Simulation for understanding what will happen in dithering

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(1) Signal of interest

a sinusoidal wave of

amplitude: **a0**

frequency: $\mathbf{f}_{sig}$

$v_{sig}(t) = a0 * \cos(2 * pi * f_{sig} * t)$

(2) White gauss noise

standard deviation of the noise = **Vn**

average of th noise = 0.0

random number generator: **gsl_ran_gaussian**

in Gnu Science Library

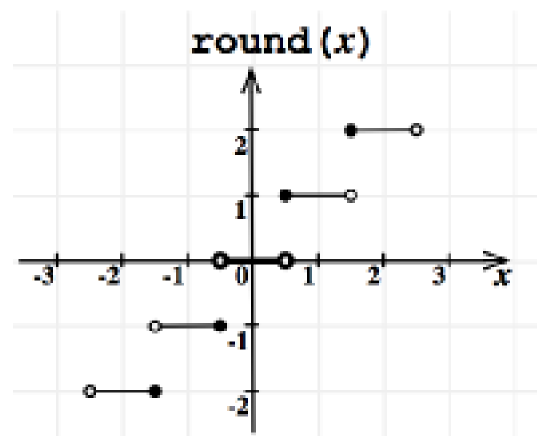
(3) Quantization

$Vq(t) = round(V(t) / \Delta) * \Delta$

Δ: quantization step

The round function in C language has a response as a left figure.

$\mathbf{f}_{sample}$: sampling frequency of the quantizer.



Simulation Example 0 :

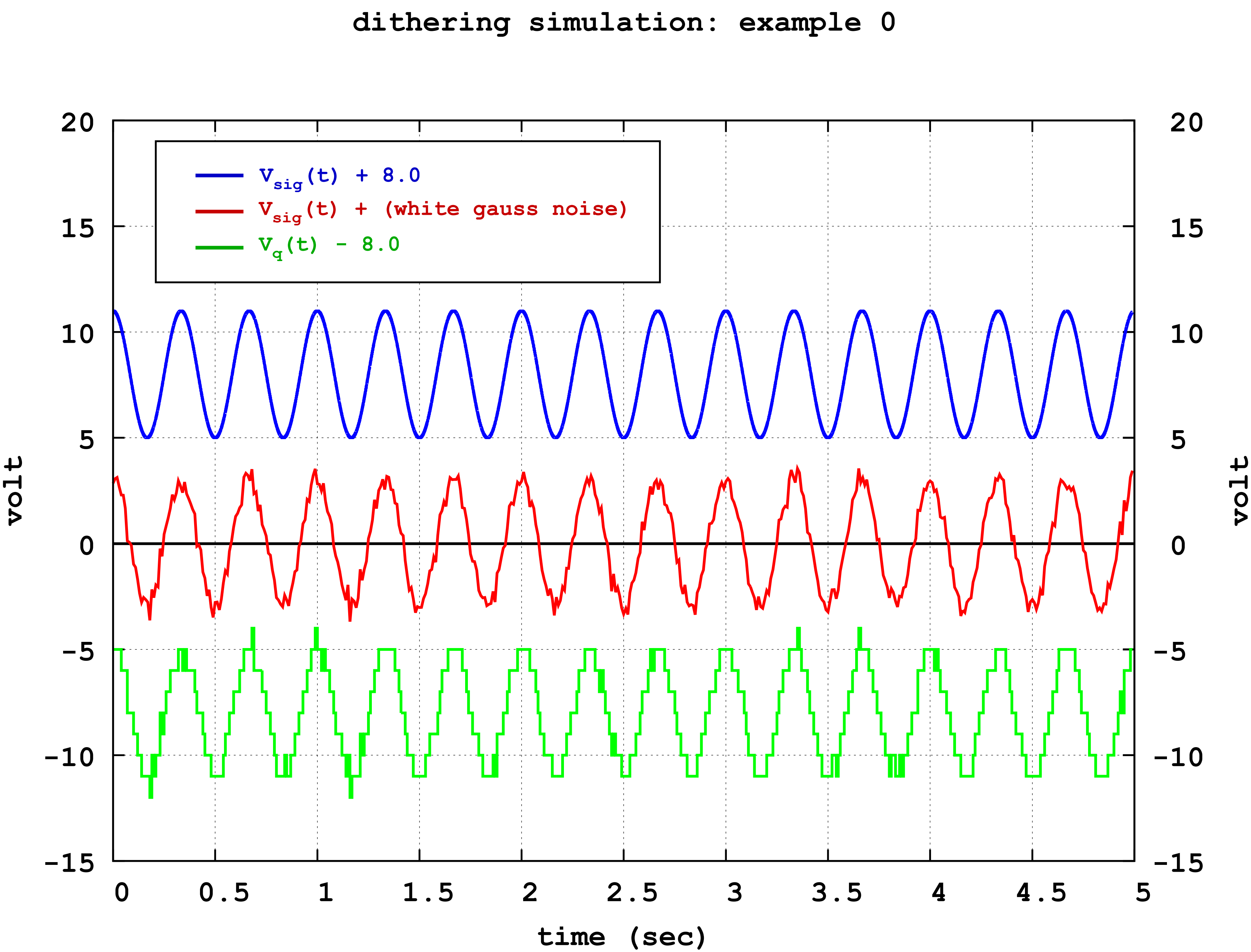
Δ = 1 volt

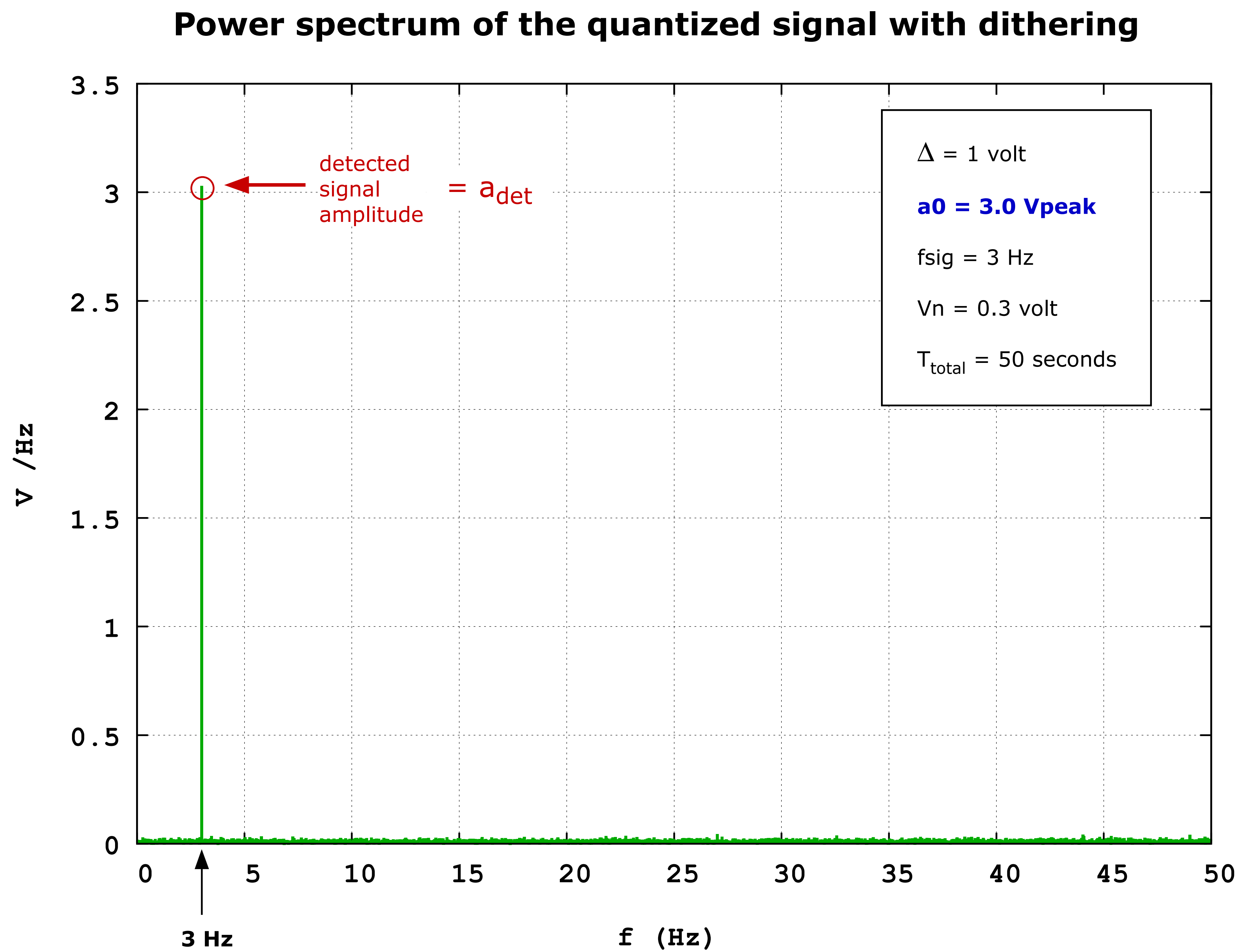
a0 = 3.0 Vpeak

fsig = 3 Hz

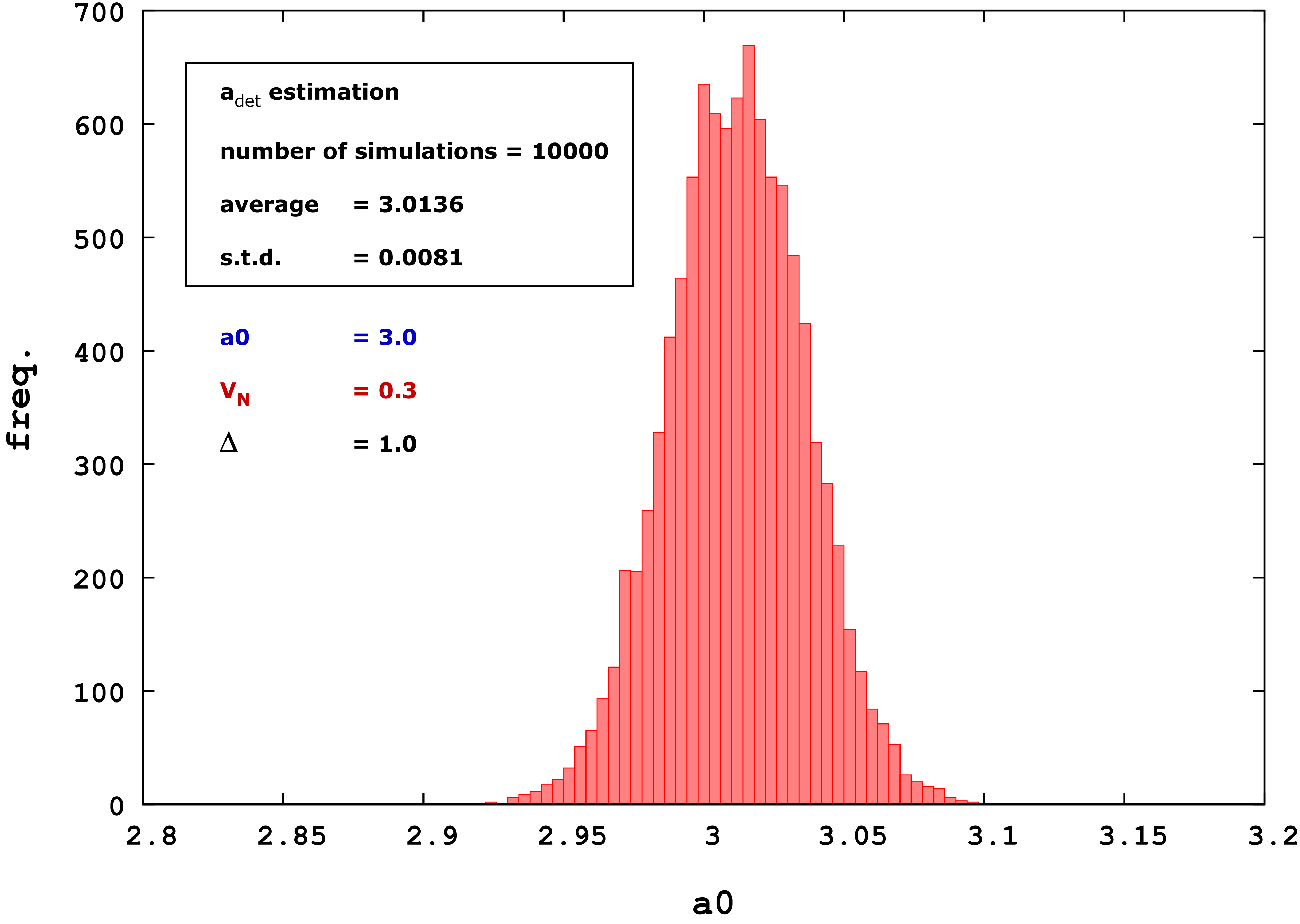
Vn = 0.3 volt

T_{total} = 50 seconds


dithering simulation: example 0
Vsig(t) + 8.0
Vsig(t) + (white gauss noise)
Vq(t) - 8.0
volt
volt
time (sec)
20
15
10
5
0
-5
-10
-15
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5



Power spectrum of the quantized signal with dithering
detected signal amplitude = a_det
Δ = 1 volt
a0 = 3.0 Vpeak
fsig = 3 Hz
Vn = 0.3 volt
T_total = 50 seconds
V /Hz
f (Hz)
3 Hz
0
0.5
1
1.5
2
2.5
3
3.5
0
5
10
15
20
25
30
35
40
45
50



a_det estimation
number of simulations = 10000
average = 3.0136
s.t.d. = 0.0081
a0 = 3.0
V_N = 0.3
Δ = 1.0
freq.
a0
0
100
200
300
400
500
600
700
2.8
2.85
2.9
2.95
3
3.05
3.1
3.15
3.2

Simulation Example 1 :

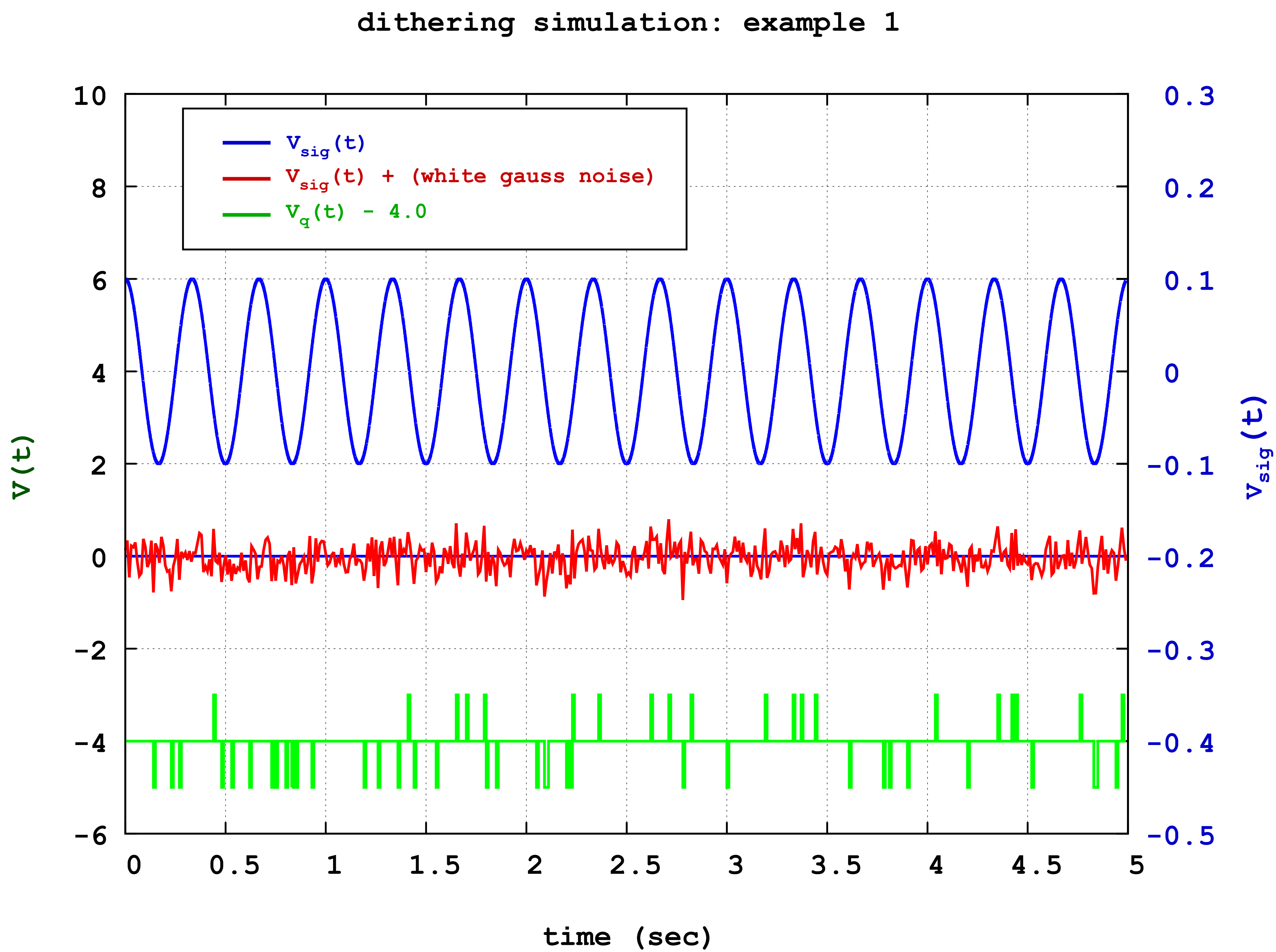
Δ = 1 volt

a0 = 0.1 Vpeak

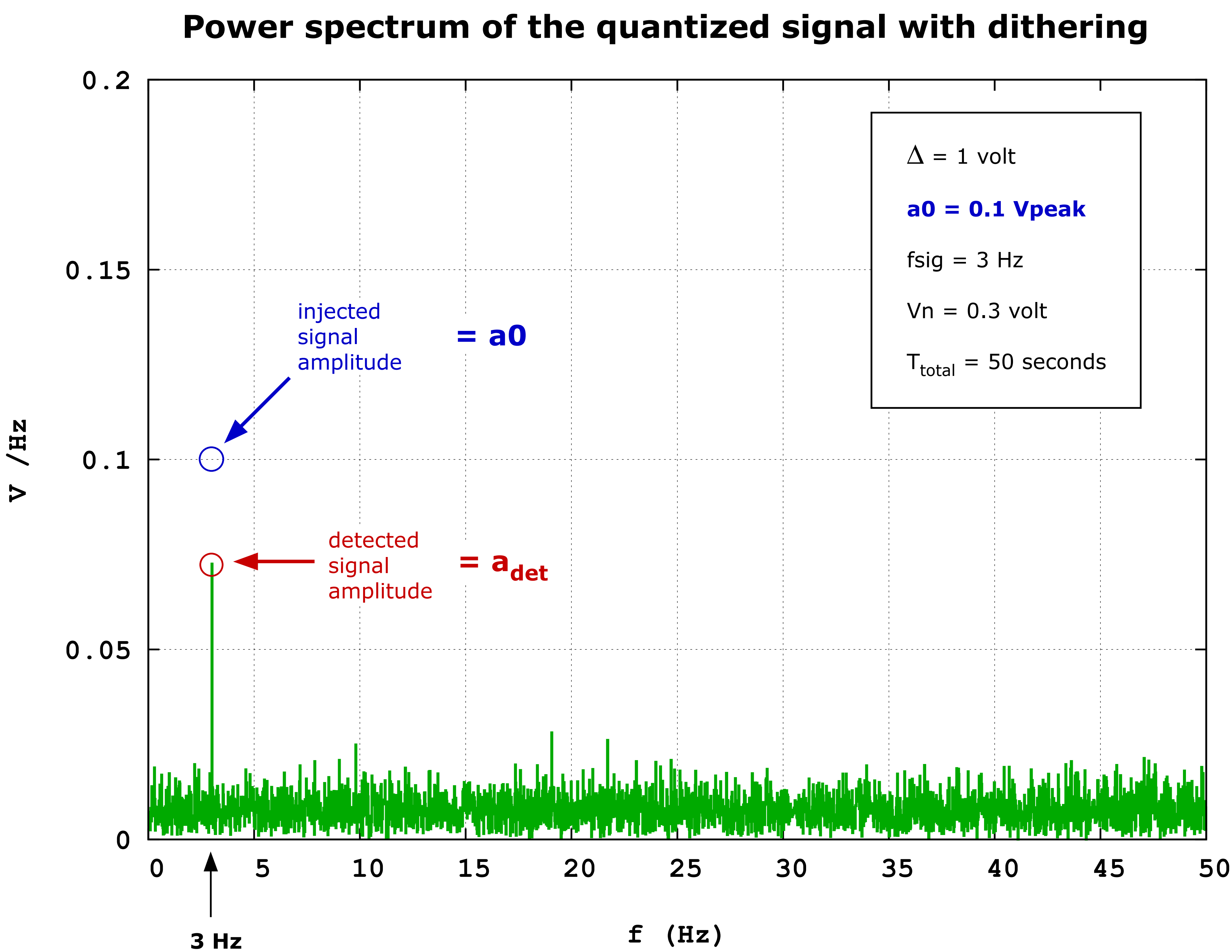
fsig = 3 Hz

Vn = 0.3 volt

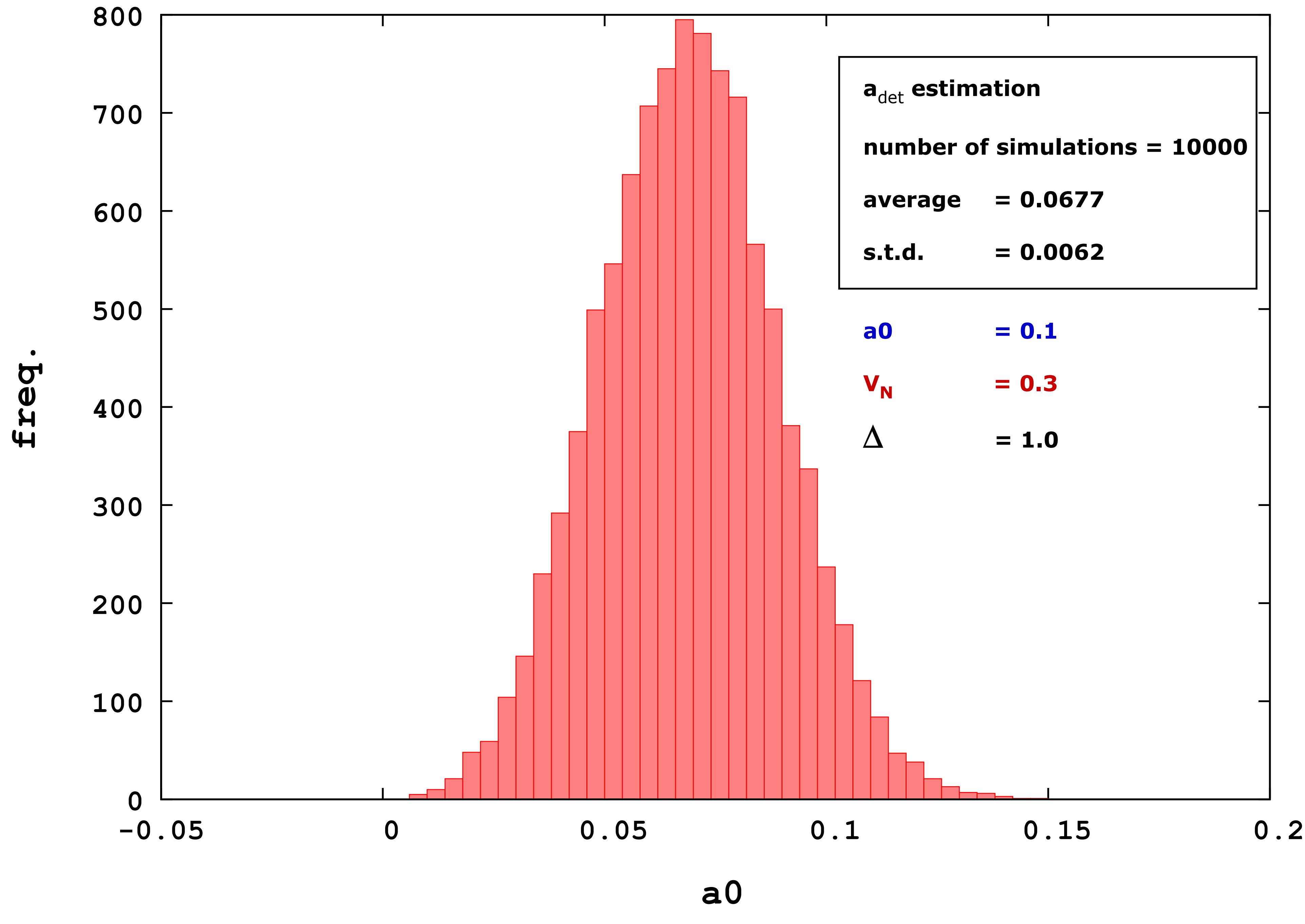
T_{total} = 50 seconds



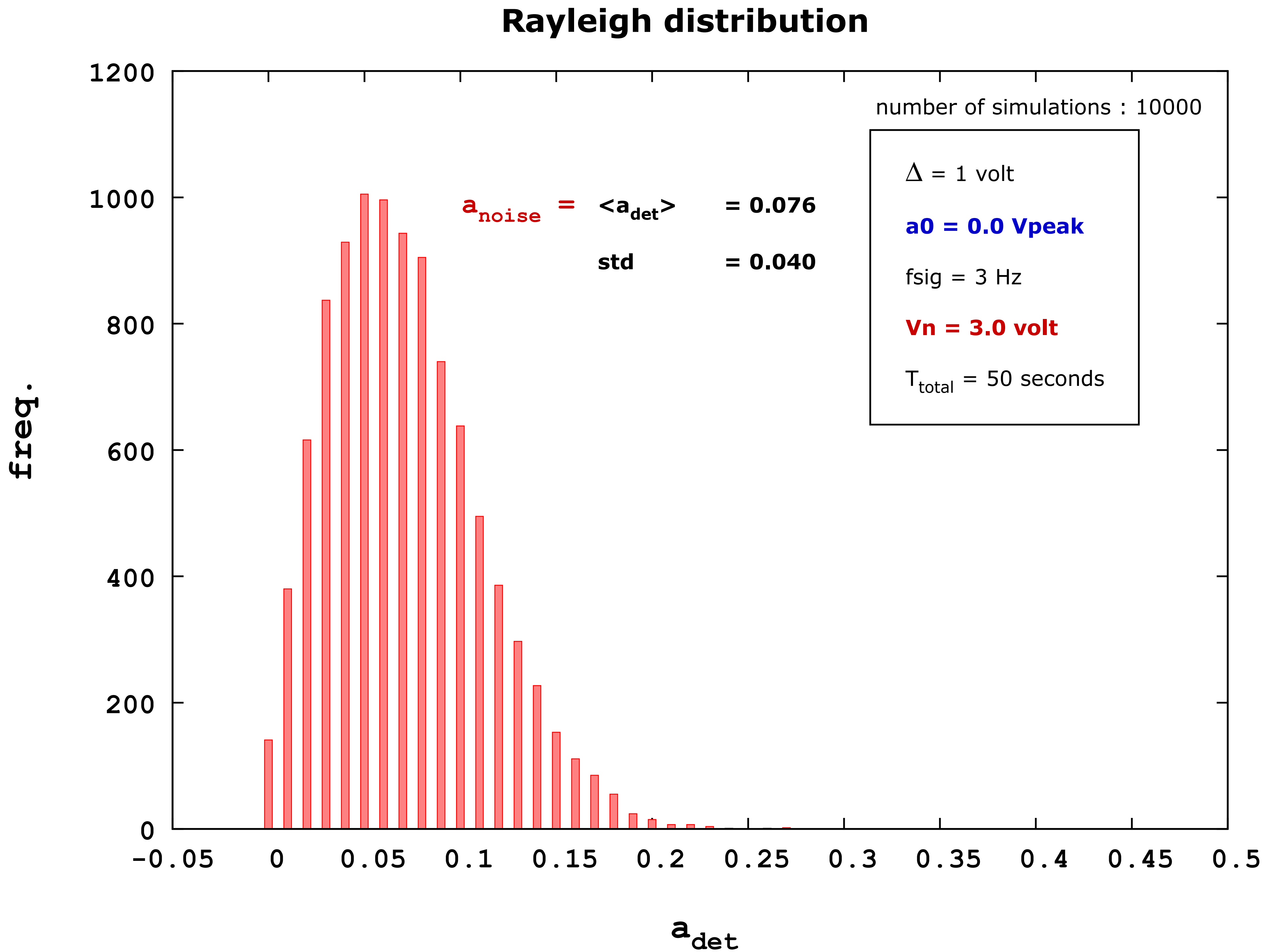
dithering simulation: example 1
V_sig(t)
V_sig(t) + (white gauss noise)
V_q(t) - 4.0
10
8
6
4
2
0
-2
-4
-6
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
V(t)
V_sig(t)
time (sec)



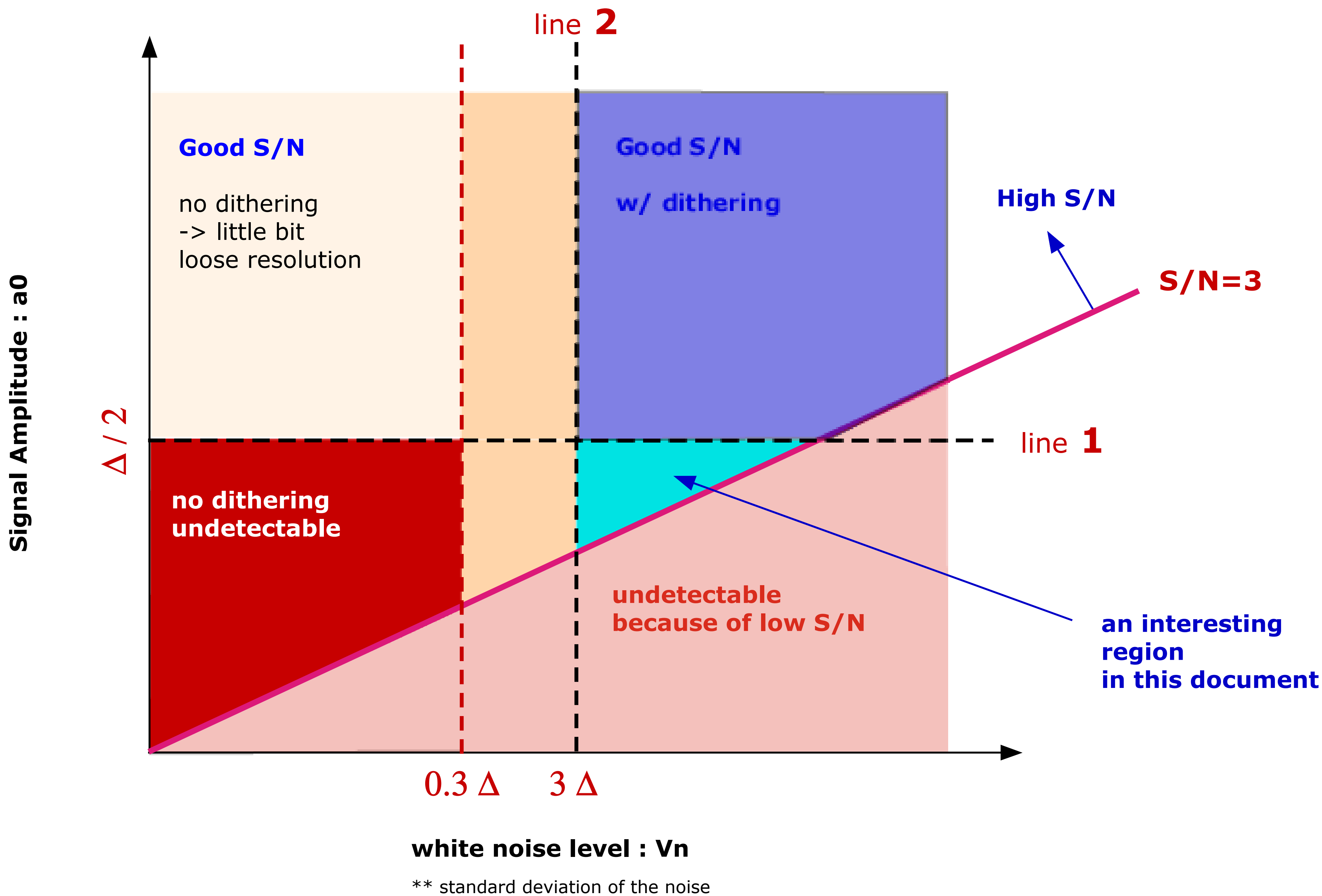
Power spectrum of the quantized signal with dithering
0.2
0.15
0.1
0.05
0
V /Hz
0
5
10
15
20
25
30
35
40
45
50
f (Hz)
3 Hz
Δ = 1 volt
a0 = 0.1 Vpeak
fsig = 3 Hz
Vn = 0.3 volt
T total = 50 seconds
injected
signal
amplitude
= a0
detected
signal
amplitude
= a det



a_{det} estimation
number of simulations = 10000
average = 0.0677
s.t.d. = 0.0062
a0 = 0.1
V_N = 0.3
Δ = 1.0
freq.
a0
0
100
200
300
400
500
600
700
800
-0.05
0
0.05
0.1
0.15
0.2



Rayleigh distribution
number of simulations : 10000
Δ = 1 volt
a0 = 0.0 Vpeak
fsig = 3 Hz
Vn = 3.0 volt
T_{total} = 50 seconds
a_{noise} = <a_{det}> = 0.076
std = 0.040
freq.
a_{det}
0
200
400
600
800
1000
1200
-0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5


line 2
Good S/N
no dithering
-> little bit
loose resolution
Good S/N
w/ dithering
High S/N
S/N=3
Signal Amplitude : a0
Δ / 2
line 1
no dithering
undetectable
undetectable
because of low S/N
an interesting
region
in this document
0.3 Δ
3 Δ
white noise level : Vn
** standard deviation of the noise

Intuitive speculations

Line 1:

It is obvious that a sinusoidal wave of amplitude less than **Δ /2** looses all of information in the case of no dithering by the quantization.

Line 2:

How large white noises are effective as dithering?

Answer;

0.3 Δ is a minimum amplitude for dithering. But it makes non-negligible signal loss.

3 Δ seems to be enough large and makes the signal loss small.

S/N=3

If there is no quantizer, we can calculate the expected S/N.

*** a_{noise} can be estimated as follows

$$a_{noise} = \widetilde{v_n}(f) \cdot \Delta f = \frac{2}{\sqrt{f_s T}} \times V_n$$

The estimations are well consistent with dithered and quantized data.

The standard deviation of the "a_{det}"

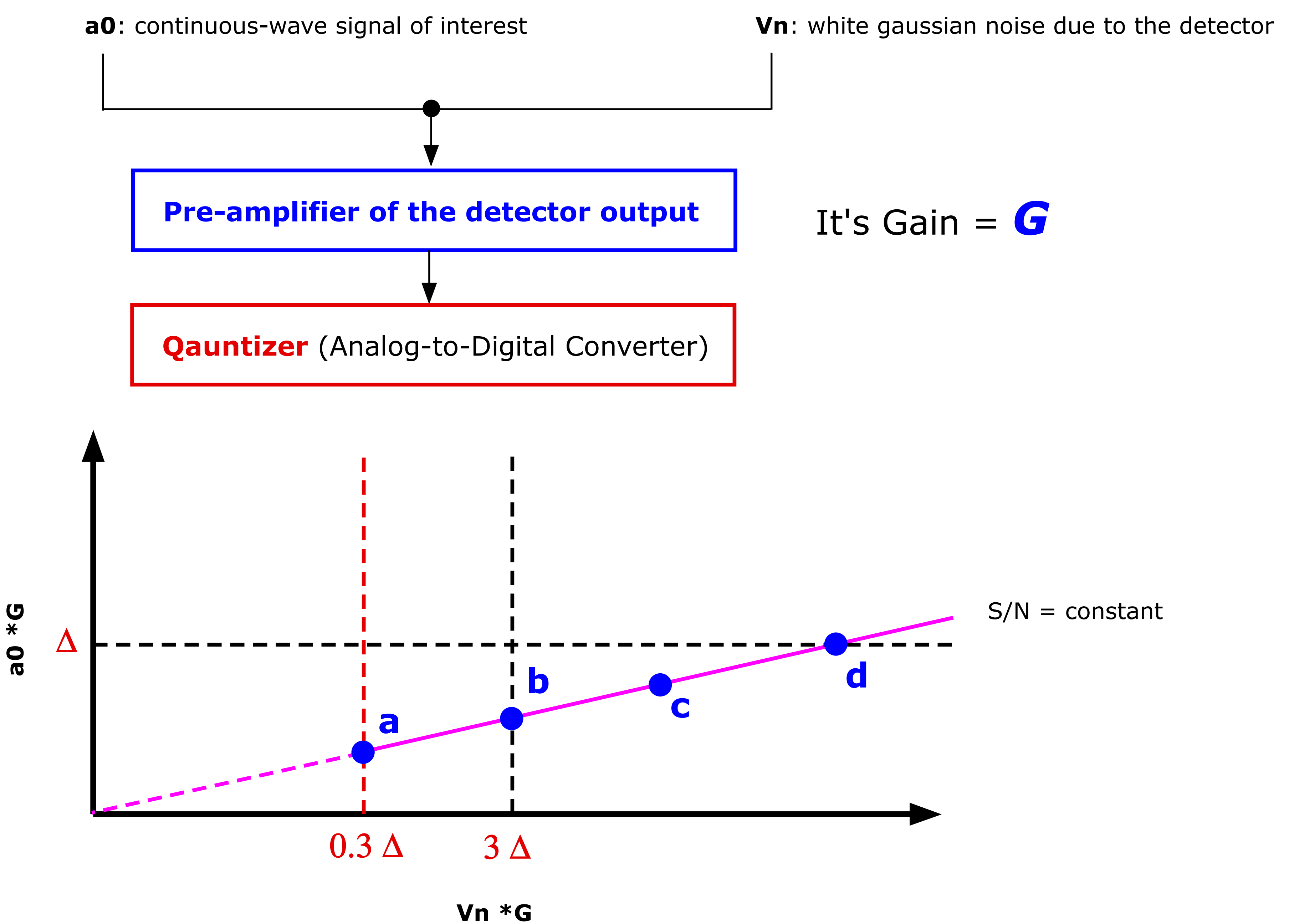
In the case of ***a*0 = 0**,

the probability density function obeys Rayleigh function.

Therefore,

$$\text{std of } a_{det} = 0.523 \times (\text{mean of } a_{det})$$
$$= 0.523 \times a_{noise}$$

Realistic situation for controlling dithering conditions



To set simulation parameters

When G=1, Vn *G = Vn = 0.3 Δ

S/N = a0 /

Intuitive speculations

Line 1:

It is obvious that a sinusoidal wave of amplitude less than **Δ /2** looses all of information in the case of no dithering by the quantization.

Line 2:

How large white noises are effective as dithering?

Answer;

0.3 Δ is a minimum amplitude for dithering. But it makes non-negligible signal loss.

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S/N=3

If there is no quantizer, we can calculate the expected S/N.

White Gaussian Noise

$$V_n^2 \stackrel{\text{def}}{=} \frac{1}{T}\int_0^T v^2(t)dt$$

$$V_n^2 = 2\int_0^{f_N} \widetilde{s_n}(f)df = 2f_N\,\widetilde{s_n} = f_s\,\widetilde{s_n}$$

f_N: Nyquist freq.

f_s : Sampling freq.

$$\widetilde{s_n} = \frac{V_n^2}{f_s}$$

Power Spectrum Density (PSD) V^2/Hz

$$\sqrt{\widetilde{s_n}} = \sqrt{\frac{V_n^2}{f_s}}$$

Liner Spectrum Density (LSD) $V/\sqrt{Hz}$

$$\widetilde{v_n}(f) = V_n\sqrt{\frac{2T}{f_N}}$$

Fourier Transform of v(t) V/Hz

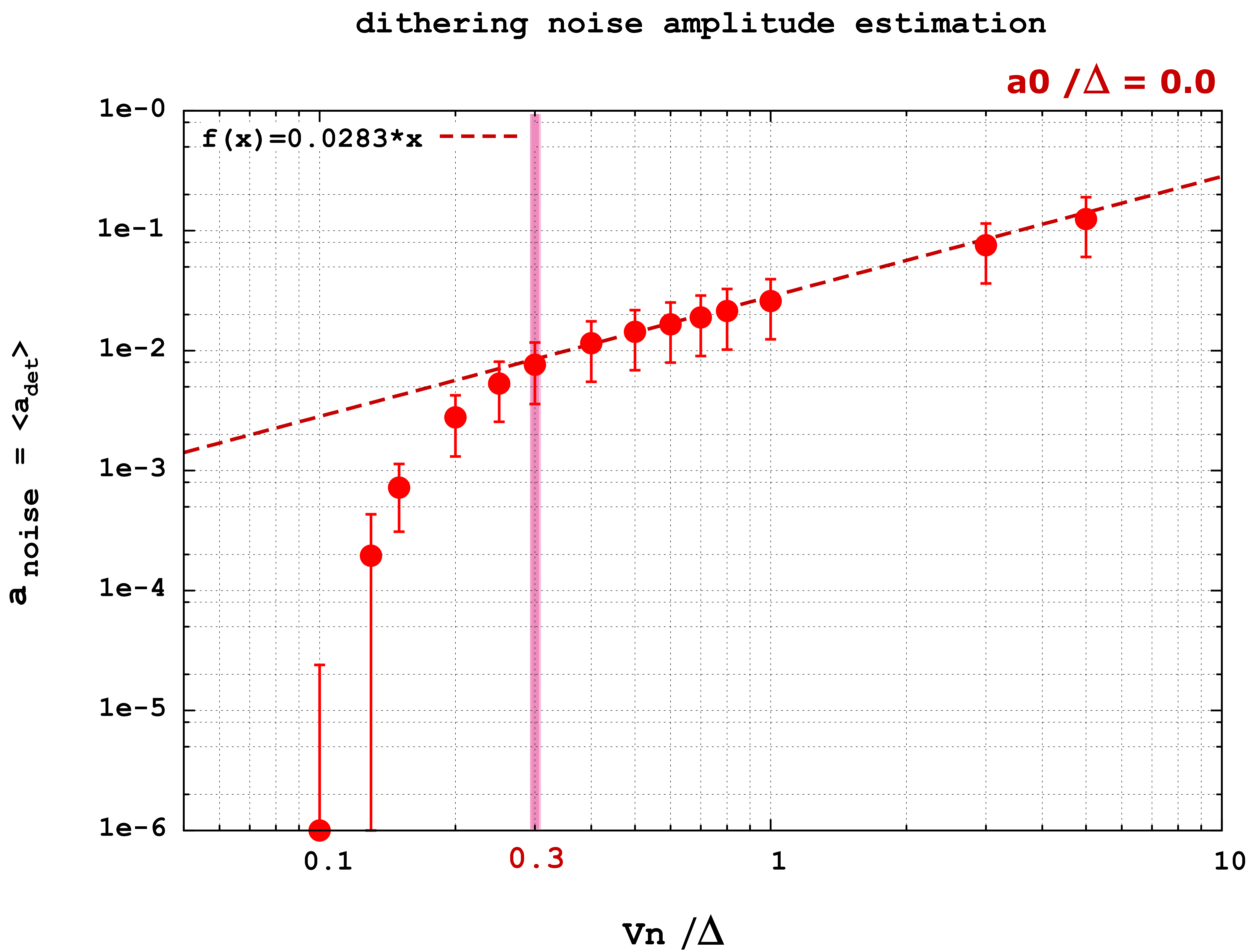
$$a_{noise} = \widetilde{v_n}(f)\cdot\Delta f = \frac{2}{\sqrt{f_s\,T}} \times V_n$$

in a unit of *Volt*

$$a_{noise} = \widetilde{v_n}(f) \cdot \Delta f = \frac{2}{\sqrt{f_s\, T}} \times V_n$$

fs = 100 Hz, *T* = 50 sec

--> $a_{noise} = 0.0283 \times V_n$



dithering noise amplitude estimation
a0 /Δ = 0.0
f(x)=0.0283*x
1e-0
1e-1
1e-2
1e-3
1e-4
1e-5
1e-6
0.1
0.3
1
10
Vn /Δ
a noise = <a det>

*** a_{noise} can be estimated as follows

$$a_{noise} = \widetilde{v_n}(f) \cdot \Delta f = \frac{2}{\sqrt{f_s T}} \times V_n$$

The estimations are well consistent with dithered and quantized data.

The standard deviation of the "a_{det}"

In the case of ***a*0 = 0**,

the probability density function obeys Rayleigh function.

Therefore,

$$\text{std of } a_{det} = 0.523 \times (\text{mean of } a_{det})$$
$$= 0.523 \times a_{noise}$$

Rayleigh function

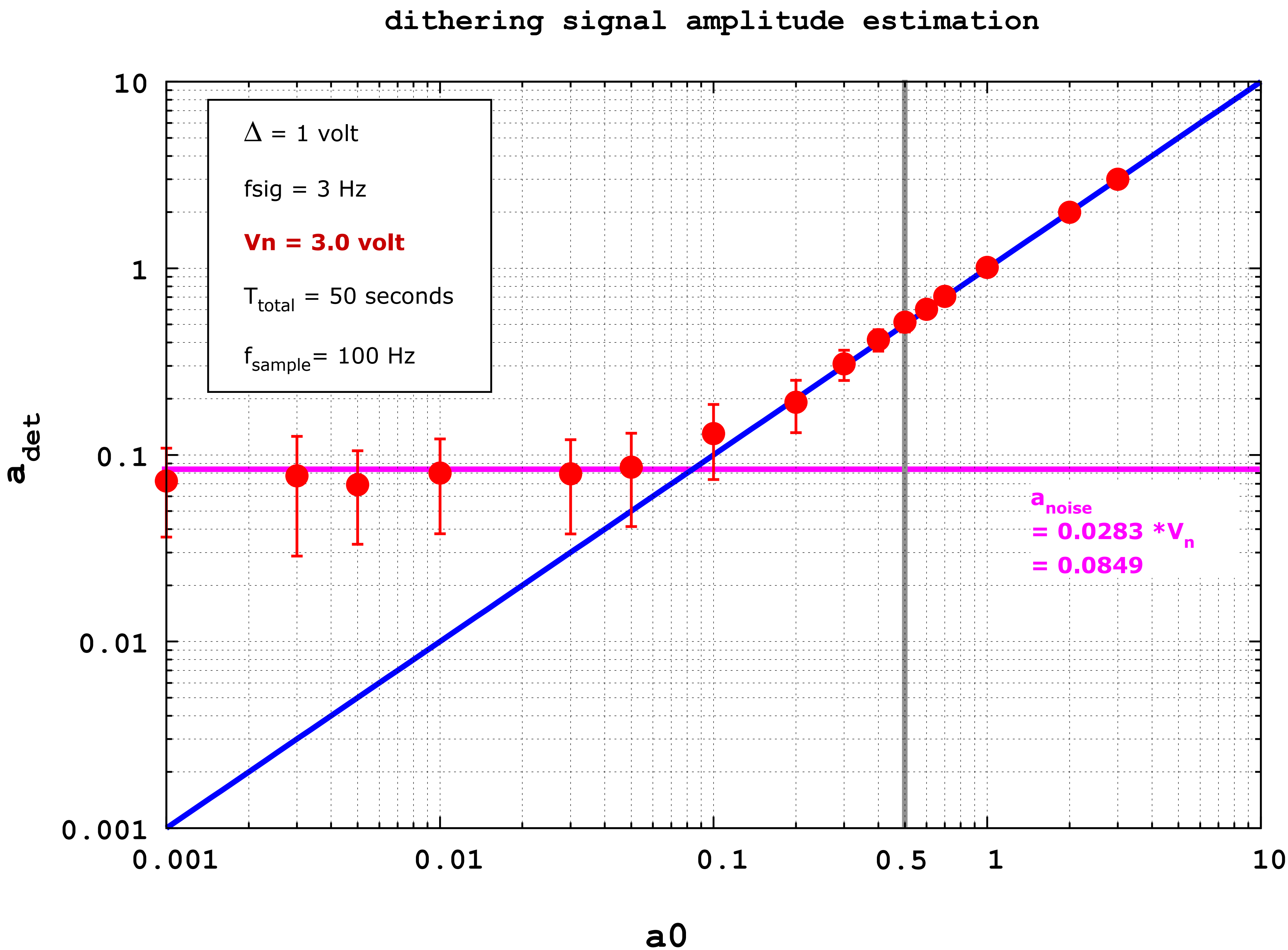
$$PDF(x,\sigma) = \frac{x^2}{\sigma^2}\ exp\left(-\frac{x^2}{\sigma^2}\right)$$

Mean = $\sqrt{\frac{\pi}{2}}\ \sigma$

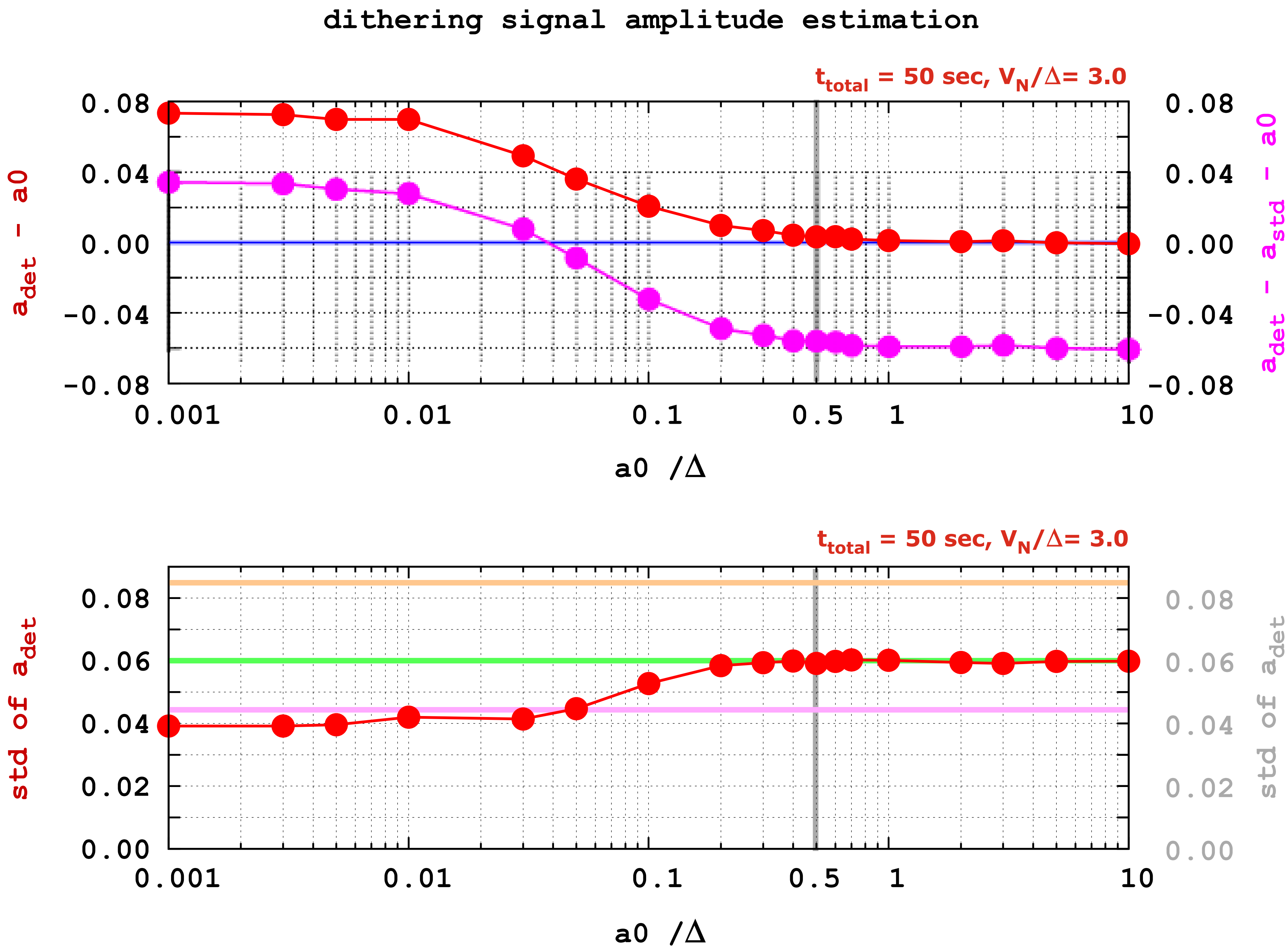
Deviation = $\left(2 - \frac{\pi}{2}\right)\sigma$

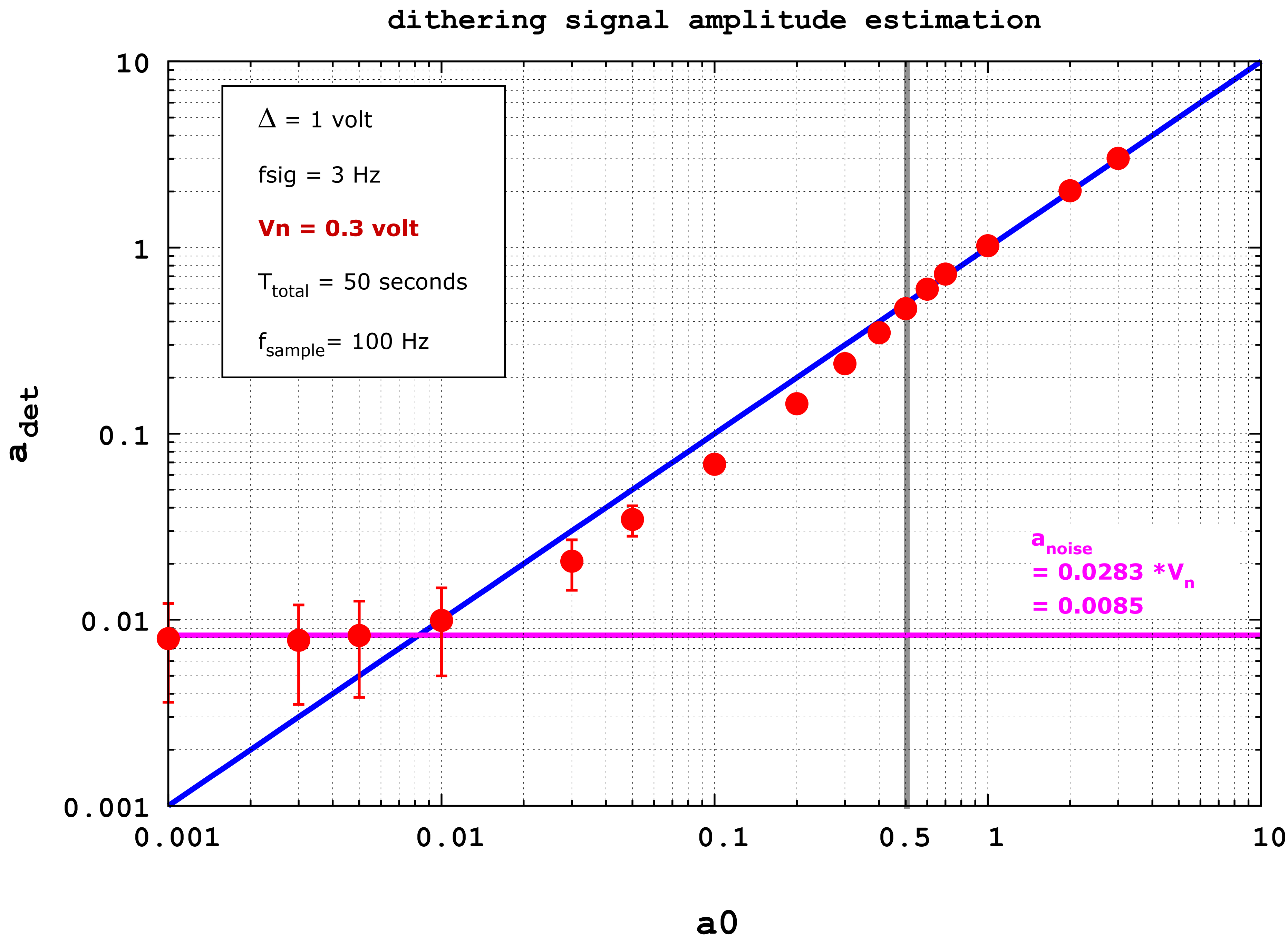
Standard deviation = $\sqrt{\frac{4}{\pi} - 1}$ x Mean = **0.523 x Mean** = σ_R

pink line	**0.523 x Mean**
green line	**0.70 x Mean**
orange line	**Mean**



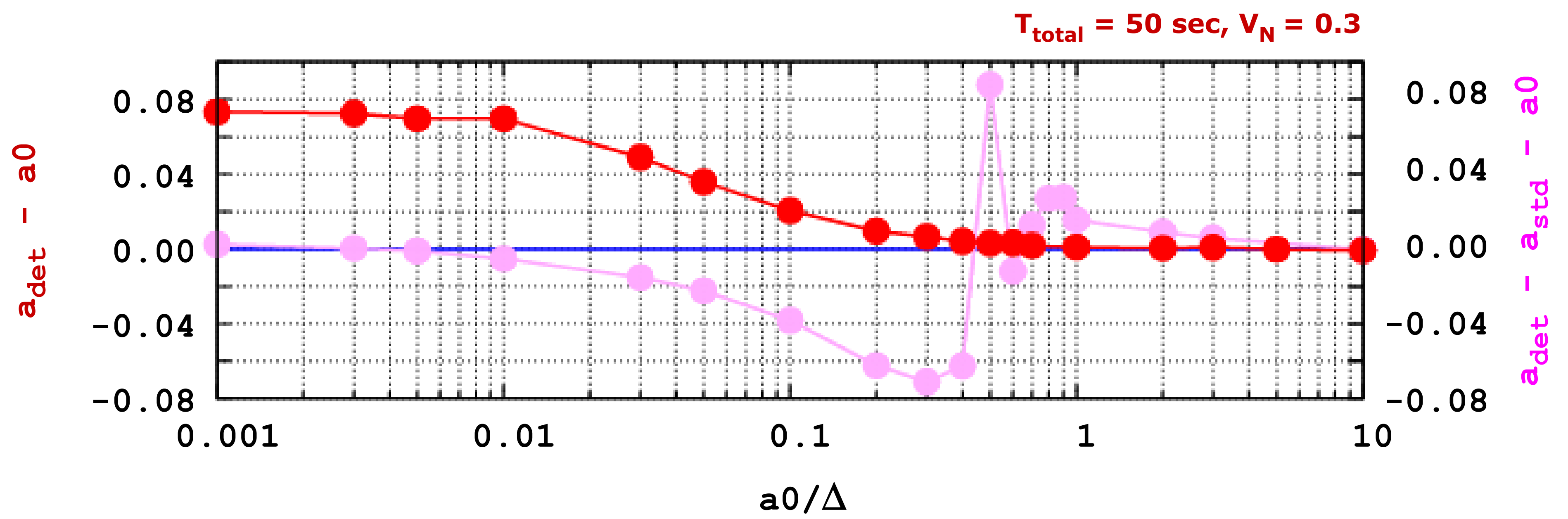
dithering signal amplitude estimation
Δ = 1 volt
fsig = 3 Hz
Vn = 3.0 volt
T total = 50 seconds
f sample= 100 Hz
a noise
= 0.0283 *V n
= 0.0849
a det
a0
10
1
0.1
0.01
0.001
0.001
0.01
0.1
0.5
1
10

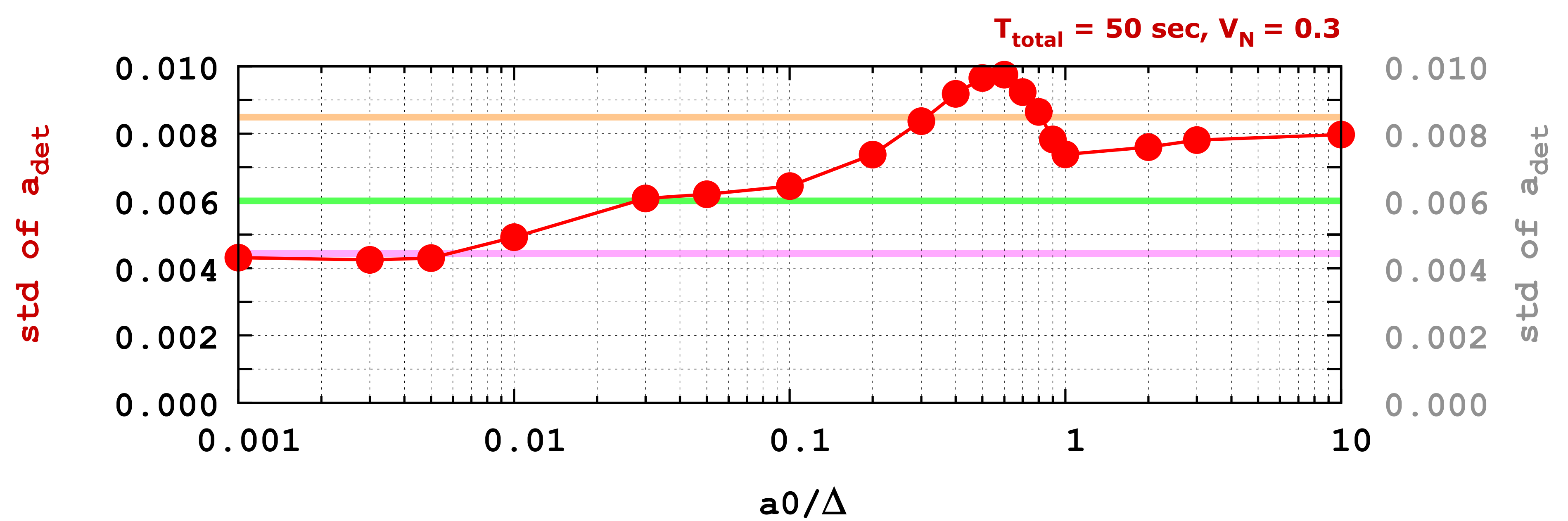

dithering signal amplitude estimation
t_{total} = 50 sec, V_N/Δ= 3.0
a_{det} - a0
a_{det} - a_{std} - a0
0.08
0.04
0.00
-0.04
-0.08
0.001
0.01
0.1
0.5
1
10
a0 /Δ
t_{total} = 50 sec, V_N/Δ= 3.0
std of a_{det}
0.08
0.06
0.04
0.02
0.00
0.001
0.01
0.1
0.5
1
10
a0 /Δ

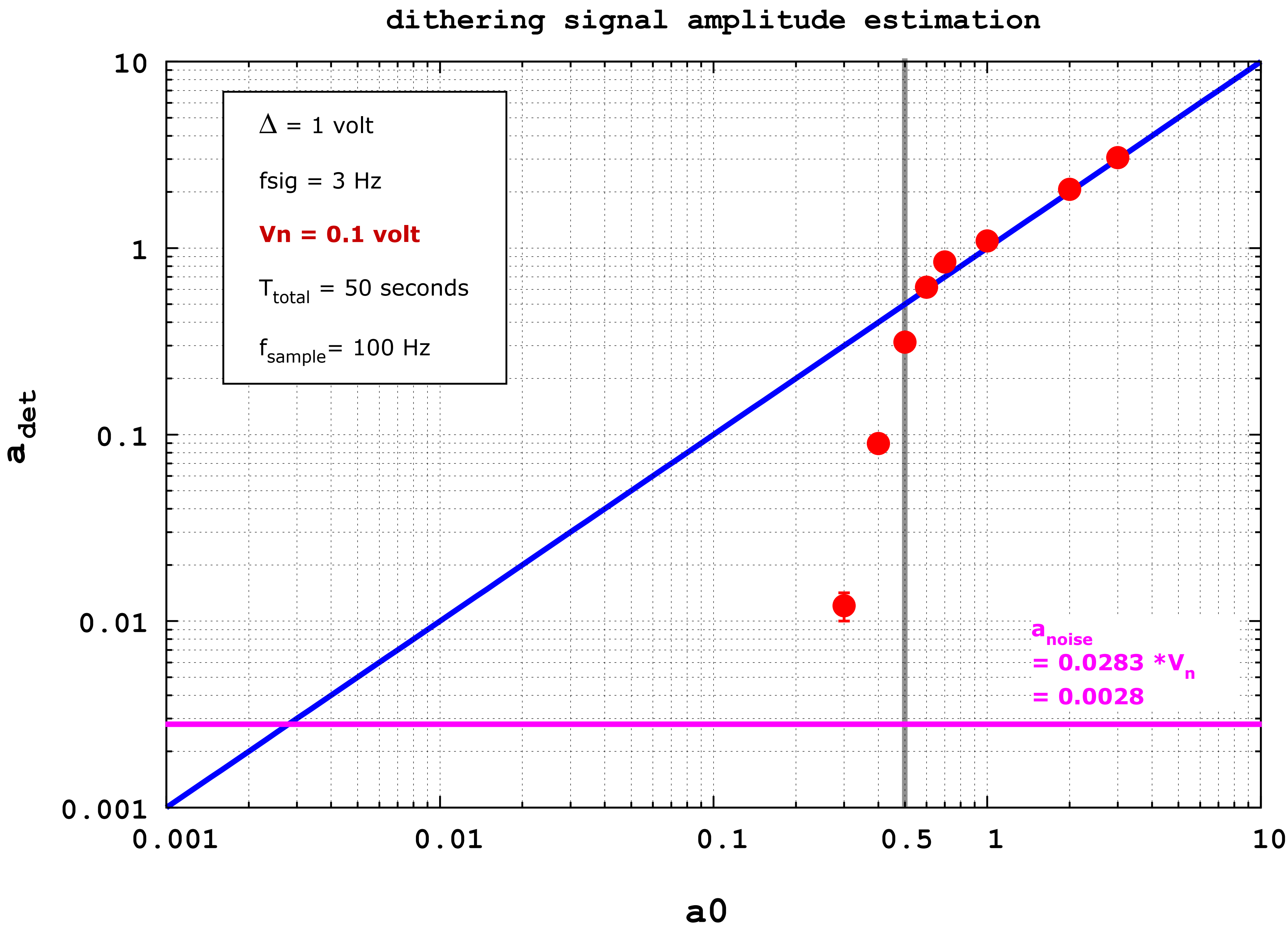


dithering signal amplitude estimation
Δ = 1 volt
fsig = 3 Hz
Vn = 0.3 volt
T_total = 50 seconds
f_sample= 100 Hz
a_noise
= 0.0283 *V_n
= 0.0085
a_det
a0
10
1
0.1
0.01
0.001
0.001
0.01
0.1
0.5
1
10

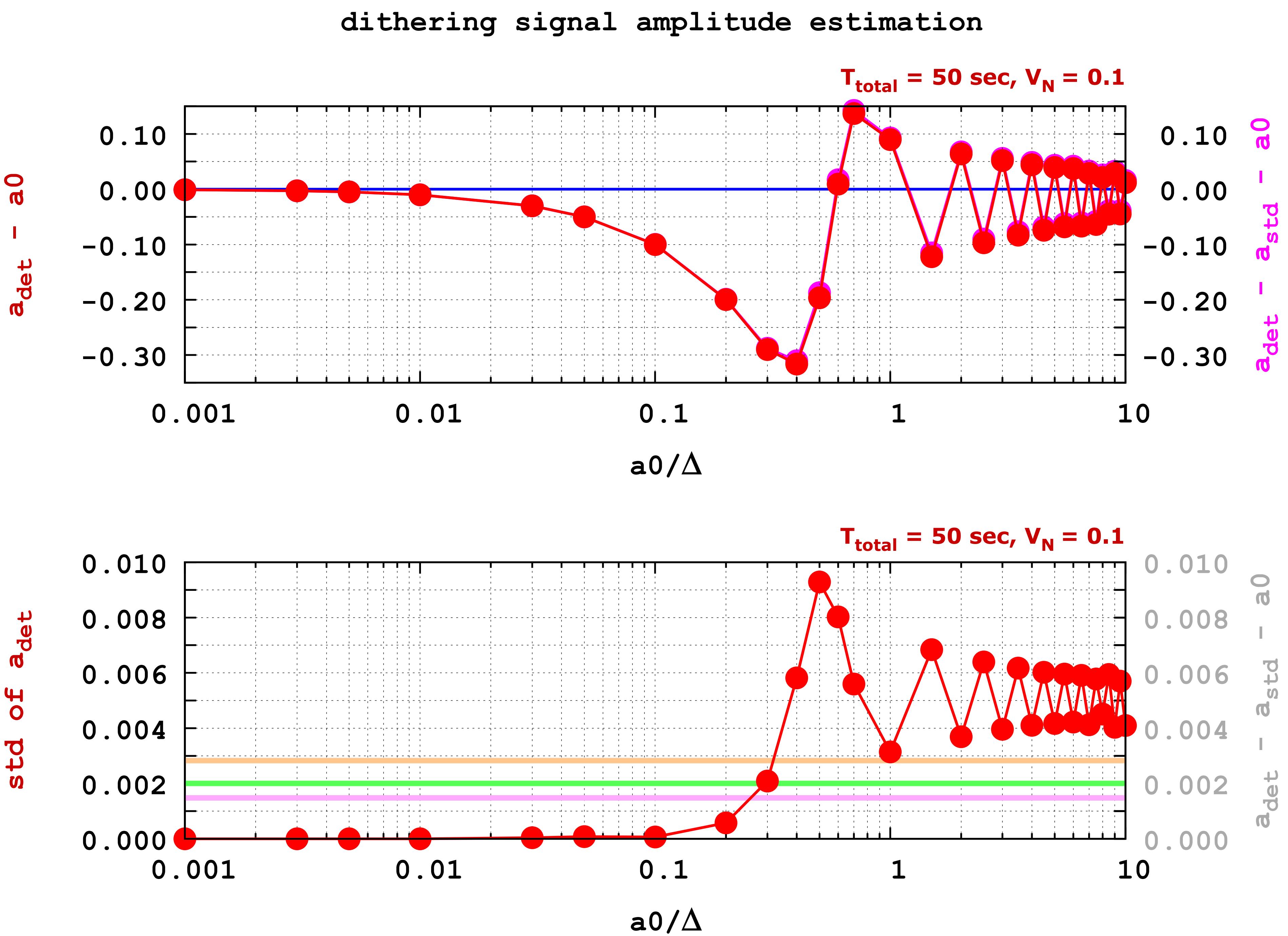
dithering signal amplitude estimation



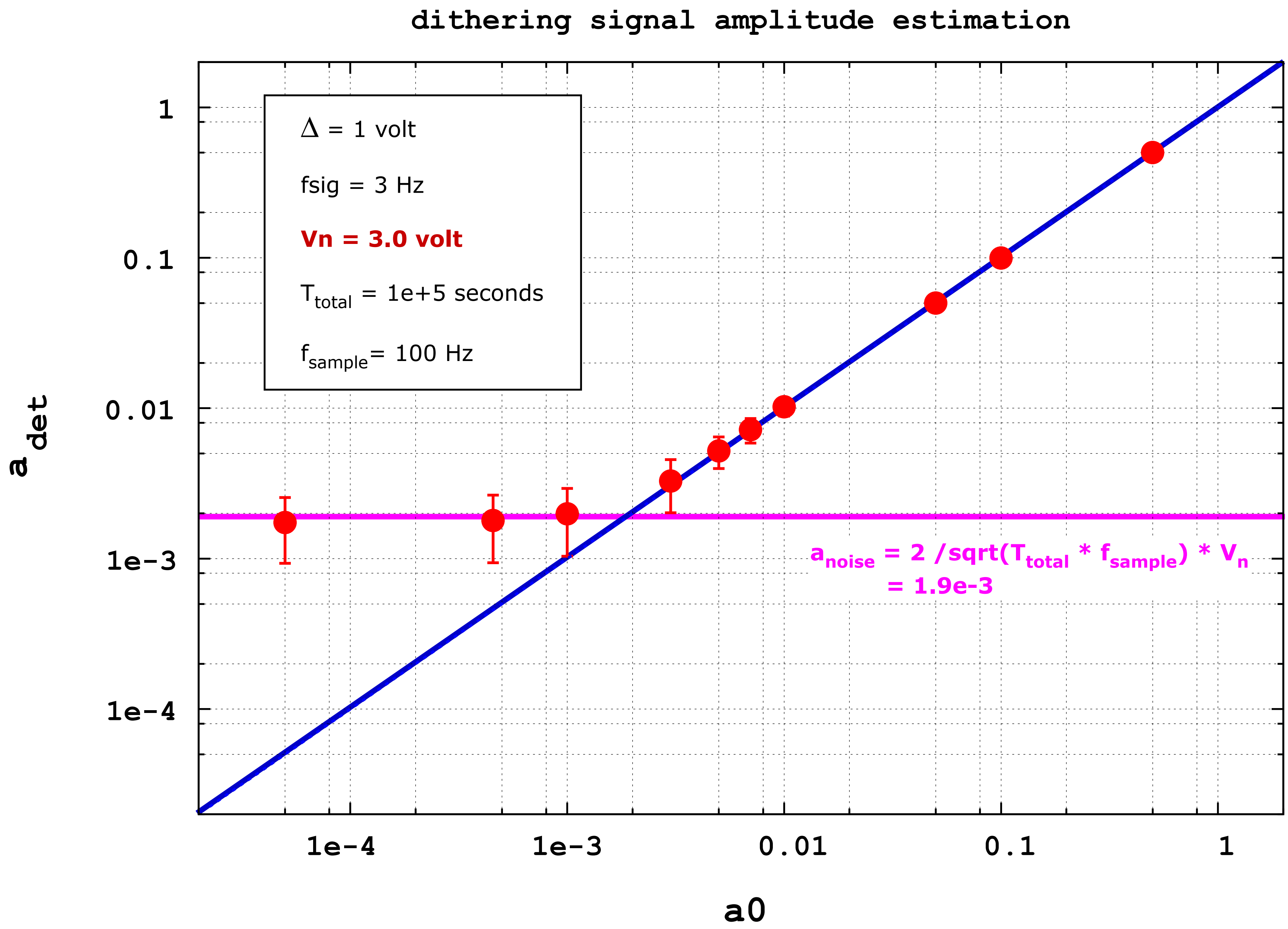




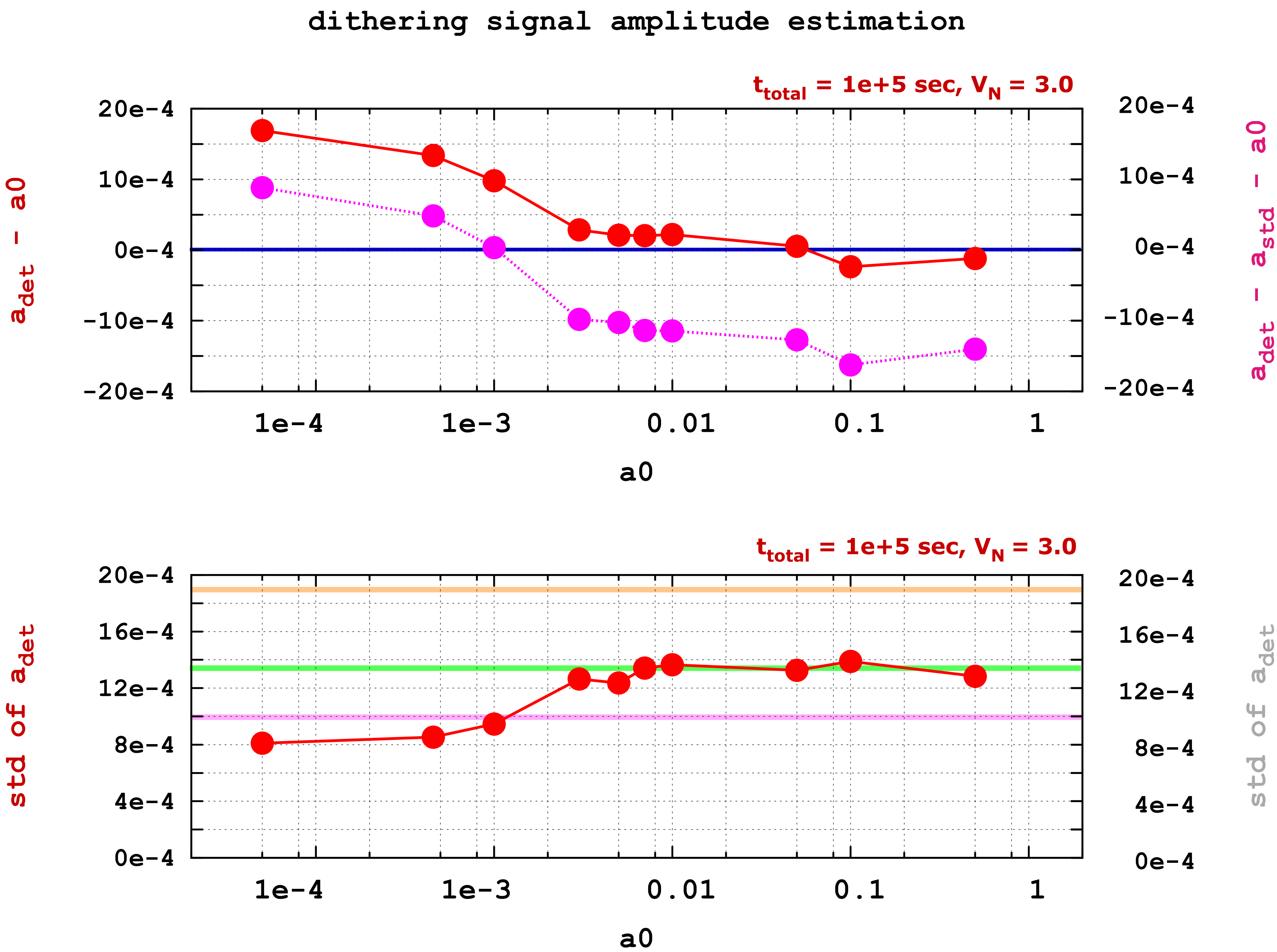
dithering signal amplitude estimation
Δ = 1 volt
fsig = 3 Hz
Vn = 0.1 volt
T total = 50 seconds
f sample = 100 Hz
a noise
= 0.0283 *V n
= 0.0028
a det
a0
10
1
0.1
0.01
0.001
0.001
0.01
0.1
0.5
1
10



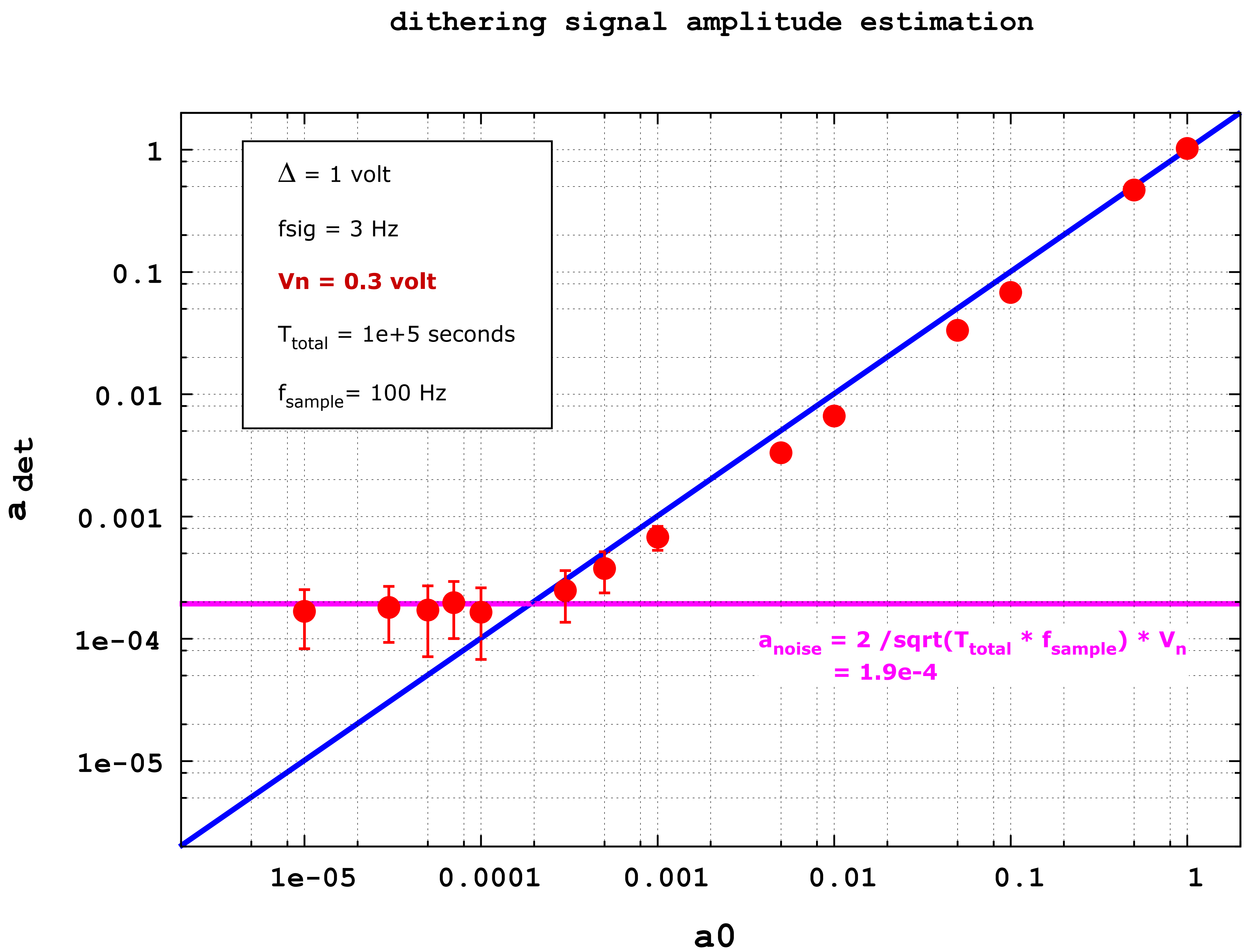
dithering signal amplitude estimation
T_total = 50 sec, V_N = 0.1
0.10
0.00
-0.10
-0.20
-0.30
a_det - a0
a_det - a_std - a0
0.001
0.01
0.1
1
10
a0/Δ
T_total = 50 sec, V_N = 0.1
0.010
0.008
0.006
0.004
0.002
0.000
std of a_det
a_det - a_std - a0
0.001
0.01
0.1
1
10
a0/Δ



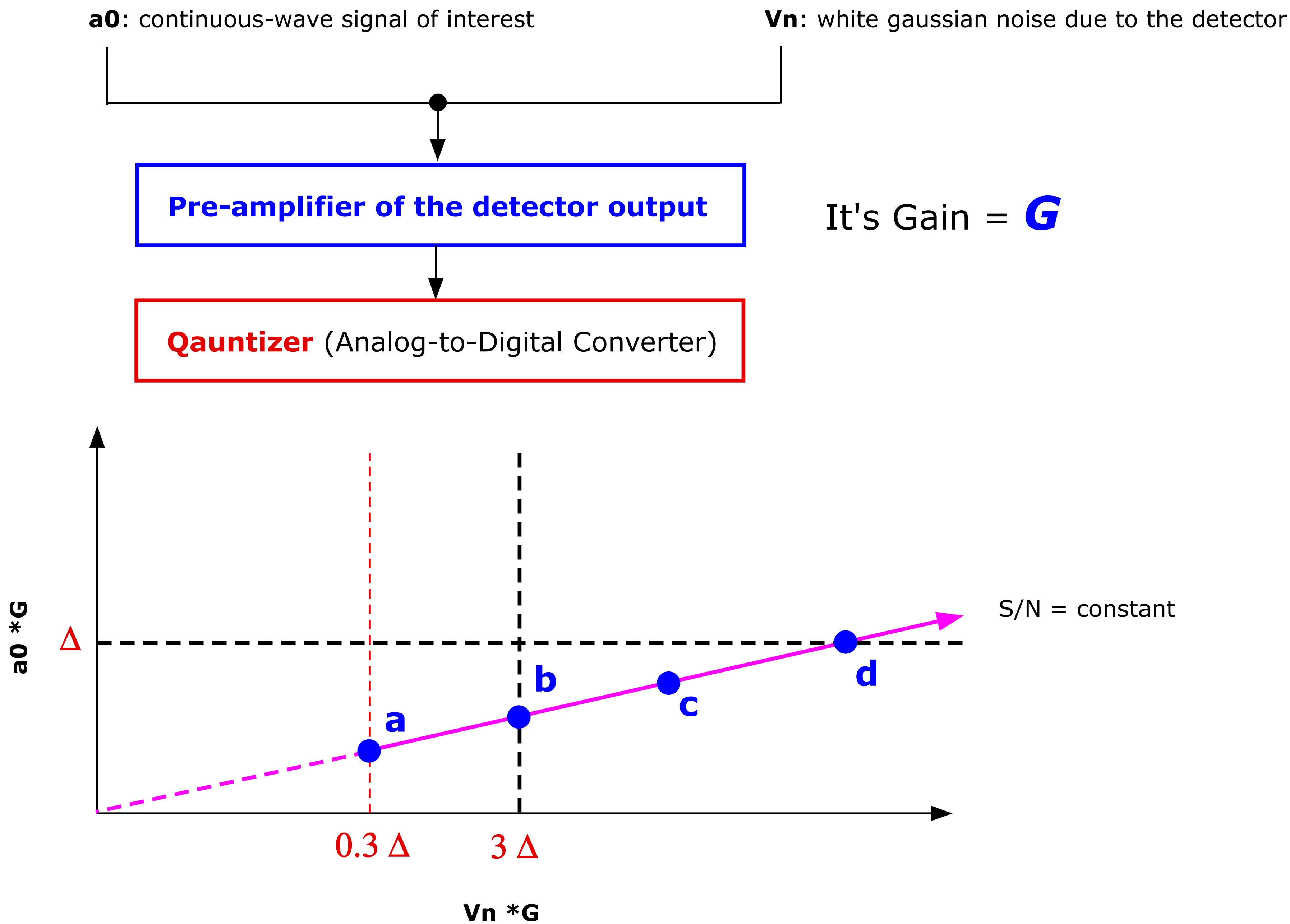
dithering signal amplitude estimation
Δ = 1 volt
fsig = 3 Hz
Vn = 3.0 volt
T_total = 1e+5 seconds
f_sample= 100 Hz
a_noise = 2 /sqrt(T_total * f_sample) * V_n
= 1.9e-3
1
0.1
0.01
1e-3
1e-4
a_det
1e-4
1e-3
0.01
0.1
1
a0



dithering signal amplitude estimation
t_total = 1e+5 sec, V_N = 3.0
20e-4
10e-4
0e-4
-10e-4
-20e-4
a_det - a0
a_det - a_std - a0
1e-4
1e-3
0.01
0.1
1
a0
t_total = 1e+5 sec, V_N = 3.0
20e-4
16e-4
12e-4
8e-4
4e-4
0e-4
std of a_det
1e-4
1e-3
0.01
0.1
1
a0


dithering signal amplitude estimation
Δ = 1 volt
fsig = 3 Hz
Vn = 0.3 volt
T_total = 1e+5 seconds
f_sample = 100 Hz
a_noise = 2 /sqrt(T_total * f_sample) * V_n
= 1.9e-4
a det
a0
1
0.1
0.01
0.001
1e-04
1e-05
1e-05
0.0001
0.001
0.01
0.1
1

Realistic situation for controlling dithering conditions



To set simulation parameters

(0) Δ = 1, T_{obs} = 1.0e+5 sec (~1 days), f_{sample} = 100 Hz

(1) When G=1, Vn *G = Vn = 0.3 Δ

(2) Set "*a0*"

$$\text{S/N} = \frac{a0}{a_{noise}} = \frac{a0\sqrt{T_{obs}\,f_s}}{2\,V_n}$$

$$a0 = \frac{2\,V_n\,(\text{S/N})}{\sqrt{T_{obs}\,f_s}}$$

(3) Make simulation data and obtain the analysis results as follows.

To set simulation parameters

(3) Make simulation data and obtain the analysis results "a_{det} /G" as follows.

3.1 Loss of the signal

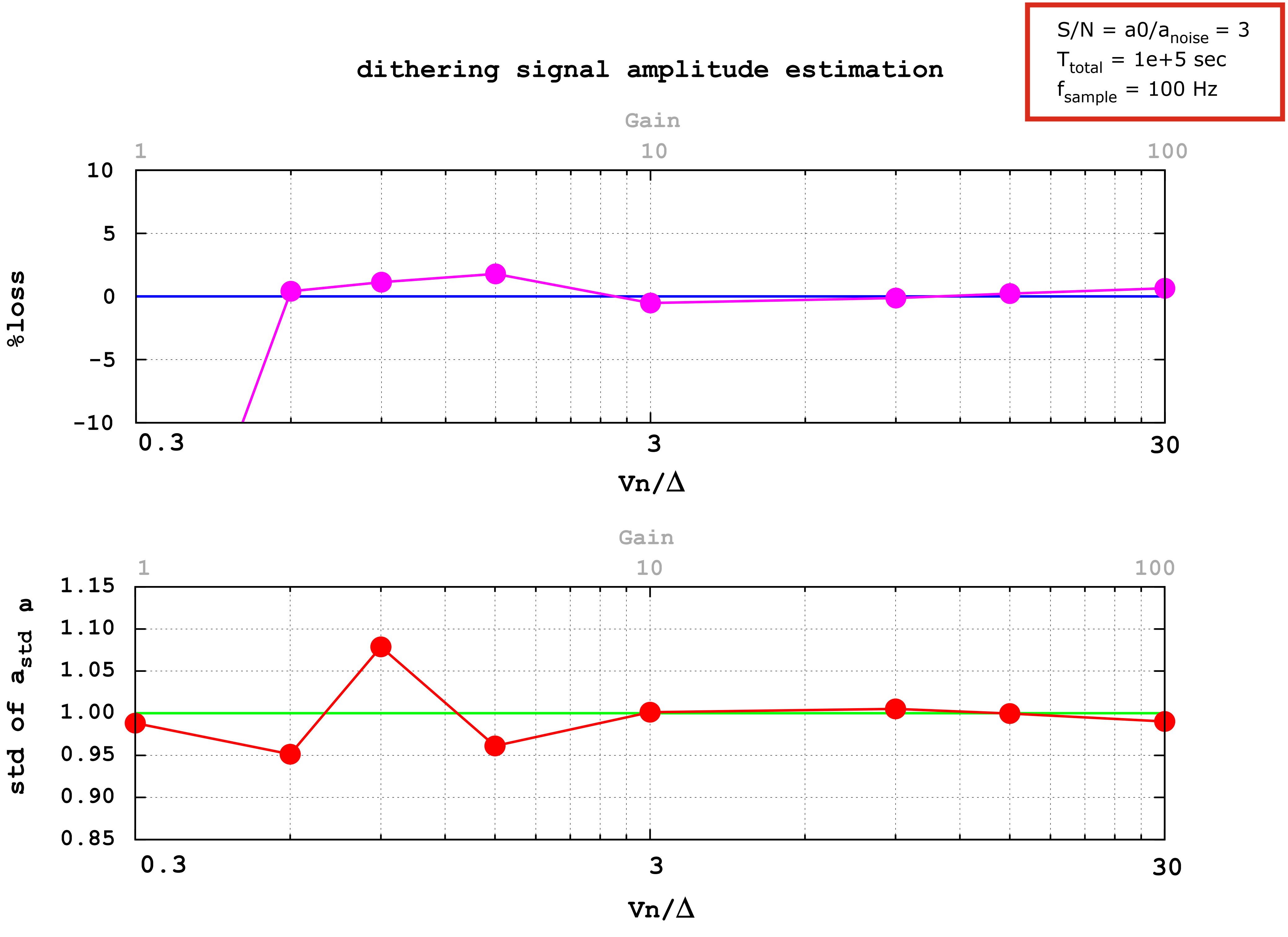
$$a' = a_{\mathrm{det}} / G$$

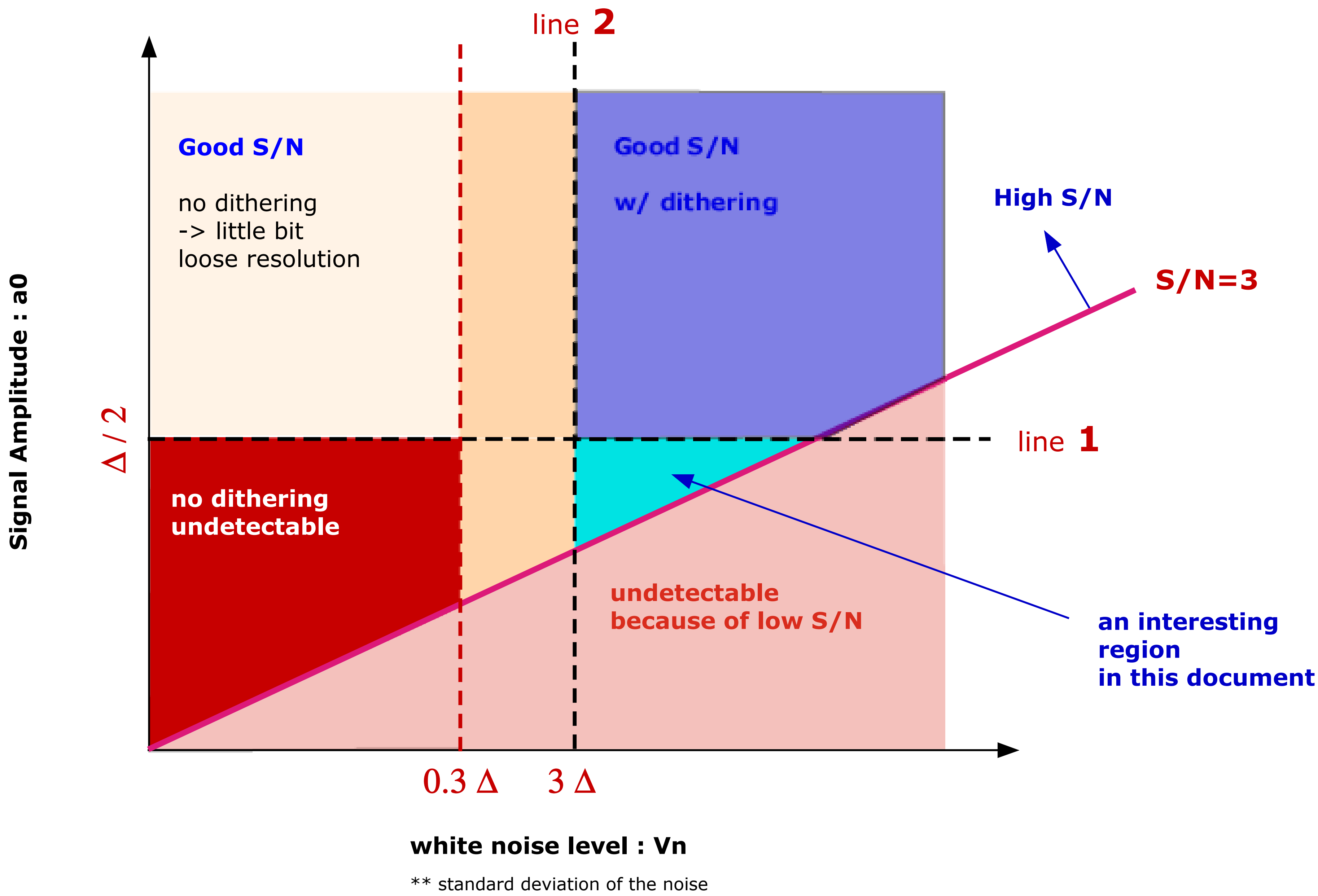
$$a_{\mathrm{measure}} = sqrt\,(\, a'_{det}{}^{2} - a'_{std}{}^{2}\,)$$

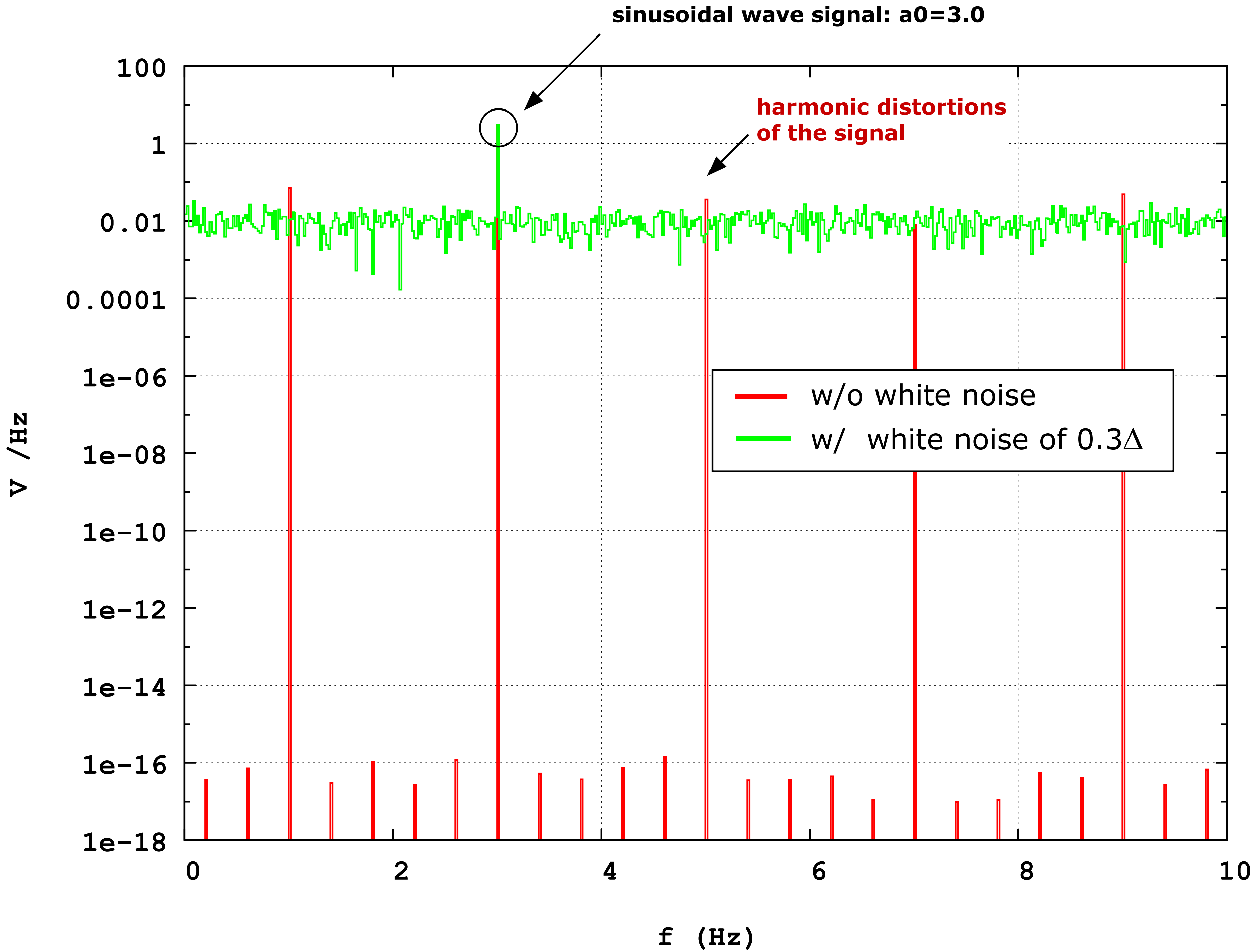
$$\%loss = \frac{a_{\mathrm{measure}} - a_0}{a_0} \times 100$$

3.2 Standard deviation

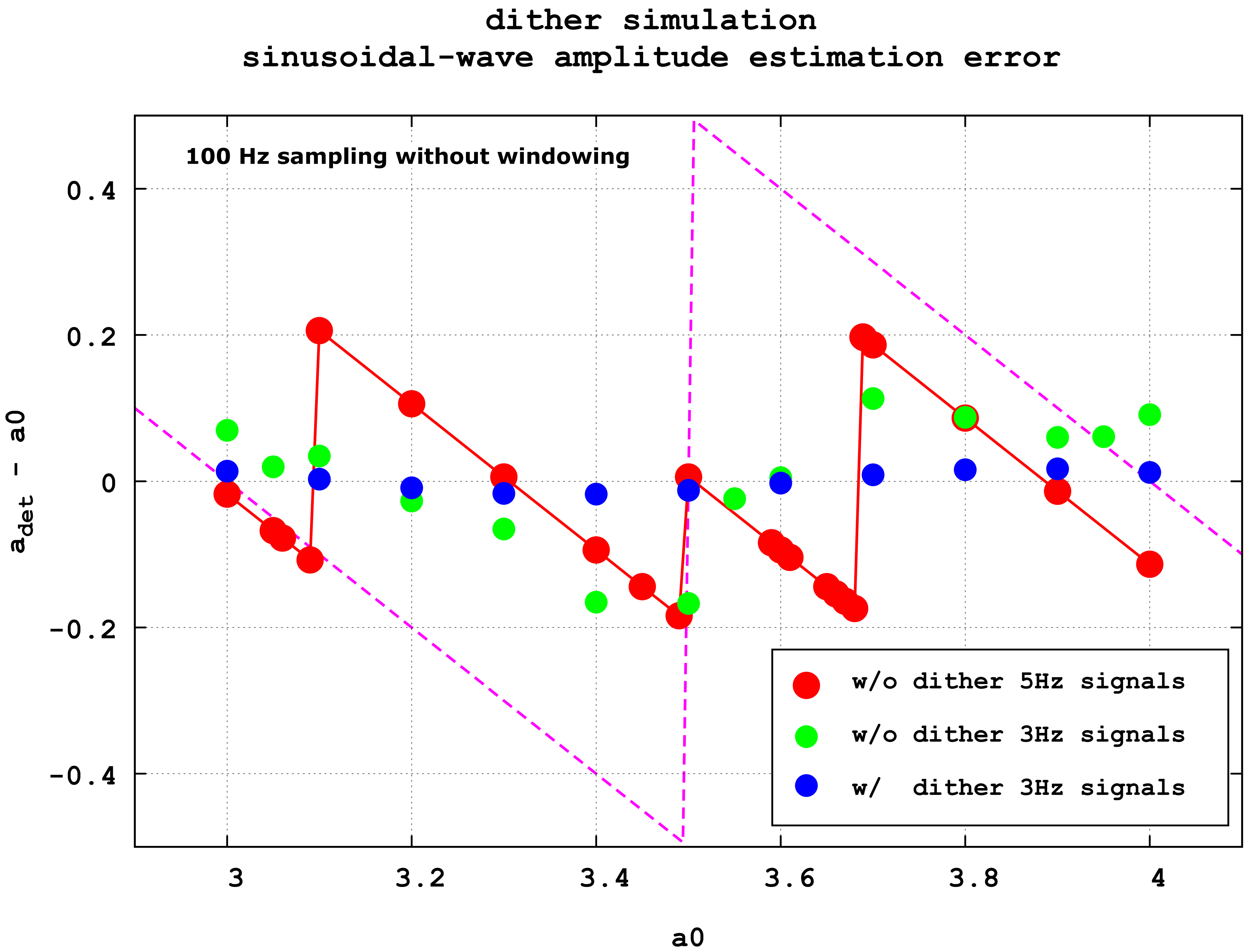
$$(\text{expected } a'_{std}) = \sqrt{\frac{2}{T_{obs} f_s}}\ V_n$$


S/N = a0/a_noise = 3
T_total = 1e+5 sec
f_sample = 100 Hz
dithering signal amplitude estimation
Gain
1
10
100
10
5
0
-5
-10
%loss
0.3
3
30
Vn/Δ
Gain
1
10
100
1.15
1.10
1.05
1.00
0.95
0.90
0.85
std of a_std a
0.3
3
30
Vn/Δ


line 2
Good S/N
no dithering
-> little bit
loose resolution
Good S/N
w/ dithering
High S/N
S/N=3
Signal Amplitude : a0
Δ / 2
line 1
no dithering
undetectable
undetectable
because of low S/N
an interesting
region
in this document
0.3 Δ
3 Δ
white noise level : Vn
** standard deviation of the noise



sinusoidal wave signal: a0=3.0
harmonic distortions of the signal
w/o white noise
w/ white noise of 0.3Δ
V /Hz
100
1
0.01
0.0001
1e-06
1e-08
1e-10
1e-12
1e-14
1e-16
1e-18
0
2
4
6
8
10
f (Hz)



dither simulation
sinusoidal-wave amplitude estimation error
100 Hz sampling without windowing
0.4
0.2
0
-0.2
-0.4
a_{det} - a0
3
3.2
3.4
3.6
3.8
4
a0
w/o dither 5Hz signals
w/o dither 3Hz signals
w/ dither 3Hz signals