

# Optical Design of the Input Mode Matching Telescope for KAGRA

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# Purpose of the IMMT

- Convert the beam size from the output of the MC to the mode of the MIF.
- Slightly bend the beam upward to align with the slope of the arms.
- Serve as the steering mirrors for the initial alignment of the beam into the MIF.
- Transmitted beam power from the IMMT1 is used as the error signal for the intensity stabilization.

## Design Procedure

- Compute the eigen-modes of the MC and the PRC
- Put the IMMT mirrors at reasonable positions in their chambers.
- Compute the propagation of the MC beam through MCo, IMMT and PRM.
- Compare the beam parameter on the PRM with the eigen mode of the PRC.
- Perform a 2D least square optimization to find the optimal values of the ROCs for the IMMT mirrors.
- Errors in the ROCs of the IMMT mirrors can be compensated by changing the positions of the IMMT mirrors. Compute how much we can tolerate the ROC error assuming that we can move the mirrors by 10cm at maximum.
- Since the beam size is smaller on IMMT1, we will use the transmission of this mirror for the monitoring of the intensity fluctuation.
- The transmissivity of IMMT1 is determined by the shot noise limit requirement for the intensity stabilization servo.
- Since the HR surfaces are highly curved, AR is better for opelevs.
- For this surface, we require a moderate reflectivity (>40%) for 670nm laser.

# Mode Matching Requirement

- A poor mode matching means less amount of light power available for the detection of GW. However, this is not a serious problem in reality because the mode matching of over 90% is easily achievable and the shot noise increase by the 10% power reduction is not a big deal.
- The real requirement comes from the shot noise of REFL port, not AS.
- If we forget about the mode matching, the amount of light power coming back to REFL is determined by the reflectivity mismatch between the PRM and the arm cavities. When we designed the LSC scheme using Optickle, the assumed mismatch gave 1.8W of carrier power coming back to REFL.
- Since we use the beat between the carrier and the RFSBs for signal extraction at REFL (for CARM and MICH), the shot noise level of these signals does not depend much on the TEM00 carrier power at REFL.
- However, the carrier higher order modes coming back to REFL caused by the mode mismatch do not contribute to the signal generation. Therefore, we have to make these much smaller than the TEM00 carrier power.
- The nominal input power to the bKAGRA is 78W. This means the reflectivity for the TEM00 carrier is  $1.8/78=2\%$ . In order to make the HOM carrier negligible to this, we set the mode mismatch to be less than 0.1%.

**Thus the mode matching has to be better than 99.9%.**

# MC Parameters

MCe ROC =  $37.3 \pm 0.1\text{m}$  (based on the measurement by E. Hirose)

MCi, MCo are flat, separated by 0.5m

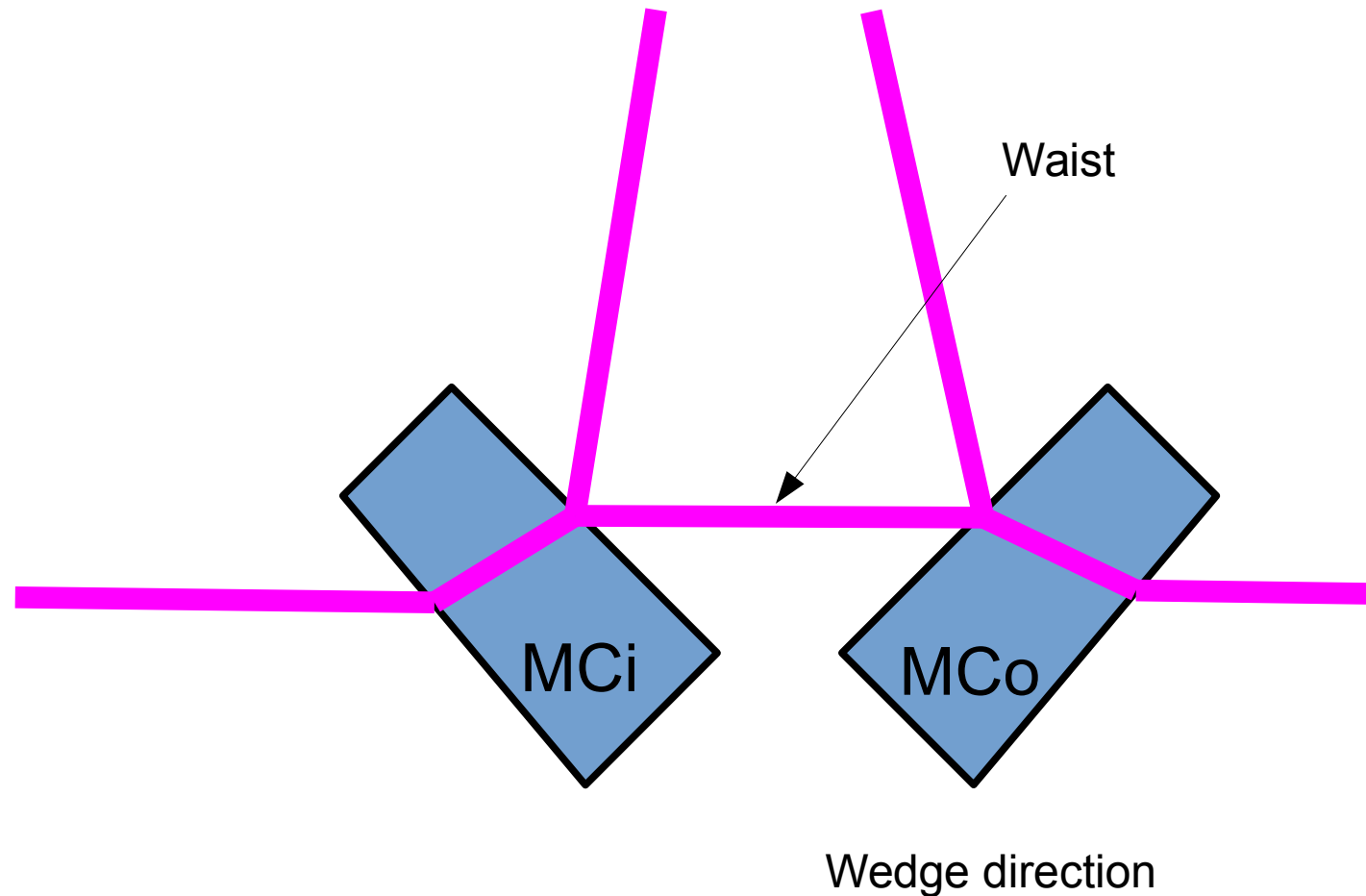
Total MC length = 26.65m

Beam size at the MC waist = 2.3887mm (slightly elliptic in reality)

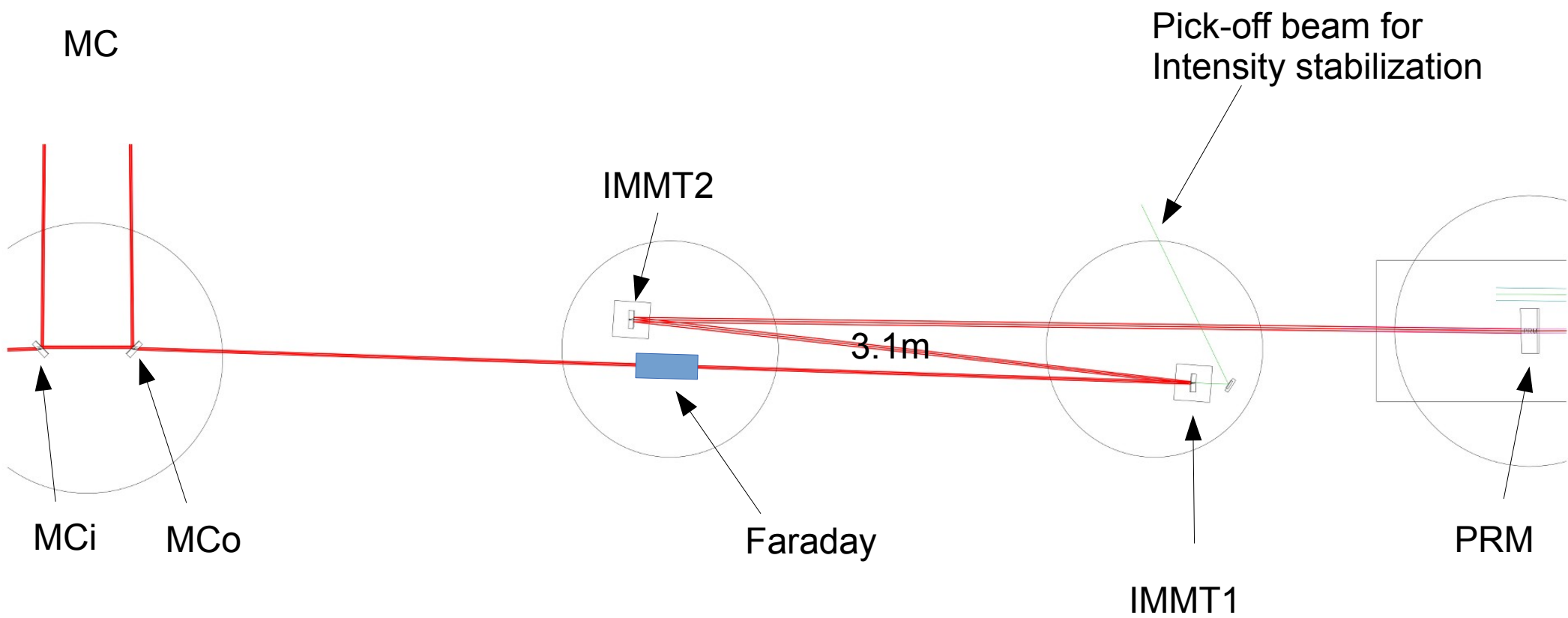
MC mirror diameter = 100mm

MC mirror thickness = 30mm (will be smaller by 1-2mm according to Mio-san)

MC wedge angle = 2.5deg

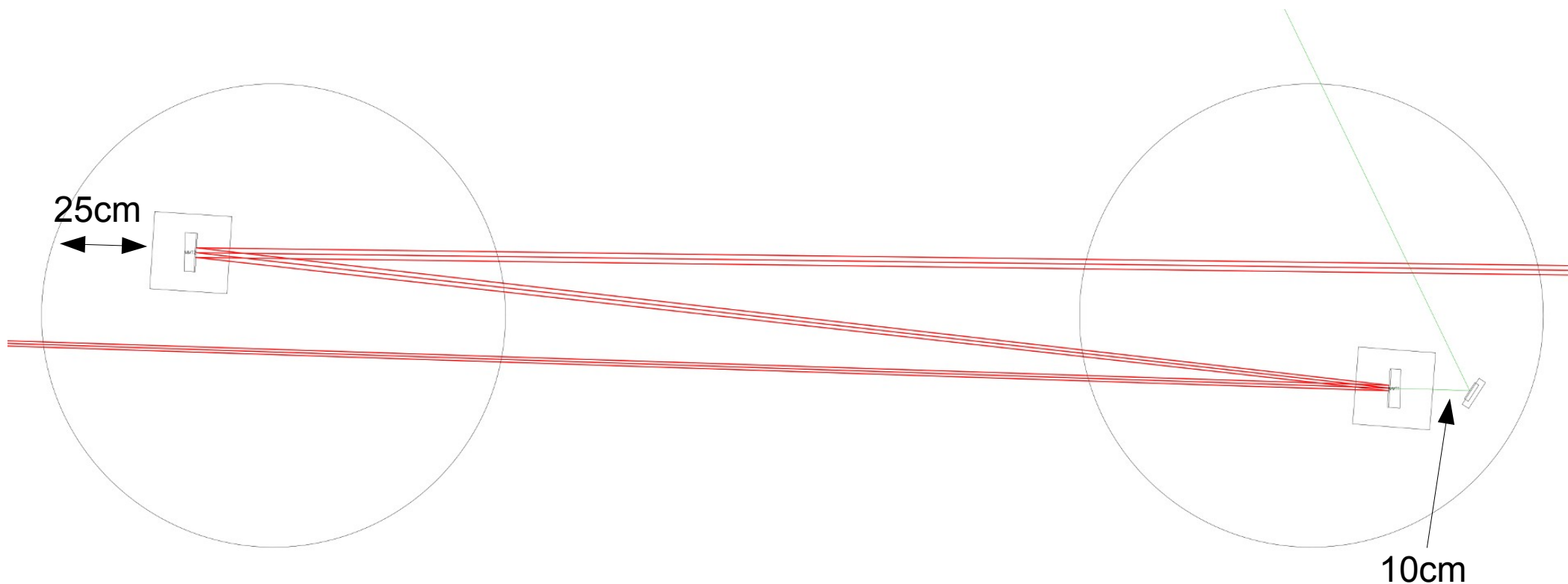


# IMMT configuration



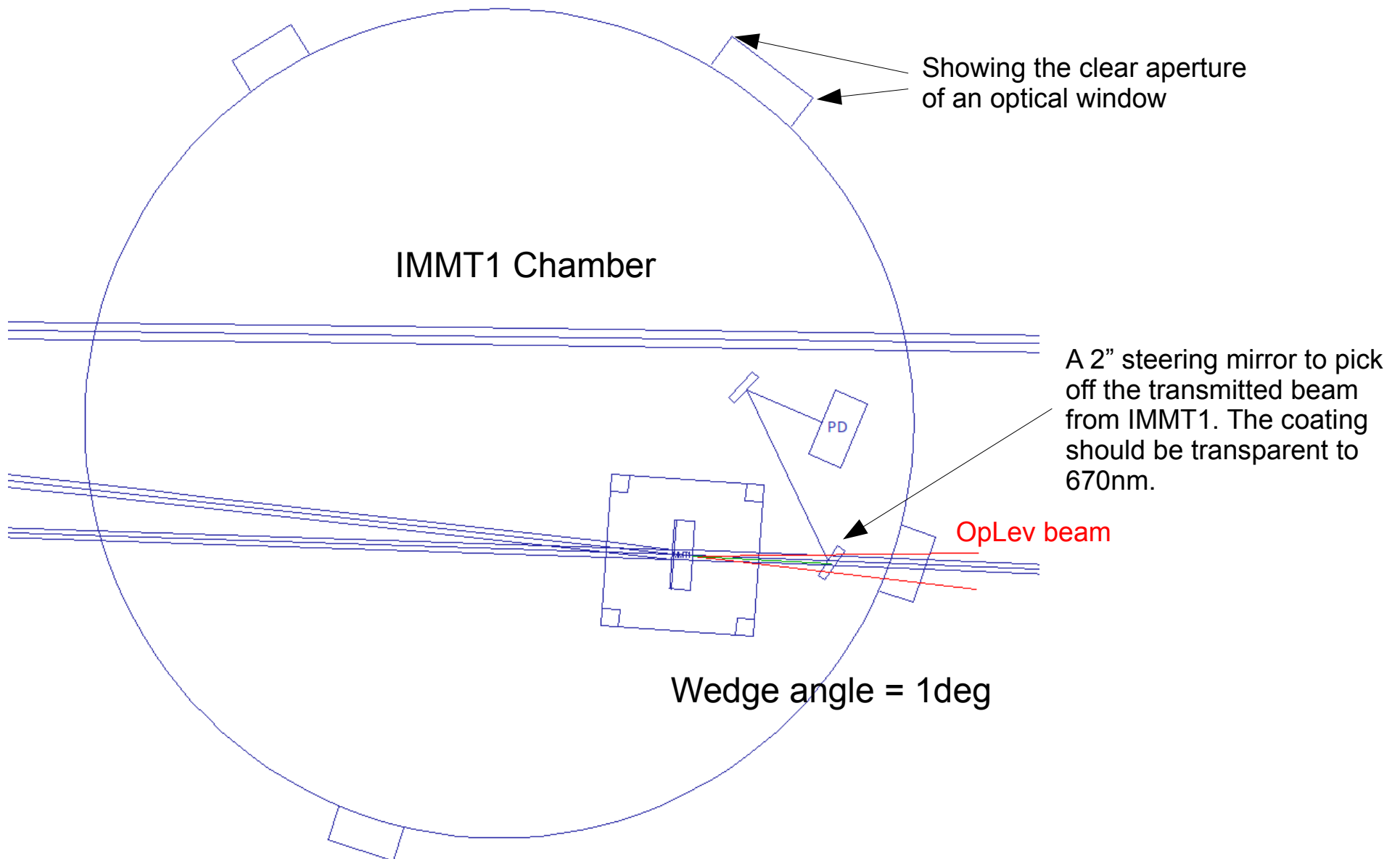
# How much can we move the mirrors ?

- As we will see in the next few slides, we have to move the mirrors from their nominal positions in order to compensate for the ROC errors.
- The suspension systems for the IMMT mirrors are the TAMA suspensions. The foot prints of the TAMA suspensions are shown as a rectangle around each mirror.
- By looking at the chamber space, the distance between the mirrors can be easily changed by  $\pm 10\text{cm}$ . We assume this is the adjustable range of the IMMT length.



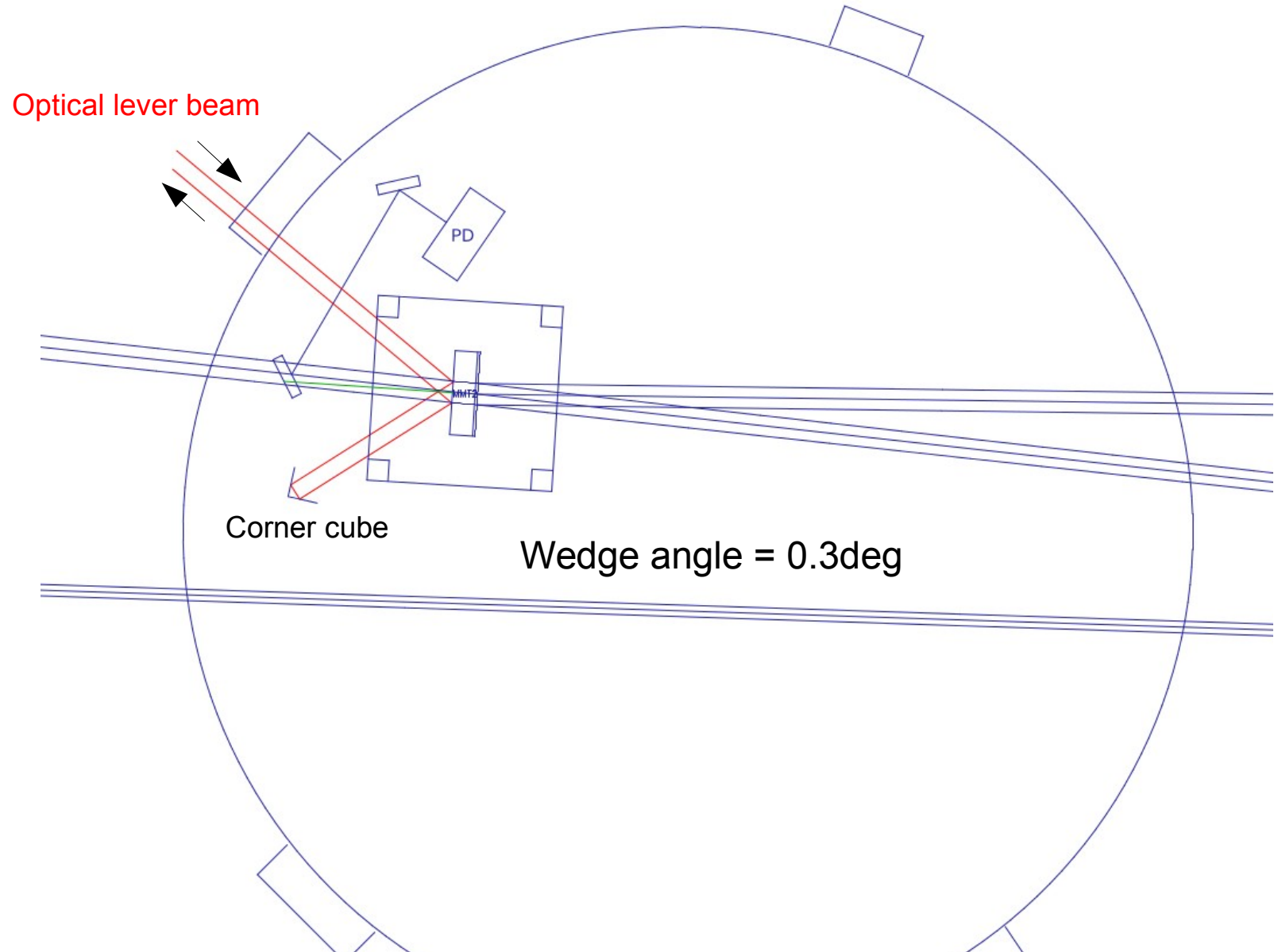
# Optical Lever for IMMT1

Because we may have to move the IMMT mirrors by +/-5cm to change the IMMT length at maximum by 10cm, the optical lever paths have to be compatible with this adjustment. The optical path gets crowded when the suspensions are moved toward the walls of the chamber. The drawing below shows when the suspension is moved towards the wall by 5cm to show the most difficult case. The same is true for the next page.



# Optical Lever for IMMT2

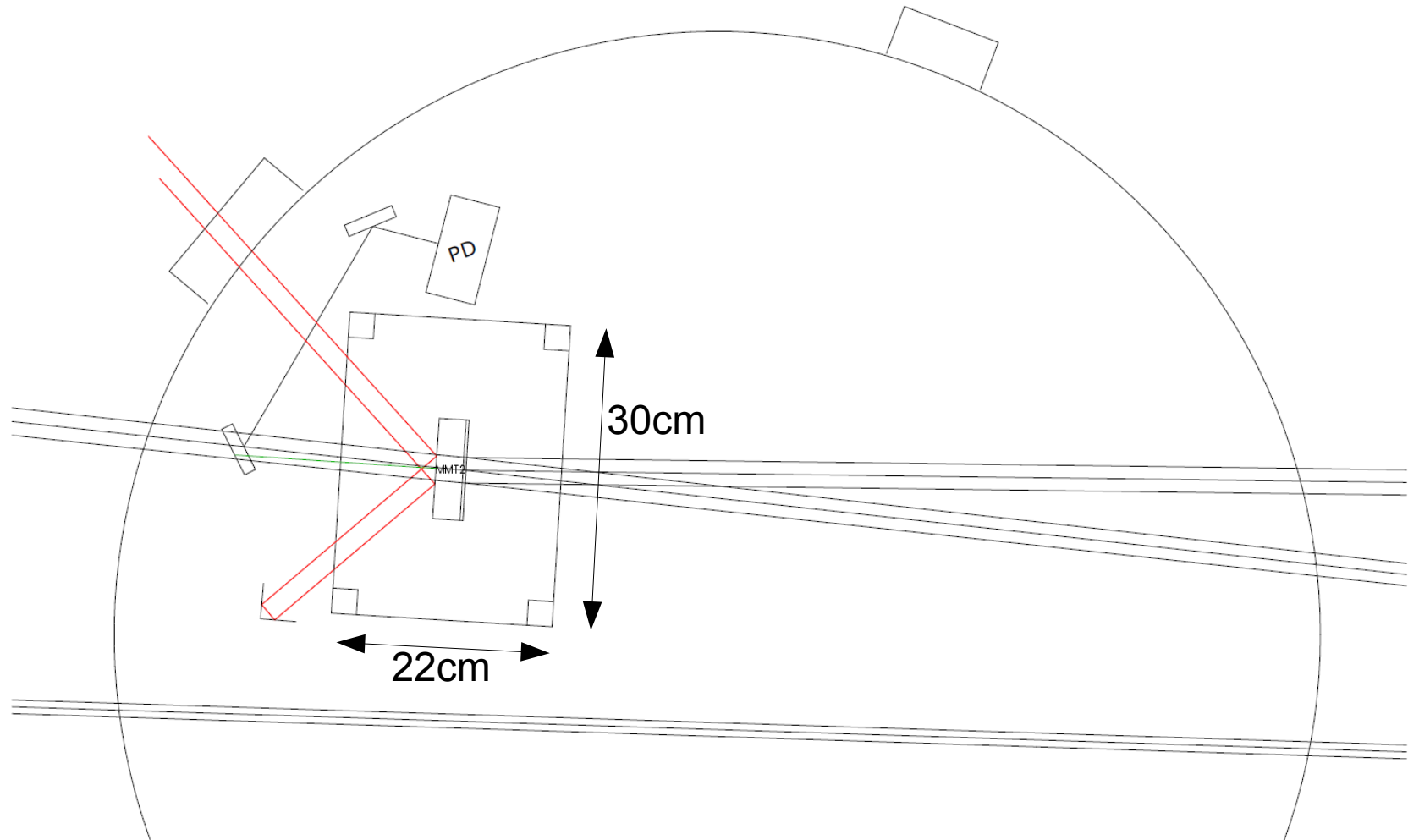
For this mirror, we cannot find a direct path. So I used a corner cube to reflect the beam back to the incident direction. The beam separation is about 2cm in this case. The beams run very close to the suspension pillars.





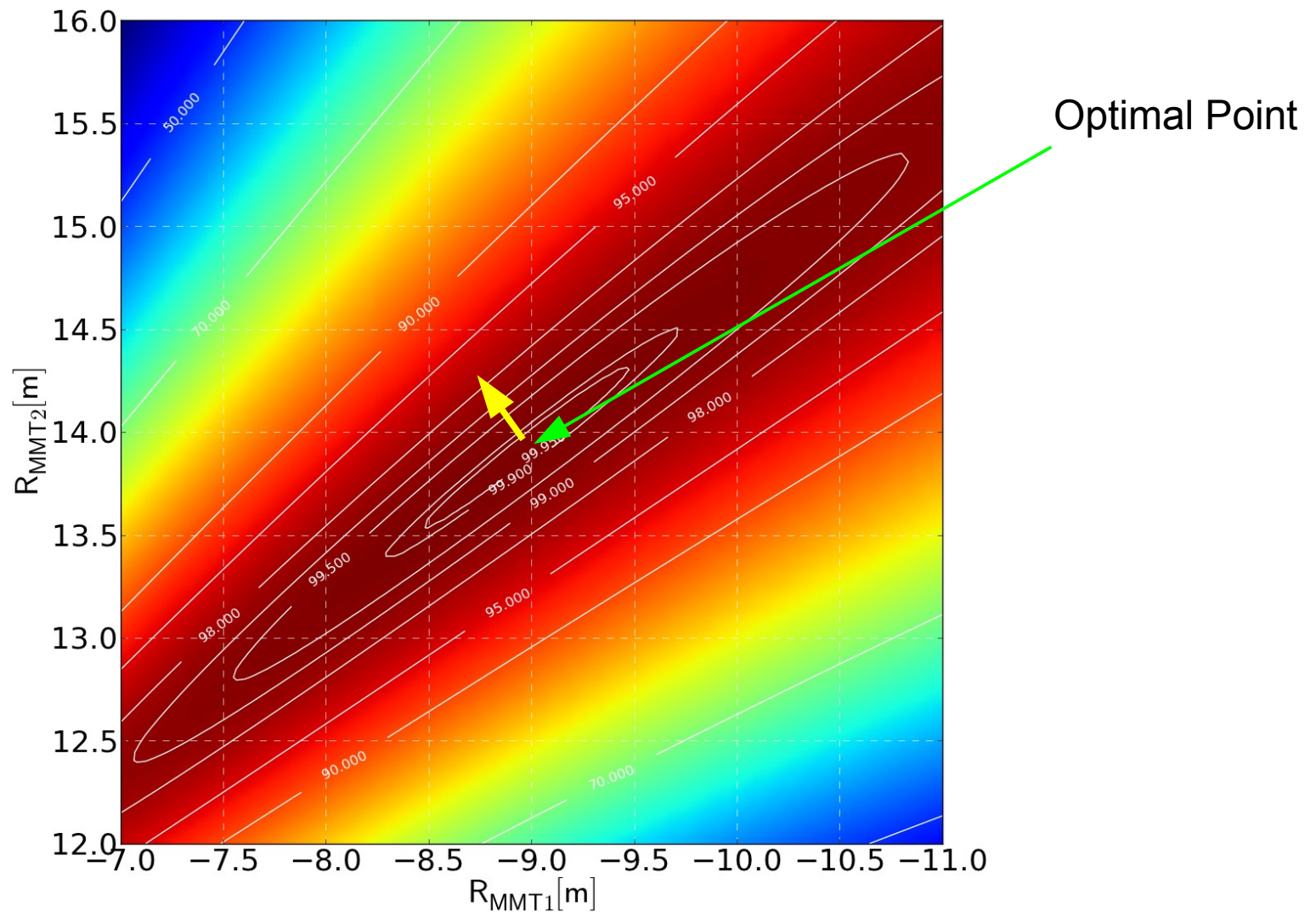
# Stretched Suspension Top Plate

- We have to also change the coil holder from + configuration to x configuration.
- Is there enough space for Faraday ?



# Optimal ROCs

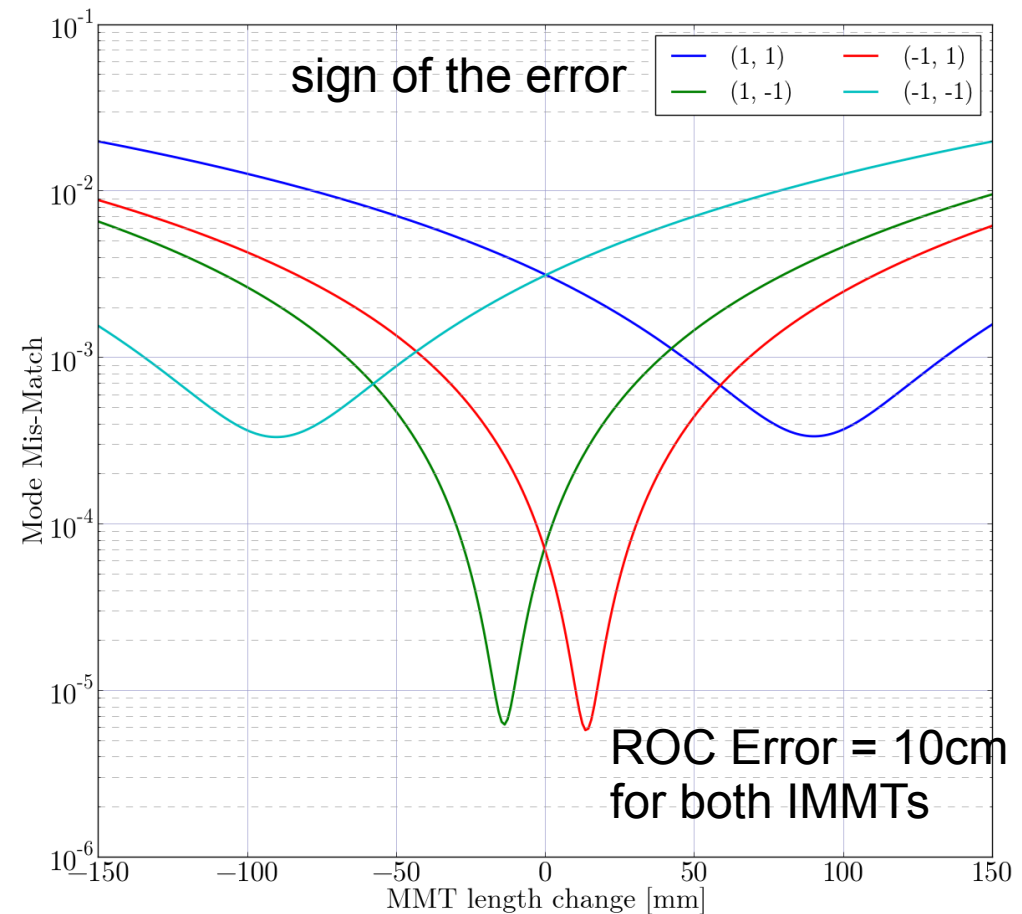
- Assuming the distance between the IMMT1 and IMMT2 is 3.1m, the mode matching rate to the MIF is computed sweeping the ROC values of the two mirrors.
- The optimal values are: IMMT1=-8.953m, IMMT2=13.910m
- The region over 99.9% mode match is highly elliptical. If the errors are in the direction indicated by the yellow arrow, the error tolerance is in the order of 10cm.



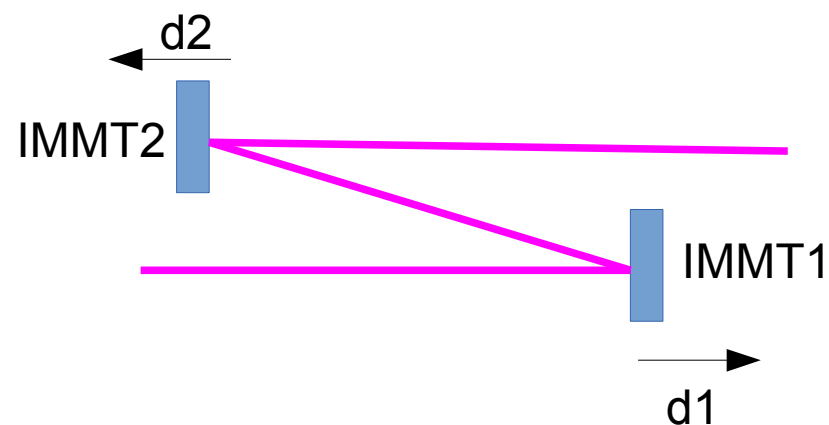
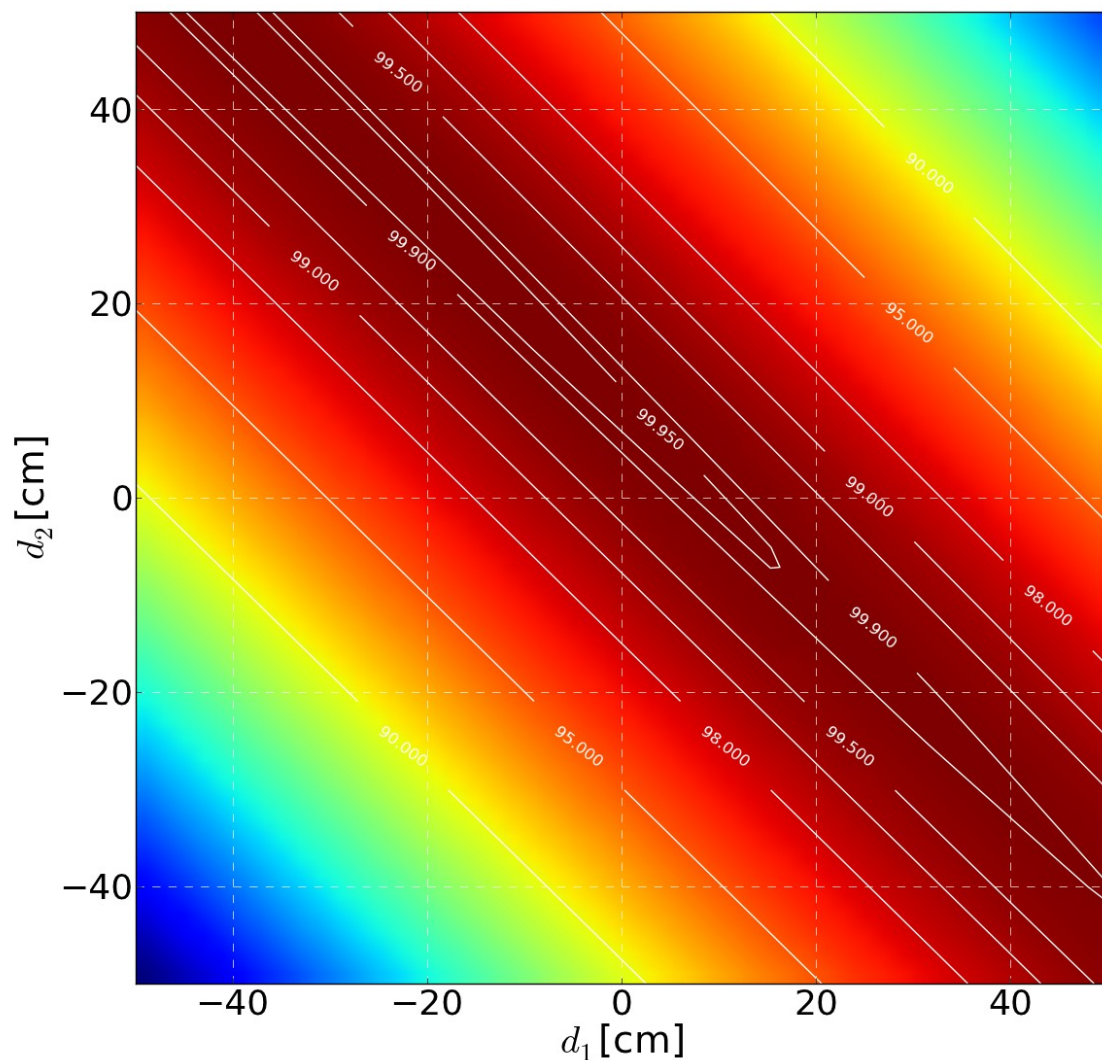
# ROC error compensation by moving the mirrors

The ROC errors of the IMMT mirrors can be compensated by moving the mirrors. Especially, the mode matching is very sensitive to the distance between the two mirrors (IMMT length). Therefore I plotted the dependence of the mode mismatch (smaller the better) as functions of the IMMT length assuming 10cm ROC errors are introduced to the mirrors. There are four curves shown in the figure corresponding to different combinations error signs. For example, (1,-1) means +10cm error is added to IMMT1 while -10cm is added to IMMT2.

Even for the worst cases (the ROC errors have the same signs), the mode matching can be recovered to more than 99.9% by changing the IMMT length by roughly the same amount as the errors (10cm in this case).

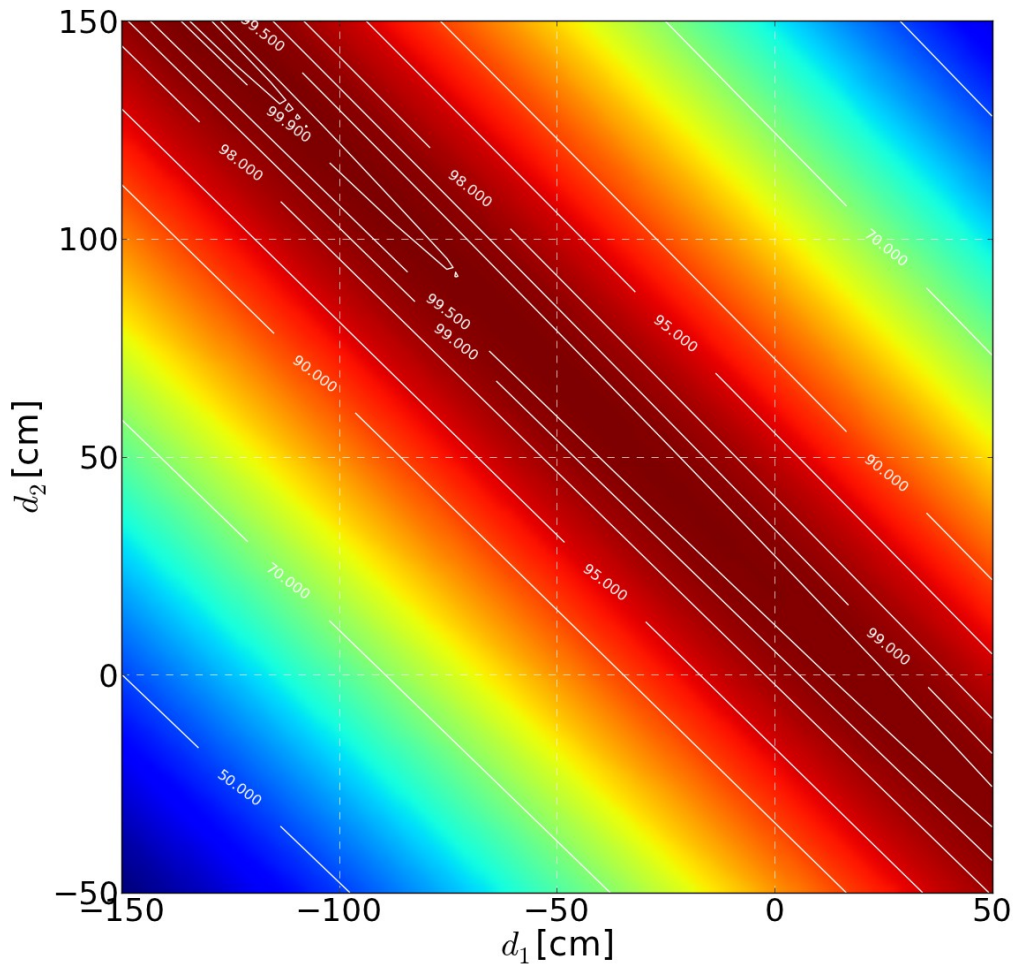


I also did a scan of the positions of the two mirrors to create a contour map of the mode matching. Here,  $d_1$  and  $d_2$  are the displacement of the IMMT1 and IMMT2 from their nominal positions, respectively. The positive directions of  $d_1$  and  $d_2$  are indicated in the drawing below. You can see that the gradient is mostly in the diagonal direction (45deg from the x-axis). This is why changing the length of the IMMT (differential displacement of the two mirrors) is the most relevant adjustment for the mode matching. However, in order to truly optimize the mode matching, we also have to move the two mirrors in a common direction.

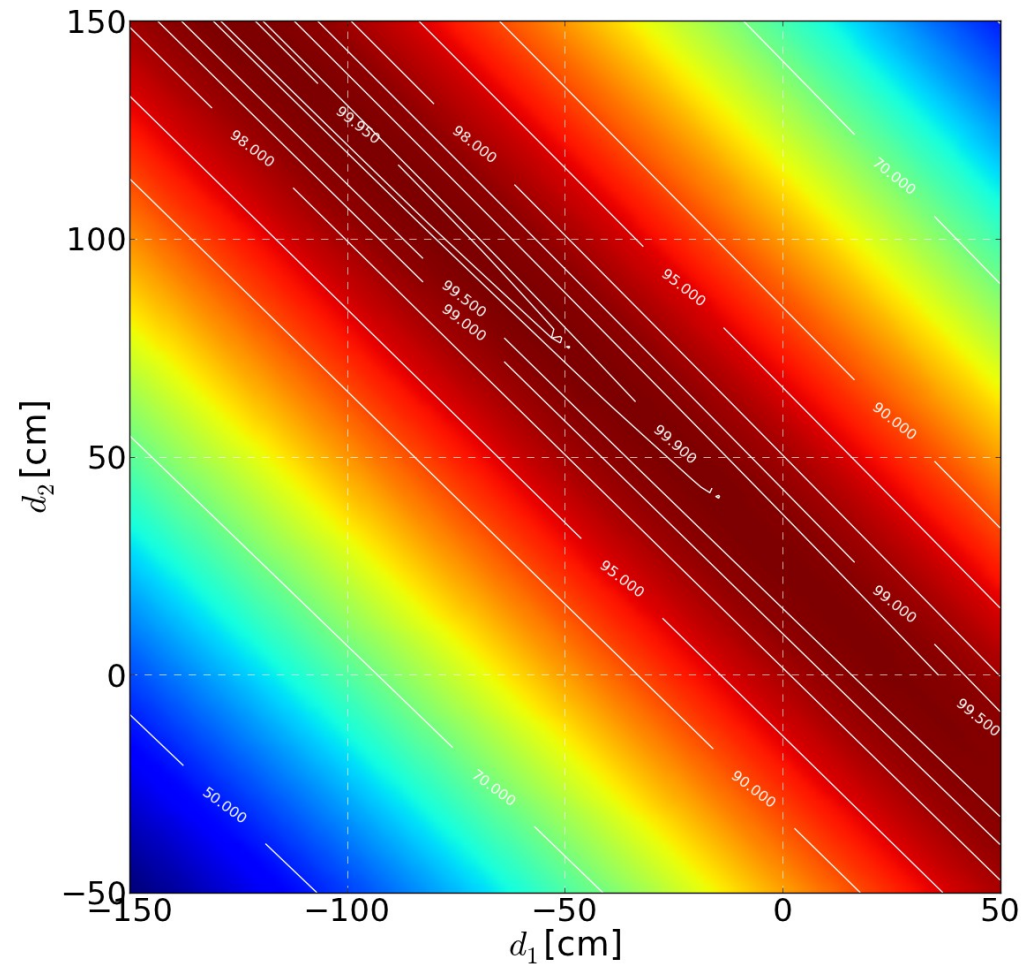


For 10cm error in both IMMT mirrors

The mode matching maps below show the cases with 50cm ROC errors for IMMT1 and IMMT2. In these cases, one cannot recover the mode matching over 99.9% by purely moving the two mirrors differentially. This tendency of the optimal point moving upper left can be seen in the 10cm error case, but it is more evident when the error is larger. Now, it is inevitable to move the mirrors more than 10cm. From this observation, it is concluded that 10cm is the maximum tolerable ROC error.



IMMT1 ROC Error = 50cm



IMMT2 ROC Error = 50cm

## RIN Requirement $\sim 2 \times 10^{-9}$ (at the input of MIF, $10 < f < 100 \text{ Hz}$ )

(a factor of 10 safety margin included, Ref. MIF design document)

Shot noise limit of an intensity stabilization servo is given by the following formula:

$$\text{RIN} = \sqrt{\frac{2h\nu P_0}{\eta P_m (P_0 - P_m)}}$$

(See Appendix for the theoretical background of this formula)

$\eta$  : quantum efficiency (nominal value = 0.9)

$P_0$  : Input power to the interferometer (75W)

$P_m$  : Power on the monitor PD (order of 100mW)

$P_m = 100 \text{ mW}$  gives  $2.4 \times 10^{-9}$  RIN limit.

For safety, I propose to make **the transmitted power of the IMMT1 be 200mW**

**→ Power Transmission = 1500ppm**

# AR Reflectivity

- Optical lever will most likely use the AR surfaces, because the HR surfaces are highly curved and the reflection angle strongly depends on where the oplev beam hits on the surface.
- Since the optical lever laser is 670nm, the AR surfaces should have at least a moderate (~40%) reflectivity at this wavelength.
- We need to specify the incident angle for the 670nm laser. However the IMMT mirrors are placed close to the edge of the chambers with the AR surfaces facing the chamber walls. This makes it difficult to directly hit the AR surfaces with oplev lasers.
- We need to consider the oplev optical configuration as soon as possible.

# HR transmittance

- We use the transmission of IMMT1 for the intensity stabilization because the beam size is smaller. The transmissivity requirement for this beam is calculated in the previous page to be 1500ppm. For the IMMT2, we still want to monitor the transmission beam position with a QPD. This should work also in a low power mode. In iKAGRA, we expect the input beam to the PRM is about 5W. Out of 5W we want to have about 1mW transmission. This requires 200ppm transmission. With the full input power of 75W, the QPD will receive 15mW.

# Conclusion

## Specs for the IMMT mirrors

	IMMT1	IMMT2
ROC	-8.953m	13.910
ROC Error Tolerance	+/-10cm	+/-10cm
HR Transmission (1064nm)	$1500 < T < 2000\text{ppm}$	$200\text{ppm} < T < 400\text{ppm}$
HR Loss (1064nm)	$L < 1000\text{ppm}$	$L < 1000\text{ppm}$
AR Reflection (1064nm)	$R < 0.1\%$	$R < 0.1\%$
AR Reflection (670nm)	$R > 40\%$ (incident angle = ?)	$R > 40\%$ (incident angle = ?)

We need to design the optical lever beam paths as soon as possible.



# Appendix

## K. Arai's email message on the method to calculate the intensity stabilized RIN limit.

\* vacuum fluctuationの正体というのは電磁場のゼロ点振動。  
だからコヒーレント状態を仮定した場合、古典場を注入している周波数・空間モードでは古典的に扱い、それ以外では場がゼロ点エネルギーで決まるRMSでランダムに振動している、と考えます。

\* このときキャリア周波数を挟んで $\pm w$ の周波数での場の振動がコモンの場合、キャリアとあわせて強度雑音を、ディファレンシャルの場合位相雑音を生み出します。個々の周波数のモードを扱うより上下セットの方が雑音の意味としてとらえやすいぞ、というのがtwo photon pictureとか言うのではなかったかと思います。（これが宗宮さんから教わった部分）

\* で、各電場を定義すると  
レーザーからくる電場

$$E_{in} = E_l + (n_{l1} / 4) * (\text{Exp}[i w t] + \text{Exp}[-i w t]) + (n_{P1} / 4) * (\text{Exp}[i w t] - \text{Exp}[-i w t])$$
  
第一項が古典場、第二項が強度雑音成分、第三項が位相雑音成分  
共通項である $\text{Exp}[i \Omega t]$ は無視しています(Phaser表示)

$n_{l1}$ と $n_{P1}$ は無相関の雑音振幅ですがRMSは同じものと考えます。  
係数4はあとで結果を綺麗にするためのファクタです。

実際、三角関数で書き換えると

$$E_{in} = E_l + 1/2 n_{l1} \text{Cos}[t w] + 1/2 i n_{P1} \text{Sin}[t w]$$
  
となり雑音の意味が分かりやすくなります。

\* BSの虚無側からくる電場は同様に

$$E_{vac} = (n_{l2} / 4) * (\text{Exp}[i w t] + \text{Exp}[-i w t]) + (n_{P2} / 4) * (\text{Exp}[i w t] - \text{Exp}[-i w t])$$

$n_{l2}$ ,  $n_{P2}$ はやはり無相関の雑音振幅で、振幅は $n_{l1}$ ,  $n_{P1}$ と同じです

\*POの反射ポート・透過ポートの電場(Eref, Etrans)は

$$E_{ref} = rBS E_{in} + tBS E_{vac}$$

$$E_{trans} = tBS E_{in} - rBS E_{vac}$$

で表されます。

\*今反射ポートで検出するパワーPrefとそのRINを考えます。

$$P_{ref} = E_{ref} E_{ref}^* = E_{in}^2 rBS^2 + E_{in} n_{l1} rBS^2 \cos[tw] + E_{in} n_{l2} rBS tBS \cos[tw]$$

ただしnl, nPの二次以上は消去しています。

第一項がDC項、第二項第三項が入射側、虚無側からの雑音寄与です。

RINは(第二項以外の振幅)/(第一項)であらわせるので

$$RIN(P_{ref}) = n_{l1}/E_{in} + (n_{l2} tBS)/(E_{in} rBS)$$

ところでこの光を用いてサーボを構成するとnl1を操作して

$$n_{l1} \rightarrow -n_{l2} tBS / rBS$$

という代入を行うことに相当します。このときRIN(Pref) = 0

\*透過ポートで同様の計算をします

$$P_{trans} = E_{trans} E_{trans}^* = E_{in}^2 tBS^2 - E_{in} n_{l2} rBS tBS \cos[tw] + E_{in} n_{l1} tBS^2 \cos[tw]$$

$$RIN(P_{trans}) = n_{l1}/E_{in} - (n_{l2} rBS)/(E_{in} tBS)$$

ここで、先ほどのサーボの効果代入すると

$$RIN(P_{trans} \text{ with servo}) = - (rBS/tBS + tBS/rBS) n_{l2} / E_{in} = - n_{l2} / (rBS tBS) / E_{in}$$

\*ちなみに全光量を検出した場合は

$$P_I = E_I E_I^* = E_{in}^2 + E_{in} n_{l1} \cos[tw]$$

$$RIN(P_I) = n_{l1}/E_{in}$$

であるから、これを直前のケースと比較するとピックアップを用いた強度制御系を組んだ場合の透過側のRINはレーザーの全光量を検出して評価したRINより、 $n = 1/(rBS tBS)$  だけ悪化する、という結果になります。