Maximum force and noise of actuators for LCGT

Kazuhiro Yamamoto
Institute for Cosmic Ray Research, the University of Tokyo,
5-1-5 Kashiwa-no-Ha, Kashiwa, Chiba 277-8582, Japan

June 1, 2011

Abstract

1 Introduction

LCGT (Large-scale Cryogenic Gravitational wave Telescope) is one of the second generation projects of interferometeric gravitational wave detectors.

maximum force and noise of actuators

2 Assumptions

In this article, some assumptions are adopted.

- All four mirrors of the Fabry-Perot cavity have actuators.
- Each mirror has four actuators ($n = 4$). The efficiency of each actuator (α) is same.
- Power spectral density of seismic motion is written as

$$\sqrt{G_{\rm seis}} = \frac{10^{-9}}{f^2}[{\rm m}/\sqrt{\rm Hz}]. \tag{1}$$

  Below 0.1 Hz,

$$\sqrt{G_{\rm seis}} = 10^{-7}[{\rm m}/\sqrt{\rm Hz}]. \tag{2}$$

3 Actuators on mirrors (along optical axis)

In this section, the actuators for mirrors are considered. These actuators are on recoil mass. Therefore, the mechanical susceptibility from actuator to mirror is the same as that of single pendulum. Only the bottom stage of suspension (below intermediate mass) is taken into account.

3.1 Maximum force of an actuator

3.1.1 Half wavelength displacement

The interval of lock points of cavity is half wavelength ($\lambda/2$=532 nm). Thus, the actuators can move mirror at least half wavelength. Since spring constant of final stage of suspension is $m{\omega_\mathrm{p}}^2$,

$$\frac{nF_\mathrm{max}}{m{\omega_\mathrm{p}}^2} > \frac{\lambda}{2}\ [\mathrm{nm}] \tag{3}$$

This equation is rewritten as

$$F_\mathrm{max} > 9.5 \times 10^{-5}\ [\mathrm{N}] \left(\frac{m}{30\ \mathrm{kg}}\right) \left(\frac{\omega_\mathrm{p}}{2\pi \times 0.79\ \mathrm{Hz}}\right)^2 \left(\frac{\lambda}{1064\ \mathrm{nm}}\right) \left(\frac{4}{n}\right). \tag{4}$$

3.1.2 Stable lock

In order to keep stable lock, the maximum motion by actuator must be larger than the root mean square (rms) of mirror displacement x_rms as follow

$$\frac{nF_\mathrm{max}}{m{\omega_\mathrm{p}}^2} > x_\mathrm{rms}. \tag{5}$$

This expression is similar to Eq. (3). According to Sec. 2, the rms is

$$x_\mathrm{rms} = \sqrt{\int_0^\infty G_\mathrm{seis} df} = 1.0 \times 10^{-7} [\mathrm{m}]. \tag{6}$$

This value is smaller than half wavelength. Therefore, if the actuators can move mirror over half wavelength, they can keep stabe lock.

3.1.3 Lock acquisition

In order to acquire lock, actuators must stop the motion of the mirror before mirror passes through the region where Pound-Drever-Hall signal can be obtained. In short, the impulse of actuators should be larfer than the momentum of the mirror as follows

$$nF_\mathrm{max} \times \frac{\lambda}{2\mathcal{F}} \frac{1}{v_\mathrm{rms}} > m v_\mathrm{rms}, \tag{7}$$

where $\mathcal{F}$ and v_rms are finesse and rms of velocity of the mirror. The v_rms is

$$v_\mathrm{rms} = \sqrt{\int_0^\infty (2\pi f)^2 G_\mathrm{seis} df} = 2.3 \times 10^{-8} [\mathrm{m/s}]. \tag{8}$$

This expression is rewritten as

$$F_\mathrm{max} > 1.1 \times 10^{-5} [\mathrm{N}] \left(\frac{4}{n}\right) \left(\frac{m}{30\ \mathrm{kg}}\right) \left(\frac{1064\ \mathrm{nm}}{\lambda}\right) \left(\frac{\mathcal{F}}{1550}\right) \left(\frac{v_\mathrm{rms}}{2.3 \times 10^{-8} [\mathrm{m/s}]}\right)^2. \tag{9}$$

Since this value is smaller than Eq. (4). Thus, if actuators can move the mirror over half wavelength, they can acquire lock.

3.2 Noise of actuators

Here, the noise caused by actuator is considered. In usual case, the maximum input voltage for actuator is about 15 V. The input voltage noise is about $10^{-9}\mathrm{V}/\sqrt{\mathrm{Hz}}$. Thus, the force noise of actuator is $6.7 \times 10^{-11} F_{\mathrm{max}}$. Above the resonant frequency of the suspension final stage, the motion of mirror by actuator noise in strain is described as

$$\sqrt{G_{\mathrm{act(h)}}} = \frac{1}{L} \frac{\sqrt{4n} \times 6.7 \times 10^{-11} F_{\mathrm{max}}}{m\omega^2}. \tag{10}$$

This formula is rewritten as

$$\sqrt{G_{\mathrm{act(h)}}} = 7.3 \times 10^{-23}\ [/\sqrt{\mathrm{Hz}}] \left(\frac{3\ \mathrm{km}}{L}\right) \left(\frac{n}{4}\right)^{1/2} \left(\frac{F_{\mathrm{max}}}{9.5 \times 10^{-5}\ [\mathrm{N}]}\right) \left(\frac{30\ \mathrm{kg}}{m}\right) \left(\frac{2\pi \times 10\ \mathrm{Hz}}{\omega}\right)^2. \tag{11}$$

This is comparable with goal sensitivity of LCGT ($\sim 2 \times 10^{-22}/\sqrt{\mathrm{Hz}}$ at 10 Hz).

4 Actuators on intermediate masses (optical axis)

The conclusion of the previous section is that the actuator noise is comparable with the goal sensitivity of LCGT if the actuators drive the mirror directly. Here, let us consider the case of actuators on intermediate masses.

4.1 Maximum force of an actuator

Maximum force depends on the amplitude in low frequency region. Even if the force is applied on the intermediate mass, mirror follows it. Therefore, the evaluation method is similar to that in the previous section. However, the mass of mirror, m, is replaced by total mass m_{total} which is total of mirror, recoil mass, and intermediate mass. The resonant frequency of final stage, ω_{p} is replaced by the resonant frequency of stage of intermediate mass, ω_{inter}.

4.1.1 Half wavelength displacement

$$F_{\mathrm{max}} > 3.1 \times 10^{-4}\ [\mathrm{N}] \left(\frac{m_{\mathrm{total}}}{120\ \mathrm{kg}}\right) \left(\frac{\omega_{\mathrm{p}}}{2\pi \times 0.70\ \mathrm{Hz}}\right)^2 \left(\frac{\lambda}{1064\ \mathrm{nm}}\right) \left(\frac{4}{n}\right). \tag{12}$$

4.1.2 Stable lock

According to Sec. 2, the rms is

$$x_{\mathrm{rms}} = \sqrt{\int_0^\infty G_{\mathrm{seis}} df} = 1.0 \times 10^{-7} [\mathrm{m}]. \tag{13}$$

This value is smaller than half wavelength. Therefore, if the actuators can move mirror over half wavelength, they can keep stabe lock.

4.1.3 Lock acquisition

$$F_{\text{max}} > 4.5 \times 10^{-5}[\text{N}] \left(\frac{4}{n}\right) \left(\frac{m_{\text{total}}}{120\ \text{kg}}\right) \left(\frac{1064\ \text{nm}}{\lambda}\right) \left(\frac{\mathcal{F}}{1550}\right) \left(\frac{v_{\text{rms}}}{2.3 \times 10^{-8}[\text{m/s}]}\right)^2 . \quad (14)$$

Since this value is smaller than Eq. (12). Thus, if actuators can move the mirror over half wavelength, they can acquire lock.

4.2 Noise of actuators

Here, the noise caused by actuator is considered. In usual case, the maximum input voltage for actuator is about 15 V. The input voltage noise is about $10^{-9}\text{V}/\sqrt{\text{Hz}}$. Thus, the force noise of actuator is $6.7 \times 10^{-11} F_{\text{max}}$. Above the resonant frequency, all masses act like free mass. The motion of mirror by actuator noise in strain is described as

$$\sqrt{G_{\text{act(h)}}} = \frac{1}{L} \frac{\sqrt{4n} \times 6.7 \times 10^{-11} F_{\text{max}}}{m_{\text{inter}} \omega^2} \left(\frac{\omega_{\text{p}}}{\omega}\right)^2 , \quad (15)$$

where m_{inter} is mass of the imtermediate mass. This formula is rewritten as

$$\begin{aligned} \sqrt{G_{\text{act(h)}}} &= 7.3 \times 10^{-25}\ [/\sqrt{\text{Hz}}] \left(\frac{3\ \text{km}}{L}\right) \left(\frac{n}{4}\right)^{1/2} \left(\frac{F_{\text{max}}}{3.1 \times 10^{-4}\ [\text{N}]}\right) \left(\frac{60\ \text{kg}}{m_{\text{inter}}}\right) \\ &\times \left(\frac{\omega_{\text{p}}}{2\pi \times 0.79\ \text{Hz}}\right)^2 \left(\frac{2\pi \times 10\ \text{Hz}}{\omega}\right)^4 . \quad (16) \end{aligned}$$

This is much less than the goal sensitivity of LCGT ($\sim 2 \times 10^{-22}/\sqrt{\text{Hz}}$ at 10 Hz).

5 Actuators on mirrors (Pitch motion)

Here, maximum pitch motion by actuators and actuator noise in pitch motion are evaluated. It is assumed that the maximum force of each actuator is the same as that in Eq. (4).

5.1 Maximum rotation

The maximum rotation is described as

$$\frac{2F_{\text{max}} r}{I {\omega_{\text{pitch}}}^2} , \quad (17)$$

where I and ω_{pitch} are inertial moment of a mirror and the resonant frequency of pitch motion of stage for mirrors. The value r is the distance between the center of flat surface of mirror and actuator. Here, let us assume that it is the radius of the mirror. This expression is rewritten as

$$3.8 \times 10^{-9}\ [\text{rad}] \frac{F_{\text{max}}}{9.5 \times 10^{-5}\ [\text{N}]} \left(\frac{r}{12.5\ \text{cm}}\right) \frac{0.17\ [\text{kgm}^2]}{I} \left(\frac{2\pi \times 31\ [\text{Hz}]}{\omega_{\text{pitch}}}\right)^2 . \quad (18)$$

This value is too small to adjust alignment of mirror.

5.2 Noise of actuator

The actuator noise in pitch motion is written as

$$\frac{\sqrt{4 \times 2} \times 6.7 \times 10^{-11} F_{\rm max} r}{I \omega^2}. \tag{19}$$

The pitch fluctuation contaminates the motion along optical axis via miscentering, d. Thus, the noise in strain sensitivity is written as

$$\frac{1}{L} \frac{\sqrt{4 \times 2} \times 6.7 \times 10^{-11} F_{\rm max} r}{I \omega^2} d \tag{20}$$

$$= 1.1 \times 10^{-24} \ [/\sqrt{\rm Hz}] \left(\frac{3 \ {\rm km}}{L}\right) \left(\frac{F_{\rm max}}{9.5 \times 10^{-5} \ [{\rm N}]}\right) \left(\frac{r}{12.5 \ {\rm cm}}\right) \left(\frac{0.17 \ [{\rm kgm}^2]}{I}\right)$$

$$\times \left(\frac{2\pi \times 10 \ [{\rm Hz}]}{\omega}\right)^2 \left(\frac{d}{1 \ [{\rm mm}]}\right) \tag{21}$$

This is much less than the goal sensitivity of LCGT ($\sim 2 \times 10^{-22}/\sqrt{\rm Hz}$ at 10 Hz).

6 Actuators on intermediate masses (Pitch motion)

Here, let us consider the case of actuators on intermediate masses.

6.1 Maximum rotation

Maximum force depends on the amplitude in low frequency region. Even if the force is applied on the intermediate mass, mirror follows it. Therefore, the evaluation method is similar to that in the previous section. However, the mass of inertial moment, I, is replaced by total moment $I_{\rm total}$ which is total of mirror, recoil mass, and intermediate mass. The resonant frequency of final stage, $\omega_{\rm pitch}$ is replaced by the resonant frequency of stage of intermediate mass, $\omega_{\rm pitch(inter)}$. The parameter r is replaced by the distance between the center of intermediate mass and actuator $r_{\rm inter}$.

$$3.7 \times 10^{-8} \ [{\rm rad}] \frac{F_{\rm max}}{3.1 \times 10^{-4} \ [{\rm N}]} \left(\frac{r_{\rm inter}}{5.05 \ {\rm cm}}\right) \frac{0.94 \ [{\rm kgm}^2]}{I_{\rm total}} \left(\frac{2\pi \times 5 \ [{\rm Hz}]}{\omega_{\rm pitch(inter)}}\right)^2. \tag{22}$$

6.2 Noise of actuators

Here, the noise caused by actuator is considered. In usual case, the maximum input voltage for actuator is about 15 V. The input voltage noise is about $10^{-9}{\rm V}/\sqrt{\rm Hz}$. Thus, the force noise of actuator is $6.7 \times 10^{-11} F_{\rm max}$. Above the resonant frequency, all masses act like free mass. The motion of mirror by actuator noise in strain is described as

$$\sqrt{G_{\rm act(h)}} = \frac{1}{L} \frac{\sqrt{4 \times 2} \times 6.7 \times 10^{-11} F_{\rm max} r_{\rm inter}}{I_{\rm inter} \omega^2} \left(\frac{\omega_{\rm pitch}}{\omega}\right)^2 d, \tag{23}$$

where $I_{\rm inter}$ is mass of the imtermediate mass. This formula is rewritten as

$$\begin{aligned}\sqrt{G_{\rm act(h)}} &= 9.9\times10^{-28}\ [/\sqrt{\rm Hz}]\left(\frac{3\ \rm km}{L}\right)\left(\frac{F_{\rm max}}{3.1\times10^{-4}\ [\rm N]}\right)\left(\frac{r_{\rm inter}}{5.05\ [\rm cm]}\right)\left(\frac{0.26\ \rm kgm^2}{I_{\rm inter}}\right)\\ &\times \left(\frac{\omega_{\rm (pitch)}}{2\pi\times31\ \rm Hz}\right)^2\left(\frac{2\pi\times100\ \rm Hz}{\omega}\right)^4\left(\frac{d}{1\ [\rm mm]}\right). \end{aligned} \tag{24}$$

This is much less than the goal sensitivity of LCGT ($\sim 3\times10^{-24}/\sqrt{\rm Hz}$ at 100 Hz).

7 Actuators on mirrors (Yaw motion)

Here, maximum yaw motion by actuators and actuator noise in yaw motion are evaluated. It is assumed that the maximum force of each actuator is the same as that in Eq. (4).

7.1 Maximum rotation

The maximum rotation is described as

$$\frac{2F_{\rm max}r}{I{\omega_{\rm yaw}}^2}, \tag{25}$$

where I and $\omega_{\rm yaw}$ are inertial moment of a mirror and the resonant frequency of pitch motion of stage for mirrors. The value r is the distance between the center of flat surface of mirror and actuator. Here, let us assume that it is the radius of the mirror. This expression is rewritten as

$$1.5\times10^{-6}\ [\rm rad]\frac{F_{\rm max}}{9.5\times10^{-5}\ [\rm N]}\left(\frac{r}{12.5\ \rm cm}\right)\frac{0.17\ [\rm kgm^2]}{I}\left(\frac{2\pi\times1.5\ [\rm Hz]}{\omega_{\rm yaw}}\right)^2. \tag{26}$$

This value is too small to adjust alignment of mirror.

7.2 Noise of actuator

The actuator noise in pitch motion is written as

$$\frac{\sqrt{4\times2}\times6.7\times10^{-11}F_{\rm max}r}{I\omega^2}. \tag{27}$$

The yaw fluctuation contaminates the motion along optical axis via miscentering, d. Thus, the noise in strain sensitivity is written as

$$\frac{1}{L}\frac{\sqrt{4\times2}\times6.7\times10^{-11}F_{\rm max}r}{I\omega^2}d \tag{28}$$

$$\begin{aligned}&= 1.1\times10^{-24}\ [/\sqrt{\rm Hz}]\left(\frac{3\ \rm km}{L}\right)\left(\frac{F_{\rm max}}{9.5\times10^{-5}\ [\rm N]}\right)\left(\frac{r}{12.5\ \rm cm}\right)\left(\frac{0.17\ [\rm kgm^2]}{I}\right)\\ &\times \left(\frac{2\pi\times10\ [\rm Hz]}{\omega}\right)^2\left(\frac{d}{1\ [\rm mm]}\right)\end{aligned} \tag{29}$$

This is much less than the goal sensitivity of LCGT ($\sim 2\times10^{-22}/\sqrt{\rm Hz}$ at 10 Hz).

8 Actuators on intermediate masses (Yaw motion)

Here, let us consider the case of actuators on intermediate masses.

8.1 Maximum rotation

Maximum force depends on the amplitude in low frequency region. Even if the force is applied on the intermediate mass, mirror follows it. Therefore, the evaluation method is similar to that in the previous section. However, the mass of inertial moment, I, is replaced by total moment I_{total} which is total of mirror, recoil mass, and intermediate mass. The resonant frequency of final stage, ω_{yaw} is replaced by the resonant frequency of stage of intermediate mass, $\omega_{\mathrm{yaw(inter)}}$. The parameter r is replaced by the distance between the center of intermediate mass and actuator r_{inter}.

$$8.9\times 10^{-7}\ [\mathrm{rad}]\frac{F_{\mathrm{max}}}{3.1\times 10^{-4}\ [\mathrm{N}]}\left(\frac{r_{\mathrm{inter}}}{15.5\ \mathrm{cm}}\right)\frac{1.4\ [\mathrm{kgm}^2]}{I_{\mathrm{total}}}\left(\frac{2\pi\times 1.4\ [\mathrm{Hz}]}{\omega_{\mathrm{yaw(inter)}}}\right)^2. \tag{30}$$

8.2 Noise of actuators

Here, the noise caused by actuator is considered. In usual case, the maximum input voltage for actuator is about 15 V. The input voltage noise is about $10^{-9}\mathrm{V}/\sqrt{\mathrm{Hz}}$. Thus, the force noise of actuator is $6.7\times 10^{-11}F_{\mathrm{max}}$. Above the resonant frequency, all masses act like free mass. The motion of mirror by actuator noise in strain is described as

$$\sqrt{G_{\mathrm{act(h)}}}=\frac{1}{L}\frac{\sqrt{4\times 2}\times 6.7\times 10^{-11}F_{\mathrm{max}}r_{\mathrm{inter}}}{I_{\mathrm{inter}}\omega^2}\left(\frac{\omega_{\mathrm{yaw}}}{\omega}\right)^2 d, \tag{31}$$

where I_{inter} is mass of the imtermediate mass. This formula is rewritten as

$$\begin{aligned}\sqrt{G_{\mathrm{act(h)}}} &= 2.7\times 10^{-26}\ [/\sqrt{\mathrm{Hz}}]\left(\frac{3\ \mathrm{km}}{L}\right)\left(\frac{F_{\mathrm{max}}}{3.1\times 10^{-4}\ [\mathrm{N}]}\right)\left(\frac{r_{\mathrm{inter}}}{15.5\ [\mathrm{cm}]}\right)\left(\frac{0.68\ \mathrm{kgm}^2}{I_{\mathrm{inter}}}\right)\\ &\times \left(\frac{\omega_{(\mathrm{yaw})}}{2\pi\times 1.4\ \mathrm{Hz}}\right)^2\left(\frac{2\pi\times 10\ \mathrm{Hz}}{\omega}\right)^4\left(\frac{d}{1\ [\mathrm{mm}]}\right).\end{aligned} \tag{32}$$

This is much less than the goal sensitivity of LCGT ($\sim 2\times 10^{-22}/\sqrt{\mathrm{Hz}}$ at 10 Hz).