Simulation for understanding what will happen in dithering

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(1) Signal of interest

a sinusoidal wave of
amplitude: **a0**
frequency: $\mathbf{f}_{sig}$

$v_{sig}(t) = a0 * \cos(2 * pi * f_{sig} * t)$

(2) White gauss noise

standard deviation of the noise = **Vn**
average of th noise = 0.0

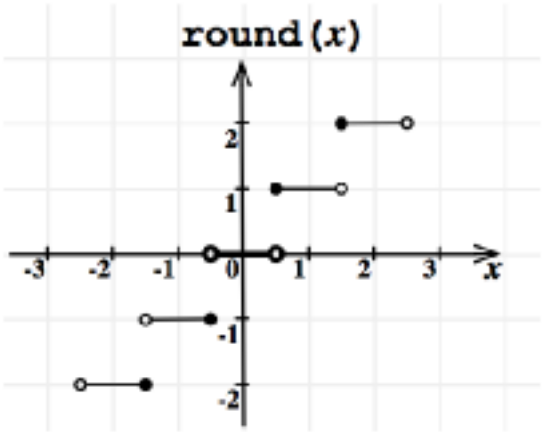
random number generator: **gsl_ran_gaussian**
in Gnu Science Library

(3) Quantization

$Vq(t) = round(V(t) / \Delta) * \Delta$

Δ: quantization step
The round function in C language has a response as a left figure.

$\mathbf{f}_{sample}$: sampling frequency of the quantizer.



Simulation Example 0 :

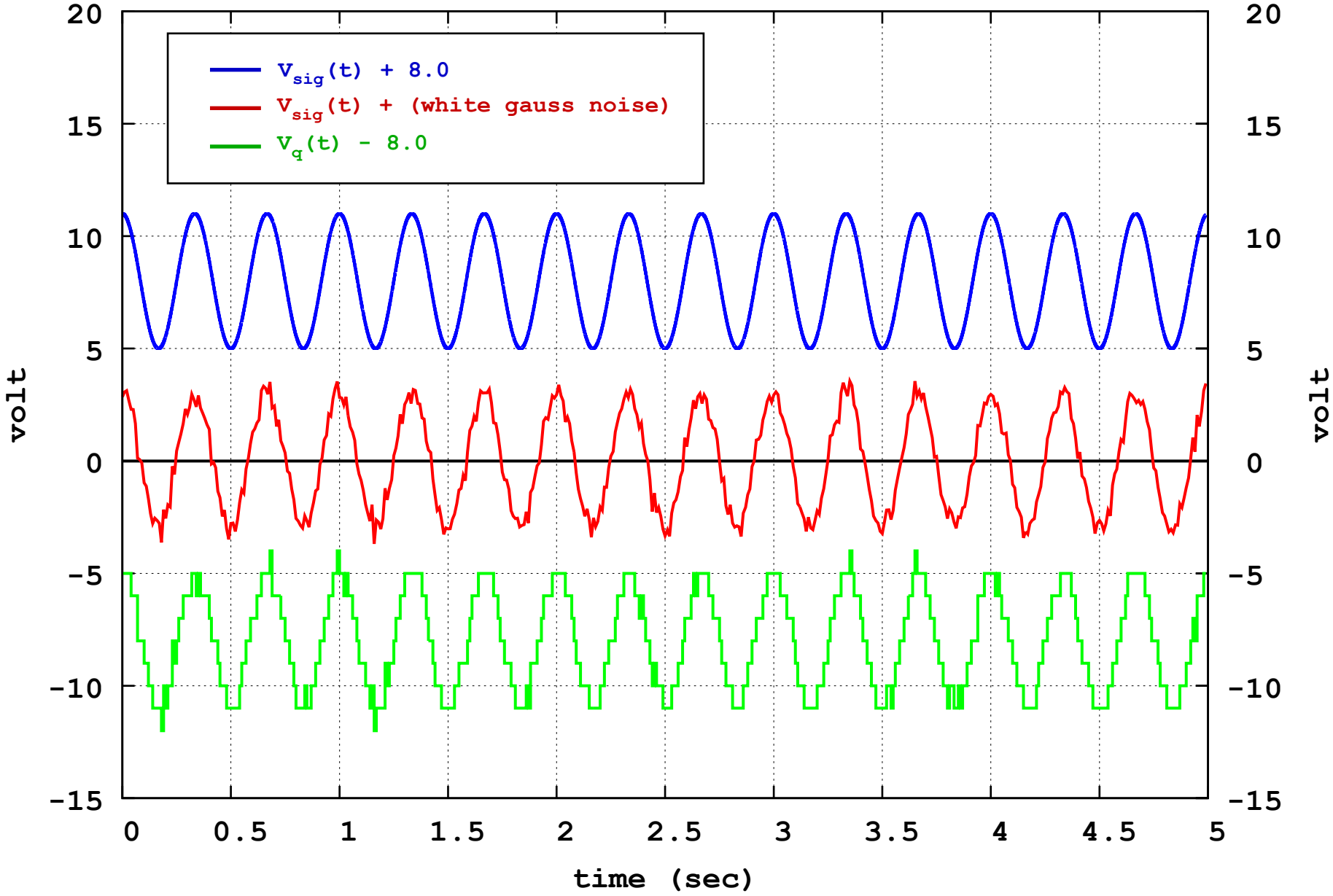
Δ = 1 volt

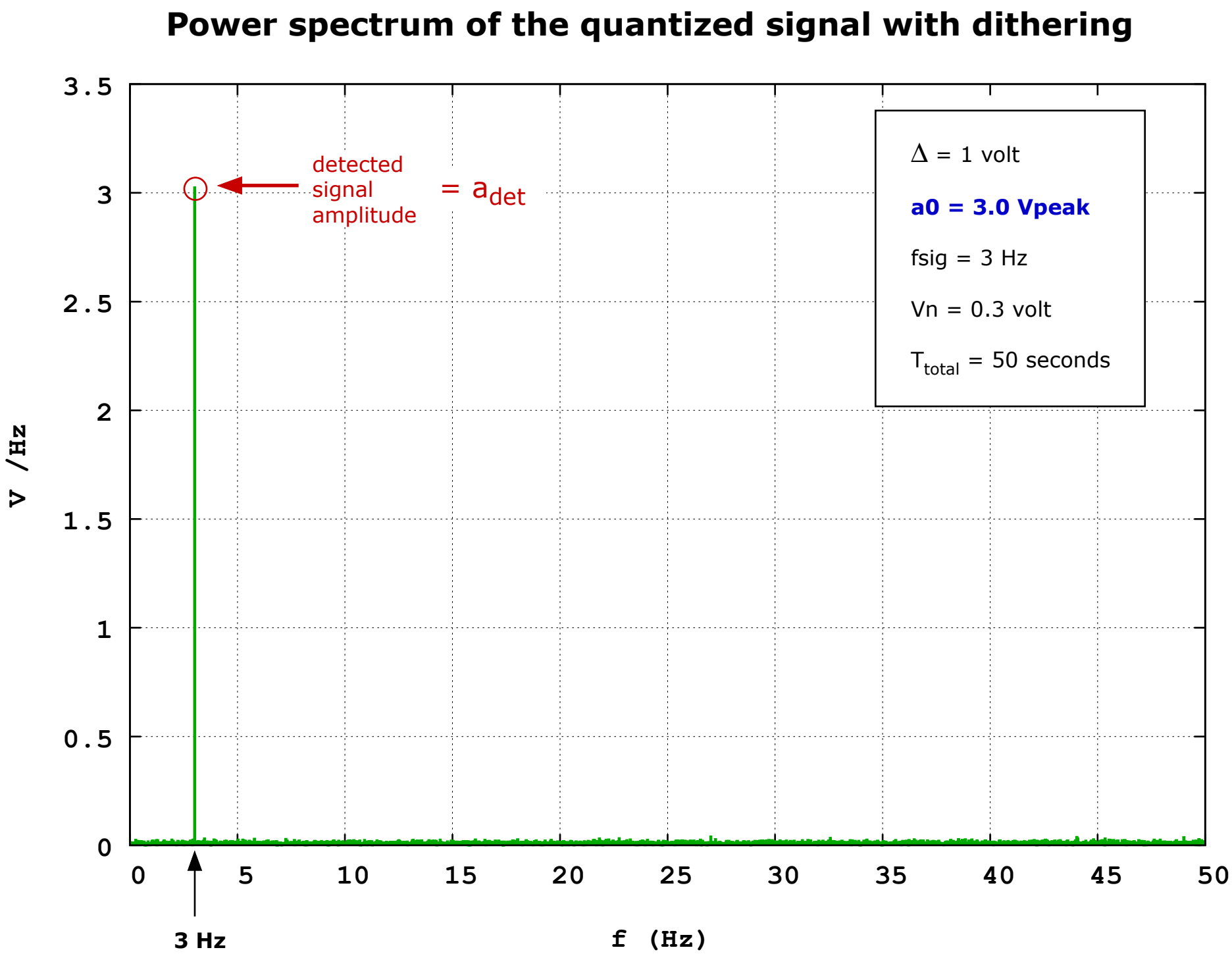
a0 = 3.0 Vpeak

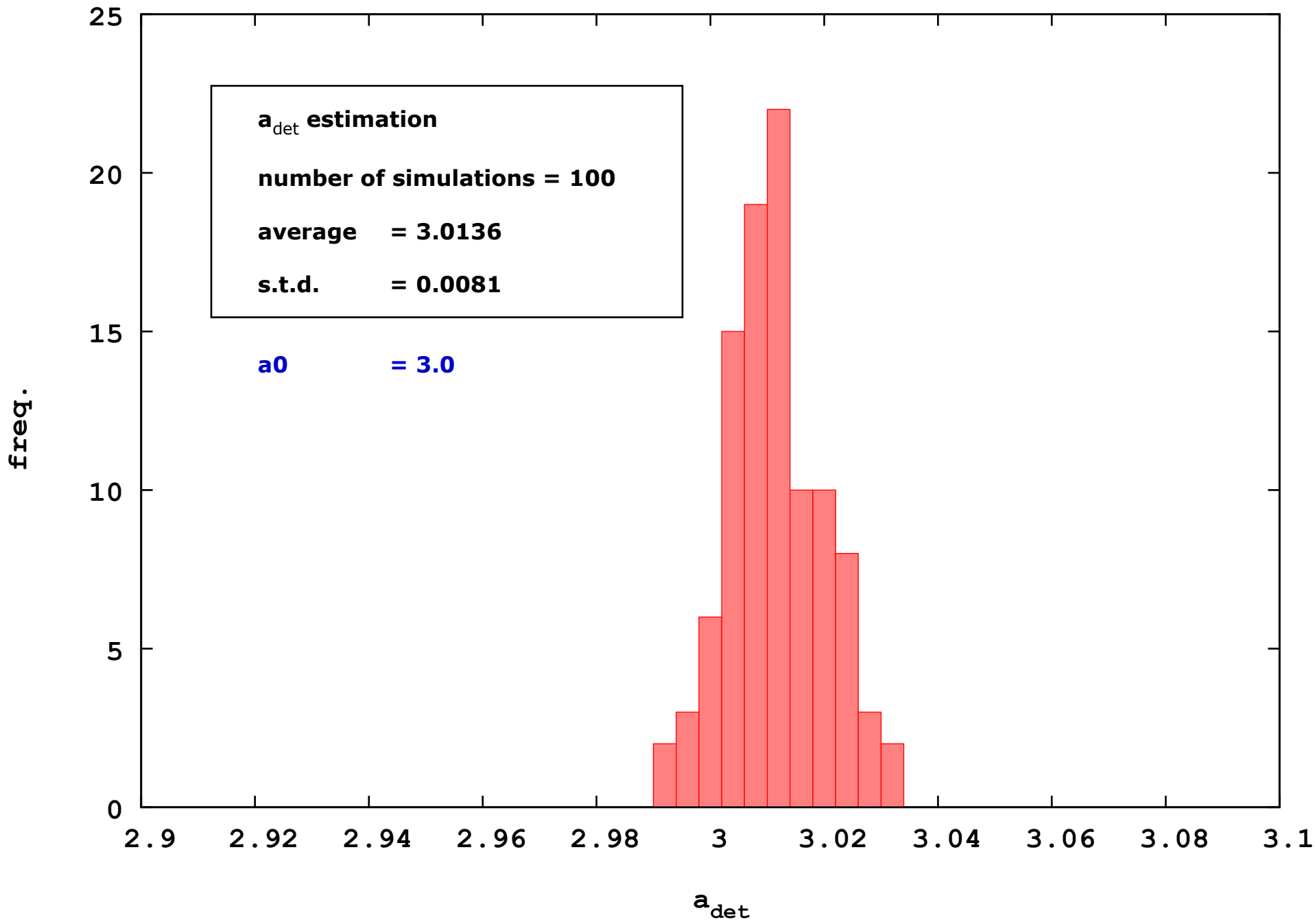
fsig = 3 Hz

Vn = 0.3 volt

T_{total} = 50 seconds


dithering simulation: example 0
$V_{sig}(t)$ + 8.0
$V_{sig}(t)$ + (white gauss noise)
$V_q(t)$ - 8.0
volt
time (sec)
20
15
10
5
0
-5
-10
-15
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Power spectrum of the quantized signal with dithering
3.5
3
2.5
2
1.5
1
0.5
0
V /Hz
0
5
10
15
20
25
30
35
40
45
50
f (Hz)
3 Hz
detected
signal
amplitude
= a_det
Δ = 1 volt
a0 = 3.0 Vpeak
fsig = 3 Hz
Vn = 0.3 volt
T_total = 50 seconds


a_det estimation
number of simulations = 100
average = 3.0136
s.t.d. = 0.0081
a0 = 3.0
freq.
a_det
0
5
10
15
20
25
2.9
2.92
2.94
2.96
2.98
3
3.02
3.04
3.06
3.08
3.1

Simulation Example 1 :

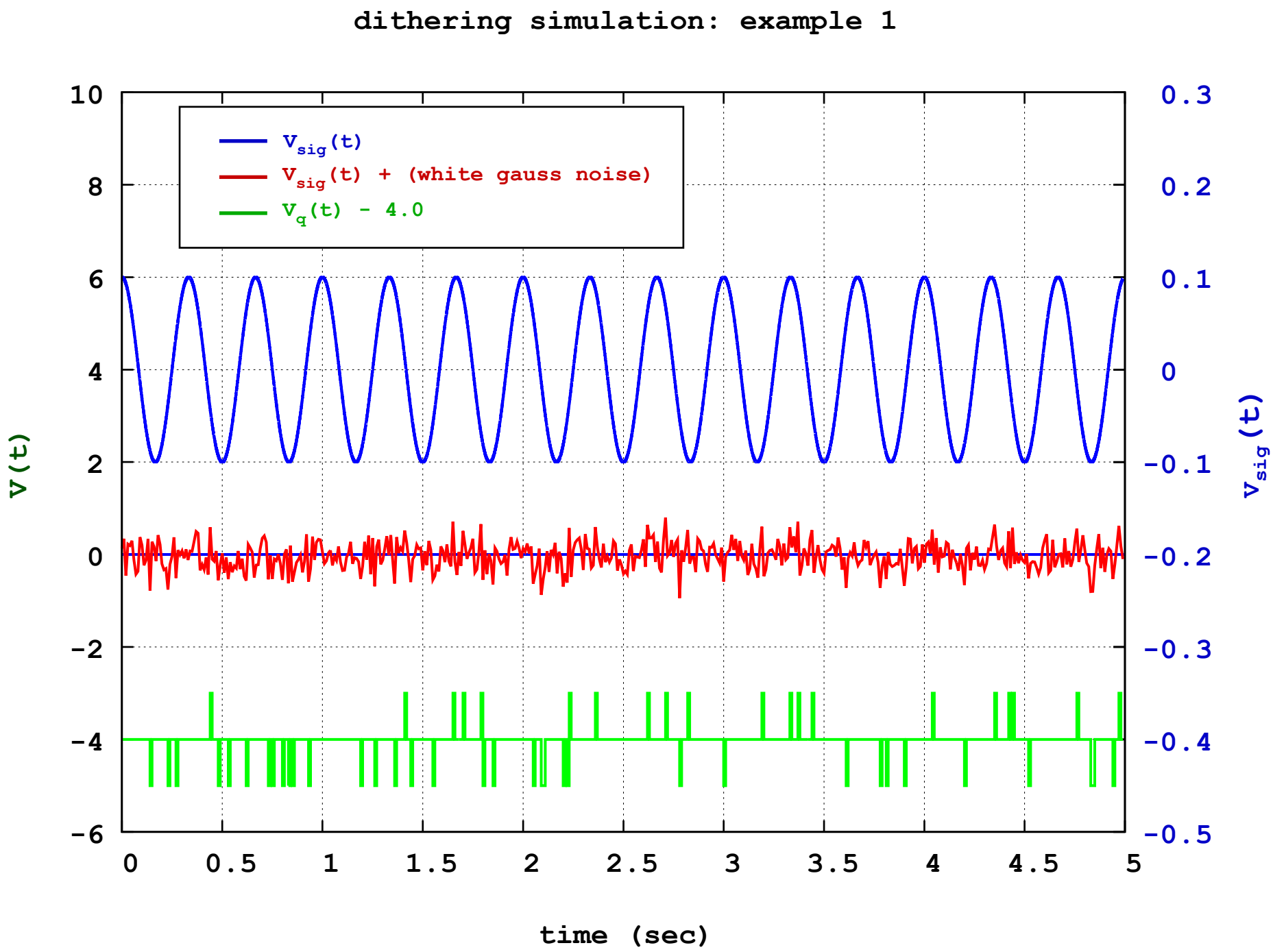
Δ = 1 volt

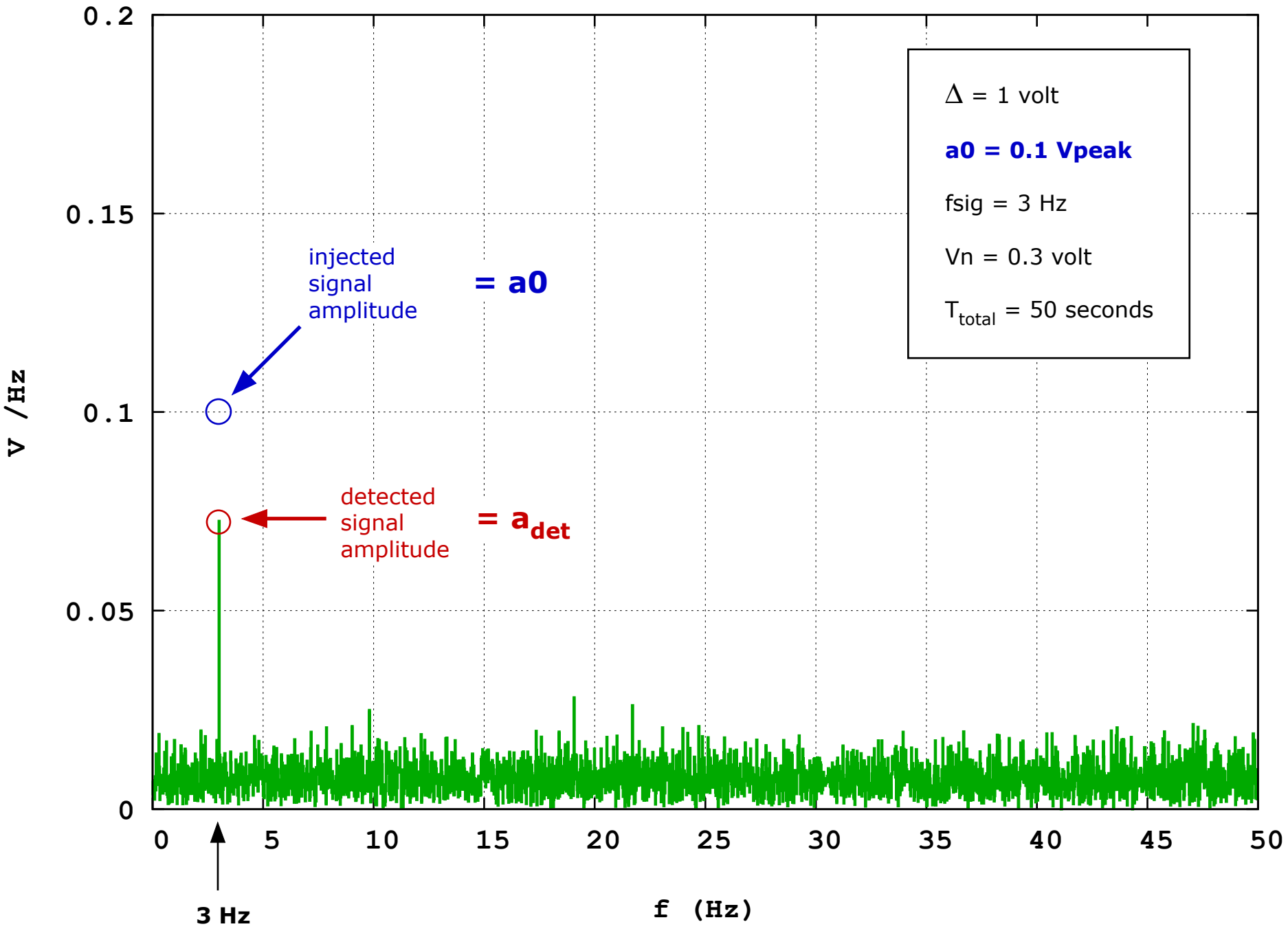
a0 = 0.1 Vpeak

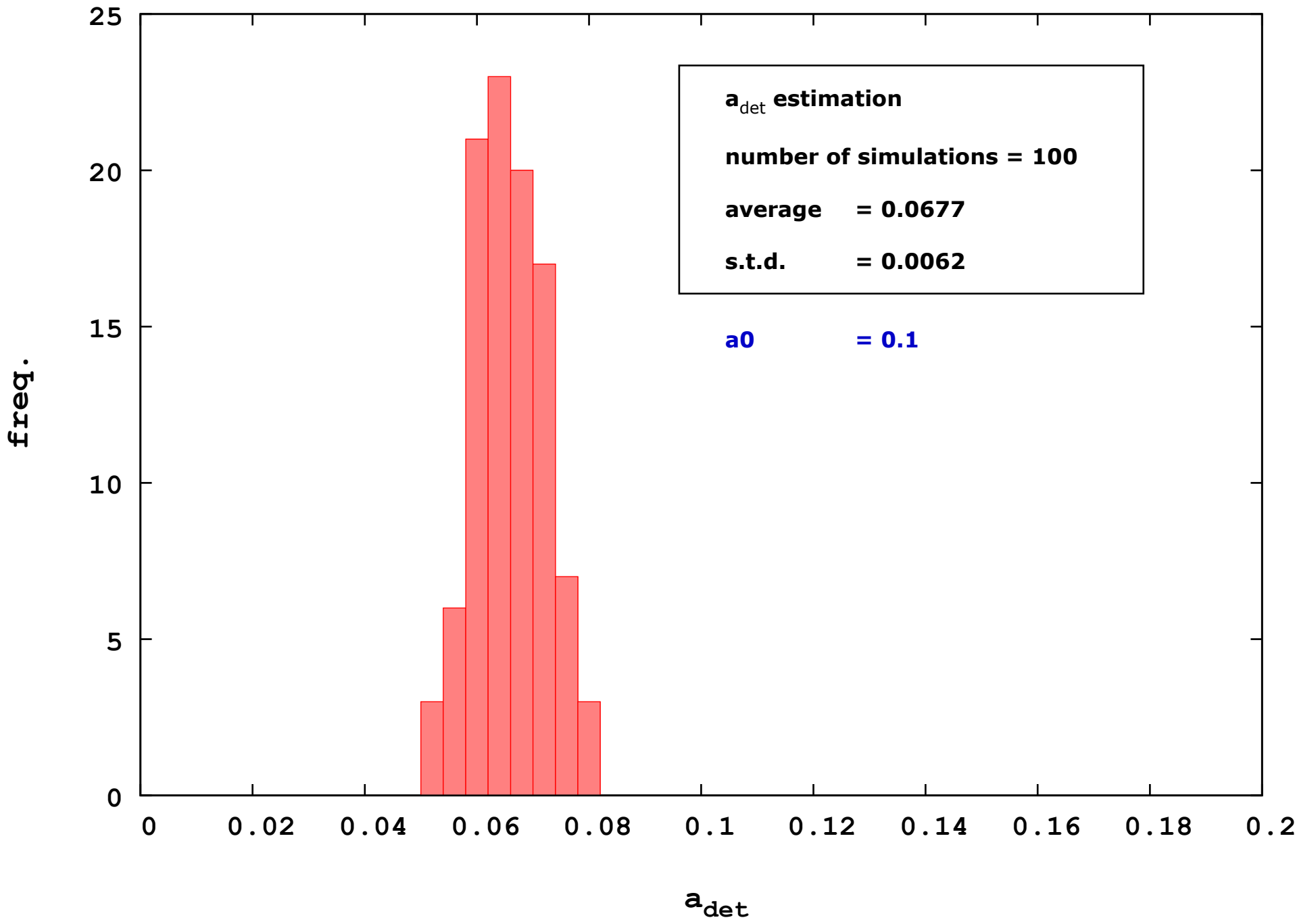
fsig = 3 Hz

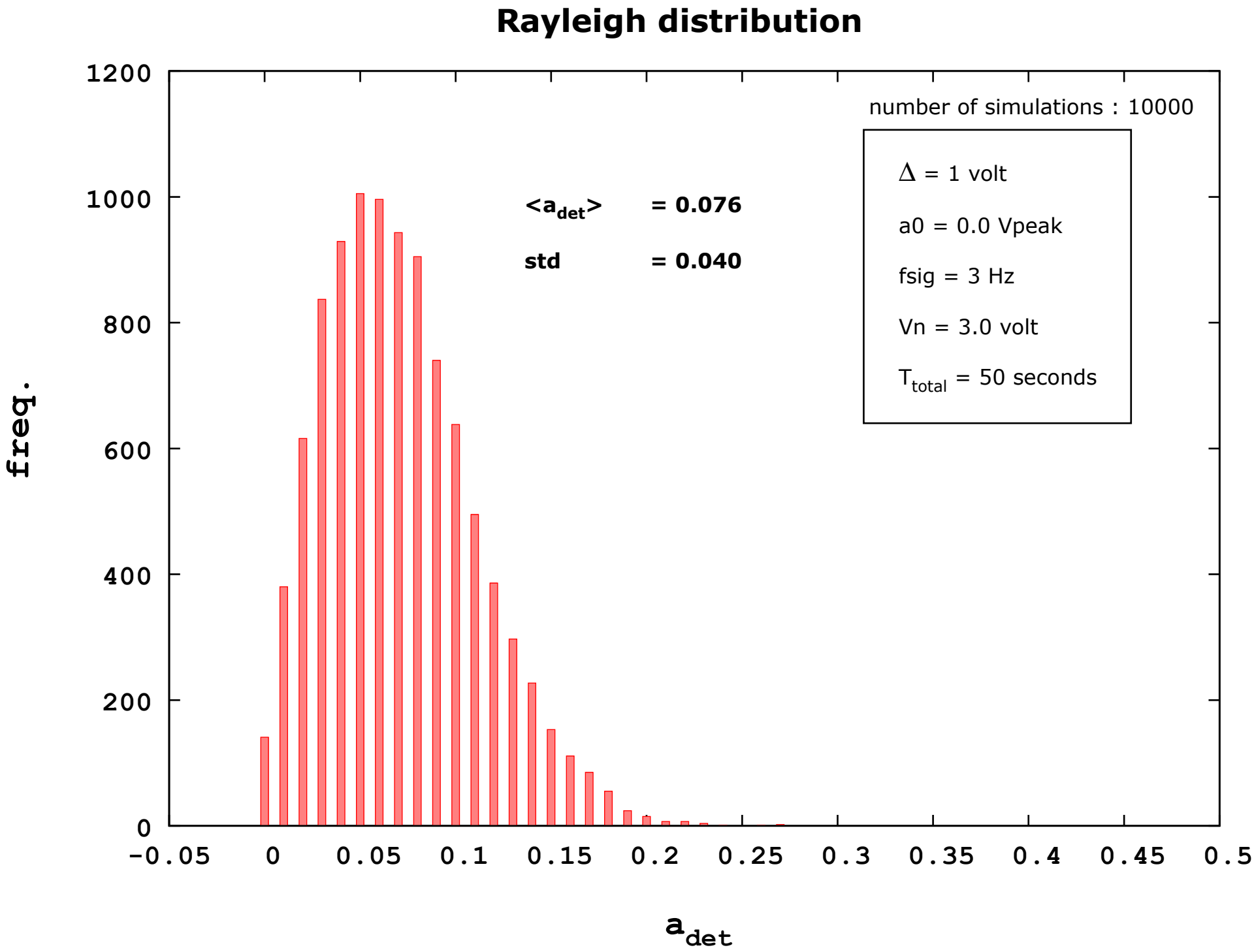
Vn = 0.3 volt

T_{total} = 50 seconds

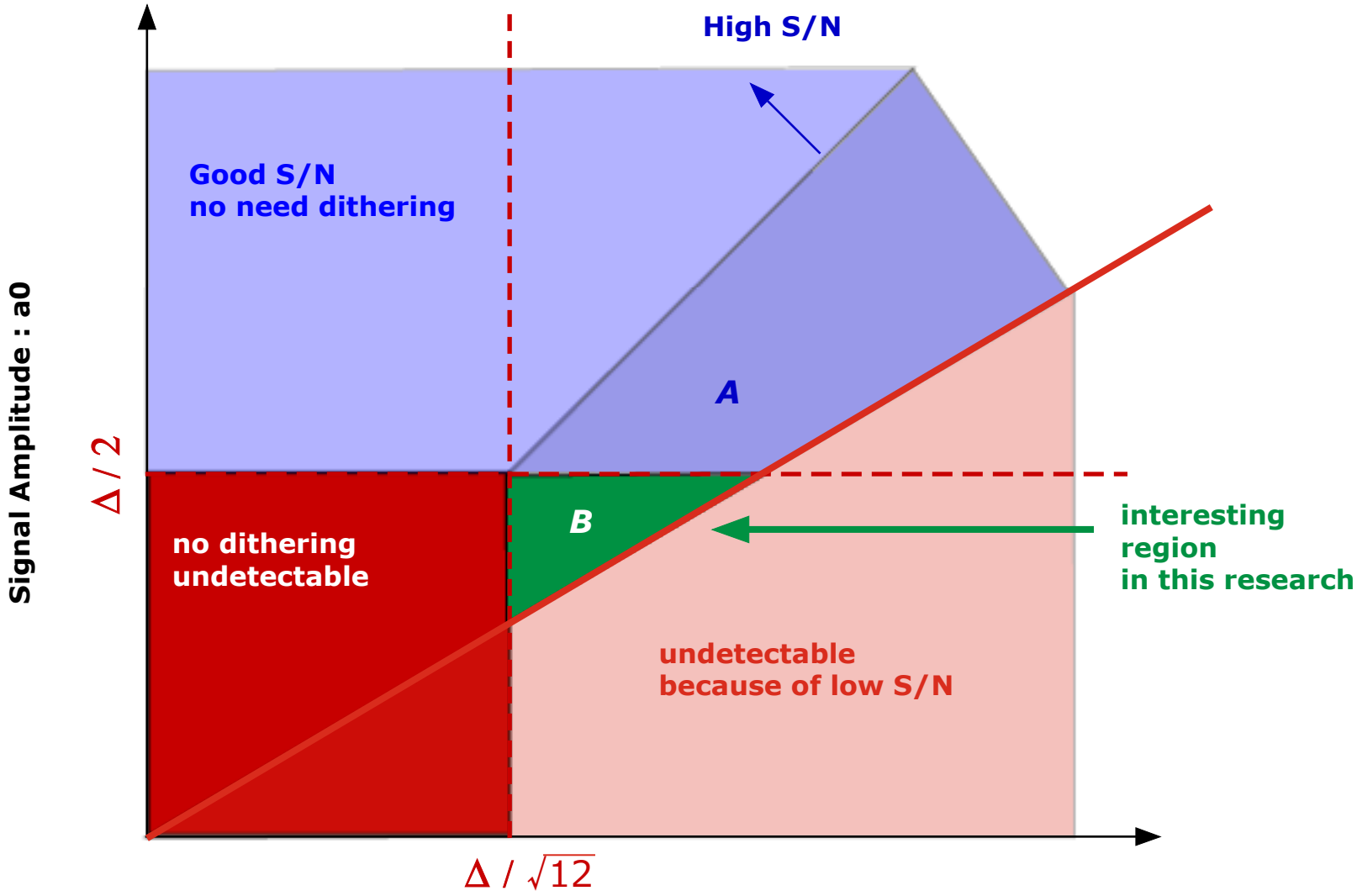

dithering simulation: example 1
$V_{sig}(t)$
$V_{sig}(t)$ + (white gauss noise)
$V_q(t)$ - 4.0
V(t)
$V_{sig}(t)$
time (sec)
10
8
6
4
2
0
-2
-4
-6
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Power spectrum of the quantized signal with dithering
0.2
0.15
0.1
0.05
0
V /Hz
0
5
10
15
20
25
30
35
40
45
50
f (Hz)
3 Hz
injected
signal
amplitude
= a0
detected
signal
amplitude
= a_det
Δ = 1 volt
a0 = 0.1 Vpeak
fsig = 3 Hz
Vn = 0.3 volt
T_total = 50 seconds

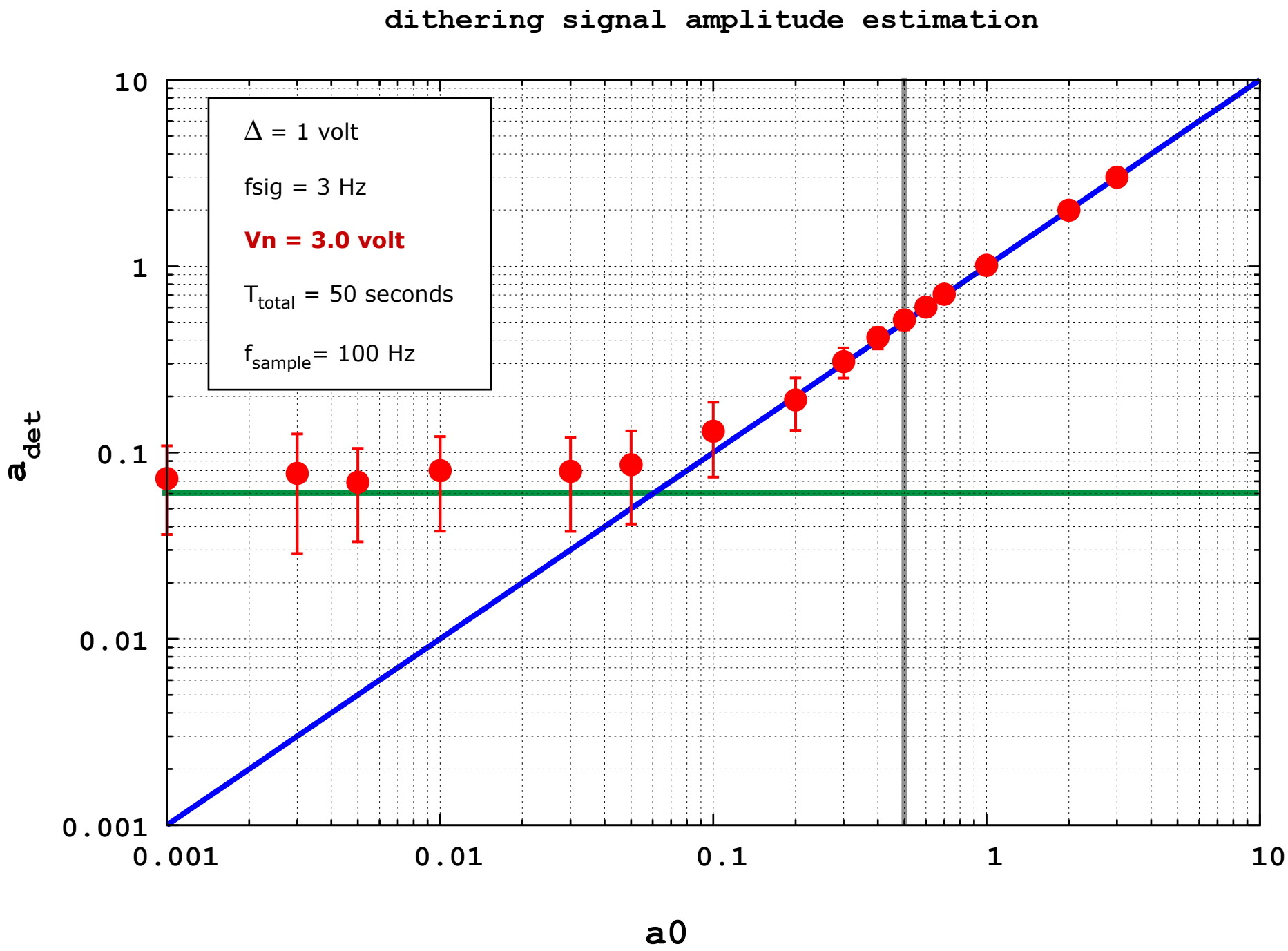

a_det estimation
number of simulations = 100
average = 0.0677
s.t.d. = 0.0062
a0 = 0.1
freq.
a_det
0
5
10
15
20
25
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2

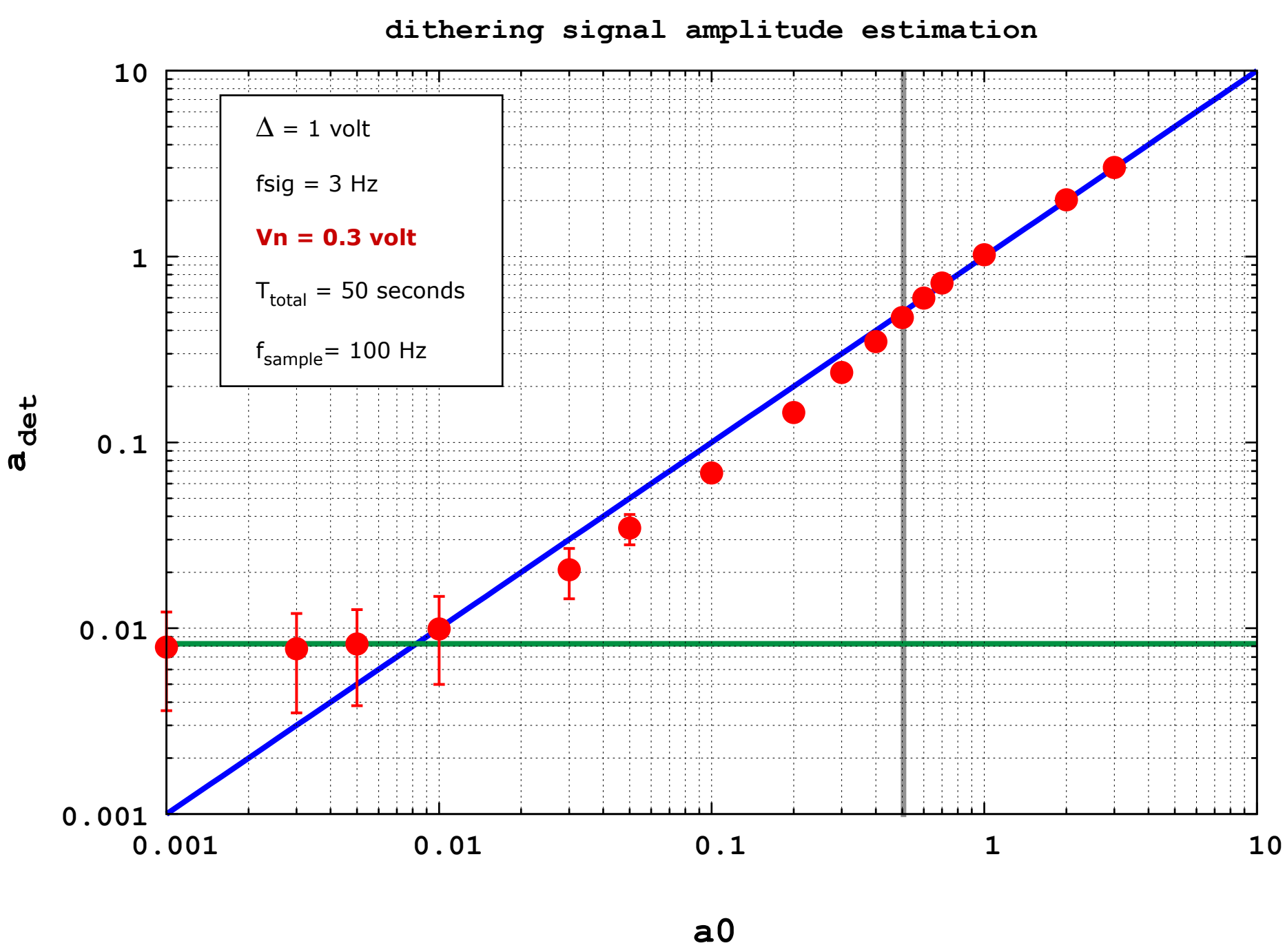

Rayleigh distribution
number of simulations : 10000
Δ = 1 volt
a0 = 0.0 Vpeak
fsig = 3 Hz
Vn = 3.0 volt
T_total = 50 seconds
<a_det> = 0.076
std = 0.040
freq.
a_det
1200
1000
800
600
400
200
0
-0.05
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5

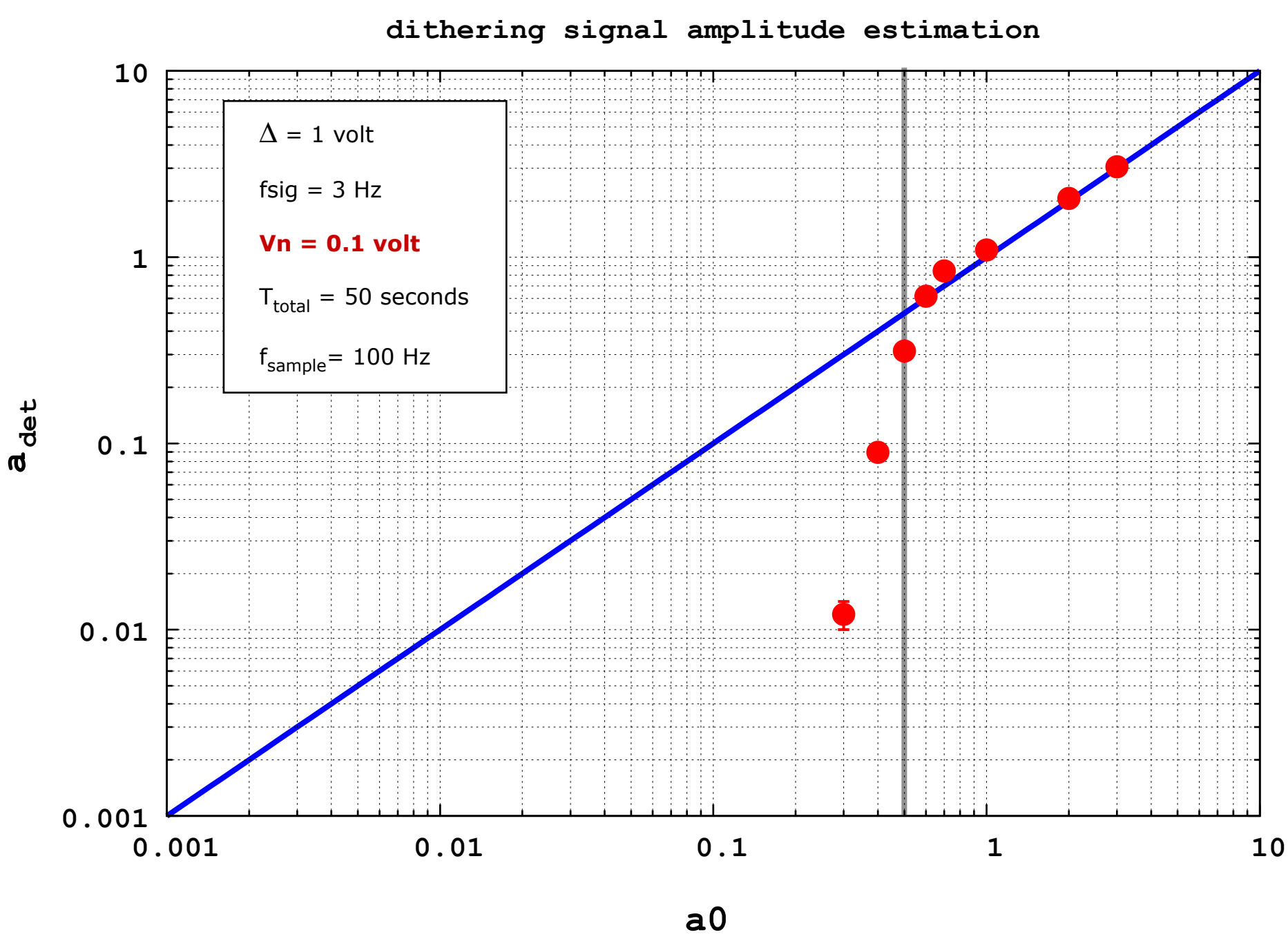
Intuitive speculations



** standard deviation


dithering signal amplitude estimation
Δ = 1 volt
fsig = 3 Hz
Vn = 3.0 volt
T total = 50 seconds
f sample = 100 Hz
a det
a0
10
1
0.1
0.01
0.001
0.001
0.01
0.1
1
10


dithering signal amplitude estimation
Δ = 1 volt
fsig = 3 Hz
Vn = 0.3 volt
T total = 50 seconds
f sample = 100 Hz
a det
a0
10
1
0.1
0.01
0.001
0.001
0.01
0.1
1
10


dithering signal amplitude estimation
Δ = 1 volt
fsig = 3 Hz
Vn = 0.1 volt
T total = 50 seconds
f sample = 100 Hz
a det
a0
10
1
0.1
0.01
0.001
0.001
0.01
0.1
1
10