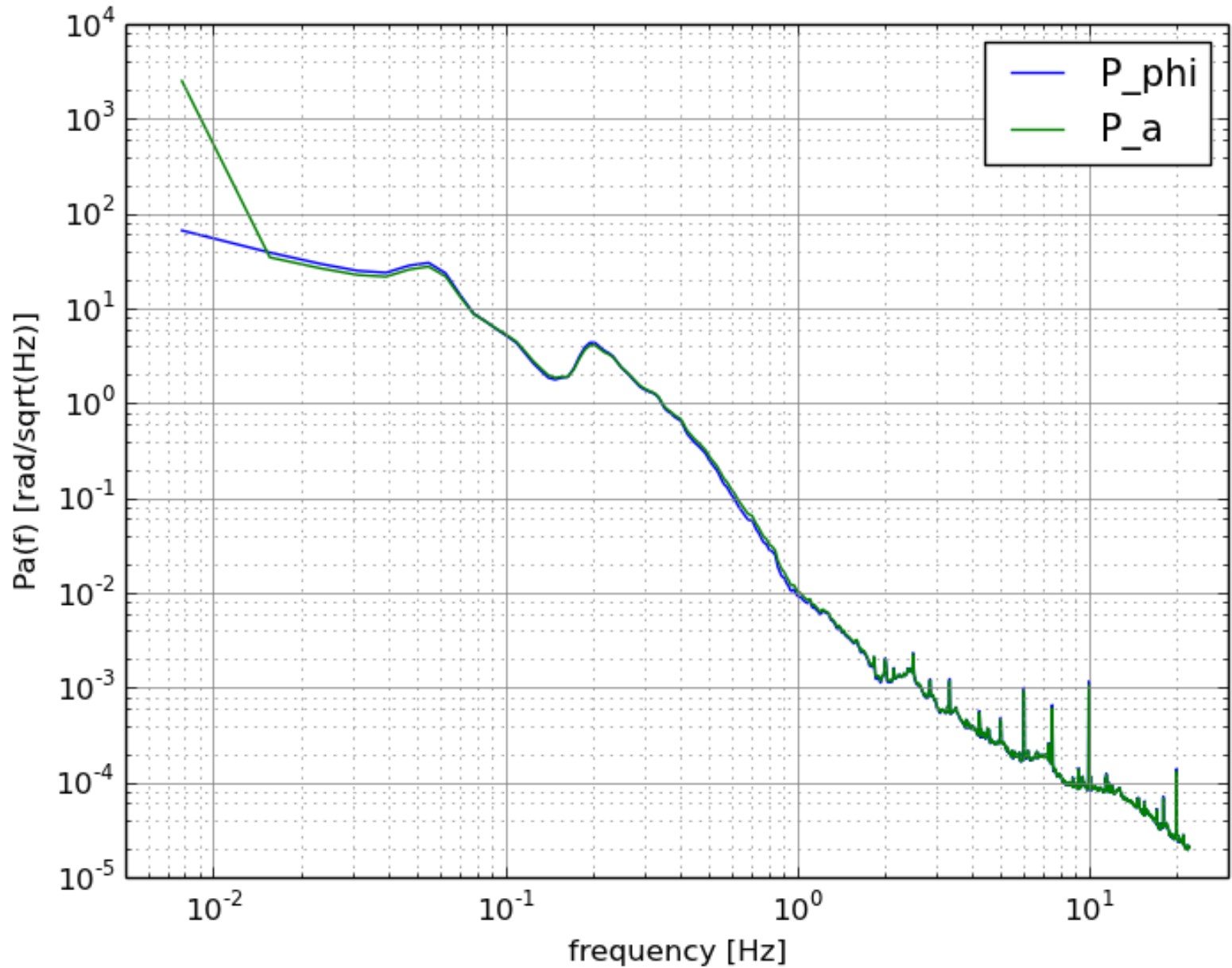
$$C_a(\tau) = \frac{1}{2} \exp\{C_{\Phi}(\tau) - C_{\Phi}(0)\}$$

$$\frac{P_a(\omega)}{\text{求めたい}} = \frac{1}{2\pi} \int_{-\infty}^{\infty} C_a(\tau) e^{-i\omega\tau} d\tau$$

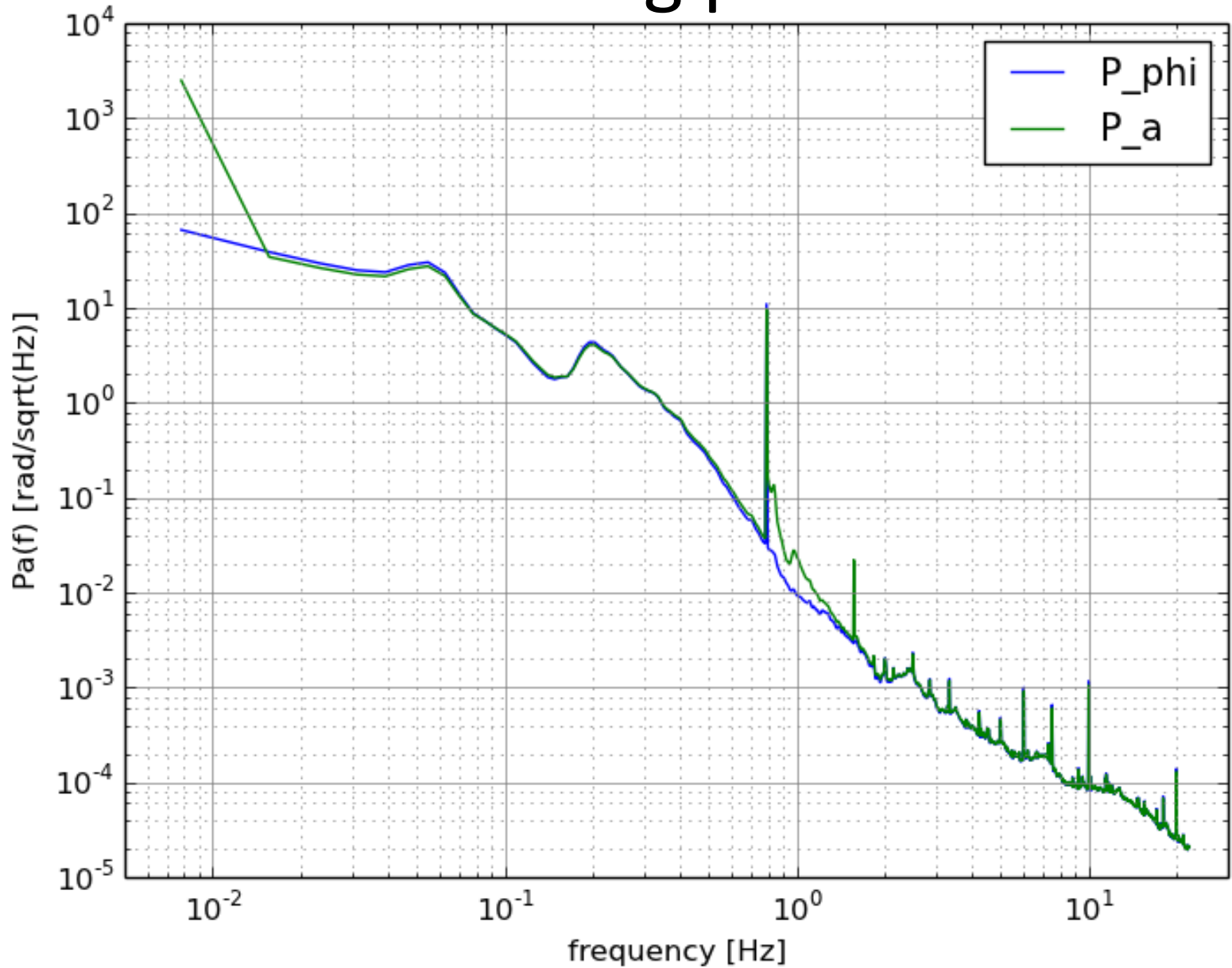
求めたい
want to know

From Aso-san slides 'ScatteringWorkshop'

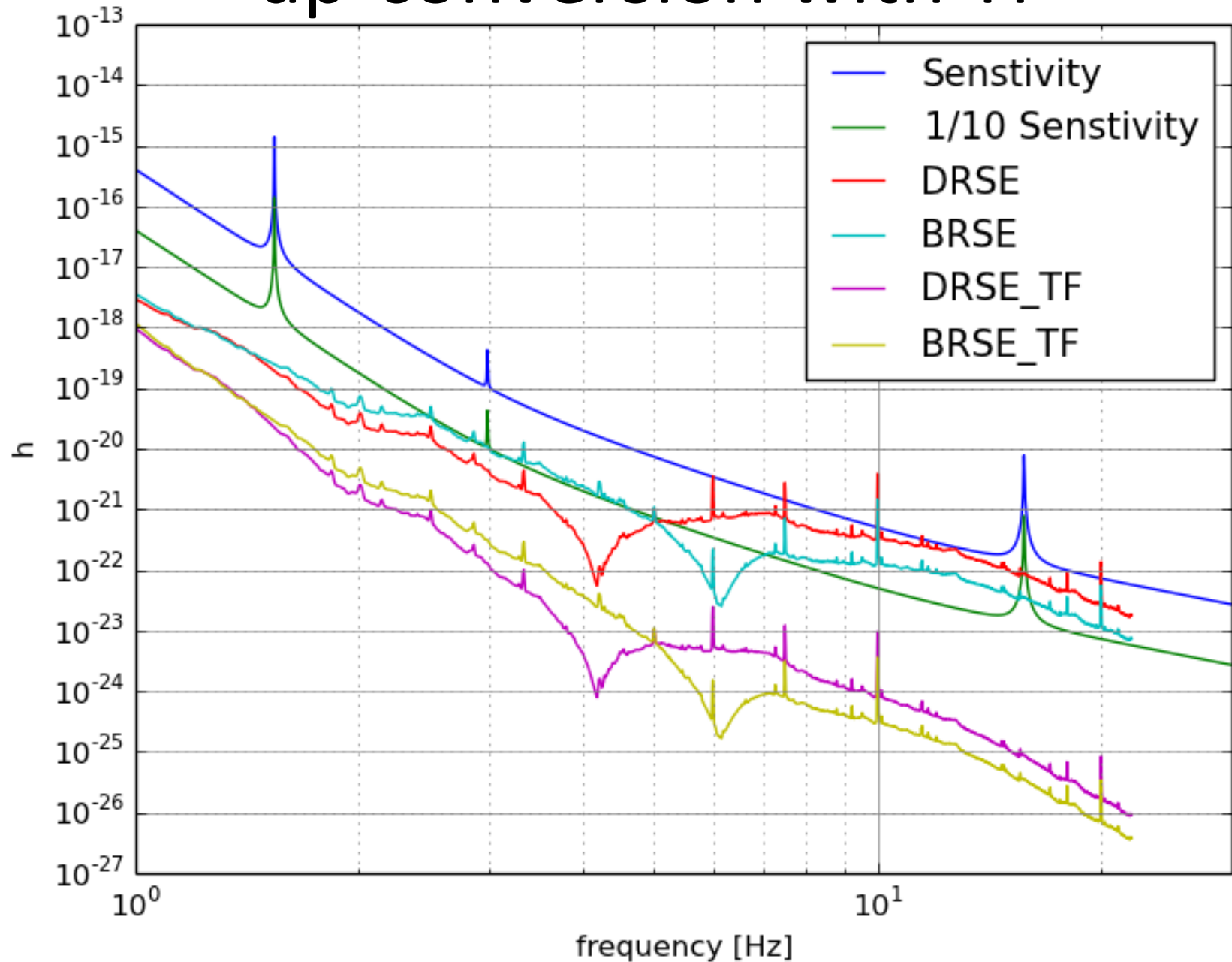
$P_\phi(\omega)$ & $P_a(\omega)$



$P_\phi(\omega)$ & $P_a(\omega)$ adding peek



up-conversion with TF

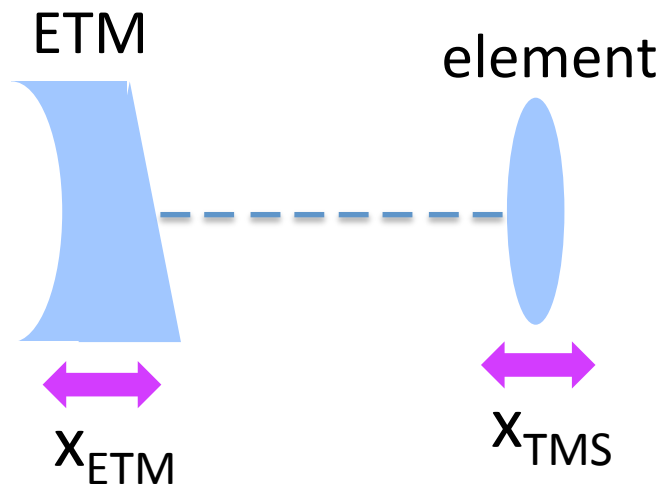


using relative motion between ETM and TMS

at low frequency :

ETM moves larger, as much as the seismic motion

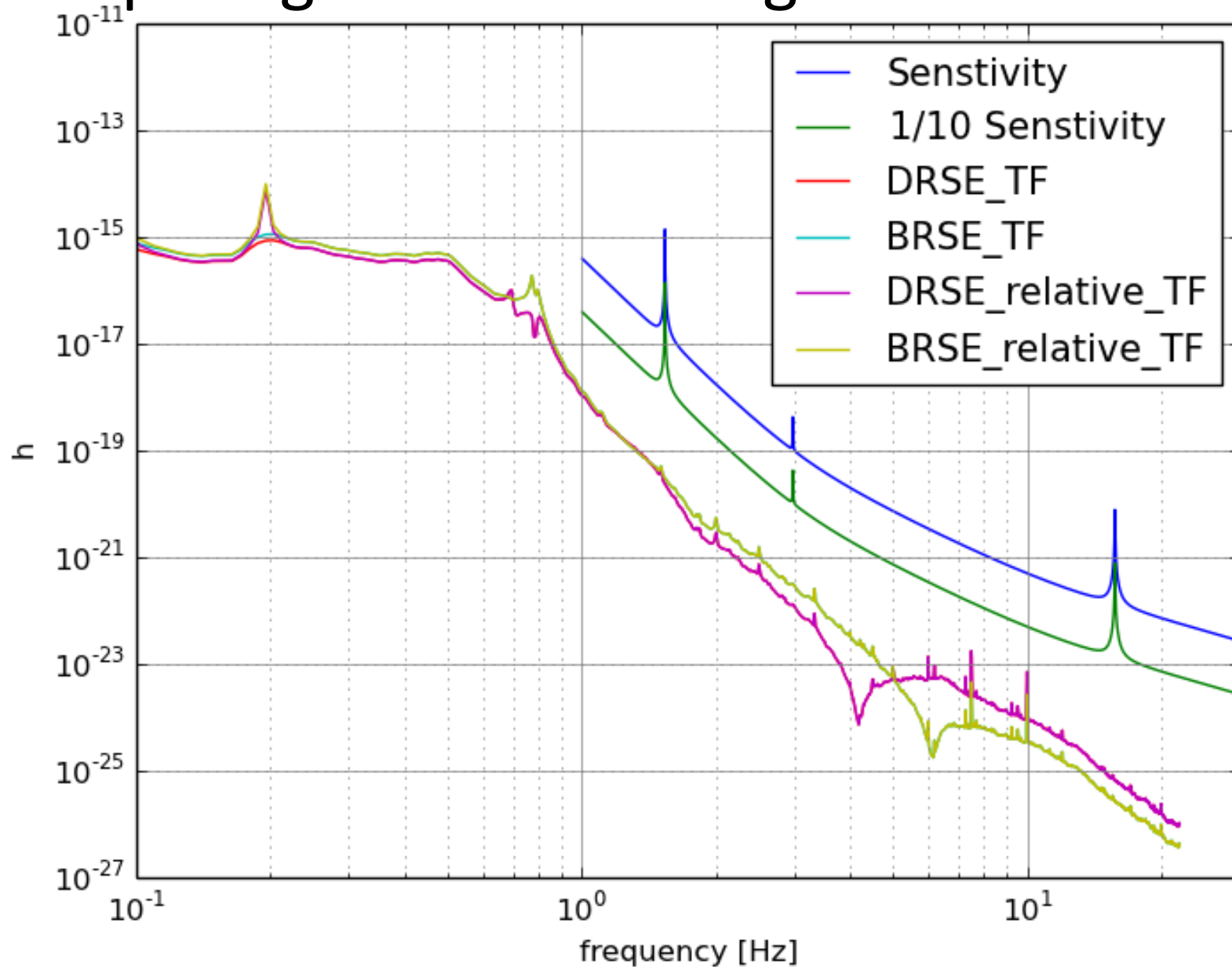
→ We should consider the relative motion between ETM and TMS elements.



$$x_{\text{relative}} = (x_{\text{ETM}}^2 + x_{\text{TMS}}^2)^{1/2}$$

up-conversion with TF

comparing normal & using relative motion

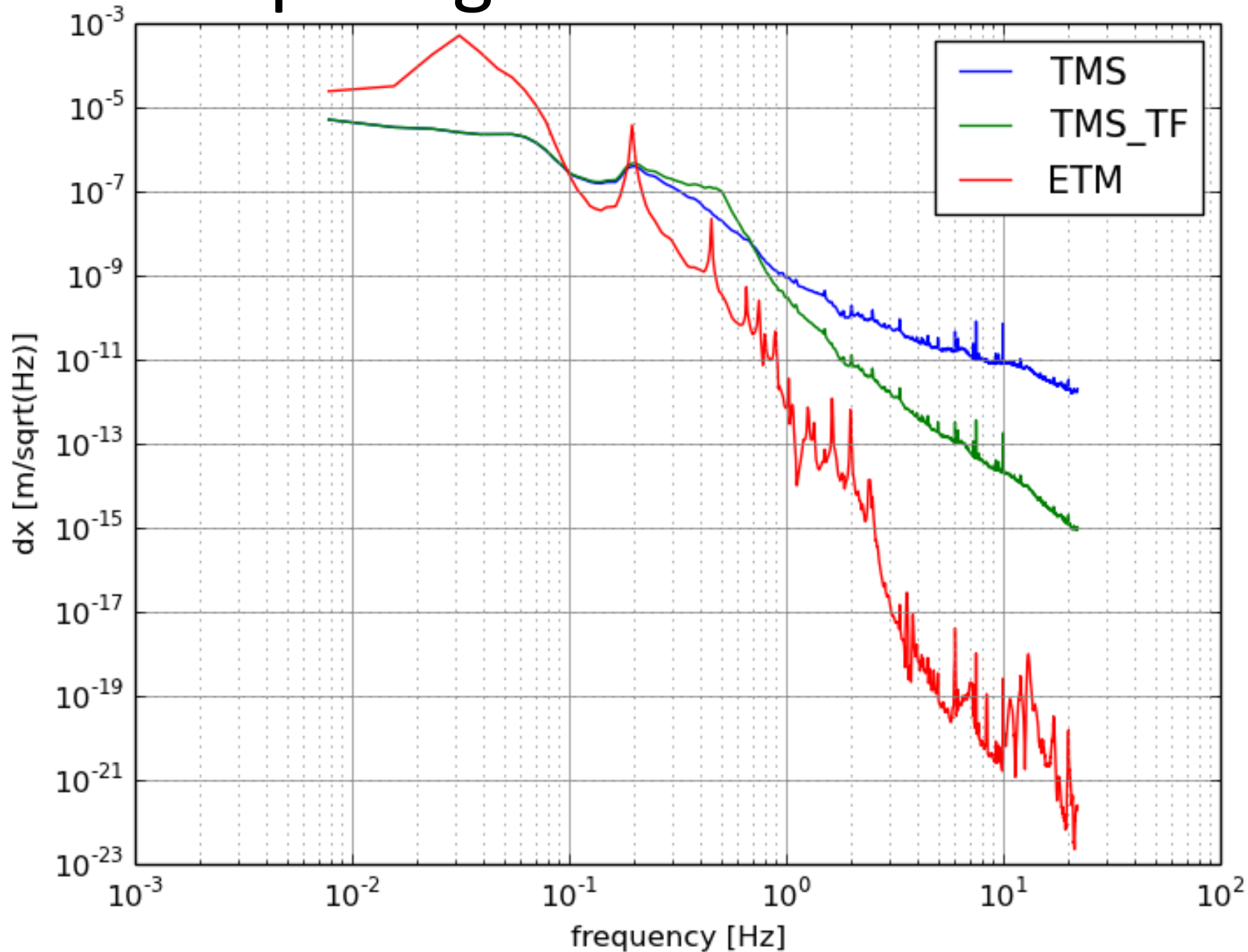


up-conversion with TF comparing normal & using relative motion

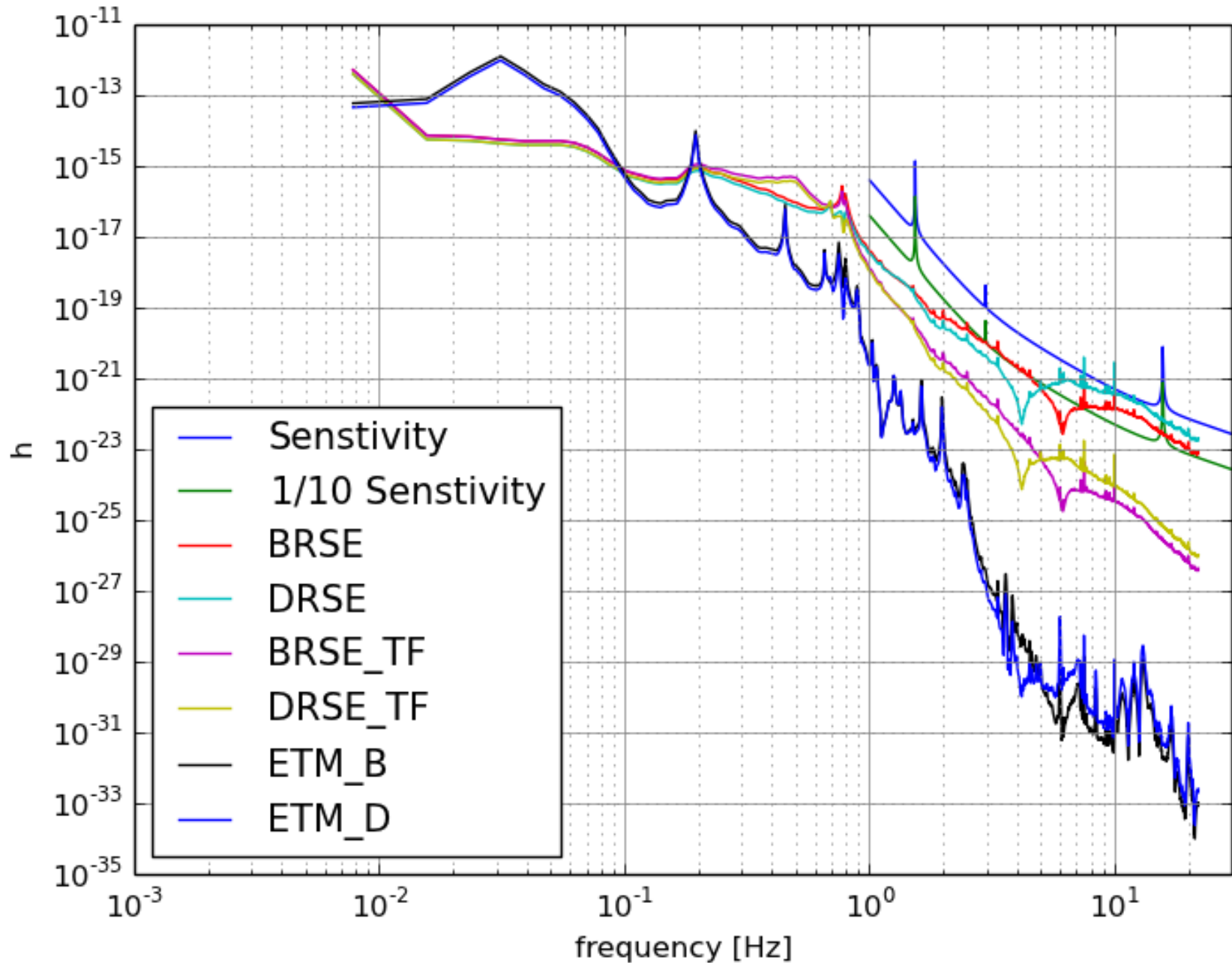
From last slide, we can say that the result computed by the relative motion has almost no effect on KAGRA sensitivity.

..we can find this reason in the next two slides.
ETM motion and its contribution to h are
enough smaller than TMS motion at > 1 Hz.

comparing ETM & TMS motion



ETM contribution to h



Conclusion

- TMS should be suspended

Simple pendulum is enough for TMS

- ETM motion is quite smaller than TMS motion
→ no need to consider the relative motion