Simulation for understanding what will happen in dithering

Daisuke TATSUM

NAOJ

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(1) Signal of interest

a sinusoidal wave of

amplitude: **a0**

frequency: $\mathbf{f}_{sig}$

$v_{sig}(t) = a0 * \cos(2 * pi * f_{sig} * t)$

(2) White gauss noise

standard deviation of the noise = **Vn**

average of th noise = 0.0

random number generator: **gsl_ran_gaussian**

in Gnu Science Library

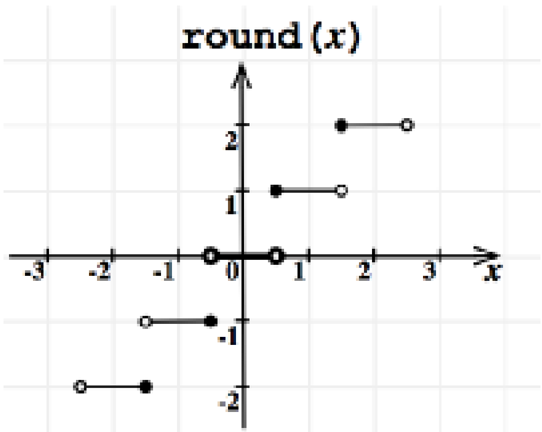
(3) Quantization

$Vq(t) = round(V(t) / \Delta) * \Delta$

Δ: quantization step

The round function in C language has a response as a left figure.

$\mathbf{f}_{sample}$: sampling frequency of the quantizer.



Simulation Example 0 :

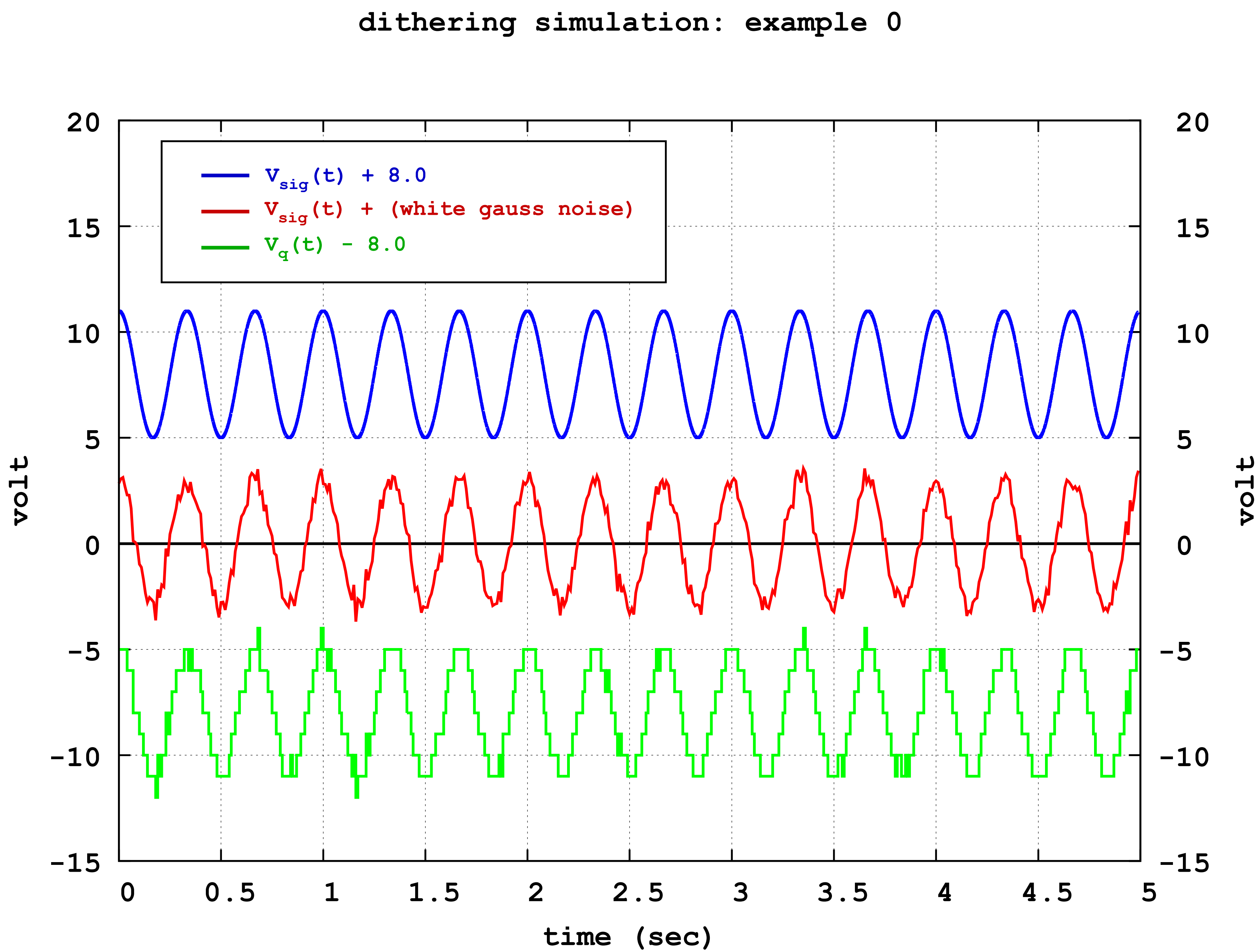
Δ = 1 volt

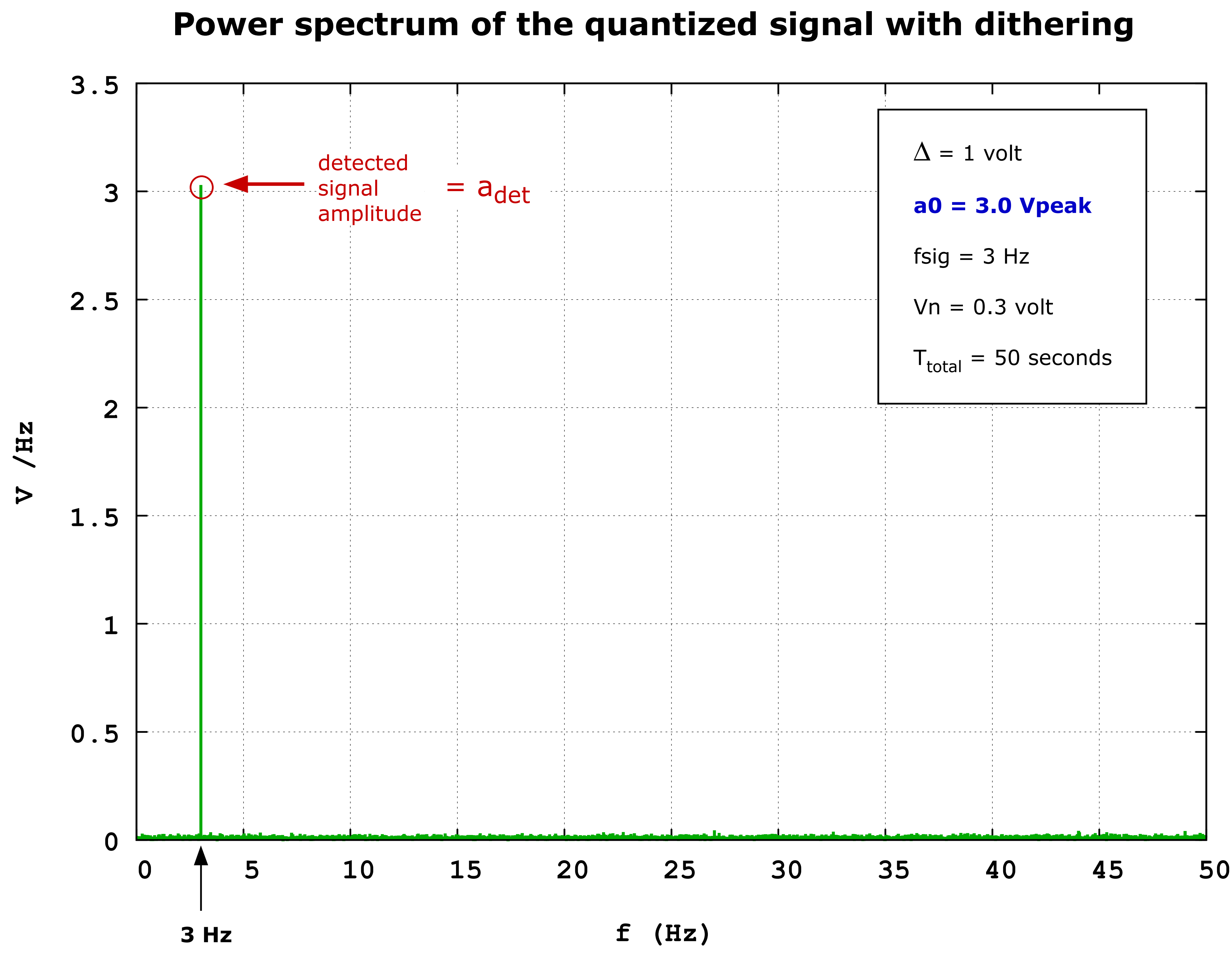
a0 = 3.0 Vpeak

fsig = 3 Hz

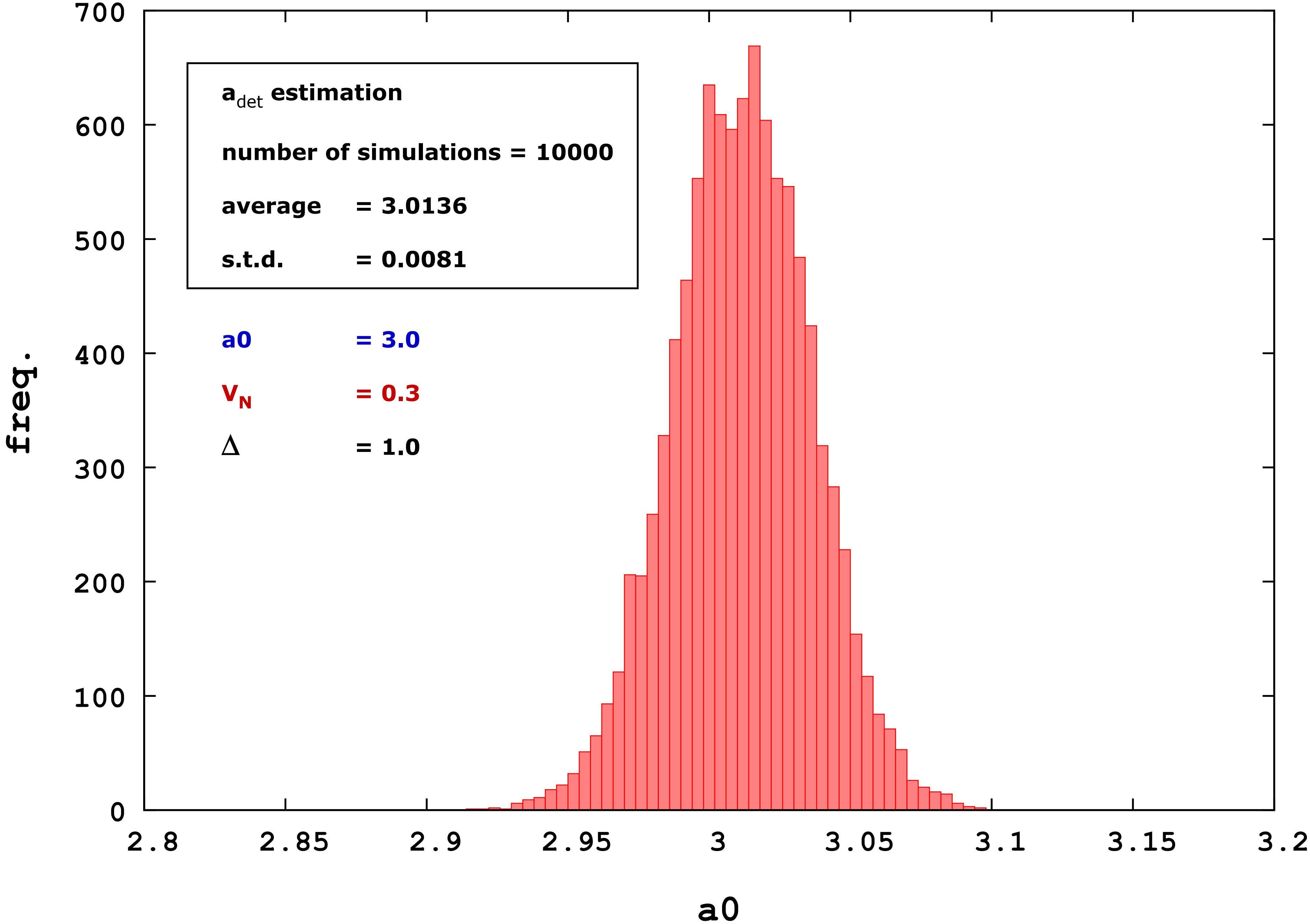
Vn = 0.3 volt

T_{total} = 50 seconds


dithering simulation: example 0
$V_{sig}(t)$ + 8.0
$V_{sig}(t)$ + (white gauss noise)
$V_q(t)$ - 8.0
volt
time (sec)
20
15
10
5
0
-5
-10
-15
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5



Power spectrum of the quantized signal with dithering
3.5
3
2.5
2
1.5
1
0.5
0
V /Hz
0
5
10
15
20
25
30
35
40
45
50
f (Hz)
3 Hz
detected signal amplitude
= a_det
Δ = 1 volt
a0 = 3.0 Vpeak
fsig = 3 Hz
Vn = 0.3 volt
T_total = 50 seconds



a_{det} estimation
number of simulations = 10000
average = 3.0136
s.t.d. = 0.0081
a0 = 3.0
V_N = 0.3
Δ = 1.0
freq.
a0
0
100
200
300
400
500
600
700
2.8
2.85
2.9
2.95
3
3.05
3.1
3.15
3.2

Simulation Example 1 :

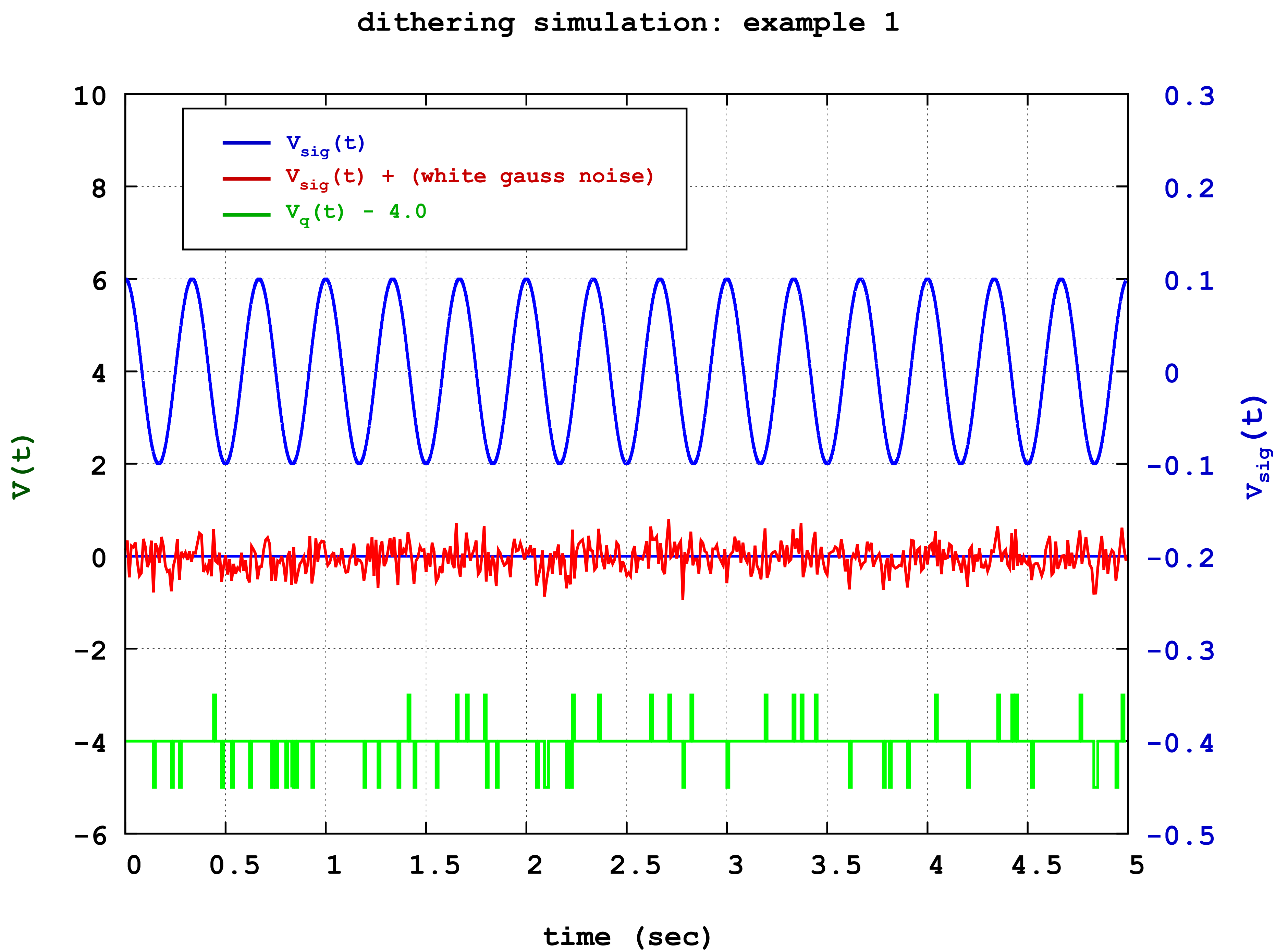
Δ = 1 volt

a0 = 0.1 Vpeak

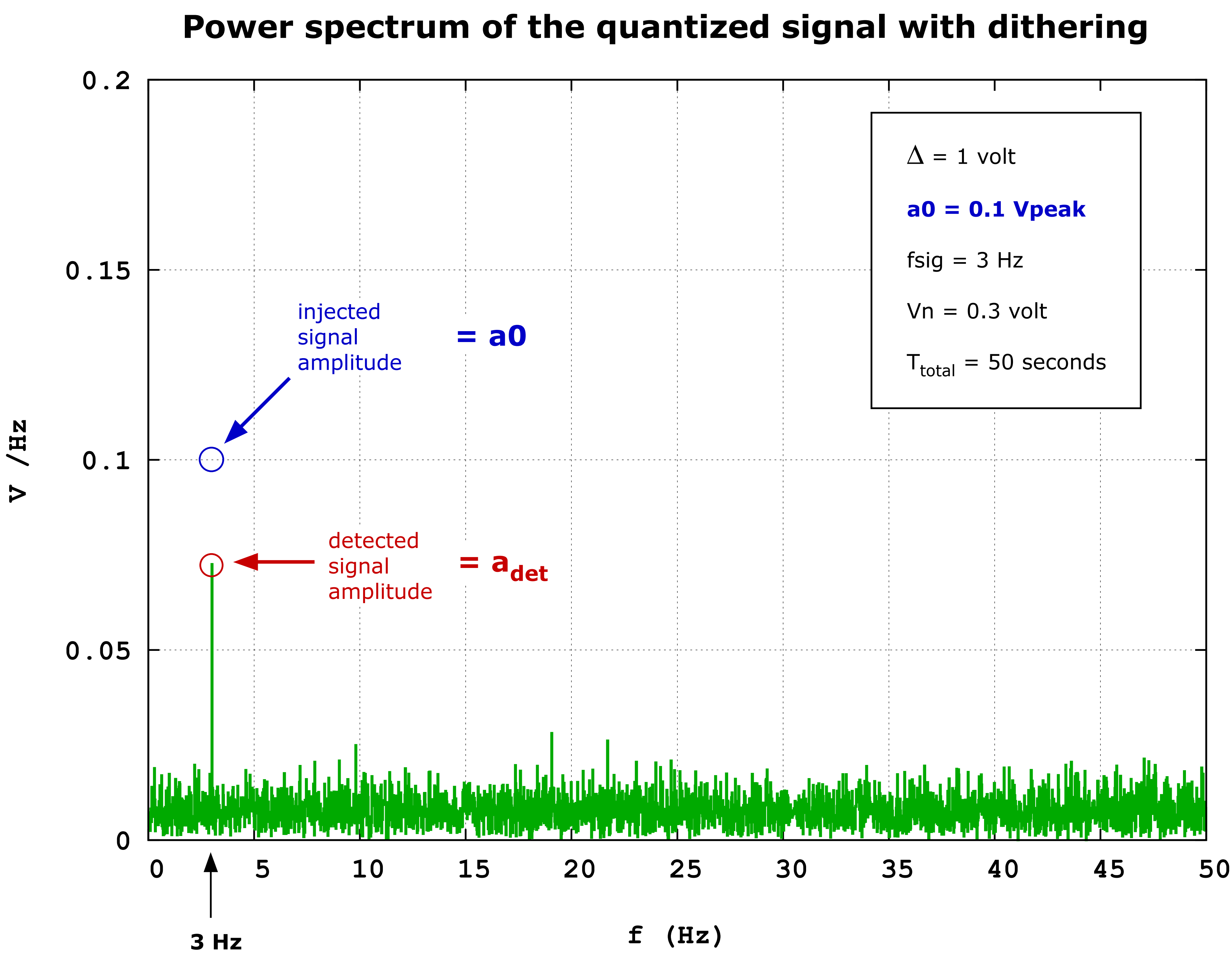
fsig = 3 Hz

Vn = 0.3 volt

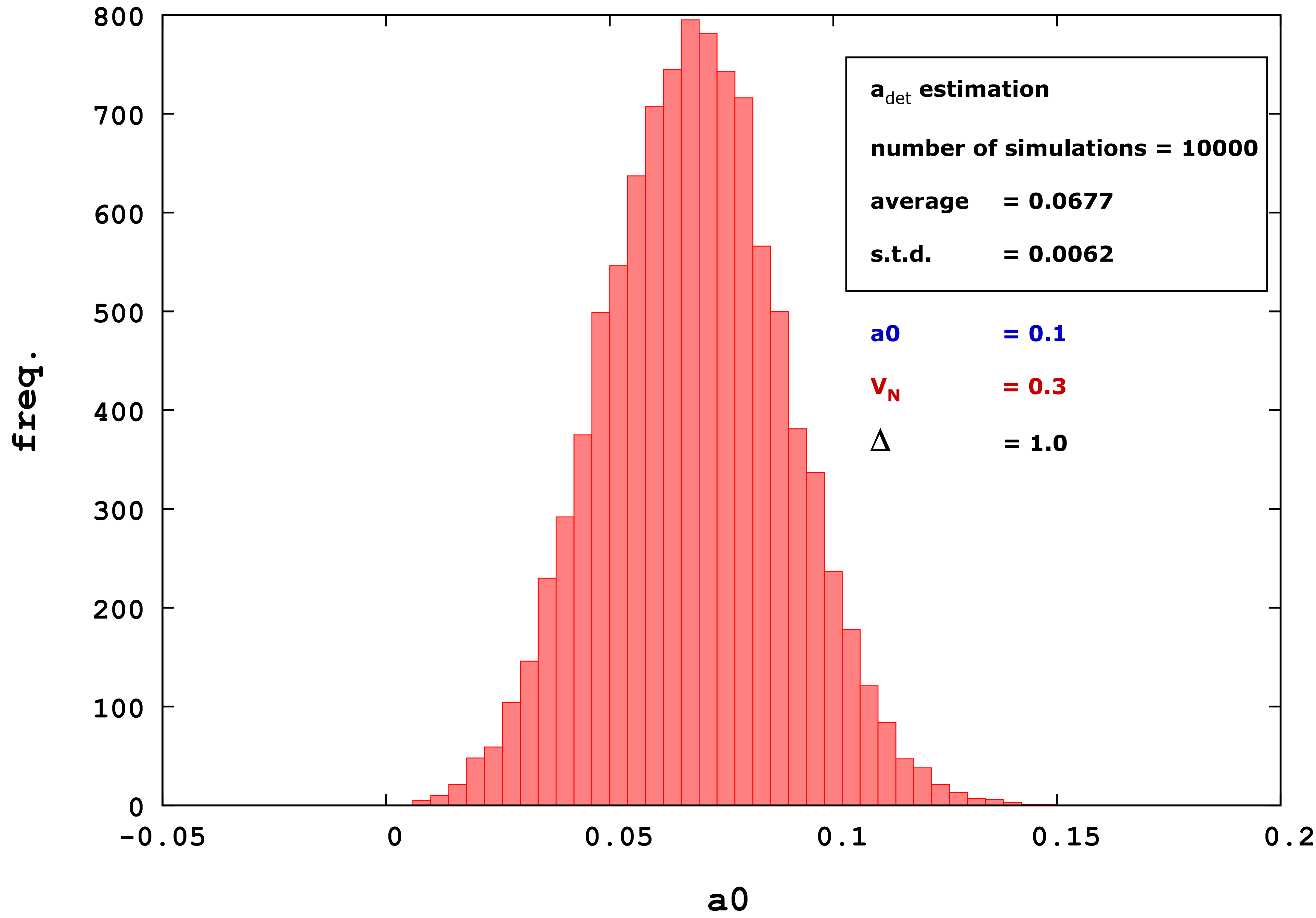
T_{total} = 50 seconds



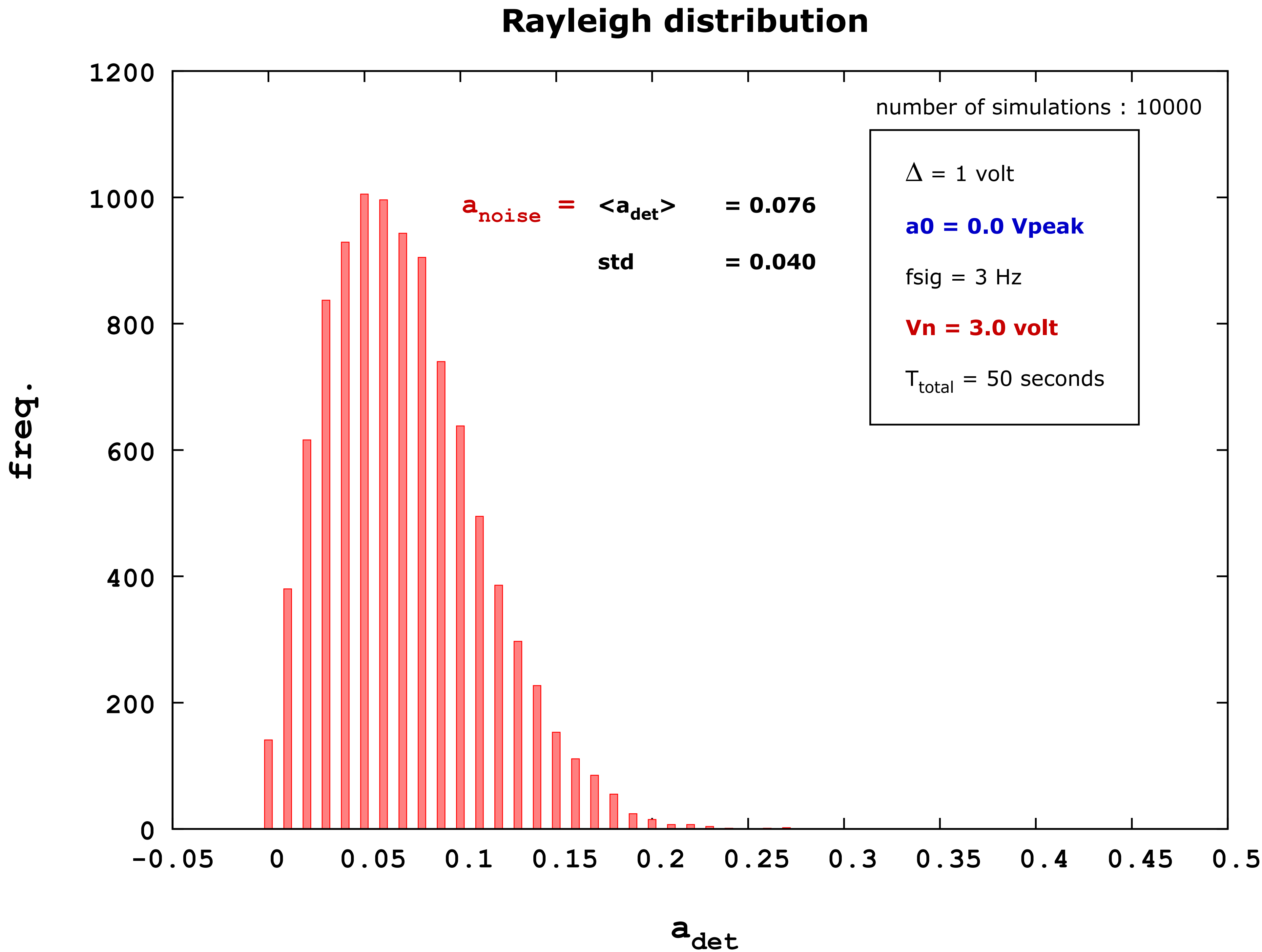
dithering simulation: example 1
$V_{sig}(t)$
$V_{sig}(t)$ + (white gauss noise)
$V_q(t)$ - 4.0
V(t)
$V_{sig}(t)$
time (sec)
10
8
6
4
2
0
-2
-4
-6
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5



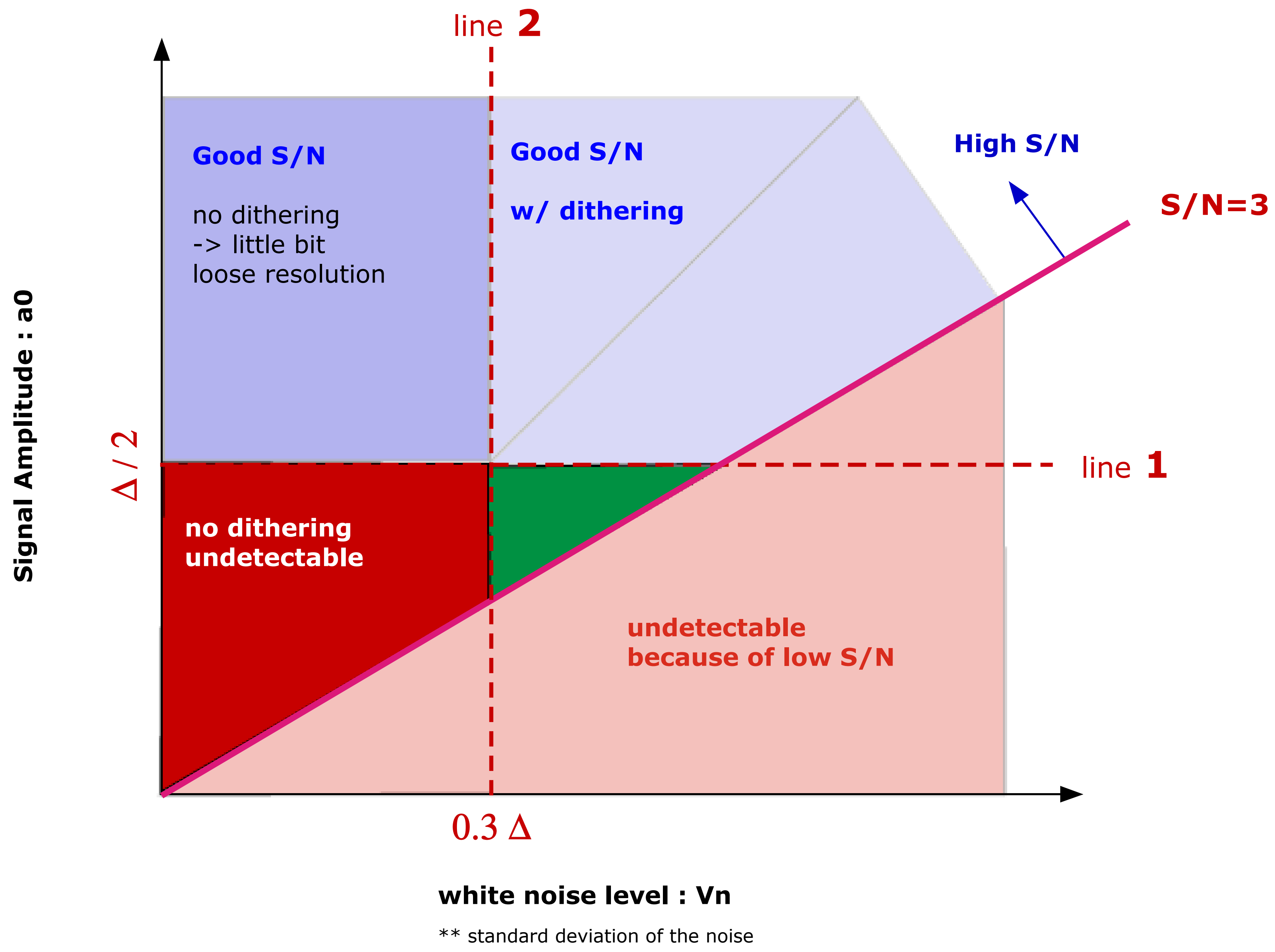
Power spectrum of the quantized signal with dithering
Δ = 1 volt
a0 = 0.1 Vpeak
fsig = 3 Hz
Vn = 0.3 volt
T total = 50 seconds
injected signal amplitude
= a0
detected signal amplitude
= a det
V /Hz
0.2
0.15
0.1
0.05
0
0
5
10
15
20
25
30
35
40
45
50
3 Hz
f (Hz)



800
700
600
500
400
300
200
100
0
-0.05
0
0.05
0.1
0.15
0.2
freq.
a0
a$_{det}$ estimation
number of simulations = 10000
average = 0.0677
s.t.d. = 0.0062
a0 = 0.1
V$_N$ = 0.3
Δ = 1.0



Rayleigh distribution
number of simulations : 10000
a_{noise} = <a_{det}> = 0.076
std = 0.040
Δ = 1 volt
a0 = 0.0 Vpeak
fsig = 3 Hz
Vn = 3.0 volt
T_{total} = 50 seconds
freq.
1200
1000
800
600
400
200
0
-0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
a_{det}


line 2
Good S/N
no dithering
-> little bit
loose resolution
Good S/N
w/ dithering
High S/N
S/N=3
Signal Amplitude : a0
Δ / 2
line 1
no dithering
undetectable
undetectable
because of low S/N
0.3 Δ
white noise level : Vn
** standard deviation of the noise

White Gaussian Noise

$$V_n^2 \stackrel{\text{def}}{=} \frac{1}{T}\int_0^T v^2(t)dt$$

$$V_n^2 = 2\int_0^{f_N} \widetilde{s_n}(f)df = 2f_N\,\widetilde{s_n} = f_s\,\widetilde{s_n}$$

f_N: Nyquist freq.

f_s : Sampling freq.

$$\widetilde{s_n} = \frac{V_n^2}{f_s}$$

Power Spectrum Density (PSD) V^2/Hz

$$\sqrt{\widetilde{s_n}} = \sqrt{\frac{V_n^2}{f_s}}$$

Liner Spectrum Density (LSD) $V/\sqrt{Hz}$

$$\widetilde{v_n}(f) = V_n\sqrt{\frac{2T}{f_N}}$$

Fourier Transform of v(t) V/Hz

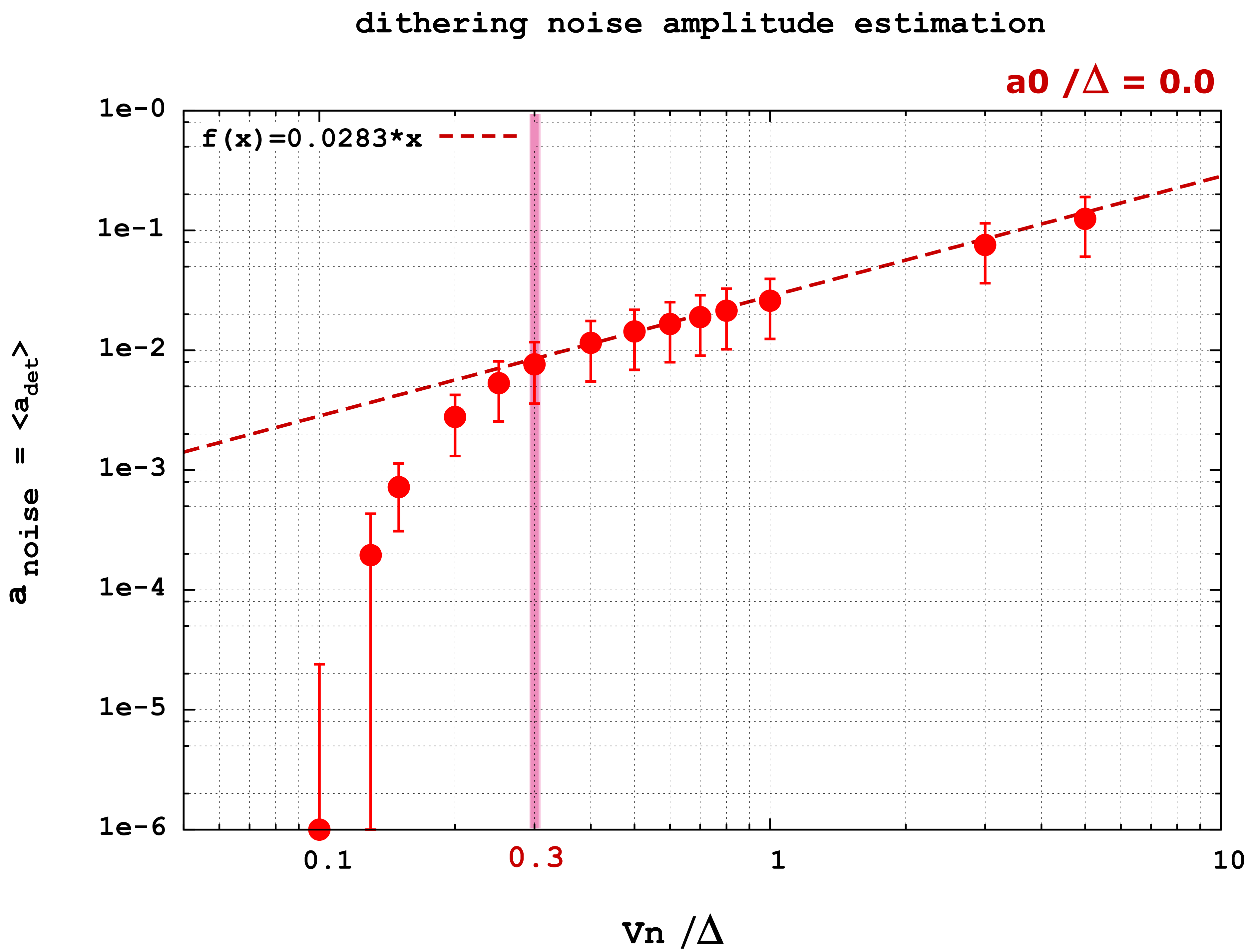
$$a_{noise} = \widetilde{v_n}(f)\cdot\Delta f = \frac{2}{\sqrt{f_s\,T}}\times V_n$$

in a unit of *Volt*

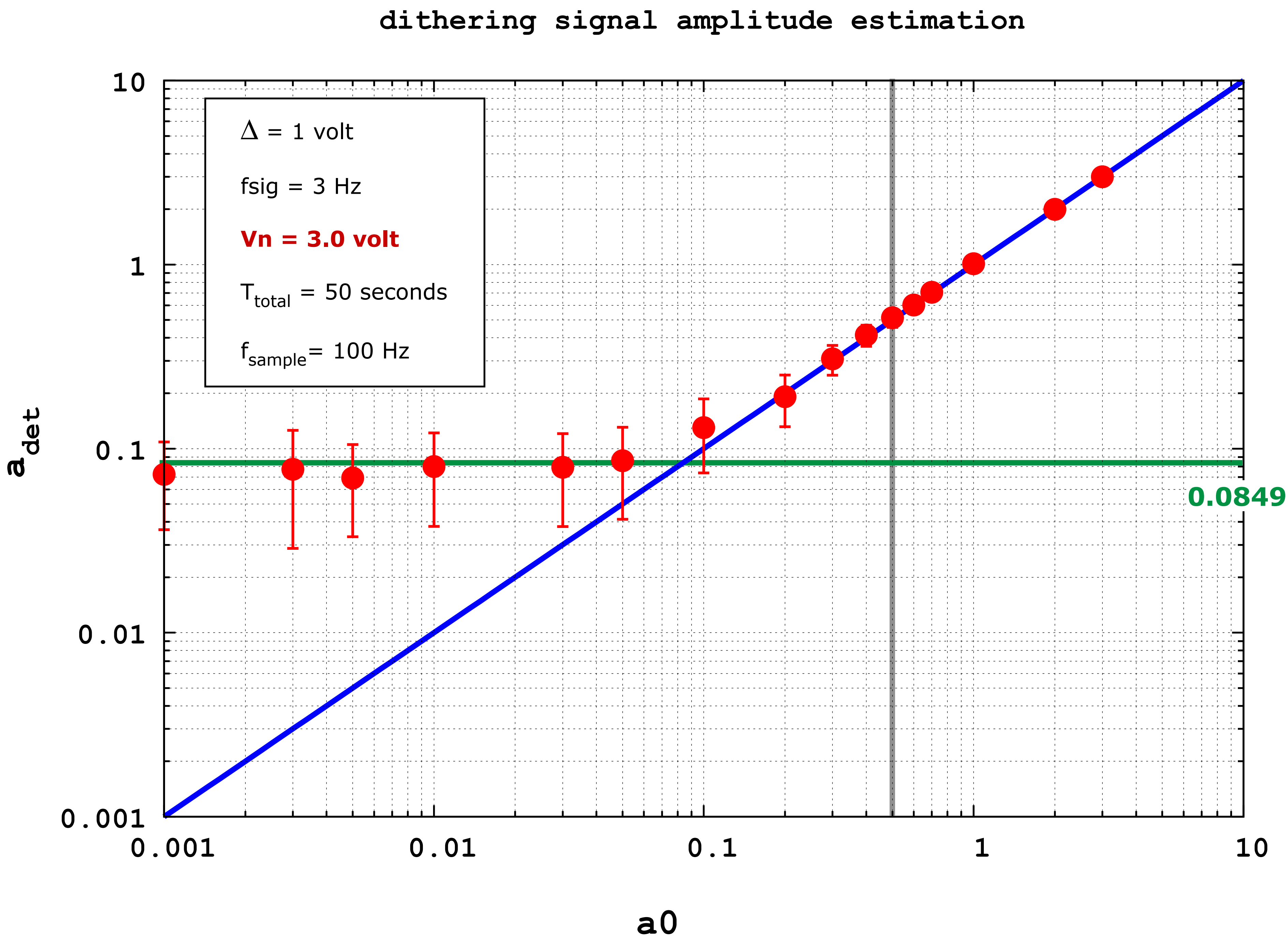
$$a_{noise} = \widetilde{v_n}(f) \cdot \Delta f = \frac{2}{\sqrt{f_s T}} \times V_n$$

fs = 100 Hz, *T* = 50 sec

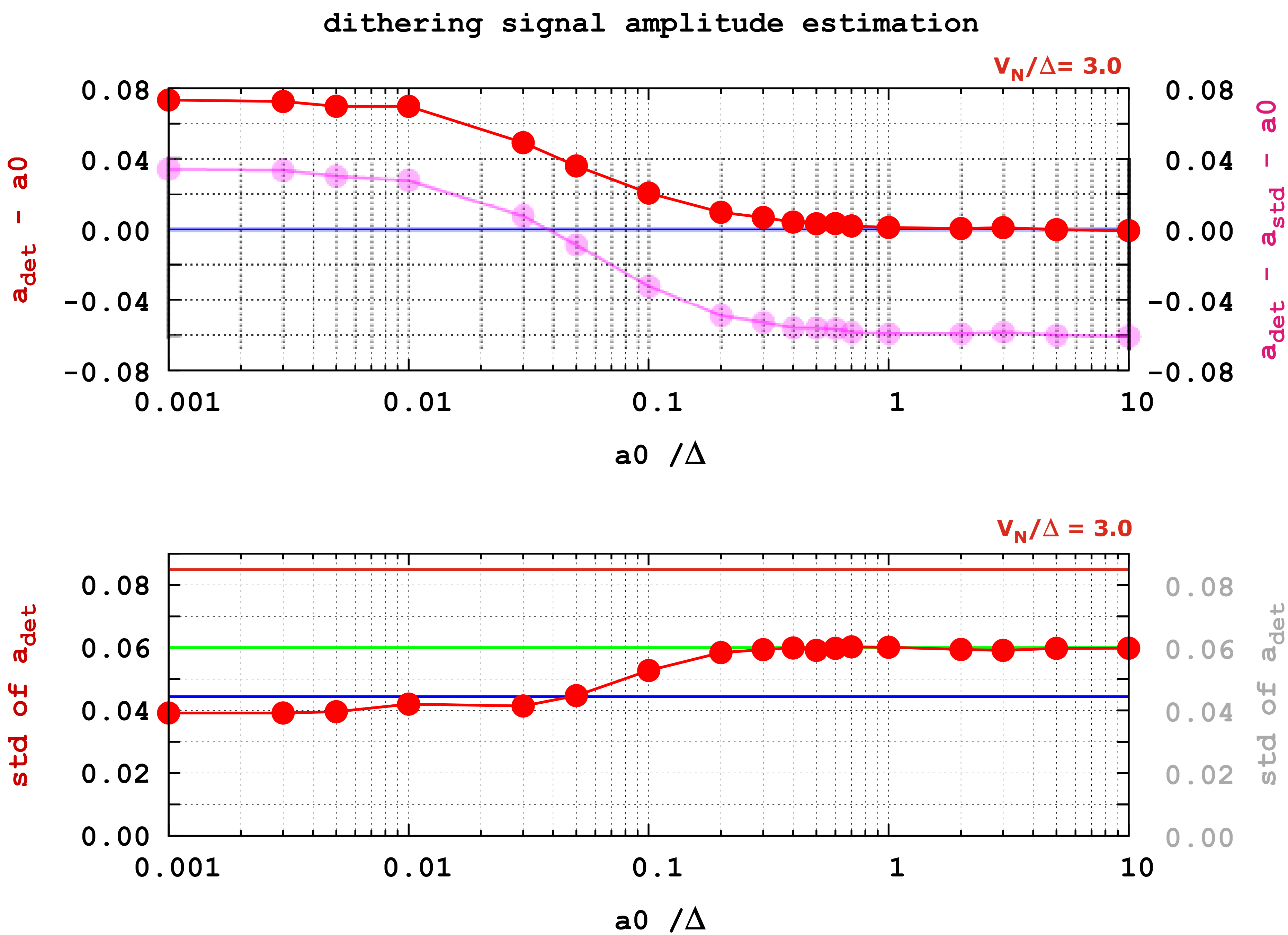
--> $a_{noise} = 0.0283 \times V_n$



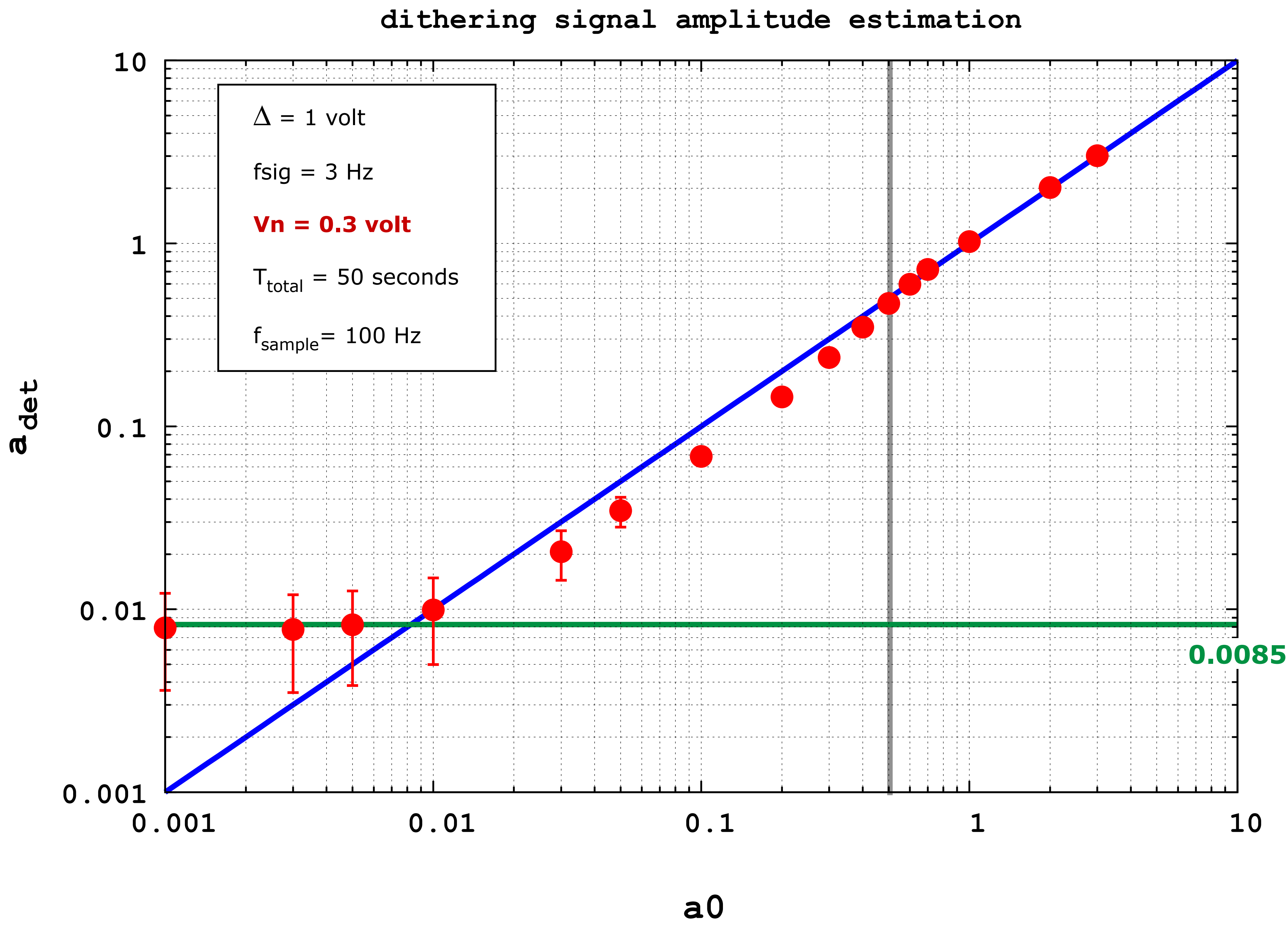
dithering noise amplitude estimation
a0 /Δ = 0.0
f(x)=0.0283*x
1e-0
1e-1
1e-2
1e-3
1e-4
1e-5
1e-6
a noise = <a det>
0.1
0.3
1
10
Vn /Δ



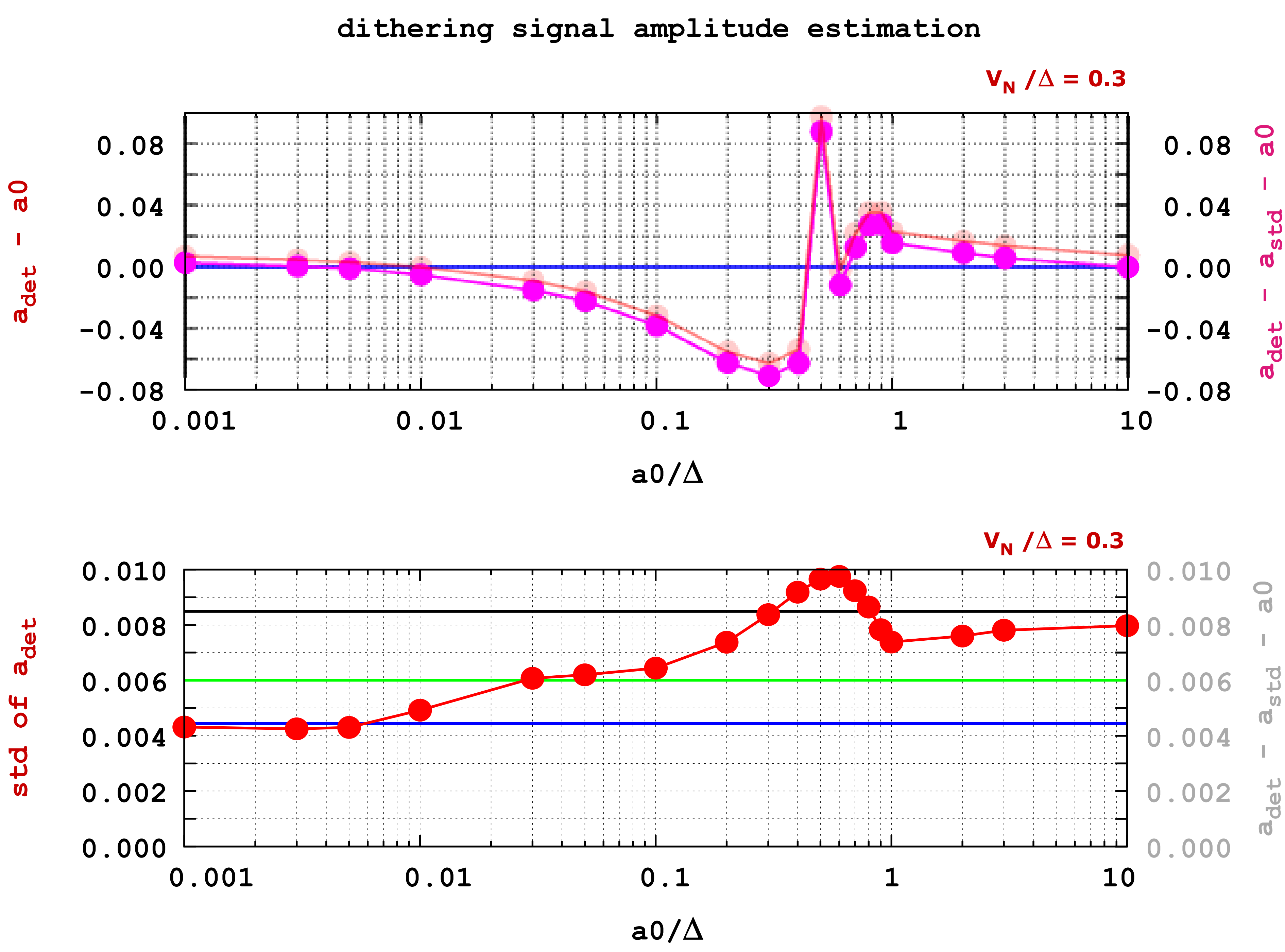
dithering signal amplitude estimation
Δ = 1 volt
fsig = 3 Hz
Vn = 3.0 volt
T_total = 50 seconds
f_sample= 100 Hz
0.0849
10
1
0.1
0.01
0.001
0.001
0.01
0.1
1
10
a_det
a0



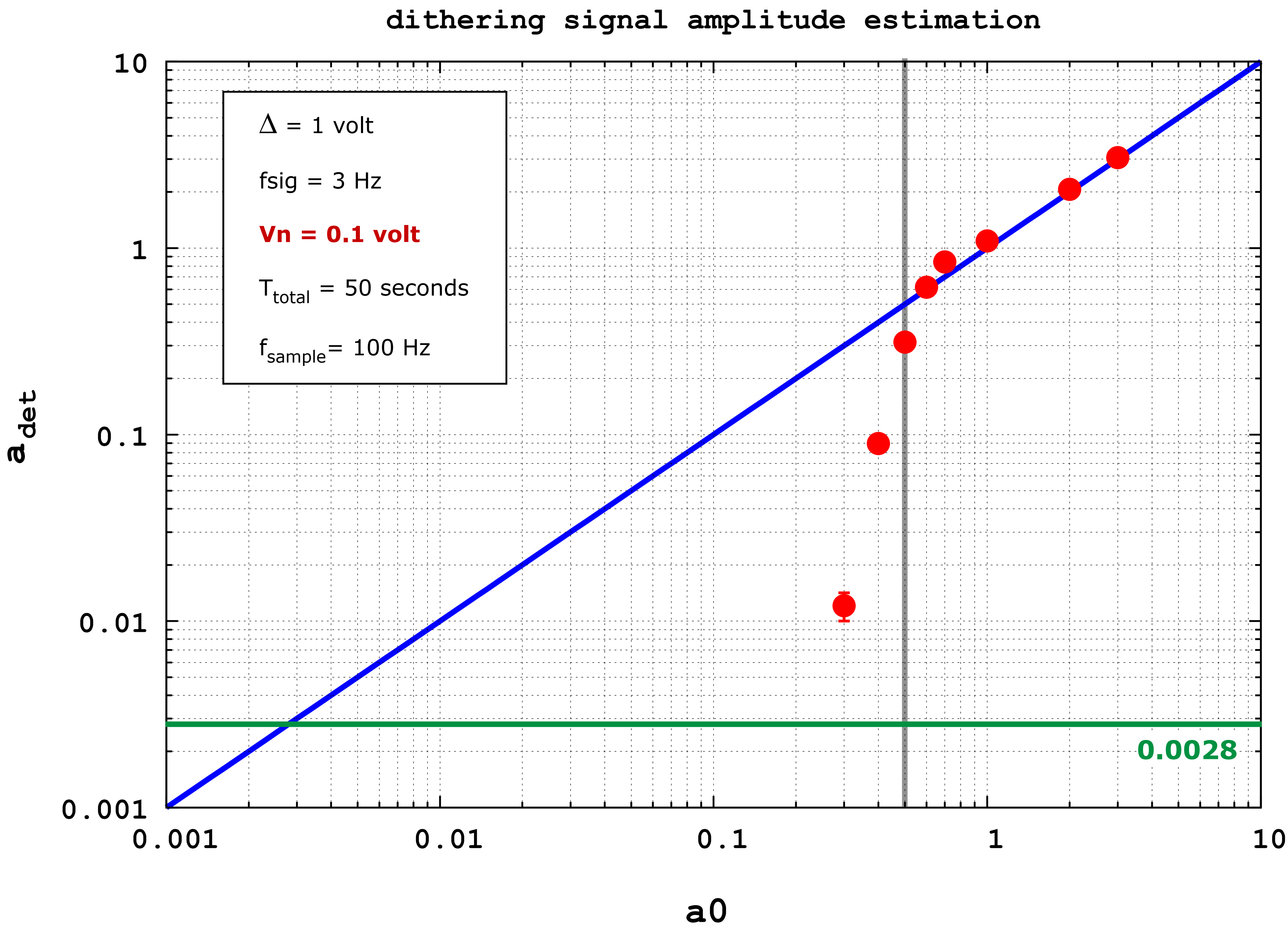
dithering signal amplitude estimation
V_N/Δ= 3.0
0.08
0.04
0.00
-0.04
-0.08
a_det - a0
a_det - a_std - a0
0.001
0.01
0.1
1
10
a0 /Δ
V_N/Δ = 3.0
0.08
0.06
0.04
0.02
0.00
std of a_det
0.001
0.01
0.1
1
10
a0 /Δ



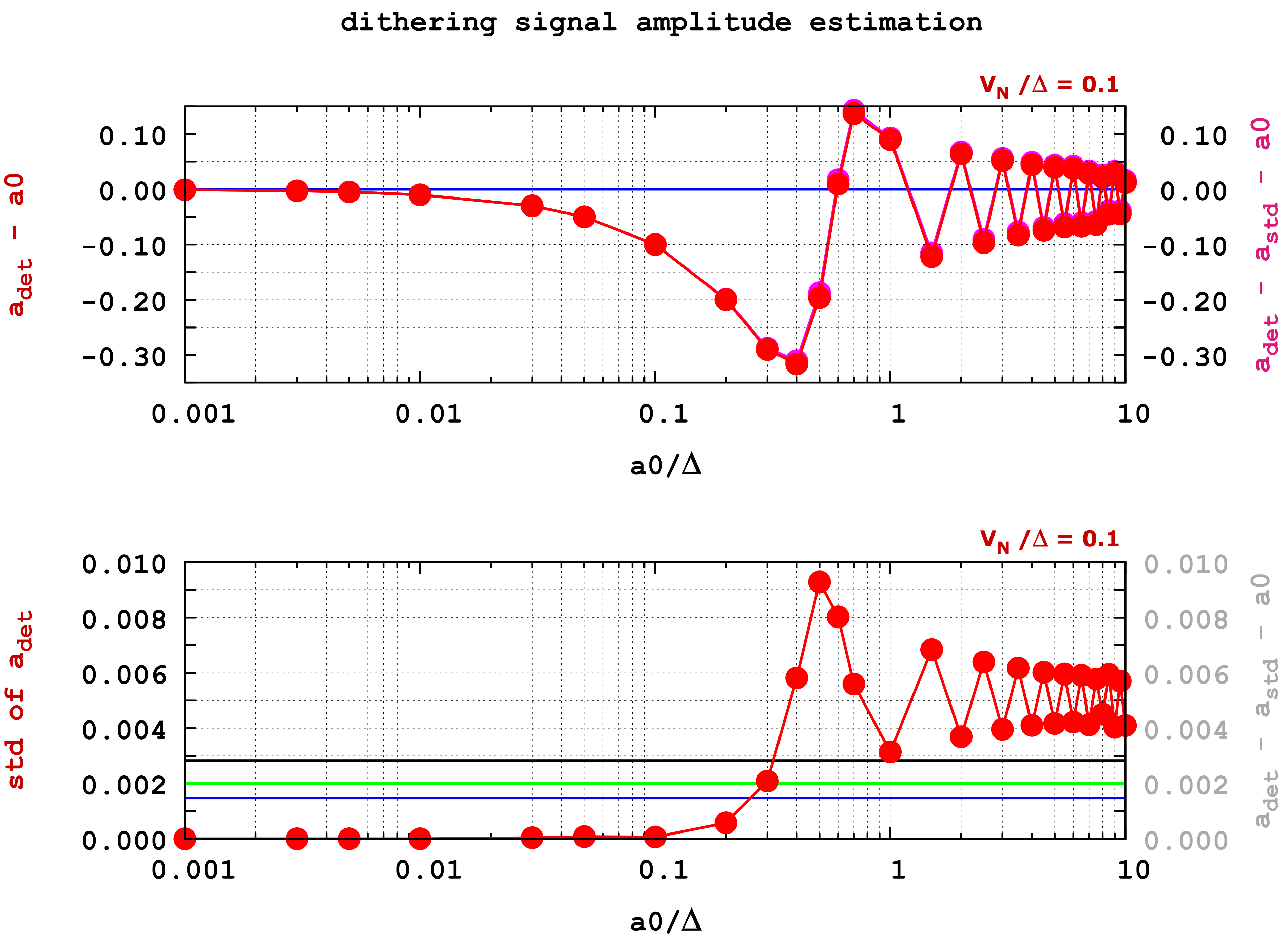
dithering signal amplitude estimation
Δ = 1 volt
fsig = 3 Hz
Vn = 0.3 volt
T_total = 50 seconds
f_sample= 100 Hz
0.0085
a_det
a0
10
1
0.1
0.01
0.001
0.001
0.01
0.1
1
10



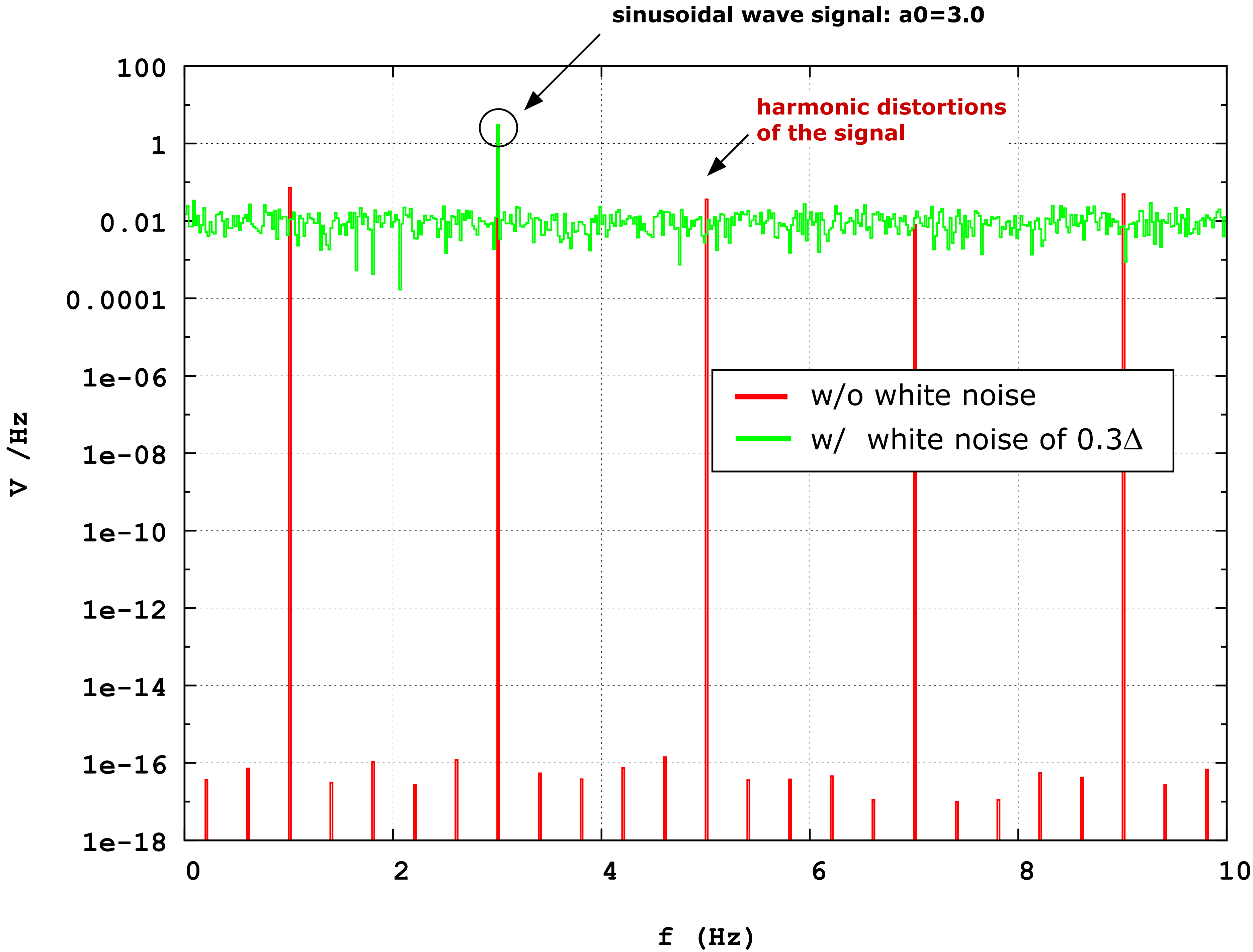
dithering signal amplitude estimation
V_N /Δ = 0.3
0.08
0.04
0.00
-0.04
-0.08
a_det - a0
a_det - a_std - a0
0.001
0.01
0.1
1
10
a0/Δ
V_N /Δ = 0.3
0.010
0.008
0.006
0.004
0.002
0.000
std of a_det
a_det - a_std - a0
0.001
0.01
0.1
1
10
a0/Δ

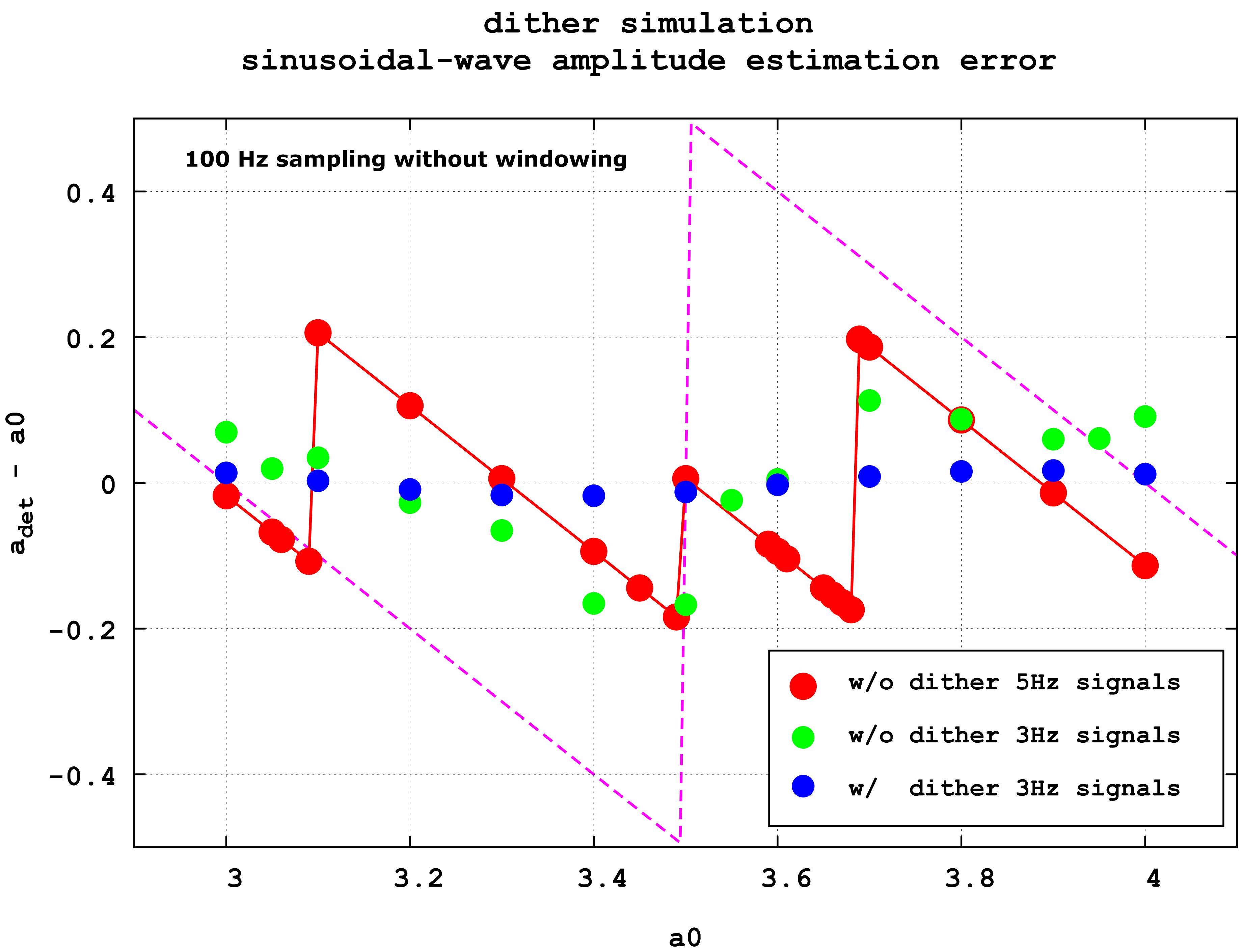


dithering signal amplitude estimation
Δ = 1 volt
fsig = 3 Hz
Vn = 0.1 volt
T_total = 50 seconds
f_sample = 100 Hz
0.0028
a_det
a0
10
1
0.1
0.01
0.001
0.001
0.01
0.1
1
10



dithering signal amplitude estimation
$V_N/\Delta = 0.1$
0.10
0.00
-0.10
-0.20
-0.30
a_{det} - a0
a_{det} - a_{std} - a0
0.001
0.01
0.1
1
10
a0/Δ
$V_N/\Delta = 0.1$
0.010
0.008
0.006
0.004
0.002
0.000
std of a_{det}
a_{det} - a_{std} - a0
0.001
0.01
0.1
1
10
a0/Δ


sinusoidal wave signal: a0=3.0
harmonic distortions
of the signal
w/o white noise
w/ white noise of 0.3Δ
V /Hz
f (Hz)
100
1
0.01
0.0001
1e-06
1e-08
1e-10
1e-12
1e-14
1e-16
1e-18
0
2
4
6
8
10



dither simulation
sinusoidal-wave amplitude estimation error
100 Hz sampling without windowing
0.4
0.2
0
-0.2
-0.4
a_{det} - a0
3
3.2
3.4
3.6
3.8
4
a0
w/o dither 5Hz signals
w/o dither 3Hz signals
w/ dither 3Hz signals