Thermal noise workshop 2013 report

2013.10.9

D.Heinart, J.Kato, S.Konisho, A.Kumeta,

M.Nakano, K.Ono, and K.Somiya

Daniel Heinert

Thermal noise workshop 2013 report

(1) Lectures on gravitational waves and suspension (G. Hammond)

The basic lectures on the field of gravitational waves and their detection gave a nice overview on how the different research topics are connected in the direction of a direct detection of gravitational waves. I realized many links within different topics of research. But the lecture also raised some questions to me.

- The multipole expansion for GW gives a quadrupole moment in lowest order. This is connected to a leading potential term of the form 1/r^3. But the amplitude of a gravitational wave scales with the inverse distance from the source 1/r (see e.g. book of Saulson).
- For bar detectors the question arose how to apply a direct noise calculation scheme following Levin to their geometry. It would also be interesting to find former publications on this field.
- Calculation of the suspension noise: Using the direct approach by Levin, Giles showed the calculating of dissipation in the fibres. There the fibre was cut into small sections with constant diameter. Each section contained an own thermoelastic loss parameter due to the cancellation proposed by Cagnoli and Willems. Further due to the displacement every section shows a different total elastic energy. Finally the total loss was obtained by dividing with a constant dilution factor. Should not this dilution factor also depend from the position in the fibre?

(2) Fluctuations in non-equilibrium steady state (Y. Shikano)

As in a cryogenic interferometer the test mass is thermally connected to the heat sink mainly via the suspension a temperature gradient will emerge along these fibres. This stationary situation does not represent an equilibrium state any more. Thus one is especially interested in how this change influences the thermal noise level of the suspension in cryogenic detectors and if and how Levin's direct approach has to be modified.

Yusuke's talk addressed the fundamentals on the fluctuations of the non-equilibrium steady state (NESS) systems. There I learned a lot about the Fokker-Planck equation as a helpful tool in statistical physics. Especially interesting is its analogy to the traditional Langevin equation. But it seems that the Fokker-Planck equation is suited to obtain the fluctuation-dissipation-theorem in the case of NESS. Also the discussion in terms of a probability flow helped me to get deeper into this topic.

(3) Thermo-optic noise by Evans et al.

In a discussion with Kentaro we found a nice explanation of how to apply virtual loads to a multilayer coating on top of a substrate. In terms of thermoelastic noise a distributed virtual force should be applied to any interface in the layer stack (probing not only the mirror surface but also the individual layer thicknesses) as well as to the substrate. But in the work of Evans et al. only a single force is applied to the top of the layer stack. Instead an additional entropy is applied to the individual coating layers. This effect cares for the thickness change of each layer and is a good approximation for thin coatings.

Thermal noise workshop report

Jumpei Kato

I got interested in fluctuation in a non-equilibrium steady state (NESS) whose lecture was given by Yutaka. Yutaka told us the fundamental knowledge of the fluctuation in an equilibrium state and in a NESS. Masayuki and I summarized Yutaka's talk on the last day of the workshop. After the workshop, I followed the Yutaka's calculation again. Then, I noticed mistakes of my talk. In this report, I will summarize the fluctuation theory again, and also correct the mistakes.

When we consider the dynamics with damping a and random force in a long-time scale, the most fundamental equation is the Langevin one.

$$\dot{X}(t) = F(X(t)) + \xi, \tag{1}$$

where $X(t)$ is a generalized position, $F(x)$ is a generalized force, and ξ is random force. When the system is in equilibrium, the Langevin equation is equivalent to the Fokker-Plank(FK) equation. The FK equation describes a probability distribution of the system.

To prove the equivalence, we need four assumptions.

1. $\xi(t)$ is white noise. That is, $\xi(t)$ is
$$\langle \xi(t) \rangle = 0, \tag{2}$$
$$\langle \xi(t)\xi(t') \rangle = D\delta(t-t'). \tag{3}$$
D is some quantity like the variance.
2. $\xi(t)$ and $X(t)$ are statistically independent. Namely, they do not have a correlation.
$$\langle \xi(t)X(t') \rangle = 0, \quad (\text{for } \forall t > t'). \tag{4}$$
3. $P(x,t)$ has a non-trivial solution. For this assumption, p needs to be 0 at the boundaries. Yutaka did not mention it but Masayuki and I did.
4. No probability flow at the boundaries. We define the probability flow $J(x,t)$ as
$$\frac{\partial J(x,t)}{\partial x} := \frac{\partial P(x,t)}{\partial t}. \tag{5}$$
At $x \to \pm\infty$, $J(x,t)$ should vanishes.

A system in an equilibrium state satisfies these assumptions. After calculation using the FK equation, we find

$$F = \frac{D}{2}\frac{\partial S(x)}{\partial x} \tag{6}$$

where $S(x) = \log P(x)$. Since we consider the equilibrium state, i.e. $t \to \infty$, P and S are independent of time. We can determine the probability distribution P from (6). However, as Yutaka said, P, and thus S, should be obtained from the result of statistical mechanics. Therefore, we should not consider (6) as the equation to determine S. This equation should be interpreted as the one to obtain D.

We are interested in the auto-correlation function of X i.e. $\langle X(t)X(t+\tau)\rangle$. $\langle X(t)X(t+\tau)\rangle$ corresponds to $\langle \dot{X}(t)\dot{X}(t+\tau)\rangle$ in the Fourier domain except for the factor $(1/2\pi f)^2$. Thus, we start from the Langevin equation.

$$\begin{aligned}\langle \dot{X}(t)\dot{X}(t+\tau)\rangle &= \langle (F(t)+\xi(t))\,(F(t+\tau)+\xi(t+\tau))\rangle \\ &= \langle F(t)F(t+\tau)\rangle + \langle F(t)\xi(t+\tau)\rangle + \langle F(t+\tau)\xi(t)\rangle + \langle \xi(t)\xi(t+\tau)\rangle \\ &= \langle F(t)F(t+\tau)\rangle + D\delta(\tau). \end{aligned} \tag{7}$$

The second and third terms vanish because F and ξ are statistically independent. Moreover, Yutaka said that the first term vanished but I forget the reason and I fail to prove that. When the first term is absent, we obtain FDT in the time domain as

$$\langle \dot{X}(t)\dot{X}(t+\tau)\rangle = \left\langle \frac{2F}{\partial S/\partial x} \right\rangle \delta(\tau). \tag{8}$$

Especially when we assume the Boltzmann system with damping, that is, $F = -m\gamma V$ and $S(v) = -mv^2/2k_bT$ where V is the average of v, we find the familiar FDT in the frequency domain,

$$S_X(f) = \frac{1}{(2\pi f)^2} 4\gamma k_b T. \tag{9}$$

Notice that this S_X is one sided power spectrum of X.

Let us consider the system in a NESS. What should be modified? In the NESS case, the FP equation is invalid because the assumption 3 and the assumption 4 will break. Therefore, D cannot be determined. However, the Langevin equation is still valid so that (7) holds in the NESS. I think $\langle F(t)F(t+\tau)\rangle$ still vanishes because F does not change from the equilibrium case but I am not sure. I hope someone can prove it for me. I said that $\langle F(t)F(t+\tau)\rangle$ would be modified in the talk but I could have been wrong. In the NESS, we lose the method to obtain D. The conclusion is worse than I said before.

For future work, I need to check the proof of equivalence of the Langevin equation and the FK equation. I could find another equation to determine D.

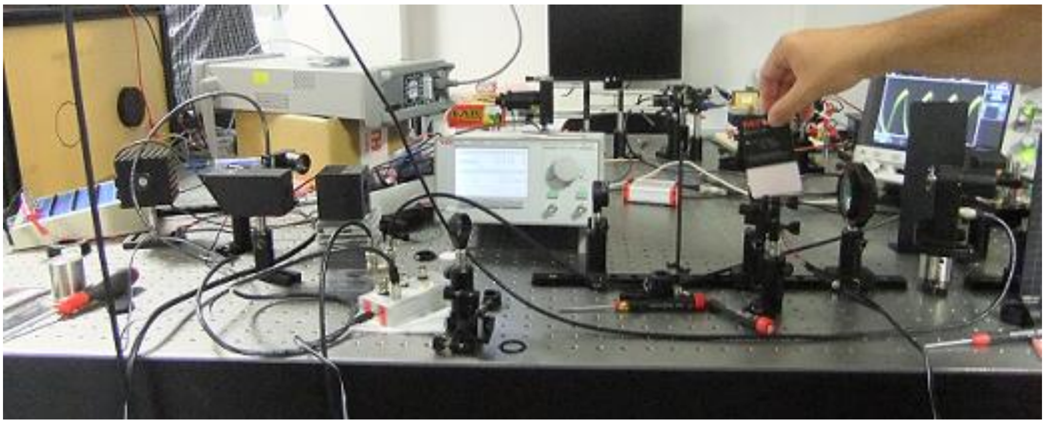
Thermal noise workshop 2013 report

Shiori Konisho

○Measuring the optical absorption: Jerome Degallaix

In the lecture, we learned how to measure the optical absorption. With two lasers the measuring point was chosen. The pump laser was locked by a chopper.

I was very interested in the lecture since it was the first time for me to learn about measuring the optical absorption. And it was a good opportunity for me to see Jerome’s experiment.



○Paper review

We read some papers as homework. And Daniel gave us a lecture about the kinds of the thermal noise. Finally we draw noise spectrum density of thermal noises.

Before the class, I didn’t catch the difference between thermo-elastic noise and thermo-refractive noise. But he told us the meanings with some pictures so I could image well. And I made a presentation of a paper. It was a nice experience for me to speak in English.

○Experiment

I and Ms. Kumeta joined with the group in which they measured the temperature distribution. And we learned how to measure the loss of substrates.

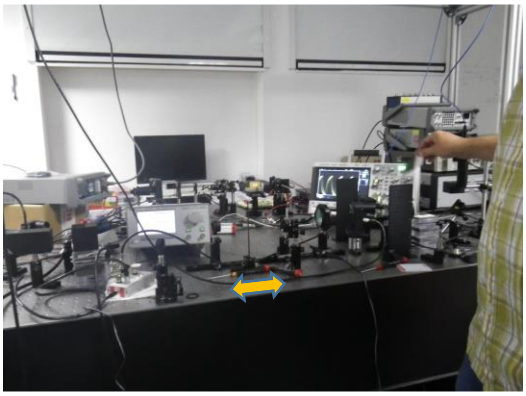
I was so interested in the difference between Jerome’s method and group’s one. In the group’s one, they used only one laser. I wish we had more time to do the experiment.

Thermal noise workshop 2013 report

Ayaka Kumeta

Before this workshop, I did not know the thermal noise at all. I had a very valuable experience.

1. Measuring the optical absorption

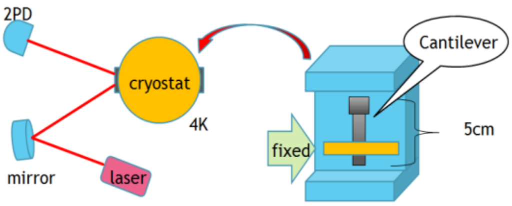


In this lecture, I learned many things. For example, what the optical absorption is, why we should take care of it, how to estimate it, and how to measure it. In particular, the method to measure the light absorption for each point of the coating by two beams was impressive. Jerome also showed me the experiment and that was very educational for me.

2. COMSOL

Simulation software "COMSOL" can calculate the temperature distribution, the resonance frequency and the loss angle of an optical component. COMSOL uses the finite element method (FEM). Not having COMSOL in Japan, I would like to do similar simulation using ANSYS.

3. Short lab work

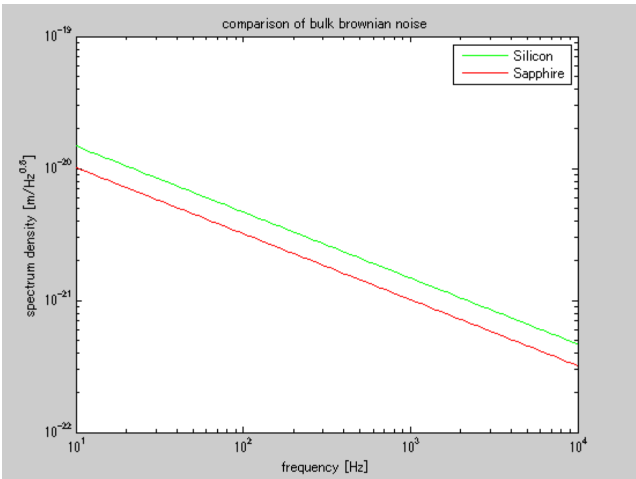


I participated in an experiment of the measurement of loss angle. In this experiment, we measured the ring-down time of a cantilever. When there is a loss, the amplitude decreases gently. This experiment was very interesting for me because it was my first time to do a cryogenic experiment. In Japan, I use a breadboard for OMC experiment. It may be necessary to do the loss measurement of the breadboard with the experiment like this or the simulation using the FEM.

4. Application to Silicon / Sapphire

After Daniel's lecture of thermal noise, we were divided into some groups and calculated the spectral density of noise. Rebecca and Rene and I calculated bulk Brownian noise of silicon and then that of sapphire to compare the noise levels.

Then Rene taught me how to use MATLAB. After that we compared our result with the results of the other teams (Bulk thermo-elastic, Coating Brownian, etc.).



bulk Brownian noise

$$S_x(f) = \frac{4k_B T}{f}\frac{1-\sigma^2}{\pi^3 E_0 r_0} I\phi\left[1 + O\left(\frac{r_0}{R}\right)\right].$$

Thermal noise workshop 2013 report

Masayuki Nakano
masayuki@icrr.u-tokyo.ac.jp

October 7, 2013

1 Thermal fluctuation in non-equilibrium steady state

We had the lecture of thermal fluctuation in non-equilibrium steady state by Yutaka Shikano. I will summarize his lecture and mention about what Jumpei and I explained at the last of workshop. And also I found some problem in our explanation, so I will mention about that. In below discussion, I possibly made a mistakes, so if you find any problem, please let me know.

The lecture started with the Langevin equation.

$$\dot{X} = F(X(t)) + \xi(t) \tag{1.1}$$

where X is displacement, velocity, or any parameter of the particle, and ξ is the fluctuation force . If you write $F(x)$ using the potential of the system $S(x)$, we can write like

$$F(x(t)) = -L\frac{\partial S}{\partial x} \tag{1.2}$$

We can calculate the response of $\dot{X}$ for the excitation force

$$\begin{aligned}
\frac{\delta \dot{X}}{\delta f_{exc}} &= \frac{\delta F(x)}{\delta f_{exc}} \\
&= L\left(\delta\left(\frac{\partial S(x)}{\partial x}\right)/\delta f_{exv}\right) \\
&= L
\end{aligned} \tag{1.3}$$

There is another important equation in our lecture. That is Fokker-Plank eq.

$$\frac{\partial P(x,t)}{\partial t} = \left(-\frac{\partial F(x)}{\partial x} + \frac{D}{2}\frac{\partial^2}{\partial x^2}\right)P(x,t) \tag{1.4}$$

If we assume $< \xi(t)\xi(t+\tau) >= D\delta(\tau)$, and the Fokker-Plank eq and Langevin eq(1.1) are equivalent, we can get the relationship between $F(x)$ and $S(x) = -\log P(x)$, where P(x) is the probability distribution,

$$F(x) = \frac{D}{2}\frac{\partial S(x)}{\partial x} \tag{1.5}$$

So in this case, $D/2 = L$. Also we have the relation between the dissipation D and the self correlation function of $\dot{X}$ like

$$< \dot{X}(t)\dot{X}(t+\tau) >=< F(t)F(t+\tau) > +D\delta(\tau). \tag{1.6}$$

if $\tau = 0$

$$< \dot{X}(t)^2 >=< F(t)^2 > +D. \tag{1.7}$$

We can get the self correlation function of $\dot{X}$ from that response,

$$\begin{aligned}
C(\tau) &=< \dot{X}(t)\dot{X}(t+\tau) > \\
&=< F(t)F(t+\tau) > +D \qquad (1.8)\\
&=< F(t)F(t+\tau) > +2R \qquad (1.9)
\end{aligned}$$

where R is the response of $\dot{X}$.

In Yutaka's lecture , he assumed the Fokker-Plank eq(1.4) and Langevin eq(1.1) are equivalent in equilibrium state and also non-equilibrium steady state. And because $< F(t)F(t+\tau) >$ is 0 in equilibrium state and is not 0 in non-equilibrium steady state, the fluctuation of $\dot{X}$ in non-equilibrium steady state become larger than in equilibrium state by $< F(t)F(t+\tau) >$. If my understanding is correct, this is the conclusion of his lecture.

But as Jumpei and I explained at last of the workshop there was the problem. For the equivalence of two equation, we need the boundary condition,

$$P(\pm\infty) = 0 \tag{1.10}$$

$$\frac{\partial P}{\partial x}(\pm\infty) = 0 \tag{1.11}$$

But in non-equilibrium steady state, this boundary condition cannot be satisfied always. In this case we cannot use the Fokker Plank eq. So in non-equilibrium steady state we cannot use equation(1.8) and we have to use the equation(1.7). Although if you compare two state in equation(1.7) the extra fluctuation in non-equilibrium steady state will come from $< F(t)F(t+\tau) >$. This is the point which Jumpei and I mentioned.

But I found this discussion has two wrong points. First one is that we should consider about the difference of response between two states. The system is different between two system, so the response of system is maybe different.

And also there is the problem in our discussion about $< F(t)F(t+\tau) >$. How we can say that this term is zero in equilibrium state? Yutaka explained, we should think about the space dependence somehow, so we should take ensemble average. And then the first term should go to zero. (Also we can see what he wrote on black board in this discussion in the Ronny's photo MG_0538). But he didn't explain why the first term go to zero if you take an ensemble average (or I just didn't understand or miss the point). I calculate this term again and my conclusion is that this term is not always zero.

You can easily calculate in the case of the Brownian particle. As Yutaka's lecture in case of Brownian particle $F(v) = -(\gamma/m)v$. So $< F(t)^2 >$ is calculated like,

$$< F(t)^2 > = \frac{\gamma^2}{m^2} < v(t)^2 > \tag{1.12}$$

And you know $< v(t)^2 >$ cannot be zero obviously.

Also I calculated $< F(t)^2 >$ by taking ensemble average. For that calculation, first of all we should think about what does "$< F(t)^2 >$" means. Actually F is the function of x and not the function of time in usual case. But if we consider about the particle, and that position (or velocity) is the function of time, then F become the function of time. So it's better to write this term down like $F(x(t))$.

Now we can consider about "time average". The time average of $F(x(t))^2$ for one particle should be same as the ensemble average of F, (if ergodicity is satisfied. And the particle has the probability distribution $P(x)$, so the ensemble average should be calculated like,

$$\overline{F(x)^2} = \int_{-\infty}^{\infty} F(x)^2 P(x) dx \tag{1.13}$$

I calculated with this equation and got same result as from eq(1.11). I put about detail of calculation in appendix.

Summarizing above discussion, I think when we think about the difference of thermal noise between two states, we have to consider two effects. One is the difference of the response of the system for external force. And another thing is the difference of $< F(t)^2 >$. I don't get anything about these differences yet. So next step is to estimate these differences somehow.

2 Appendix

The equation of motion of Browning particle is

$$m\ddot{x} = -\gamma x + \xi(t) \tag{2.1}$$

and from this EoM, you can get the Langevin eq for velocity.

$$\dot{v} = -\frac{\gamma}{m}v + \frac{1}{m}\xi \tag{2.2}$$

from eq(2.2), the spectrum of v is

$$\dot{v}(\omega) = \frac{1}{i\omega + (\gamma/m)}\xi(\omega)^2 \tag{2.3}$$

we assume ξ is white noise and the power is D/m^2, so the power spectrum density of v is

$$|v(\omega)|^2 = \frac{1}{\omega^2 + (\gamma/m)^2}\frac{D}{m^2} \tag{2.4}$$

Using Percival theorem we can get the time average of v^2

$$\begin{aligned} < v^2 > &= \frac{1}{2\pi}\int_{-\infty}^{\infty} |v(\omega)|^2 d\omega \\ &= \frac{D}{2m\gamma} \end{aligned} \tag{2.5}$$

From eq(1.11) we get $< F(t)^2 >$

$$< F(t)^2 >= \frac{D\gamma}{2m^3} \tag{2.6}$$

And next I will calculate from eq(1.12). The Brownian particle have the velocity distribution

$$P(v) = \sqrt{\frac{m}{2\pi k_B T}} e^{-\frac{mv^2}{2k_B T}} \tag{2.7}$$

So we can calculate $< F(t)^2 >$ like

$$\begin{aligned} < F(t)^2 > &= \overline{F(x)^2} \\ &= \int_{-\infty}^{\infty} F(x)^2 P(x) dx \\ &= \int_{-\infty}^{\infty} (-\frac{\gamma}{m}v)^2 \sqrt{\frac{m}{2\pi k_B T}} e^{-\frac{mv^2}{2k_B T}} dx \\ &= \frac{\gamma^2 k_B T}{m^3} \end{aligned} \tag{2.8}$$

As Yutaka7s lecture $D = 2k_B T\gamma$. So the result of two calculation are same.

Kenji Ono

THERMAL NOISE WORKSHOP 2013 REPORT

FEA(Comsol):Daniel and short lab work

In this lecture, I learned Finite Element Analysis(FEA); overview of the theory and methods of analysis. In the analysis part, we used FEA software,named Comsol.

In my experiment in Japan, I often evaluate some transfer functions and thermal noise in a prototype KAGRA-SAS system. However, I didn't have a calculation method in order to evaluate loss angle in this system. So It's a very useful method for me to estimate thermal noise.

I wanted to know and get further experience to use Comsol, so I chose a simulation course in short lab work. In this work, I constructed a sapphire mirror model using Comsol and evaluated the elastic energy in my model. In this short lab work, Daniel helped me for many things. For example, he taught me how to get a good result using a sectioned mesh. I would like to thank Daniel.

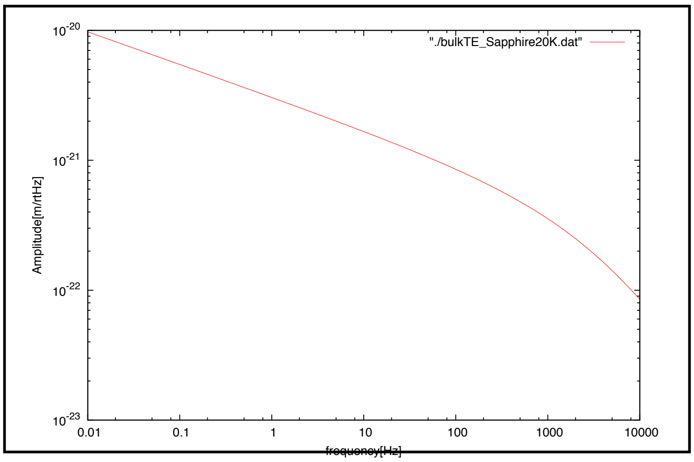
Application to Silicon/Sapphire

Amang 5 way to evaluate bulk/coating thermal noise, We evaluated LIGO and KAGRA mirror thermal noise in this section. I evaluated Bulk-thermoelastic noise with Ronny.

I used Mathematica to evaluate this thermal noise. However, my computer program had some mistakes, so I did not get correct data at first.

When I could not find my mistakes, Ronny suggested me many good methods to find where the error is. And he taught me some ways to get data efficientry

Thanks to his good advice, I god a correct data like the right figure, and learned how to make a computer program more efficient. I wish to thank Ronny very muth.



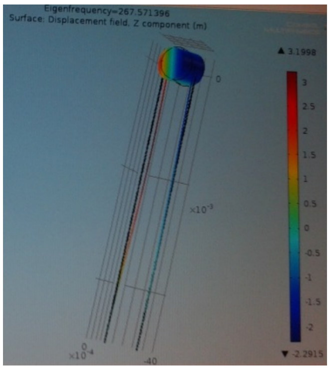
Thermal noise workshop 2013 report

Kentaro Somiya

(1) COMSOL tutorial

Daniel's tutorial was helpful for us to start using such a complicated program. In addition to the tutorial, Stuart gave us a short lecture for ANSYS, which is also helpful for us in Tokyo Tech where we have a licence only for ANSYS.

Here I leave some notes for what I found in the COMSOL calculation for the mechanical resonances of a suspension system. First, it was needed to put rectangular ear to connect a cylindrical mirror and a cylindrical fiber of different materials. Overlapping the volume can cause a trouble defining its properties. I did not check if but it might be ok if we define all the segments first and then change the unoverlapping parts later. Second, I could not find a way to add gravity for a pendulum. There are functions to add mass or load, but the one found are for a static deformation, not for the gravitational restoring force. There should be a way though.



(2) Coating thermoelastic noise in 20K

I was in the group of thermo-refractive noise in the noise analysis challenge, but I found it interesting to reconsider coating TE noise of a cryogenic mirror. Let me leave some notes of the thinking. See the figure below. The left panel represents substrate TE noise and the right panel represents coating TE noise. The temperature fluctuations at A and B can be averaged only if their distance is smaller than the beam radius. The fluctuations at A and C can be also averaged for substrate TE noise in the distance smaller than the beam radius, but they will not so much averaged out for coating TE noise as the thermal properties are different in different coating layers. It can be explained better with Levin's method. Expansion due to a generalized force causes different temperature in different coating materials to generate dissipations. Let us change the temperature to 20K. For the substrate, as is reported by Cerdonio, the heat flow is so fast that the temperature difference is small while the deformation is large for the high thermal expansion. For the coatings, the difference in the heat flow has little to do with the translational fluctuation but with the transvertial fluctuation. I will check it out.