How to design feedback filters for a given system?

October 23, 2013

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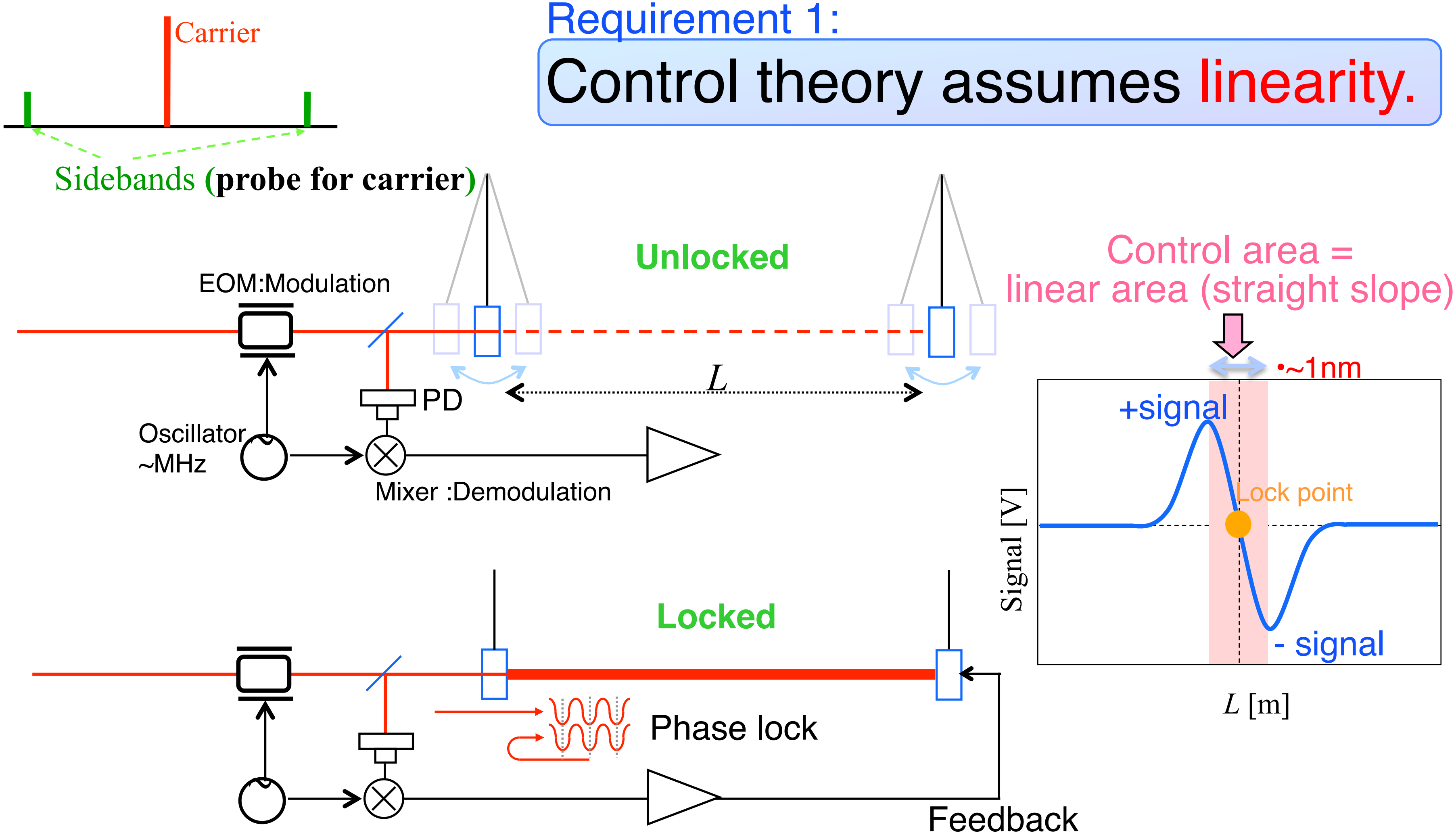
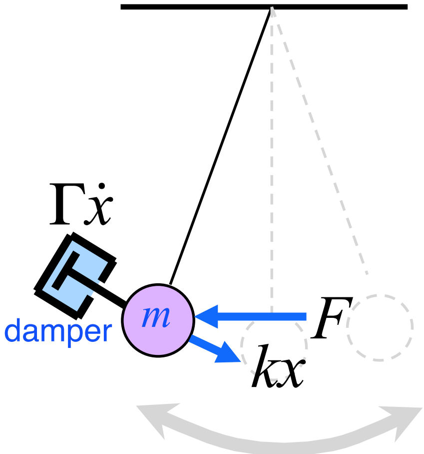
Requirement 1: Assumption of Linearity

Requirement 1:

Control theory assumes linearity.



Pendulum in Physics



$$F = m\frac{d^2x}{dt^2} + \Gamma\frac{dx}{dt} + kx$$

$x = Ae^{i\omega t}$: complex number

$|x| = A$: amplitude

$\mathrm{Arg}(x) = \omega t = 2\pi ft$: angle

$$\frac{dx}{dt} = i\omega x$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

Laplace transform $\bar{f}(s) = L[f(t)] \equiv \int_0^\infty f(t)e^{-st}dt$

$$L[F] = m \times L\left[\frac{d^2x}{dt^2}\right] + \Gamma \times L\left[\frac{dx}{dt}\right] + k \times L[x]$$

$$\bar{F} = ms^2\bar{x} + \Gamma s\bar{x} + k\bar{x}$$

$$\frac{\bar{x}}{\bar{F}} = \frac{1}{ms^2 + \Gamma s + k} = \frac{1}{\left(s - \left(\frac{-\Gamma - \sqrt{\Gamma^2 - 4mk}}{2m}\right)\right)\left(s - \left(\frac{-\Gamma + \sqrt{\Gamma^2 - 4mk}}{2m}\right)\right)} \text{; 2 poles}$$

$$s \equiv i\omega$$

$$\bar{F} = -m\omega^2\bar{x} + i\Gamma\omega\bar{x} + k\bar{x}$$

$$\frac{\bar{x}}{\bar{F}} = \frac{1}{-m\omega^2 + i\Gamma\omega + k}$$

Interpretation in Physics

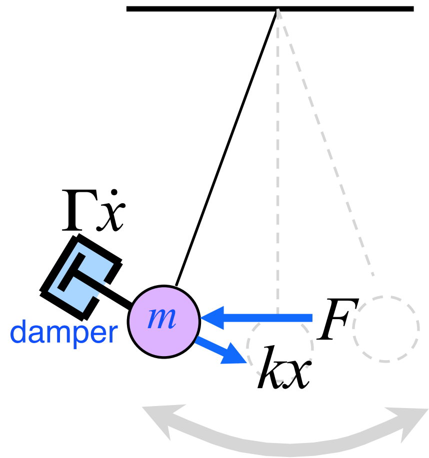
resonant anglar frequency: $\omega_0 = 2\pi f_0 = \sqrt{k/m}$

quality factor $Q \Rightarrow 1/\text{energy loss}$: $Q = m\omega_0/\Gamma$

Transfer function from force to position :

$$\frac{\bar{x}}{\bar{F}/m} = \frac{1}{-\omega^2 + i\omega\omega_0/Q + \omega_0^2}$$

Bode plot : frequency vs. gain, phase



$$\frac{\bar{x}}{\bar{F}/m} = \frac{1}{-\omega^2 + i\omega\omega_0/Q + \omega_0^2}$$

$$\text{if } \omega << \omega_0 \Rightarrow \frac{\bar{x}}{\bar{F}/m} \rightarrow \frac{1}{\omega_0^{\ 2}} \quad \text{: constant}$$

$$\text{if } \omega >> \omega_0 \Rightarrow \frac{\bar{x}}{\bar{F}/m} \rightarrow \frac{1}{-\omega^2} \quad : f^{-2} \text{ slope}$$

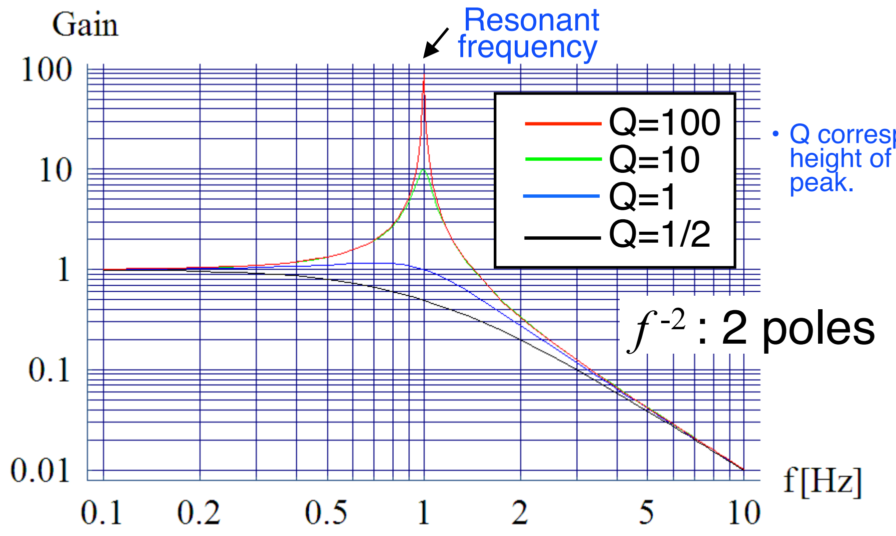
$$\text{if } \omega = \omega_0 \Rightarrow \frac{\bar{x}}{\bar{F}/m} \rightarrow \frac{Q}{i\omega_0^{\ 2}} \quad \text{: resonance}$$

$$\text{if } \omega = \omega_0, \text{Q} \rightarrow \infty \Rightarrow \frac{\bar{x}}{\bar{F}/m} \rightarrow \infty$$

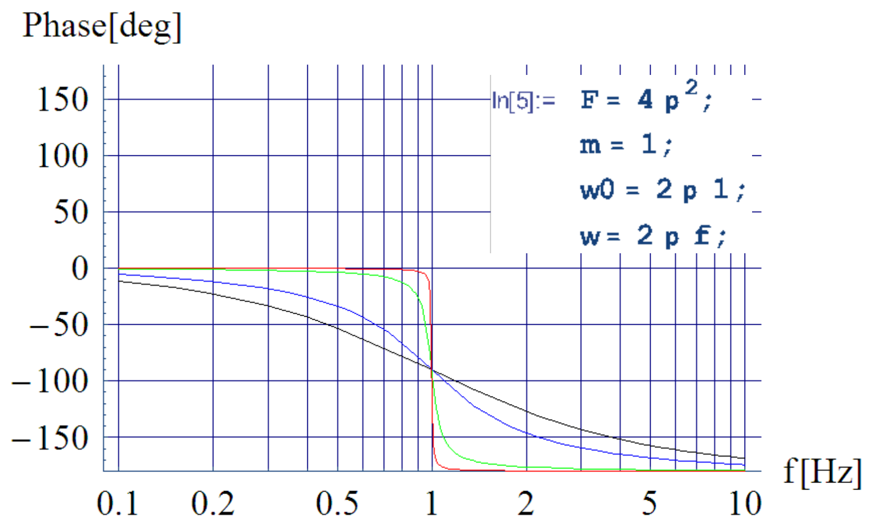
: resonance, no damp

if $\text{Q} = 1/2 \Rightarrow$ cretical damping

if $\text{Q} < 1/2 \Rightarrow$ over damping

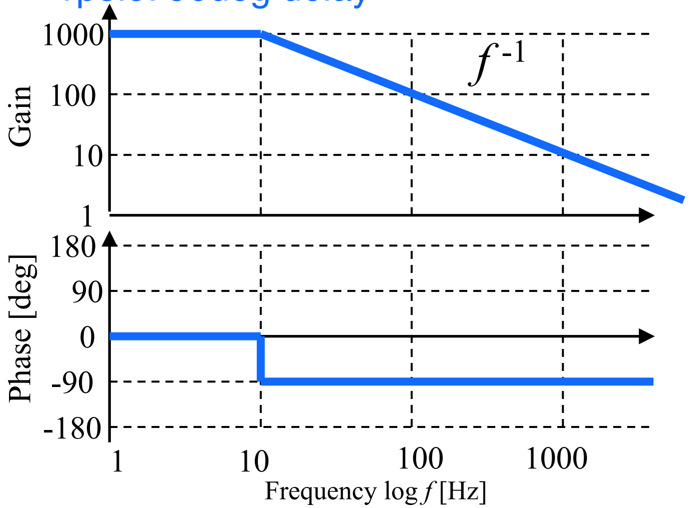


• Q corresponds to height of resonant peak.

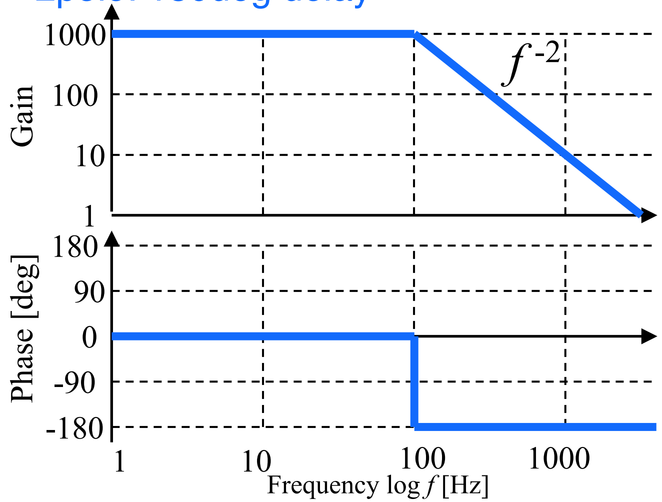


Basic concept : relationship between gain and phase

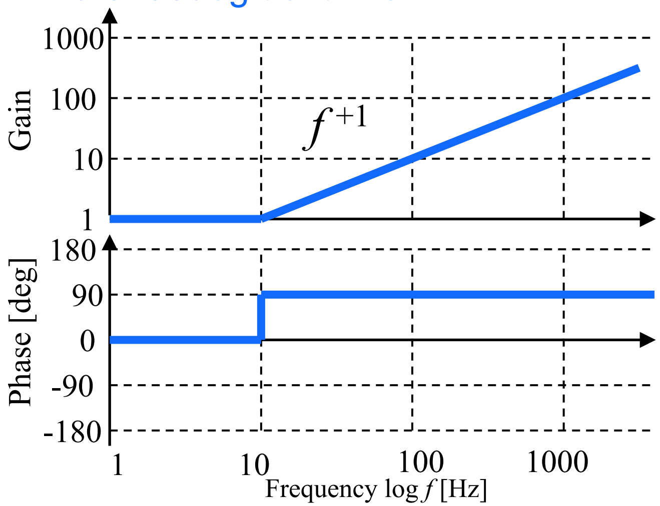
- 1pole: 90deg delay



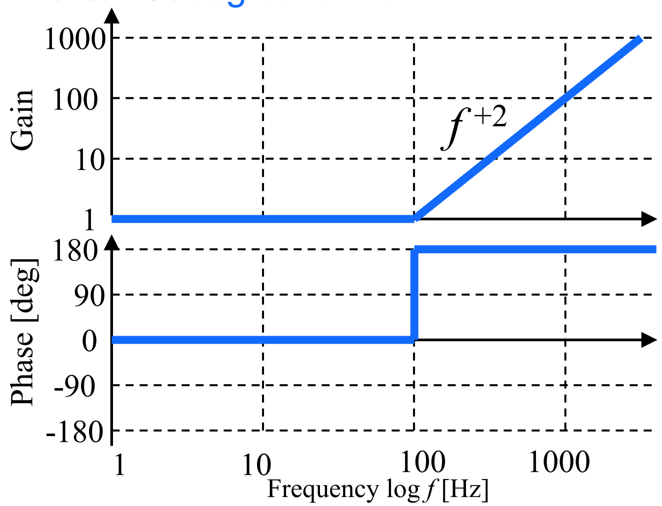
- 2pole: 180deg delay



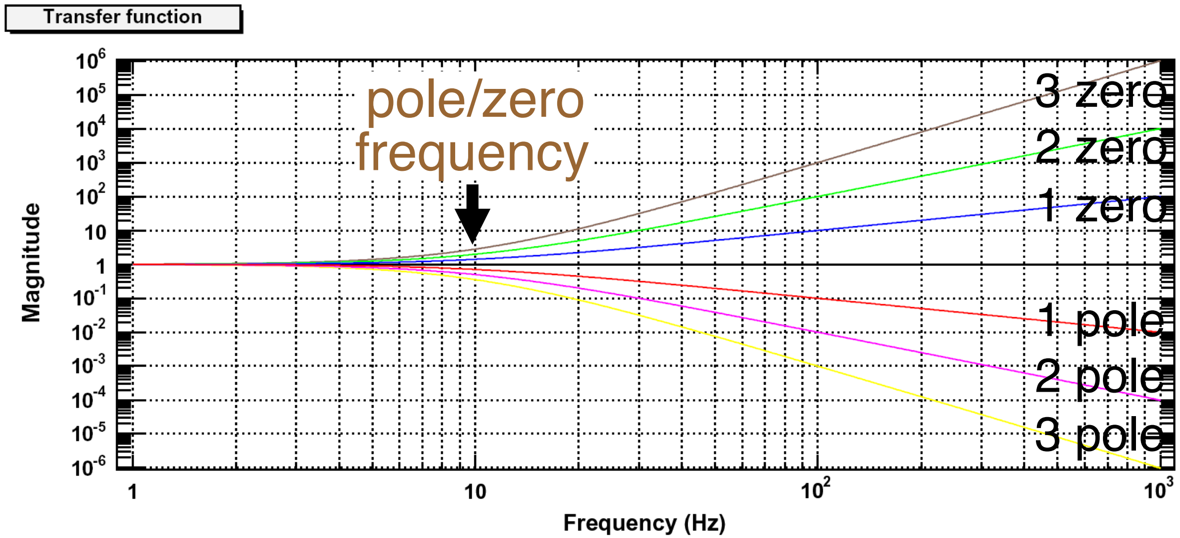
- 1zero: 90deg advance

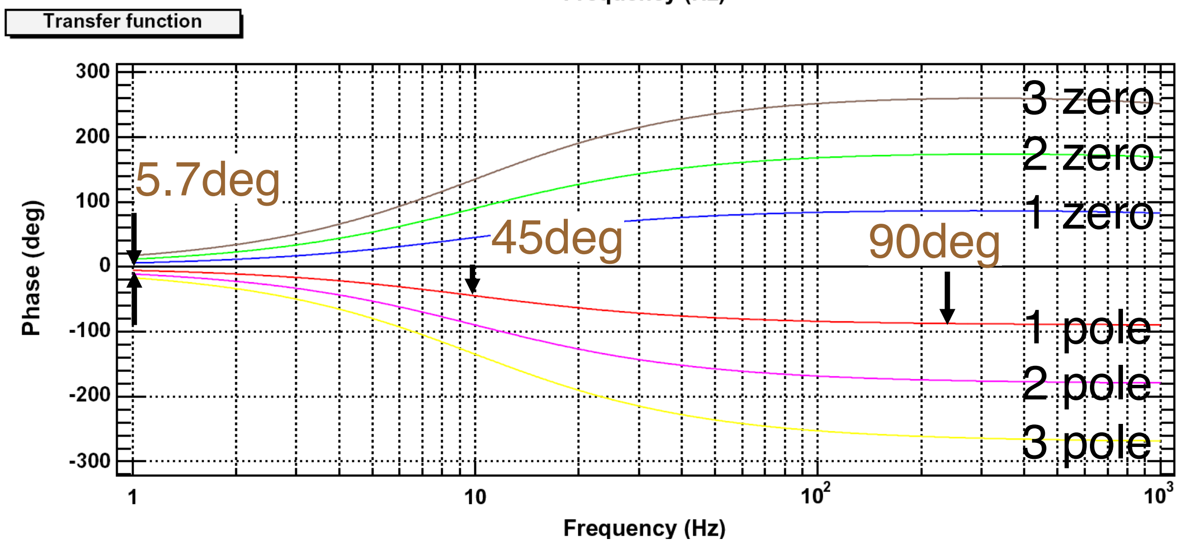


- 2zero: 180deg advance



Actual pole and zero with phase





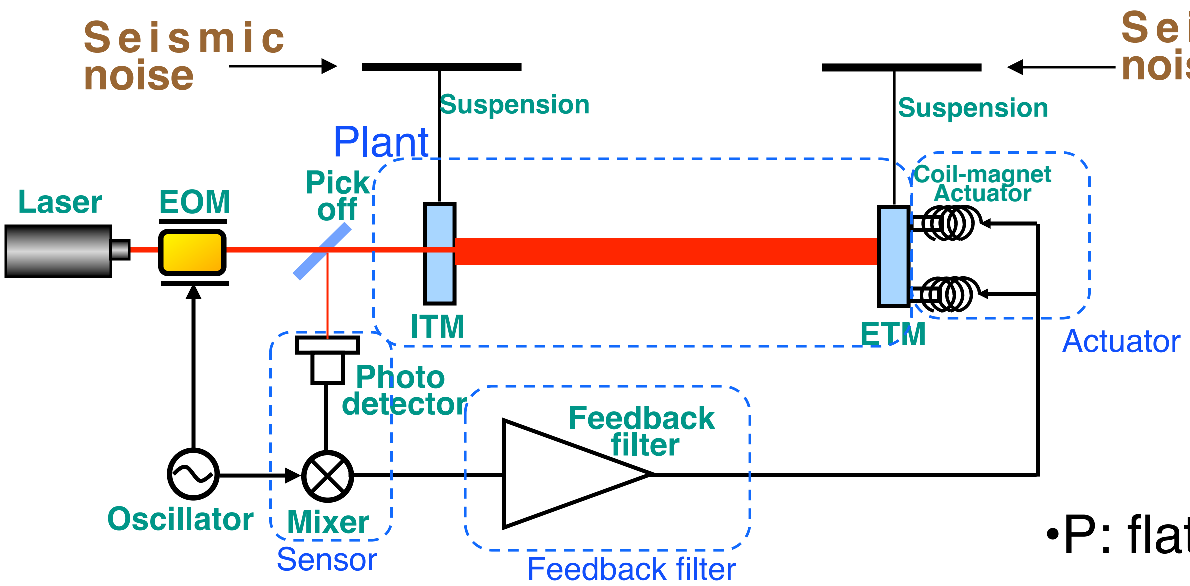
Actual gain and phase slope change smoothly.

Phase above pole (zero) frequency has N x 90degree phase delay (advance).

Phase at pole (zero) frequency has N x 45degree phase delay (advance).

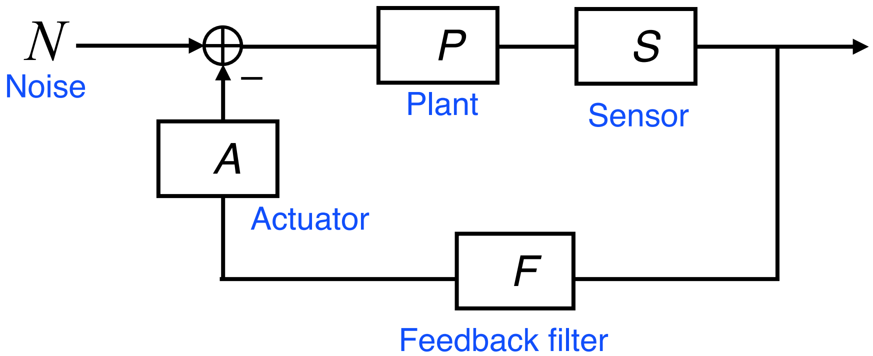
Phase at 10times lower frequency of pole (zero) frequency has N x 5.7degree phase delay (advance).

Modeling system: Single Fabry-Perot cavity

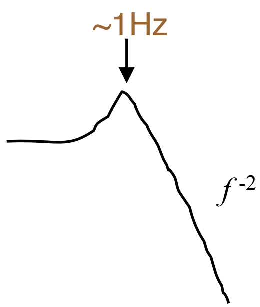


Mission:

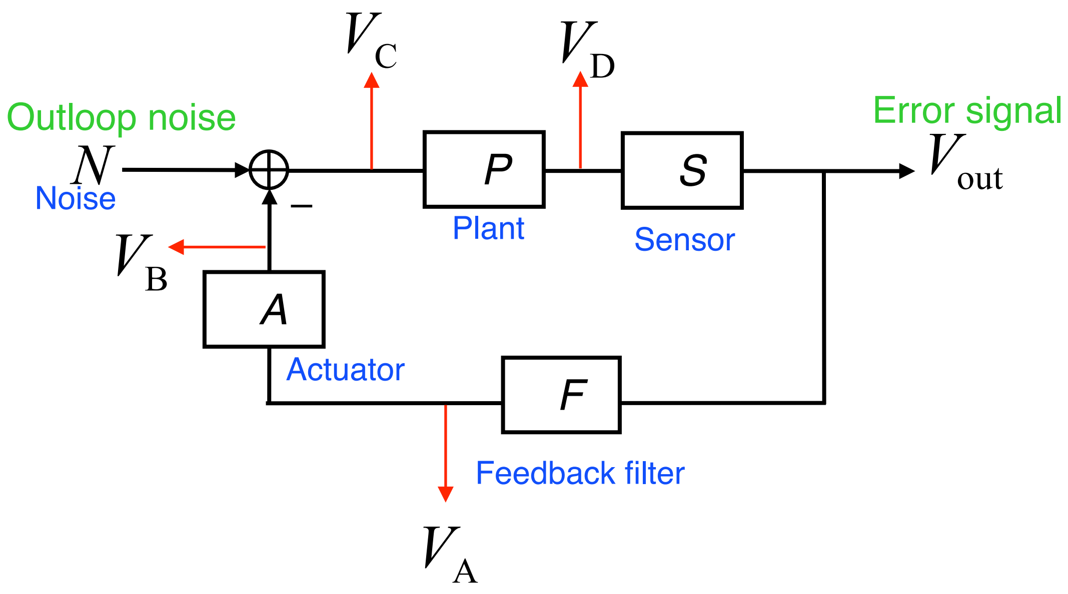
Make a feedback filter to keep distance between ITM and ETM constant !



- P: flat around resonance
- S: flat
- F: ??
- A: suspension TF



Transfer function from Noise N to error signal V_{out}



$$V_A = F \times V_{OUT}$$

$$V_B = A \times V_A$$

$$V_C = N - V_B$$

$$V_D = P \times V_C$$

$$V_{OUT} = S \times V_D$$

$$V_{OUT} = S \times P \times (N - A \times F \times V_{OUT})$$

$$(1 + SPAF) V_{OUT} = SP \times N$$

$$\therefore V_{OUT} = \frac{SP}{(1 + SPAF)} \times N = \frac{SP}{(1 + G)} \times N$$

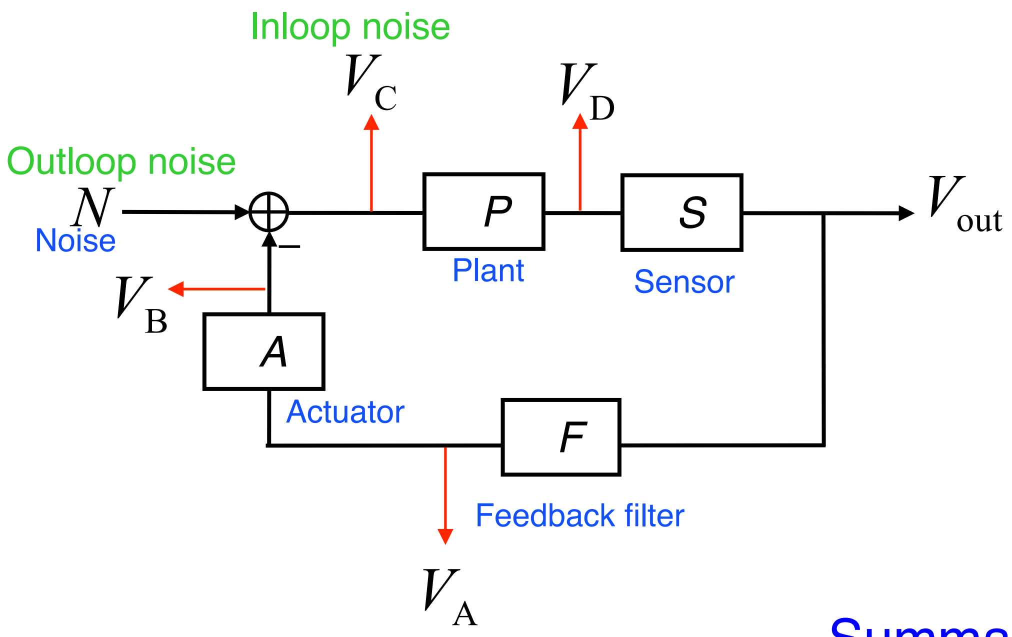
Open loop transfer function :

$$G = SPAF$$

Summary:

Relationship between "Error signal" V_{out} and "Outloop noise" N can be written with "Open loop transfer function" G and "transfer function" P,S.

Transfer function from outloop noise to inloop noise



$$V_C = \frac{V_{out}}{SP} = \frac{1}{SP}\frac{SP}{(1+G)} \times N = \frac{1}{(1+G)} \times N$$

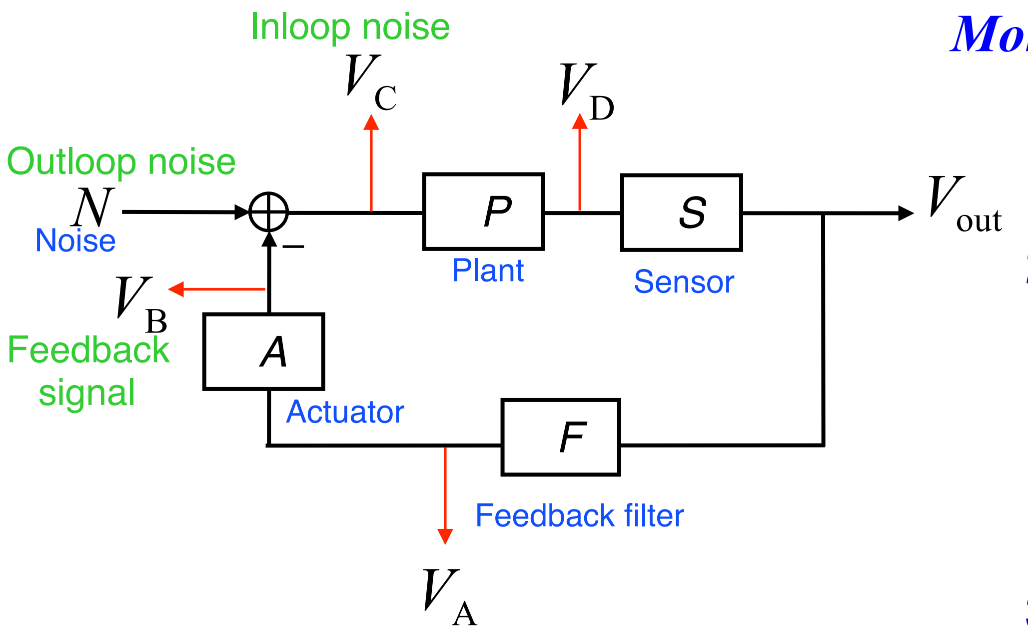
$$\text{if } G >> 1, V_C \approx \frac{N}{G} << N; \text{supression}$$

$$\text{if } G << 1, V_C \approx N; \text{ no suppression}$$

Summary:

"Outloop noise" N is suppressed by Open loop transfer function G (if G>>1) into "Inloop noise" $N/(1+G)$, then it is multiplied by transfer functions SP through output port.

Feed back signal



More example;

1. $$V_D = \frac{P}{(1+G)} N$$

2. $$V_B = \frac{PSFA}{(1+G)} N = \frac{G}{(1+G)} N$$

$$\text{if } G >> 1, V_B \approx \frac{G}{G} N = N$$

$$\text{if } G << 1, V_B \approx GN << N$$

3. $$N - V_B = N - \frac{G}{(1+G)} N$$

$$= \frac{N(1+G) - GN}{(1+G)}$$

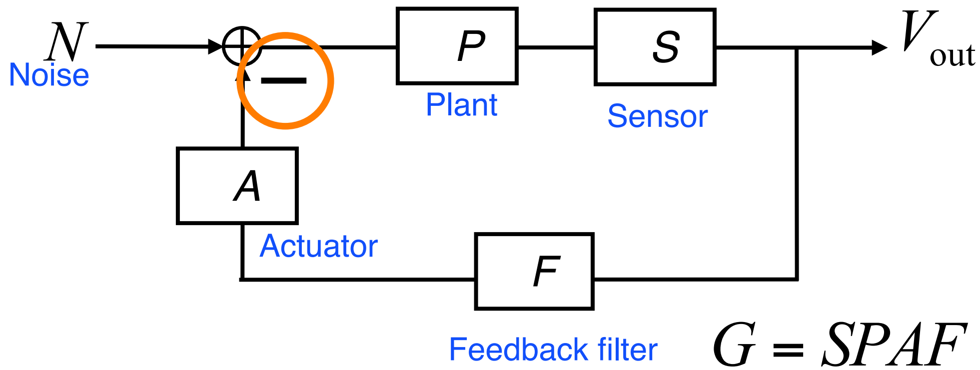
$$= \frac{N + GN - GN}{(1+G)}$$

$$= \frac{N}{(1+G)} \Rightarrow V_C$$

Summary:

"Outloop noise" N is canceled out by "Feedback signal" $G/(1+G)$ N, then "Inloop noise" becomes $N/(1+G)$.

Requirement 2: Negative feedback



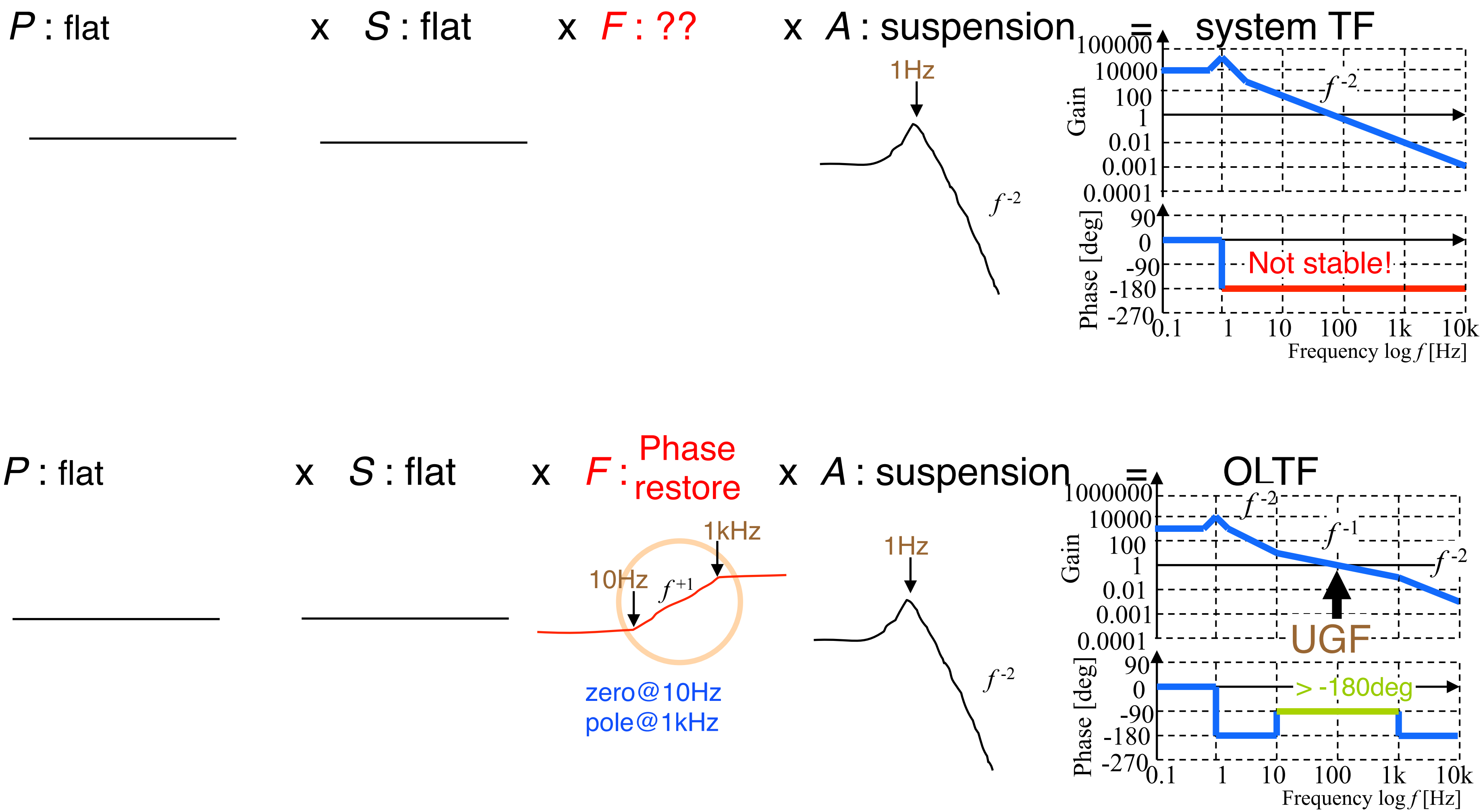
$G = SPAF$: Open loop transfer function

Requirement 2:

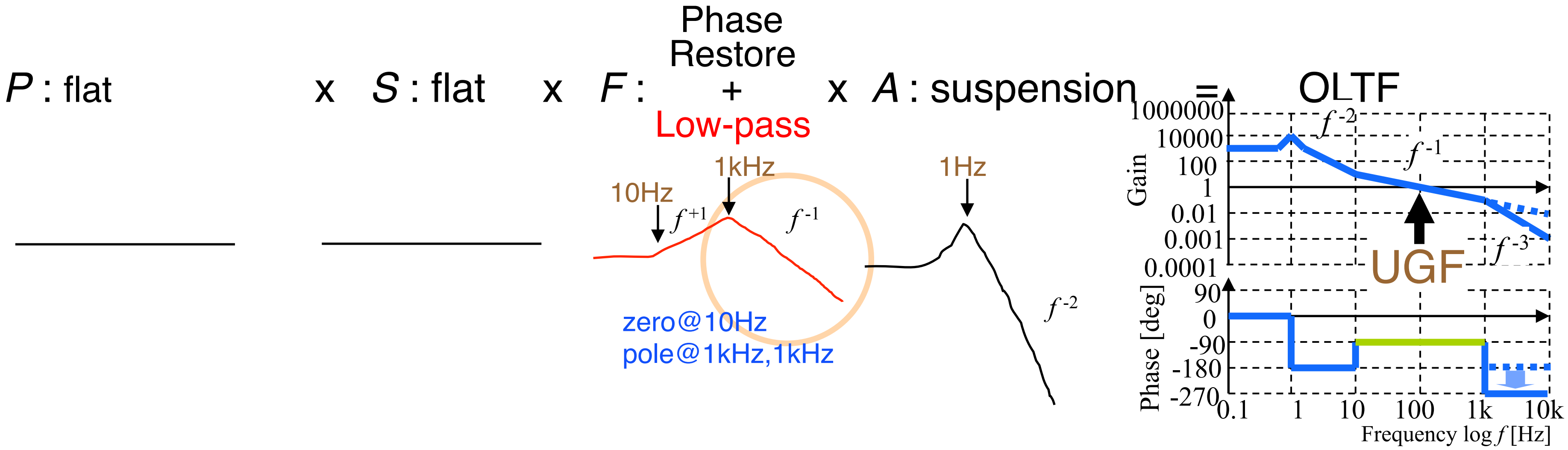
Phase delay of open loop transfer function at unity gain frequency (frequency at gain = 1) must be less than 180 degree.

You must have negative feedback if total gain (OLG) is larger than 1.

Open Loop TF and Unity Gain Frequency



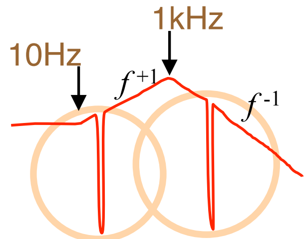
Low-pass filter to avoid feed back noise



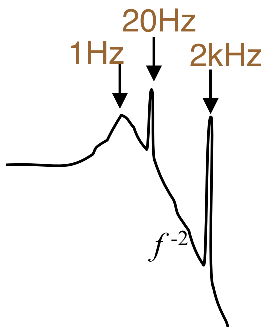
Too high feedback signal at high frequency will be not only noise but also saturation, not desired mechanical resonance, then can break control!

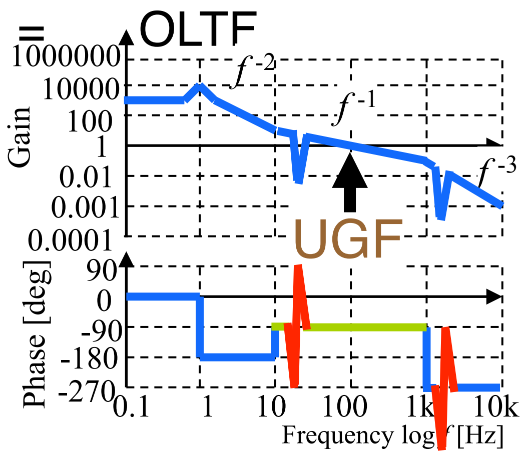
Notch filter to avoid mechanical resonance

P : flat x *S* : flat x *F* : Phase Restore + Low-pass + notch x *A* : Suspension + Mechnical resonance =



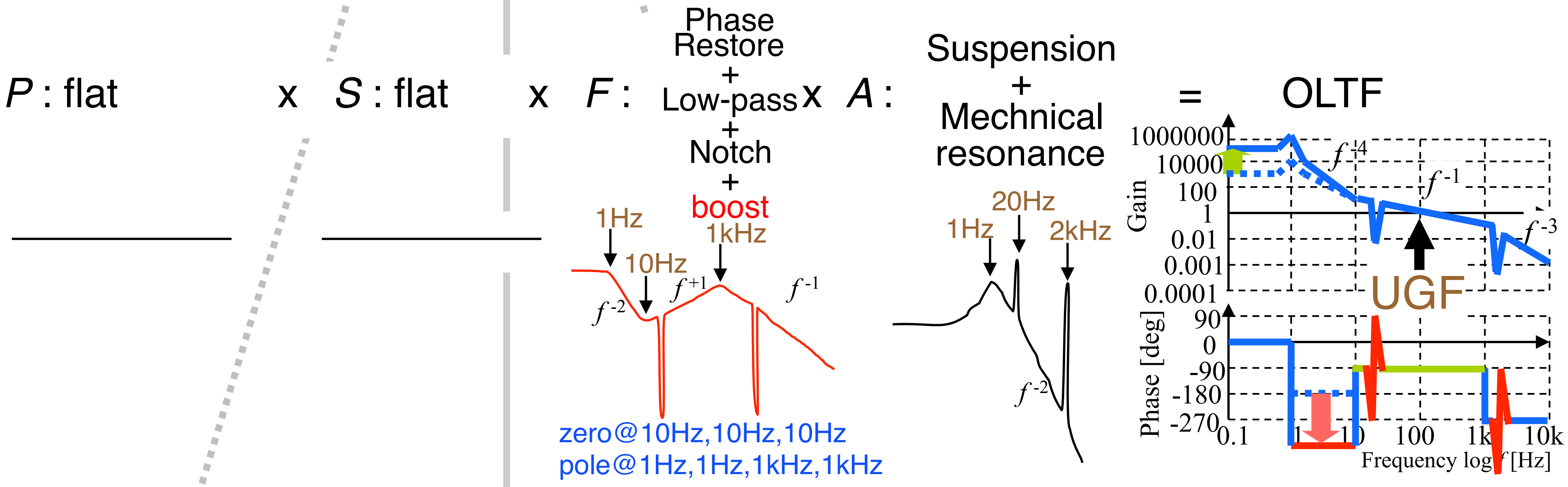
zero@10Hz
pole@1kHz,1kHz
notch@20Hz, 2kHz

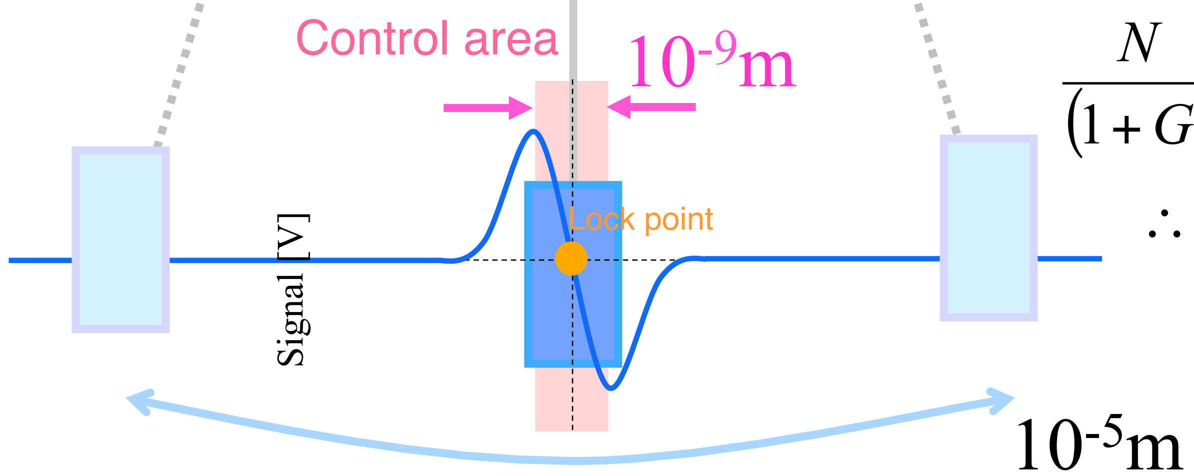




- Mechanical resonance can make another UGF which has more than 180 degree phase delay.
- Notch filter has big phase delay and advance in very narrow band, so it can be used both above and below UGF, but not too close to UGF.

Boost filter; engaged after lock acquisition

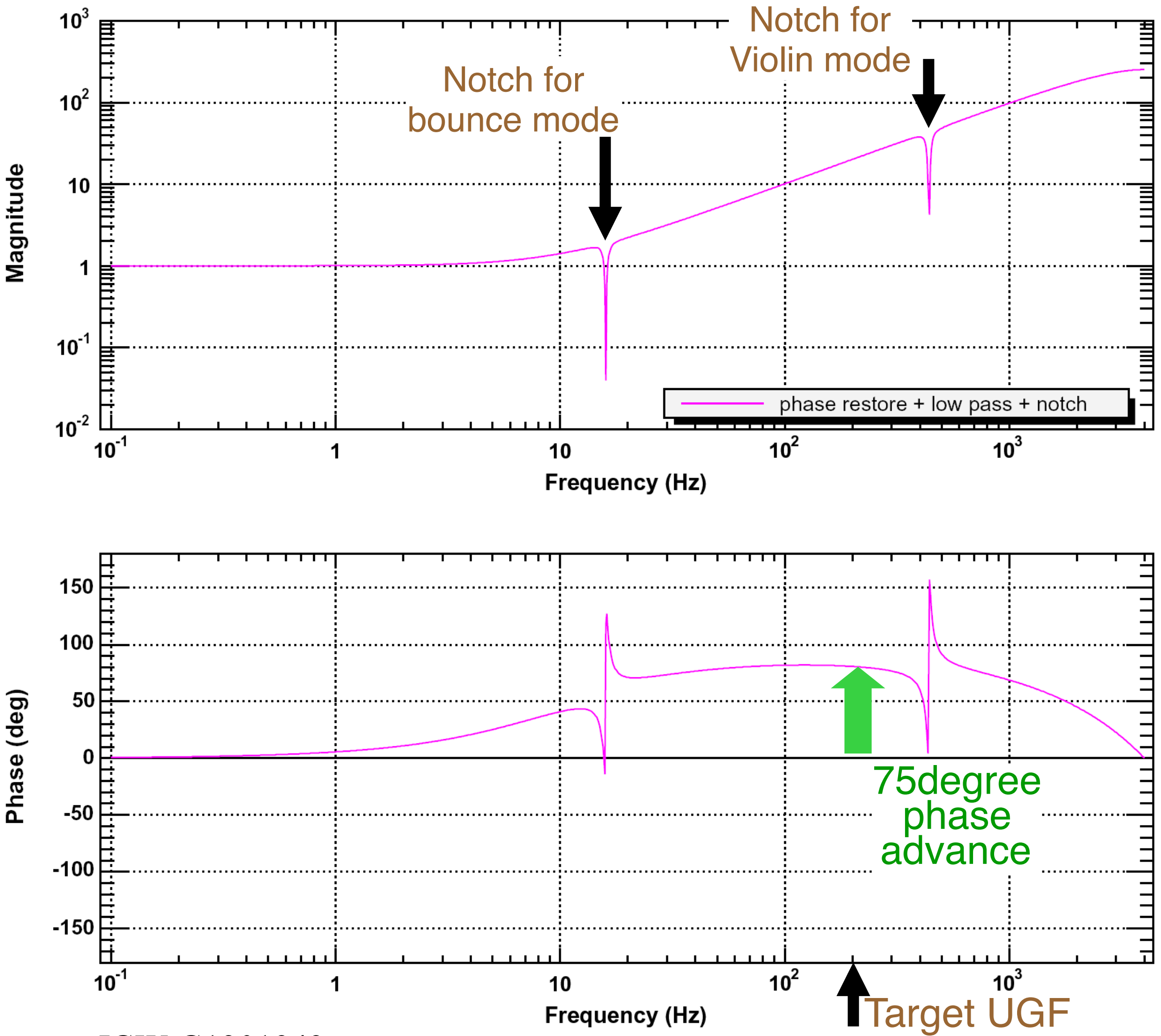




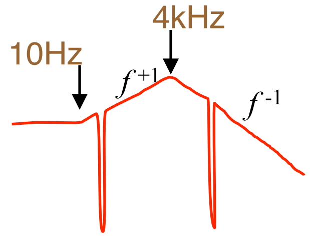
$$\frac{N}{(1+G)} = \frac{10^{-5}\,m}{(1+G)} < 10^{-9}\,m@1Hz$$

$$\therefore G > 10^{4}$$

Example of Feedback filter

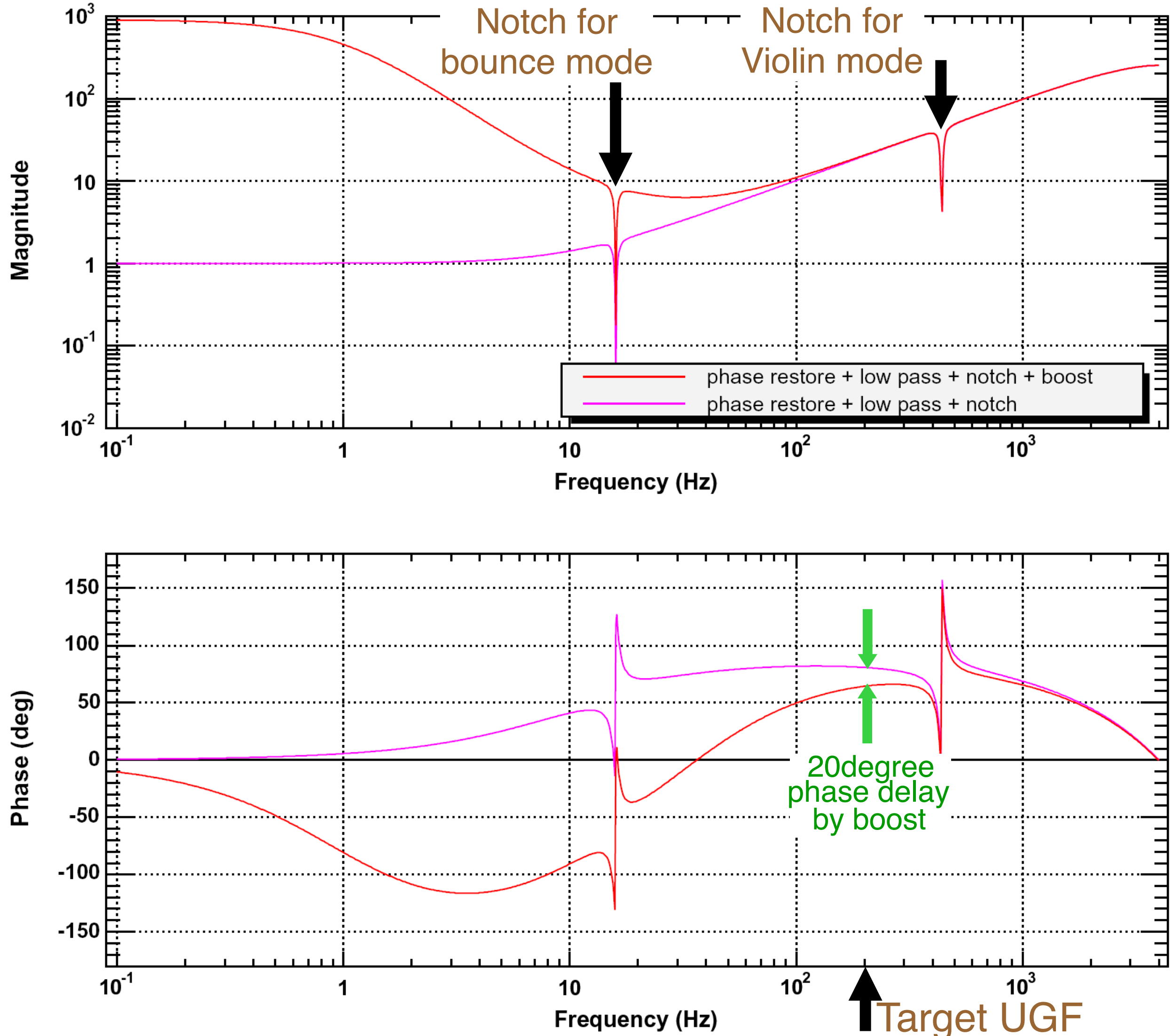


F : Phase Restore + Low-pass + notch

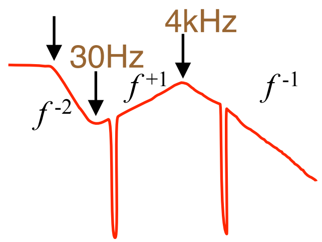


zero@10Hz
pole@4kHz,4kHz
notch@16Hz, 440Hz

Example of Boost

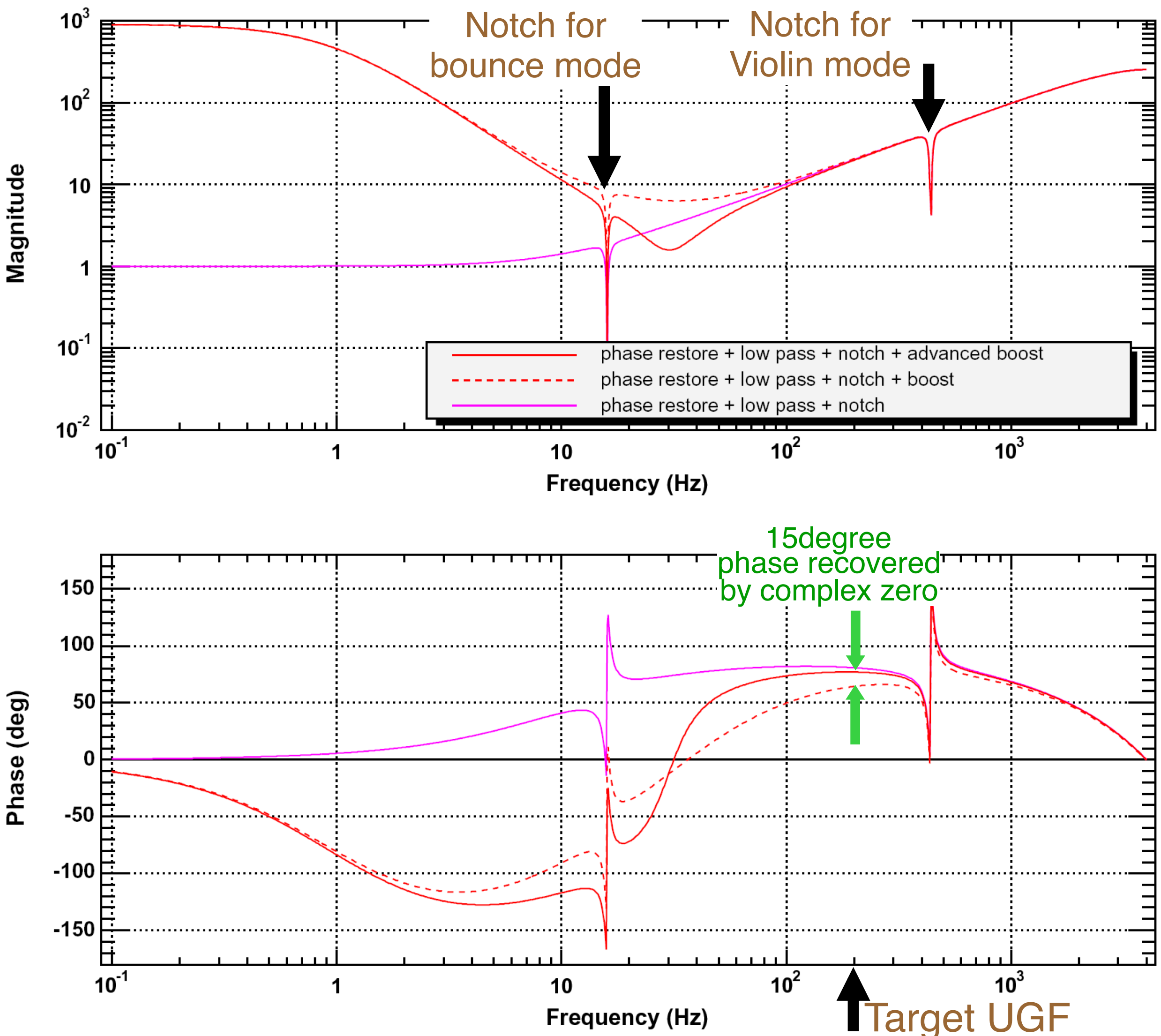


F : Phase Restore + Boost

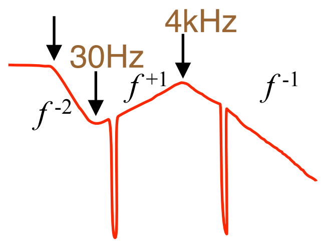


zero@10Hz,30Hz,30Hz
pole@1Hz,1Hz,4kHz,4kHz

Advanced Boost using complex pole/zero



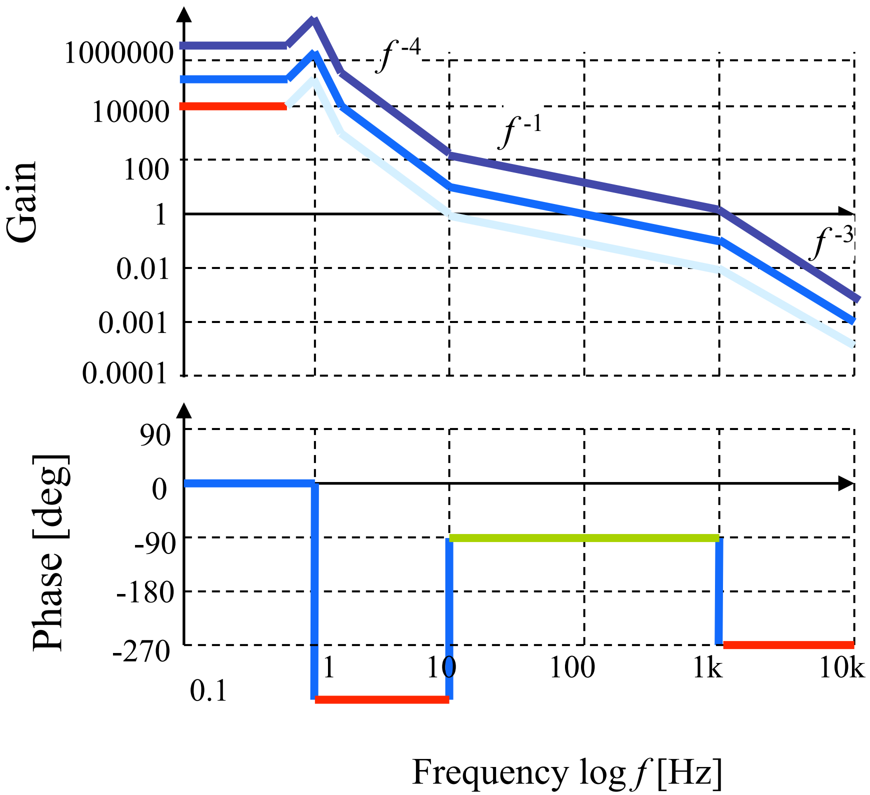
F : Phase Restore + Boost



zero@10Hz
complex zero@30Hz
pole@1Hz,1Hz,4kHz,4kHz

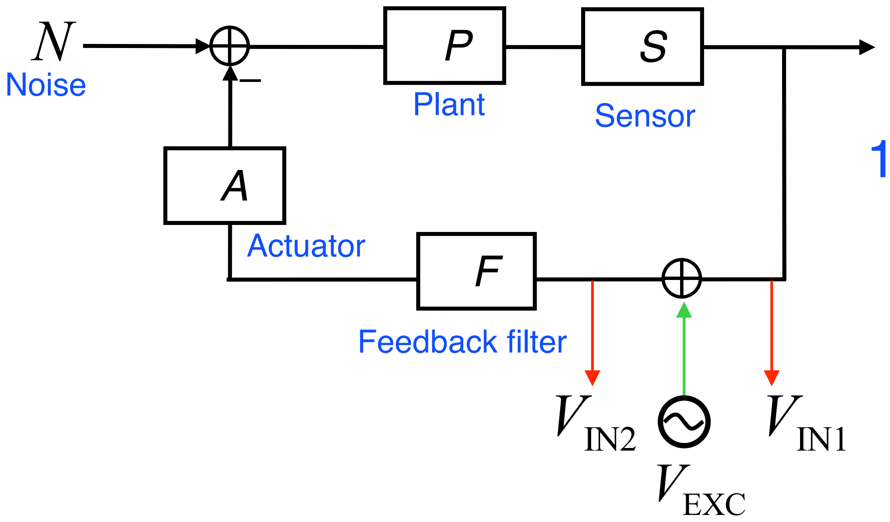
How to adjust feedback gain?

High gain: limited by phase delay due to slow circuit, DAC/ADC time delay etc., or by feedback noise, or by saturation
Low gain : limited by phase delay of boost, or by too low DC gain.



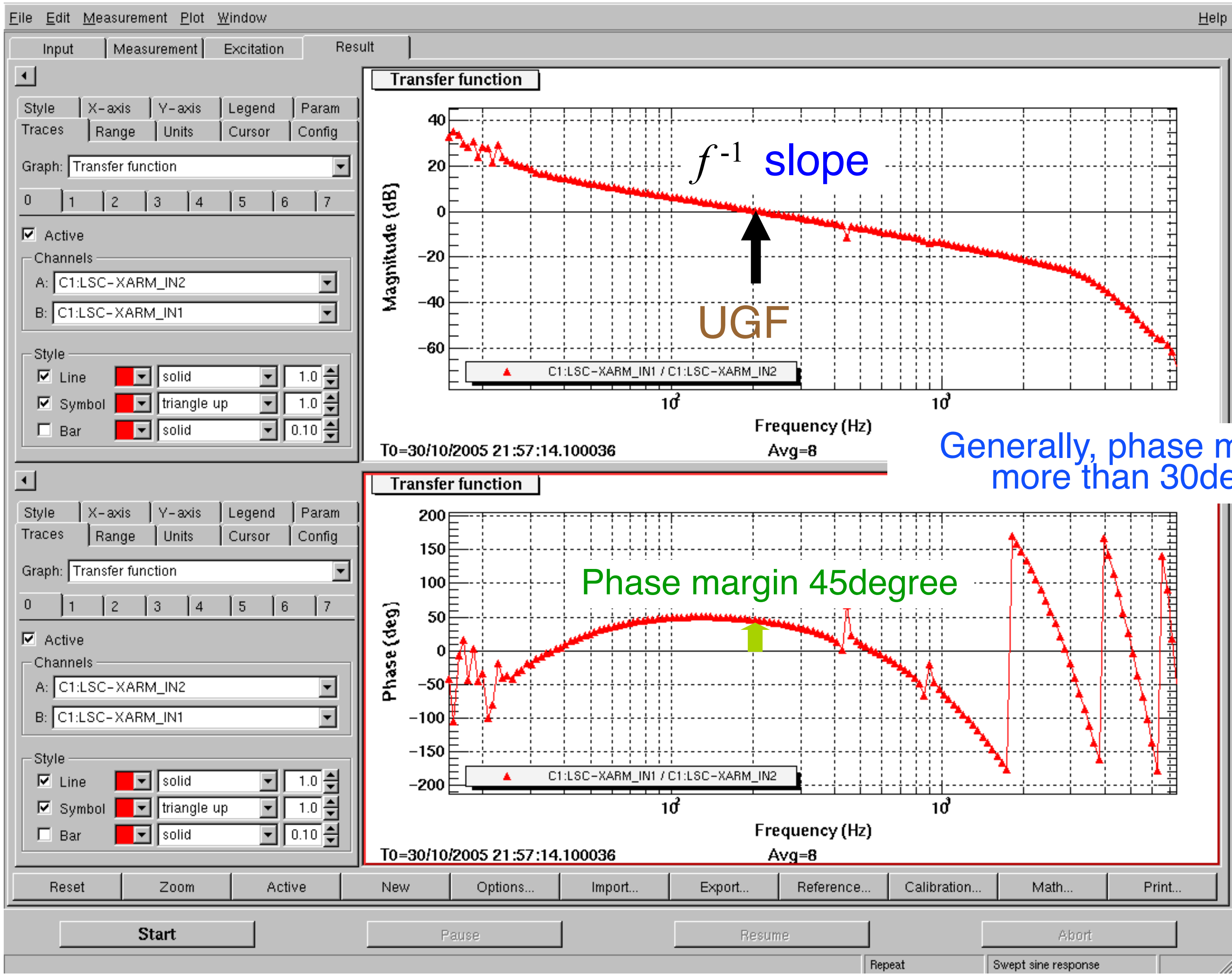
Once you get stable operation, it is very important to measure open loop TF to see how stable!

How to measure Open Loop TF?



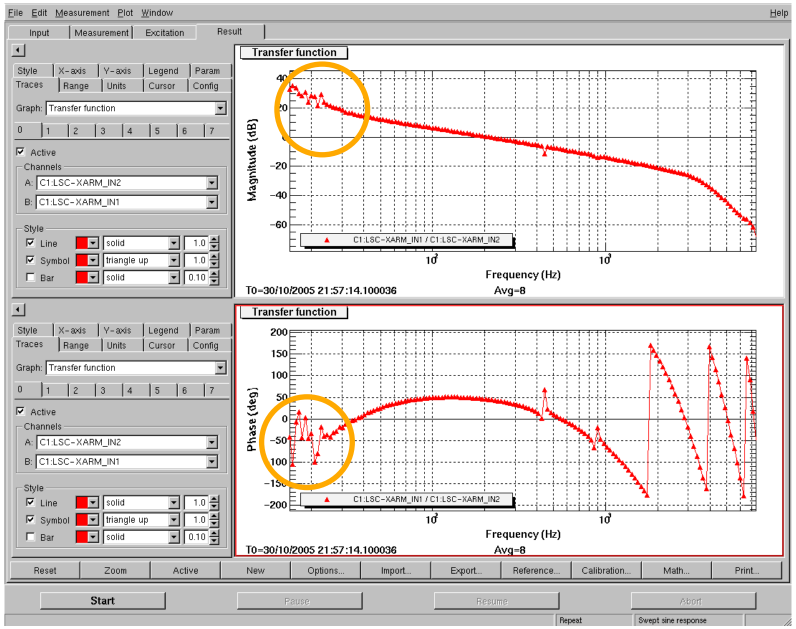
1. Use closed transfer function: $C \equiv \frac{1}{(1+G)}$
   - Measure $\frac{V_{\mathrm{IN2}}}{V_{\mathrm{EXC}}} = \frac{1}{(1+G)}$
   - Calculate G $G = \frac{1}{C} - 1$
2. Measure OLTF directly
   - Measure $\frac{V_{\mathrm{IN1}}}{V_{\mathrm{IN2}}} = FAPS = G$

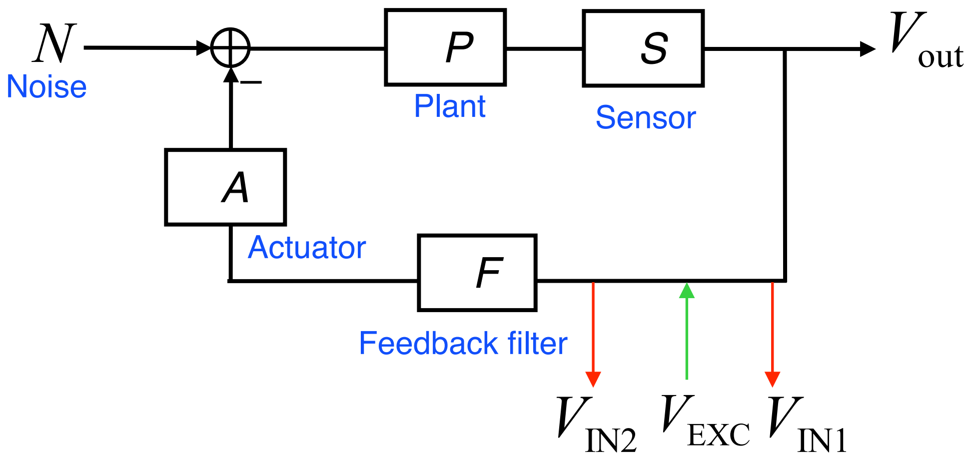
Example of measured Open loop TF



Generally, phase margin should be more than 30degree at least.

Why low frequency measurement is dirty?



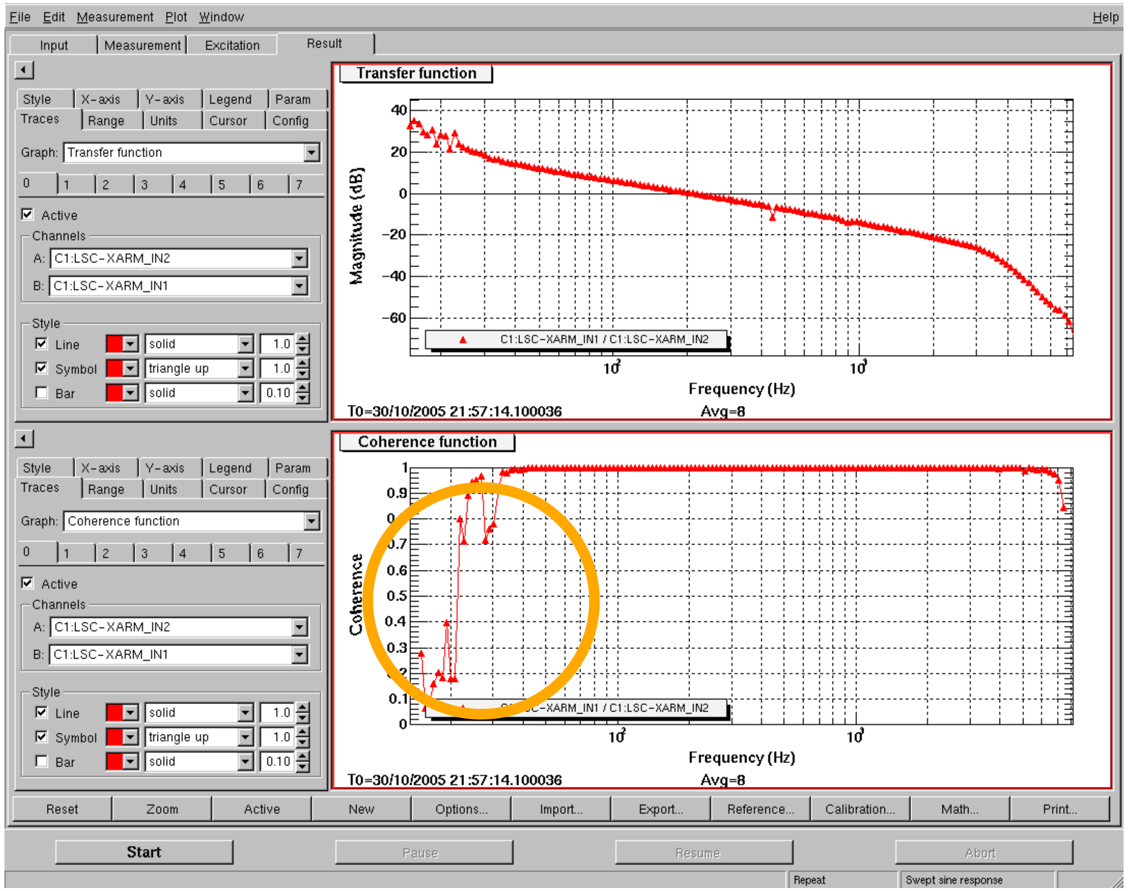


- Excitation signal is suppressed a lot with very high gain *G*>>1 at low frequency.

$$V_{\mathrm{IN2}} = \frac{1}{(1+G)} V_{\mathrm{EXC}} << \mathrm{Noise}$$

LIGO

What is Coherence?



- Coherence: coh(f)

  How much related between input and output

$$\mathrm{coh}(h) = \frac{|W_{xy}(f)|}{\sqrt{W_{xx}(f)\cdot W_{yy}(f)}}$$

$W_{xy}(f)$: cross spectrum of x and y

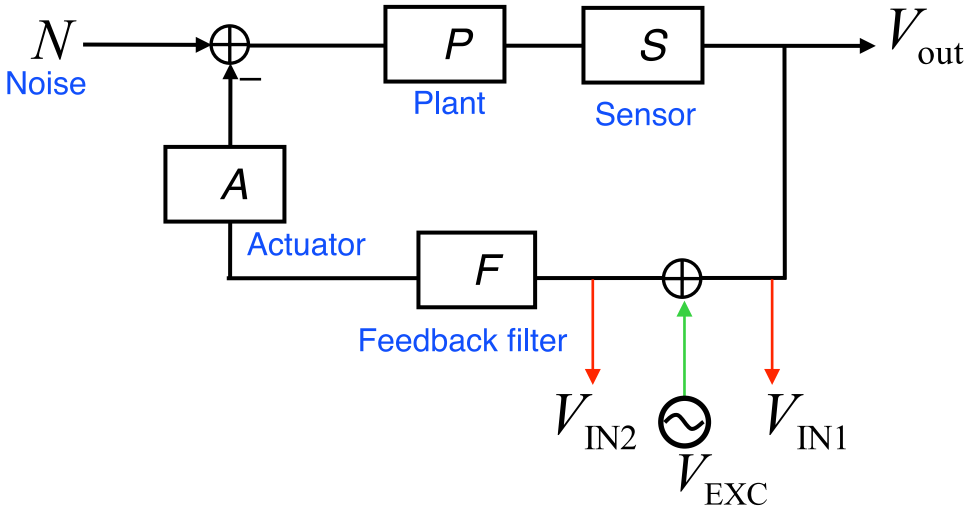
$W_{xx}(f), W_{yy}(f)$: power spectrum of x and y

- $0 < \mathrm{coh}(f) < 1$

- Coherence is sometimes convenient to estimate whether the measurement is reliable.

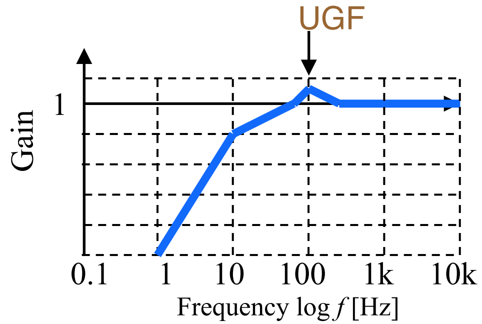
- Generally, If coherence is smaller than 0.8 the measurement is not good.

What does Closed Loop TF mean?

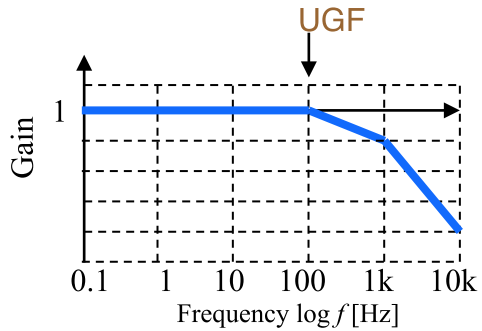


There are two definitions for closed loop TF

$$C_1 \equiv \frac{1}{(1+G)} \qquad C_2 \equiv \frac{G}{(1+G)}$$

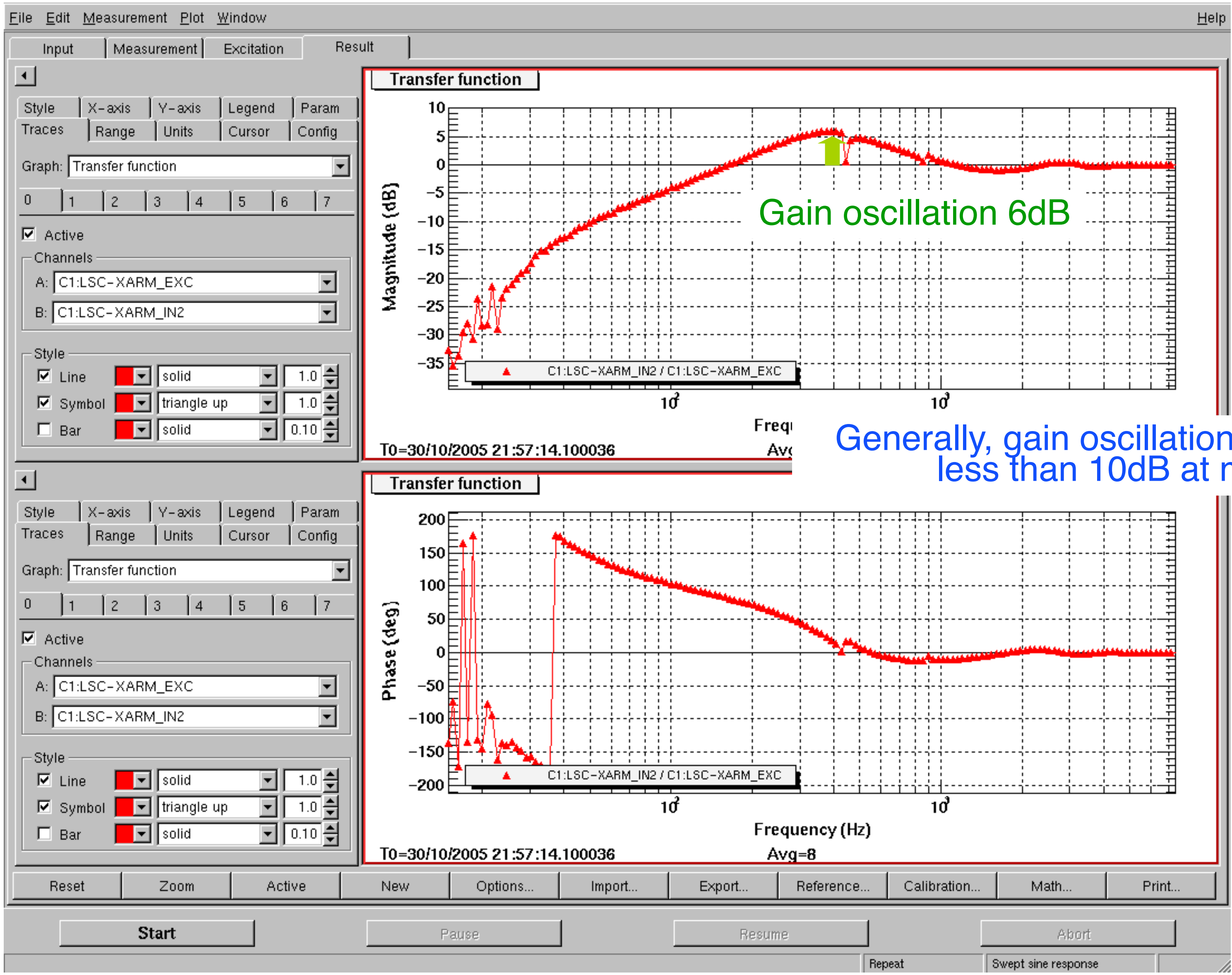


1. C_1 :used to estimate gain oscillation
   - Measure $\dfrac{V_{\mathrm{IN2}}}{V_{\mathrm{EXC}}} = \dfrac{1}{(1+G)}$
   - Gain oscillation is caused by small phase margin at UGF.



2. C_2 :
   - Measure $\dfrac{V_{\mathrm{IN1}}}{V_{\mathrm{EXC}}} = \dfrac{G}{(1+G)}$

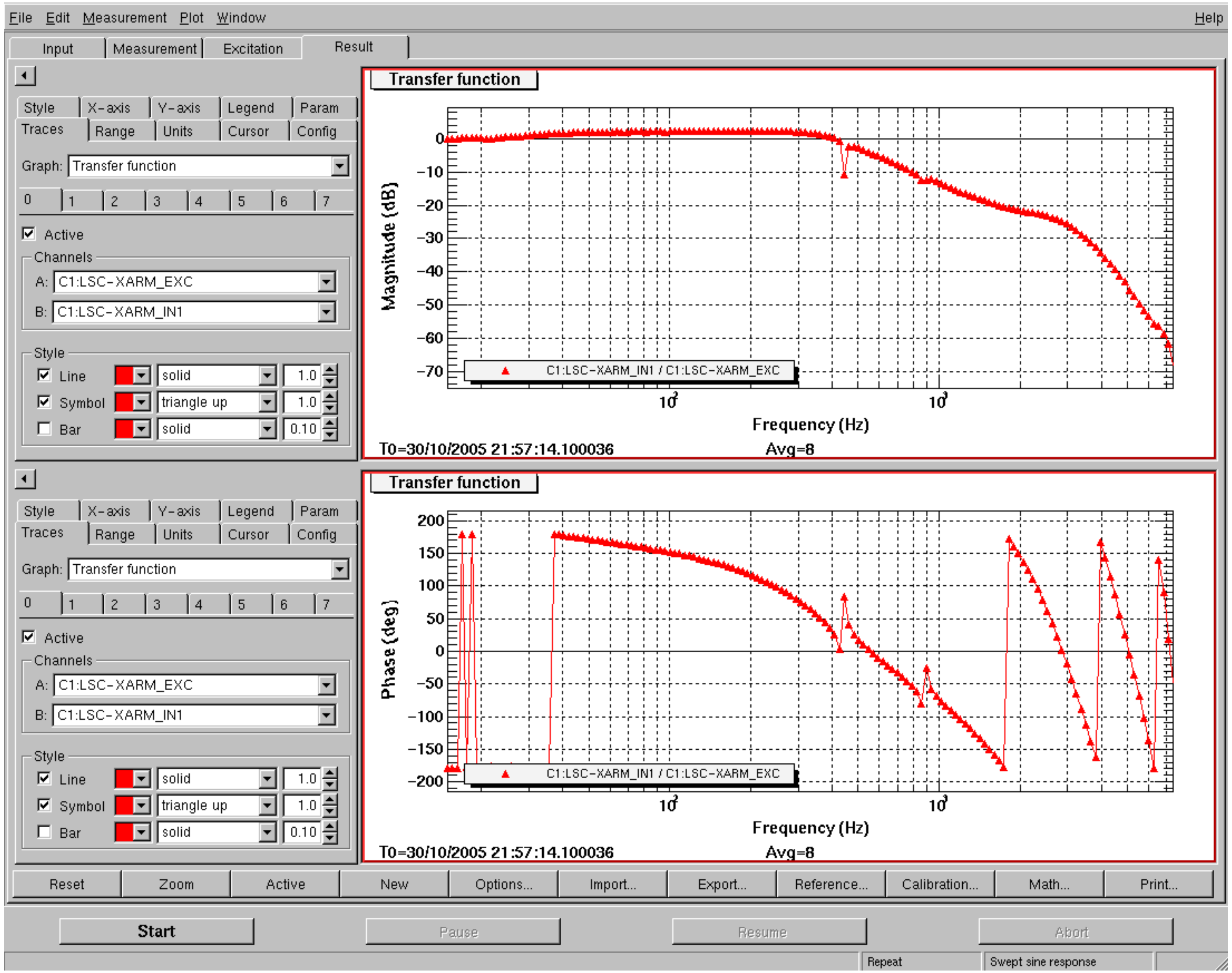
Example of measured Closed loop TF



Gain oscillation 6dB

Generally, gain oscillation should be less than 10dB at most.

Example of measured Closed loop TF



Summary of designing feedback filters

1. Look at target system carefully and estimate each transfer function even if you cannot measure it directly.
2. Design a feedback filter by putting pole and zero to have f^{-1} slope around UGF.
3. Close loop with a designed feedback filter
4. Adjust gain of feedback filter if system is not stable.
5. If system is stable, measure open loop transfer function in order to see how stable (don't forget to see closed loop TF and coherence function also).
6. Put boost, notch, low-pass filters if you need.