

# Optical Design of the Input Mode Matching Telescope for KAGRA

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# Purpose of the IMMT

- Convert the beam size from the output of the MC to the mode of the MIF.
- Slightly bend the beam upward to align with the slope of the arms.
- Serve as the steering mirrors for the initial alignment of the beam into the MIF.
- Transmitted beam power from the IMMT1 is used as the error signal for the intensity stabilization.

## Design Procedure

- Compute the eigen-modes of the MC and the PRC
- Put the IMMT mirrors at reasonable positions in their chambers.
- Compute the propagation of the MC beam through MCo, IMMT and PRM.
- Compare the beam parameter on the PRM with the eigen mode of the PRC.
- Perform a 2D least square optimization to find the optimal values of the ROCs for the IMMT mirrors.
- Errors in the ROCs of the IMMT mirrors can be compensated by changing the positions of the IMMT mirrors. Compute how much we can tolerate the ROC error assuming that we can move the mirrors by 10cm at maximum.
- Since the beam size is smaller on IMMT1, we will use the transmission of this mirror for the monitoring of the intensity fluctuation.
- The transmissivity of IMMT1 is determined by the shot noise limit requirement for the intensity stabilization servo.
- Since the HR surfaces are highly curved, AR is better for opelevs.
- For this surface, we require a moderate reflectivity (>40%) for 670nm laser.

# Mode Matching Requirement

- A poor mode matching means less amount of light power available for the detection of GW. However, this is not a serious problem in reality because the mode matching of over 90% is easily achievable and the shot noise increase by the 10% power reduction is not a big deal.
- The real requirement comes from the shot noise of REFL port, not AS.
- If we forget about the mode matching, the amount of light power coming back to REFL is determined by the reflectivity mismatch between the PRM and the arm cavities. When we designed the LSC scheme using Optickle, the assumed mismatch gave 1.8W of carrier power coming back to REFL.
- Since we use the beat between the carrier and the RFSBs for signal extraction at REFL (for CARM and MICH), the shot noise level of these signals does not depend much on the TEM00 carrier power at REFL.
- However, the carrier higher order modes coming back to REFL caused by the mode mismatch do not contribute to the signal generation. Therefore, we have to make these much smaller than the TEM00 carrier power.
- The nominal input power to the bKAGRA is 78W. This means the reflectivity for the TEM00 carrier is  $1.8/78=2\%$ . In order to make the HOM carrier negligible to this, we set the mode mismatch to be less than 0.1%.

**Thus the mode matching has to be better than 99.9%.**

# MC Parameters

MCe ROC =  $37.3 \pm 0.1\text{m}$  (based on the measurement by E. Hirose)

MCi, MCo are flat, separated by 0.5m

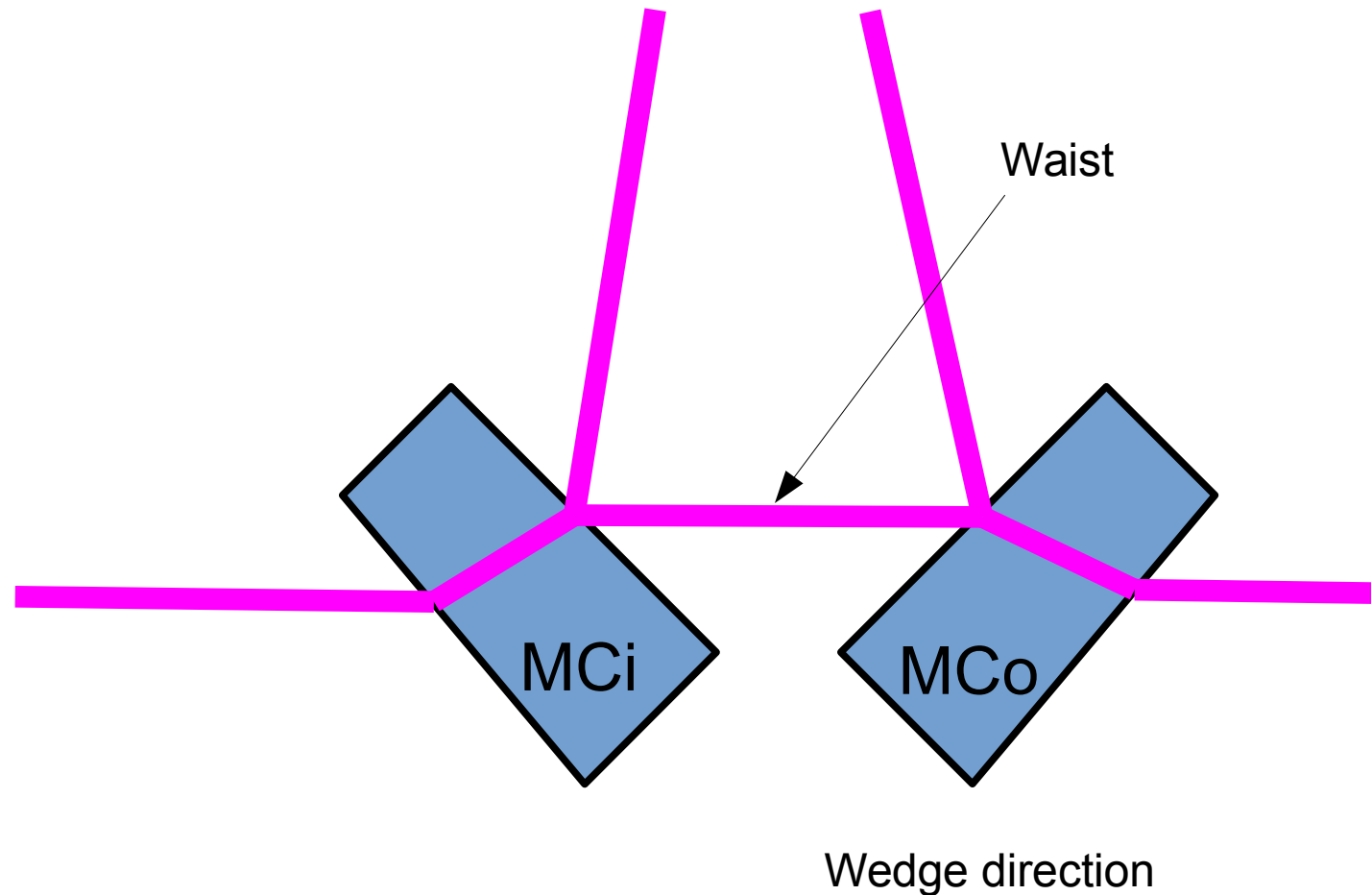
Total MC length = 26.65m

Beam size at the MC waist = 2.3887mm (slightly elliptic in reality)

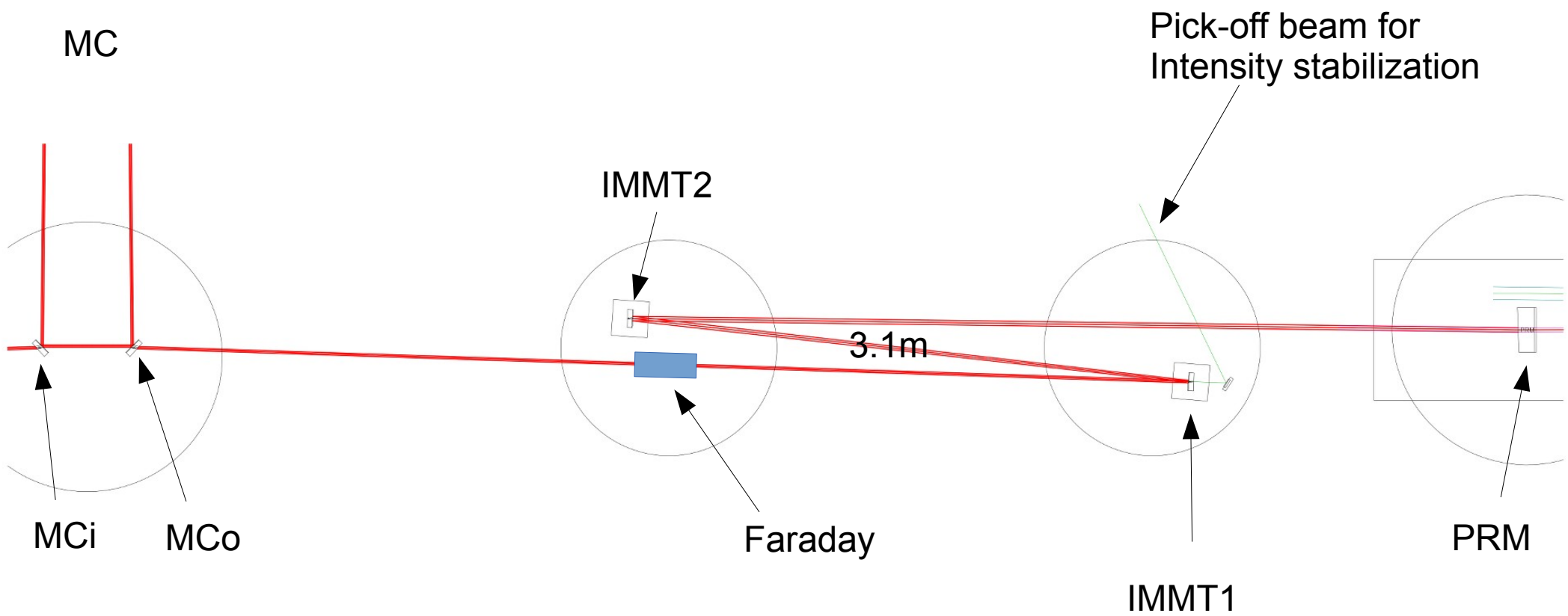
MC mirror diameter = 100mm

MC mirror thickness = 30mm (will be smaller by 1-2mm according to Mio-san)

MC wedge angle = 2.5deg

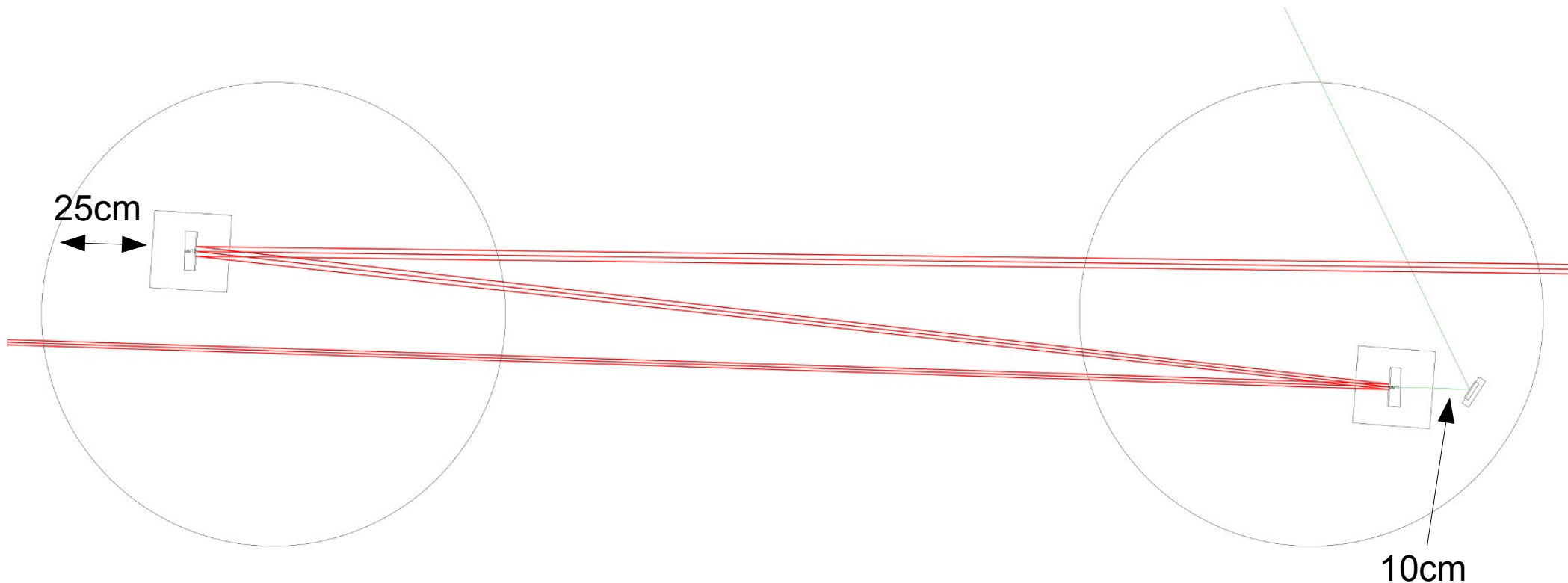


# IMMT configuration



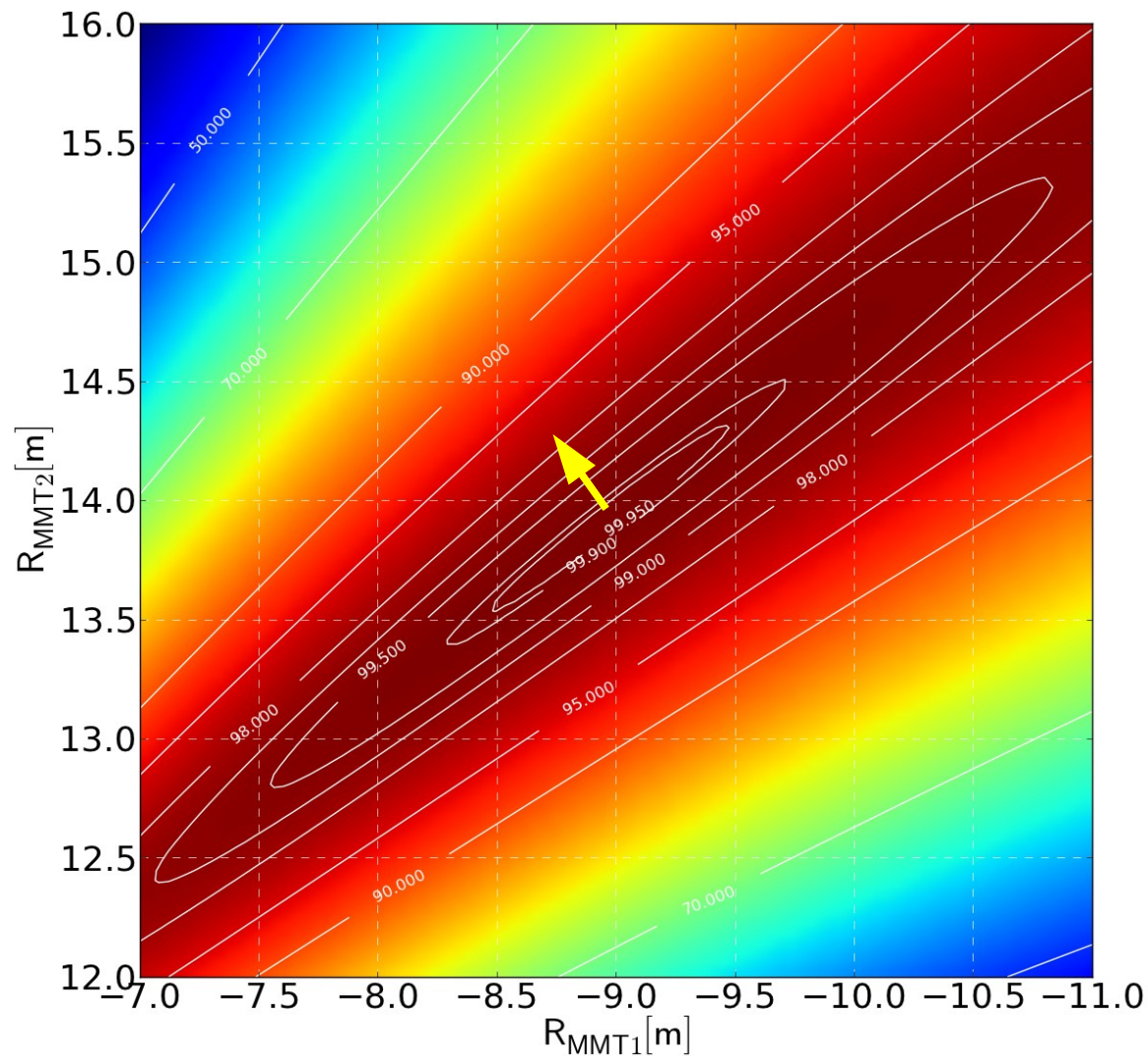
# How much can we move the mirrors ?

- As we will see in the next few slides, we have to move the mirrors from their nominal positions in order to compensate for the ROC errors.
- The suspension systems for the IMMT mirrors are the TAMA suspensions. The foot prints of the TAMA suspensions are shown as a rectangle around each mirror.
- By looking at the chamber space, the distance between the mirrors can be easily changed by  $\pm 10\text{cm}$ . We assume this is the adjustable range of the IMMT length.

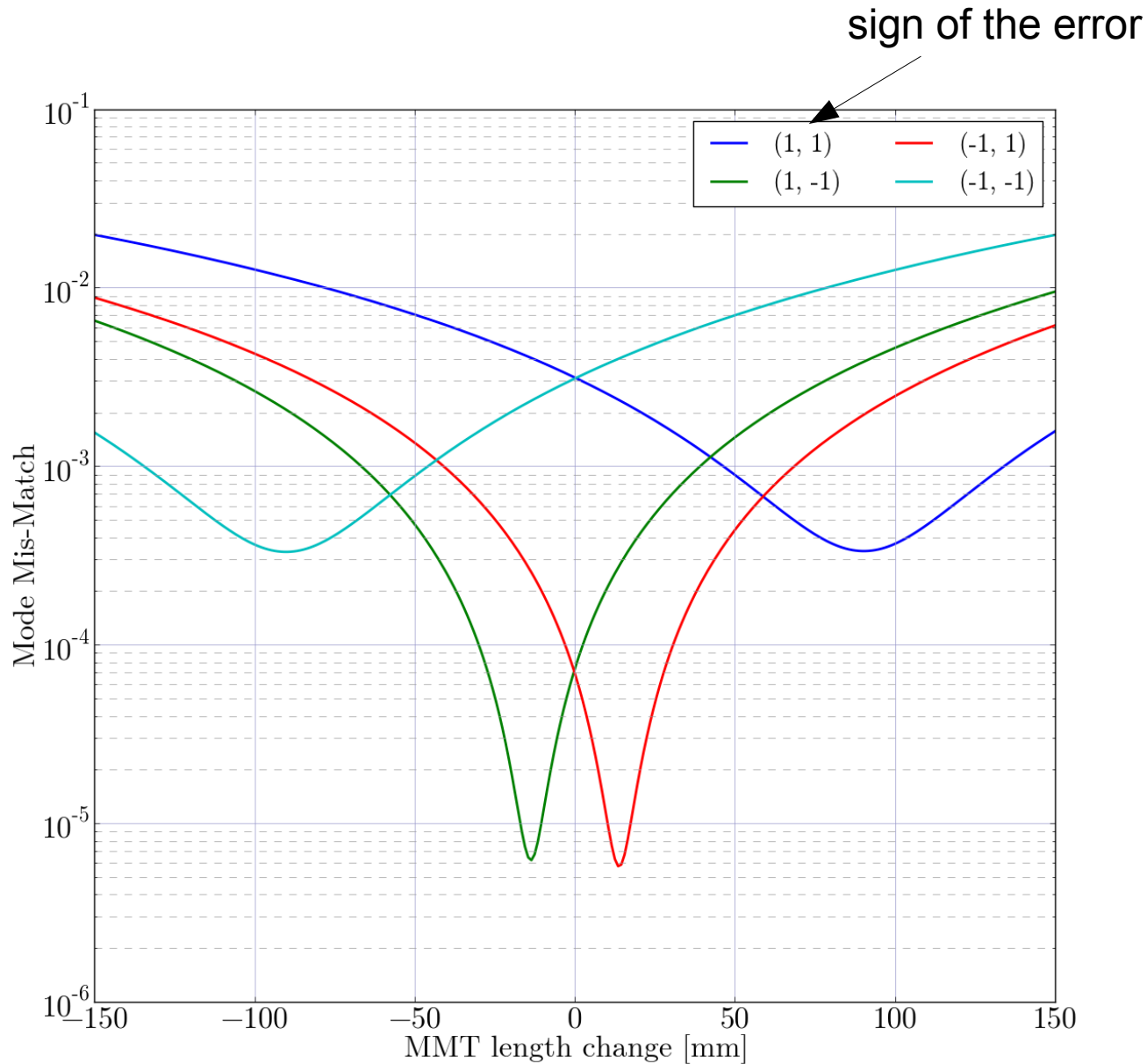


# Optimal ROCs

- Assuming the distance between the IMMT1 and IMMT2 is 3.1m, the mode matching rate to the MIF is computed sweeping the ROC values of the two mirrors.
- The optimal values are: IMMT1=-8.953m, IMMT2=13.910m
- The region over 99.9% mode match is highly elliptical. If the errors are in the direction indicated by the yellow arrow, the error tolerance is in the order of 10cm.



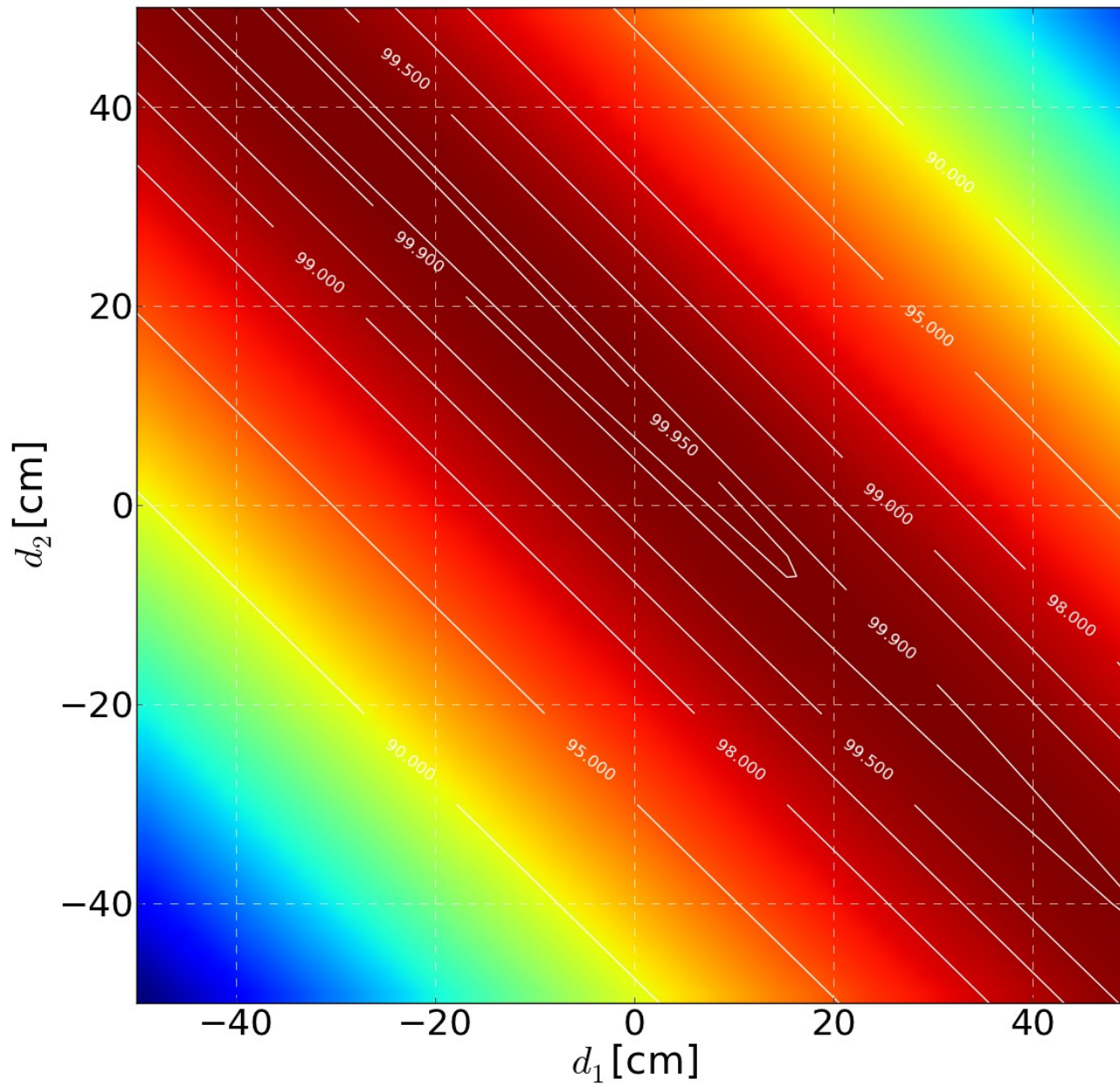
ROC Error = 10cm for both IMMTs



ROC error can be compensated by moving the IMMT mirrors.  
In this figure, two mirrors are moved by the same amount in the opposite direction.

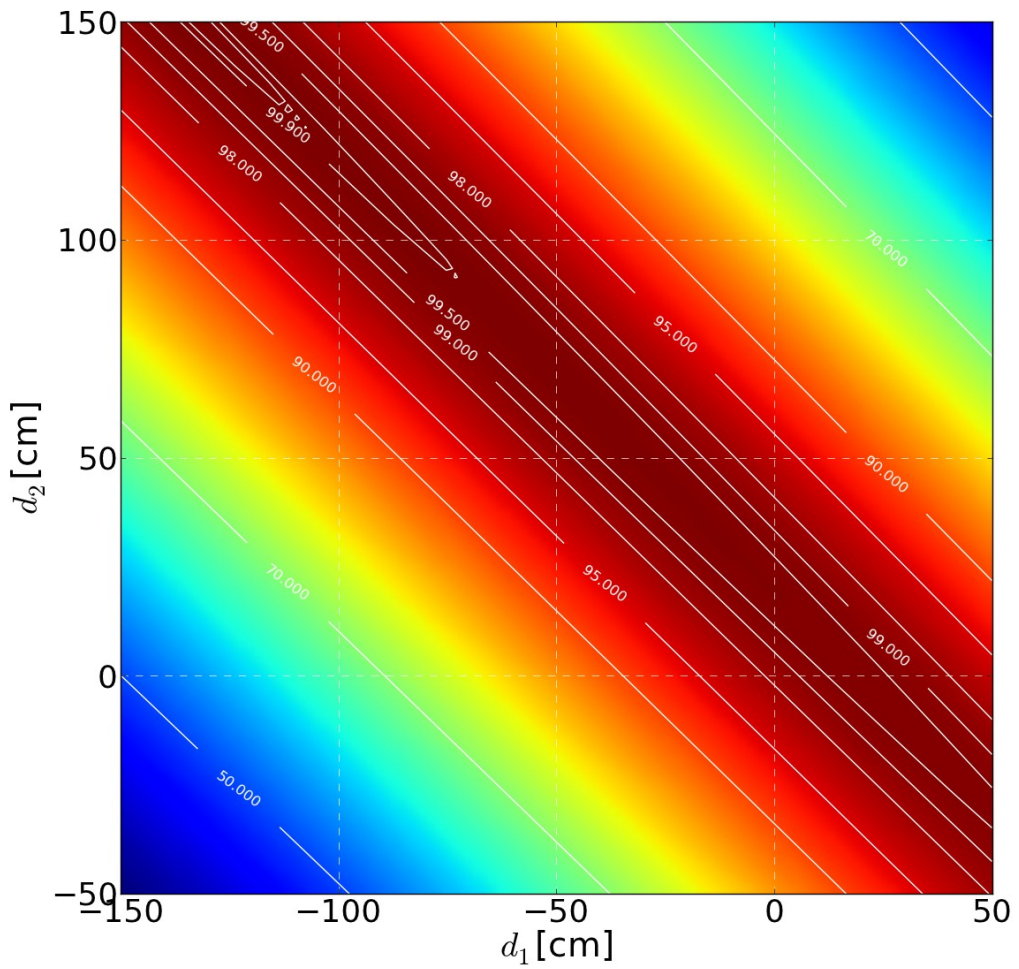


# Scanning the IMMT positions

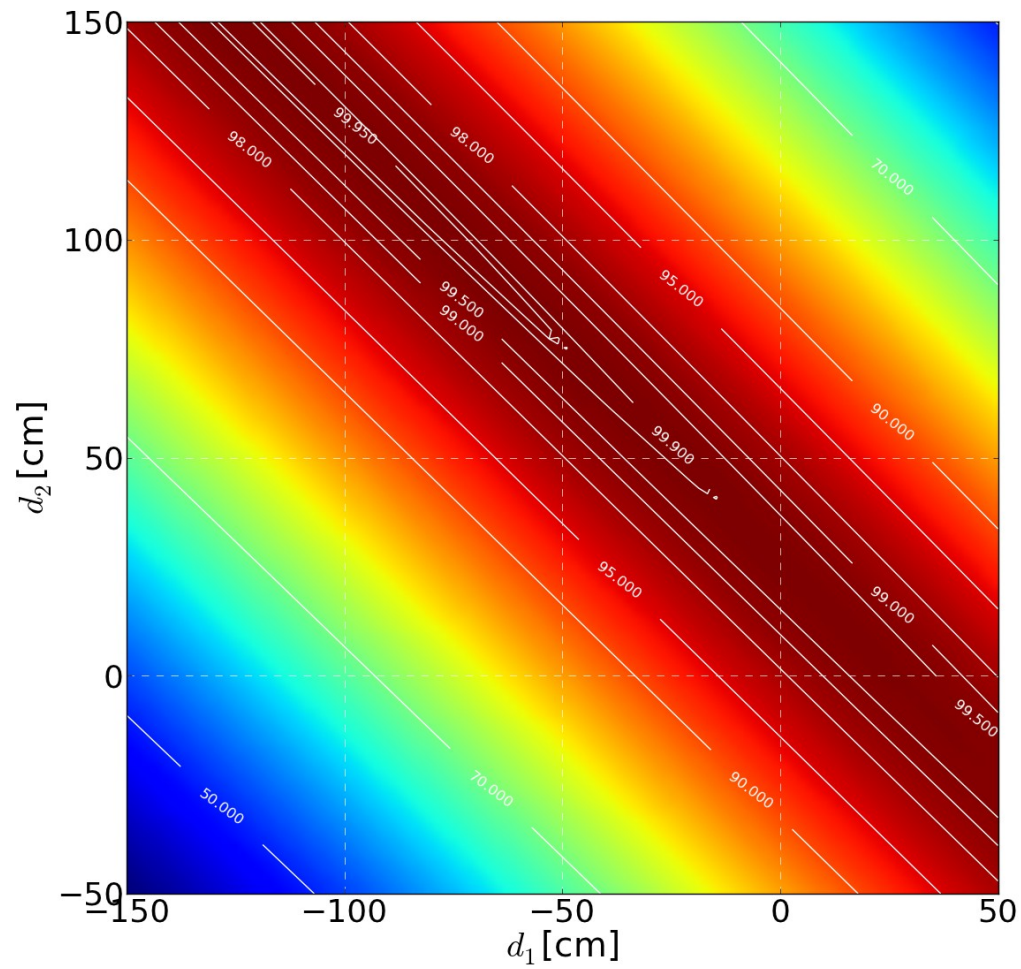


For 10cm error in both IMMT mirrors

IMMT1 ROC Error = 50cm



IMMT2 ROC Error = 50cm



# RIN Requirement $\sim 2 \times 10^{-9}$ (at the input of MIF, $10 < f < 100 \text{ Hz}$ )

(a factor of 10 safety margin included, Ref. MIF design document)

Shot noise limit of an intensity stabilization servo is given by the following formula:

$$\text{RIN} = \sqrt{\frac{2h\nu P_0}{\eta P_m (P_0 - P_m)}}$$

(See Appendix for the theoretical background of this formula)

$\eta$  : quantum efficiency (nominal value = 0.9)

$P_0$  : Input power to the interferometer (75W)

$P_m$  : Power on the monitor PD (order of 100mW)

$P_m = 100 \text{ mW}$  gives  $2.4 \times 10^{-9}$  RIN limit.

For safety, I propose to make **the transmitted power of the IMMT1 be 200mW**

**→ Power Transmission = 1500ppm**

# Conclusion

## Specs for the IMMT mirrors

	IMMT1	IMMT2
ROC	-8.953m	13.910
ROC Error Tolerance	+/-10cm	+/-10cm
HR Transmission (1064nm)	$1500 < T < 2000\text{ppm}$	$T < 500\text{ppm}$
HR Loss (1064nm)	$L < 1000\text{ppm}$	$L < 1000\text{ppm}$
AR Reflection (1064nm)	$R < 0.1\%$	$R < 1\%$
AR Reflection (670nm)	$R > 40\%$	$R > 40\%$



# Appendix

## K. Arai's email message on the method to calculate the intensity stabilized RIN limit.

\* vacuum fluctuationの正体というのは電磁場のゼロ点振動。  
だからコヒーレント状態を仮定した場合、古典場を注入している周波数・空間モードでは古典的に扱い、それ以外では場がゼロ点エネルギーで決まるRMSでランダムに振動している、と考えます。

\* このときキャリア周波数を挟んで $\pm w$ の周波数での場の振動がコモンの場合、キャリアとあわせて強度雑音を、ディファレンシャルの場合位相雑音を生み出します。個々の周波数のモードを扱うより上下セットの方が雑音の意味としてとらえやすいぞ、というのがtwo photon pictureとか言うのではなかったかと思えます。（これが宗宮さんから教わった部分）

\* で、各電場を定義すると  
レーザーからくる電場

$$E_{in} = E_I + (n_{I1} / 4) * (\text{Exp}[i w t] + \text{Exp}[-i w t]) + (n_{P1} / 4) * (\text{Exp}[i w t] - \text{Exp}[-i w t])$$
  
第一項が古典場、第二項が強度雑音成分、第三項が位相雑音成分  
共通項である $\text{Exp}[i \Omega t]$ は無視しています(Phaser表示)

$n_{I1}$ と $n_{P1}$ は無相関の雑音振幅ですがRMSは同じものと考えます。  
係数4はあとで結果を綺麗にするためのファクタです。

実際、三角関数で書き換えると

$$E_{in} = E_I + 1/2 n_{I1} \text{Cos}[t w] + 1/2 i n_{P1} \text{Sin}[t w]$$
  
となり雑音の意味が分かりやすくなります。

\* BSの虚無側からくる電場は同様に

$$E_{vac} = (n_{I2} / 4) * (\text{Exp}[i w t] + \text{Exp}[-i w t]) + (n_{P2} / 4) * (\text{Exp}[i w t] - \text{Exp}[-i w t])$$

$n_{I2}$ ,  $n_{P2}$ はやはり無相関の雑音振幅で、振幅は $n_{I1}$ ,  $n_{P1}$ と同じです

\*POの反射ポート・透過ポートの電場(Eref, Etrans)は

$$E_{ref} = rBS E_{in} + tBS E_{vac}$$

$$E_{trans} = tBS E_{in} - rBS E_{vac}$$

で表されます。

\*今反射ポートで検出するパワーPrefとそのRINを考えます。

$$P_{ref} = E_{ref} E_{ref}^* = E_{in}^2 rBS^2 + E_{in} n_{l1} rBS^2 \cos[tw] + E_{in} n_{l2} rBS tBS \cos[tw]$$

ただしnl, nPの二次以上は消去しています。

第一項がDC項、第二項第三項が入射側、虚無側からの雑音寄与です。

RINは(第二項以外の振幅)/(第一項)であらわせるので

$$RIN(P_{ref}) = n_{l1}/E_{in} + (n_{l2} tBS)/(E_{in} rBS)$$

ところでこの光を用いてサーボを構成するとnl1を操作して

$$n_{l1} \rightarrow -n_{l2} tBS / rBS$$

という代入を行うことに相当します。このときRIN(Pref) = 0

\*透過ポートで同様の計算をします

$$P_{trans} = E_{trans} E_{trans}^* = E_{in}^2 tBS^2 - E_{in} n_{l2} rBS tBS \cos[tw] + E_{in} n_{l1} tBS^2 \cos[tw]$$

$$RIN(P_{trans}) = n_{l1}/E_{in} - (n_{l2} rBS)/(E_{in} tBS)$$

ここで、先ほどのサーボの効果代入すると

$$RIN(P_{trans} \text{ with servo}) = - (rBS/tBS + tBS/rBS) n_{l2} / E_{in} = - n_{l2} / (rBS tBS) / E_{in}$$

\*ちなみに全光量を検出した場合は

$$P_I = E_{in} E_{in}^* = E_{in}^2 + E_{in} n_{l1} \cos[tw]$$

$$RIN(P_I) = n_{l1}/E_{in}$$

であるから、これを直前のケースと比較するとピックアップを用いた強度制御系を組んだ場合の透過側のRINはレーザーの全光量を検出して評価したRINより、 $n = 1/(rBS tBS)$ だけ悪化する、という結果になります。