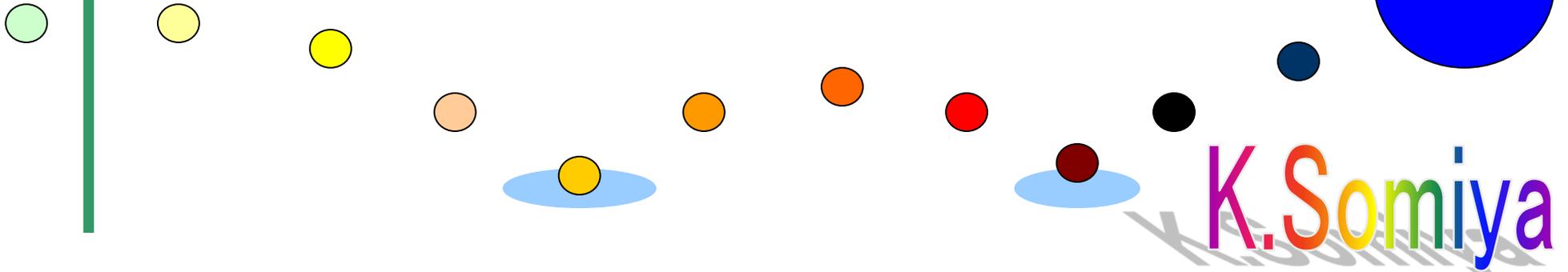


# Introduction to laser interferometric gravitational wave telescope

**KAGRA summer school 2013**  
**July 31, 2013**

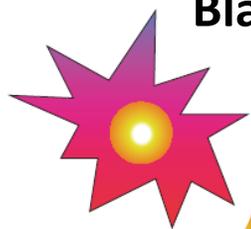
Tokyo Inst of Technology  
Kentaro Somiya



# Interferometric GW detector

Far Galaxy

Supernova explosion,  
Black hole binaries, etc.



Gravitational Waves



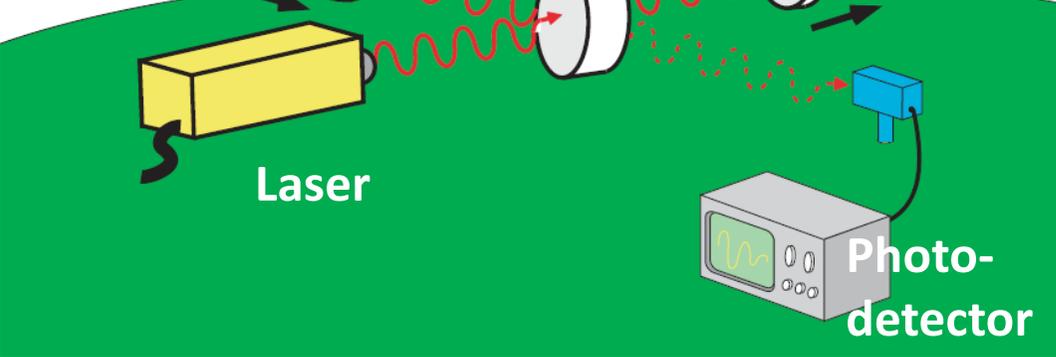
Shrink



Expand



Laser



Earth

Massive Astronomical events



Distance of two objects changes

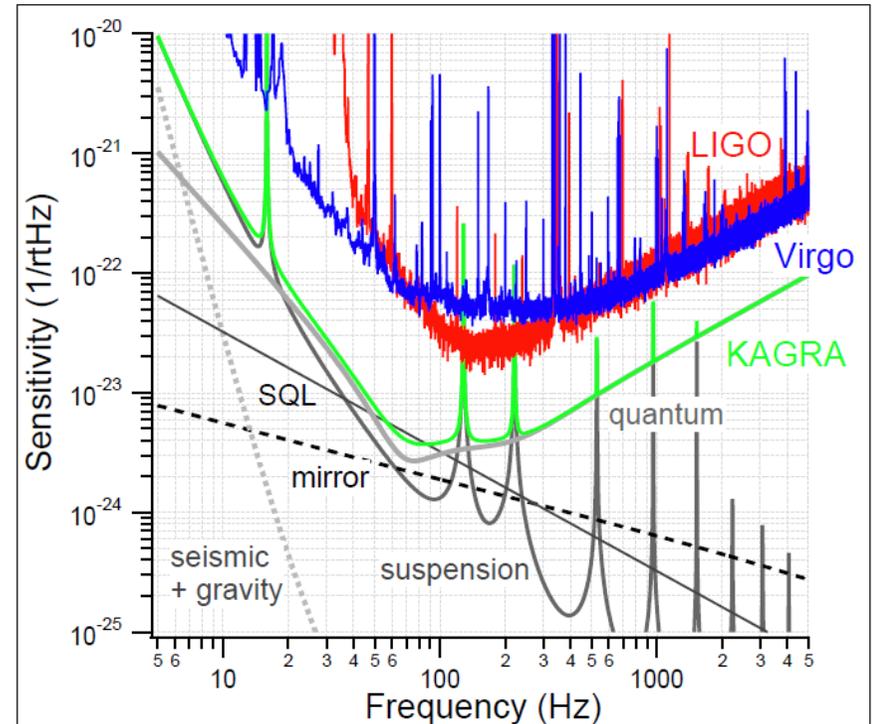
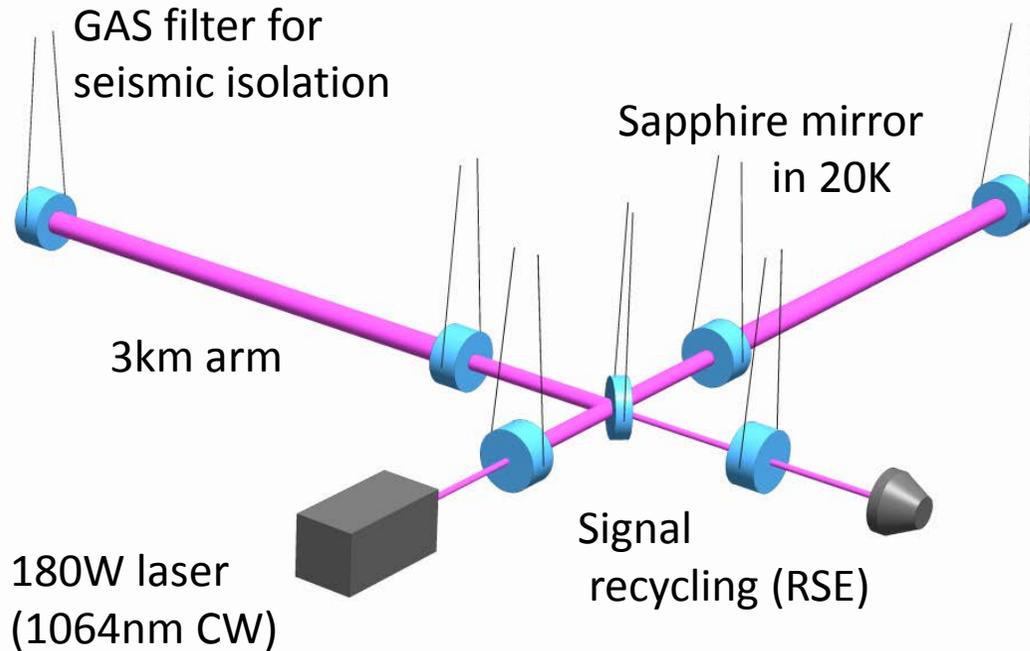


Observe the change with  
big high-power interferometers

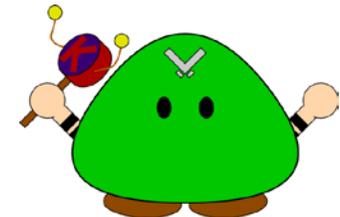
- LIGO in US [4km]
- Virgo in Italy [3km]
- GEO in Germany [600m]
- KAGRA in Japan [3km]

# KAGRA

*Location: Kamioka, Gifu*



- **Underground + Cryogenic + Quantum non-demolition**
- **To be complete in 2017~18**
- **~10 events per year**



## Purpose of this lecture

To obtain precedent knowledge for the f2f meeting

People will use many technical terms:

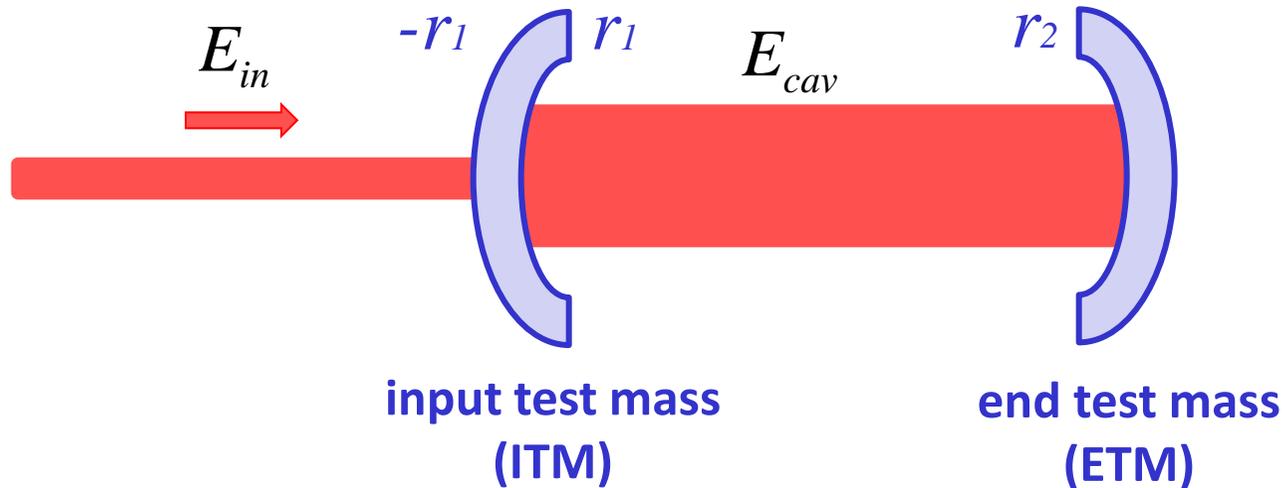
*RSE, Q-phase, Schnupp asymmetry, Gouy phase, g-factor, mechanical loss, dissipation, ... etc.*

**None of them are too difficult.**

**One just needs some precedent knowledge.**

# Optical resonator (cavity)

GWD is an interferometer with a number of **cavities**:  
*arm cavities, power recycling, signal recycling, mc, omc.*



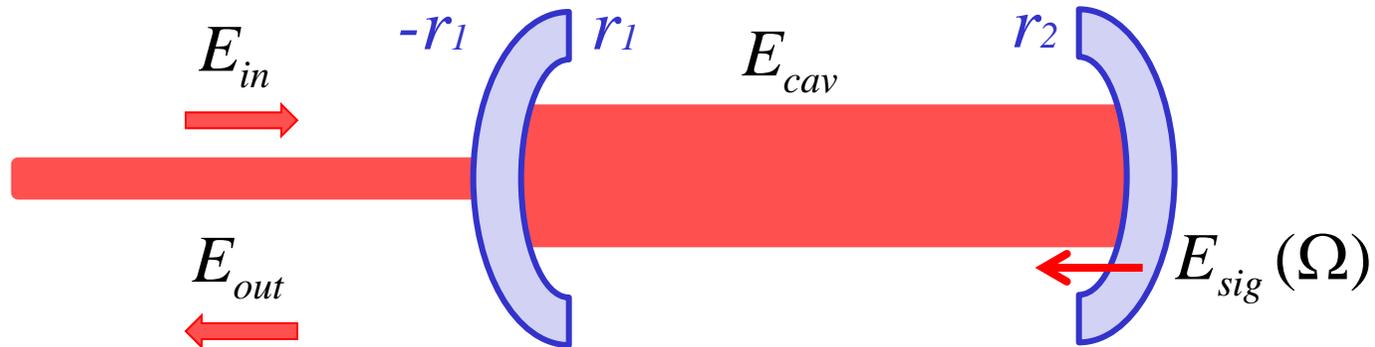
$$F = \frac{2\pi}{t_1^2}$$

$$E_{cav} = t_1 E_{in} + t_1 r_2 r_1 E_{in} + t_1 r_2 r_1 r_2 r_1 E_{in} + \dots = \frac{t_1}{1 - r_1 r_2} E_{in}$$

The power increases in the cavity by  $\sim F$  (finesse).

# Optical resonator (cavity)

GWD is an interferometer with a number of **cavities**:  
*arm cavities, power recycling, signal recycling, mc, omc.*



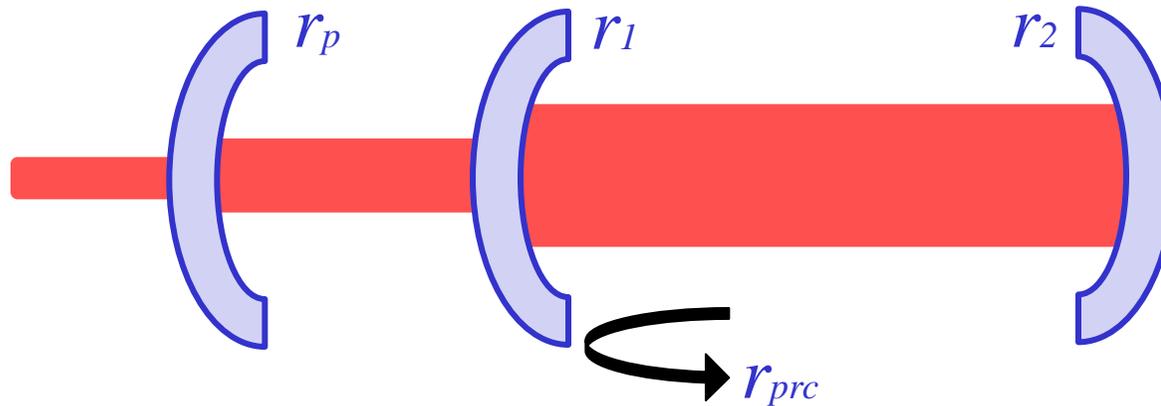
$$E_{out} = \frac{r_2 - r_1}{1 - r_1 r_2} E_{in} + \frac{t_1}{1 - r_1 r_2 e^{i2L\Omega/c}} E_{sig}$$

$$\gamma = \frac{\pi c}{2FL}$$

The signal increases as well,  
but then decays after  $\Omega > \gamma$  (cavity pole)

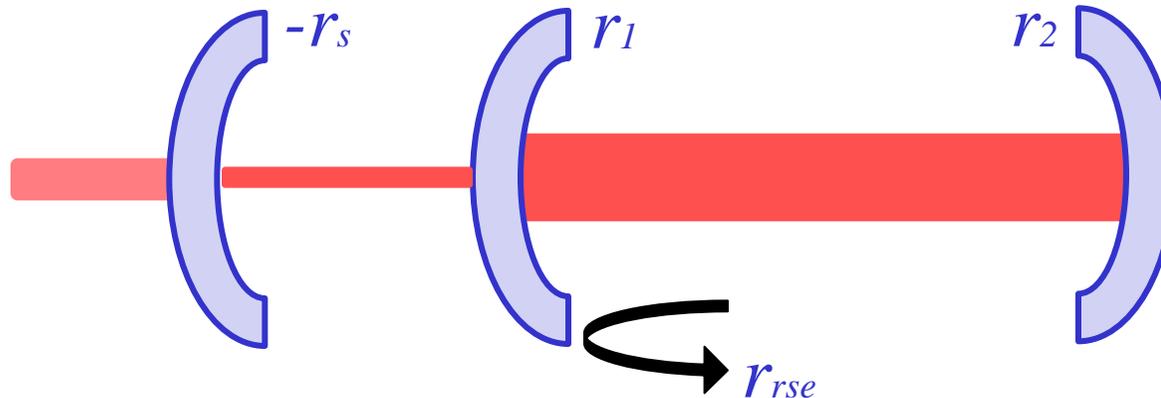
# Recycling cavities

We would like to increase the power as much as possible.



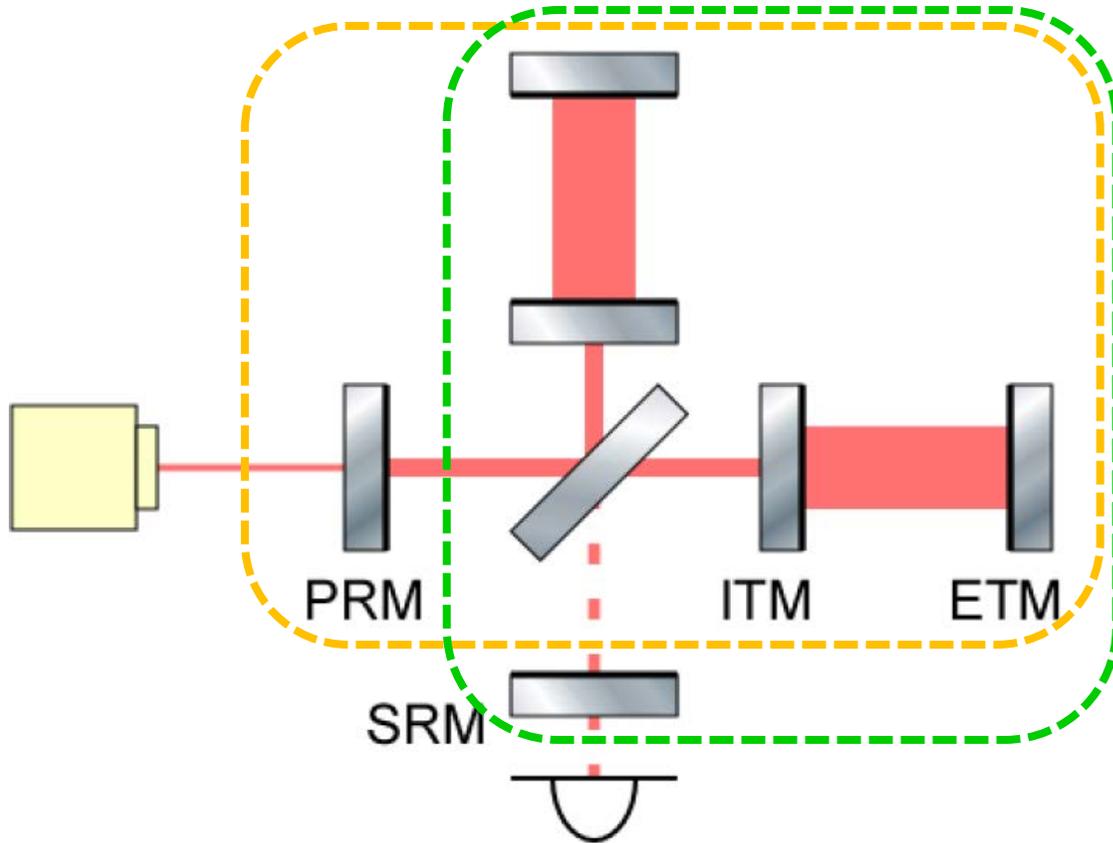
$$r_{prc} = \frac{r_1 + r_p}{1 + r_1 r_p} > r_1$$

There is an appropriate gain (bandwidth) for the signals.



$$r_{rse} = \frac{r_1 - r_s}{1 - r_1 r_s} < r_1$$

# KAGRA's recycling cavities



Arm cavity alone:

$$F = 1500$$

PRC: anti-resonant

>>PR-Arm cavity:

$$F = 16000$$

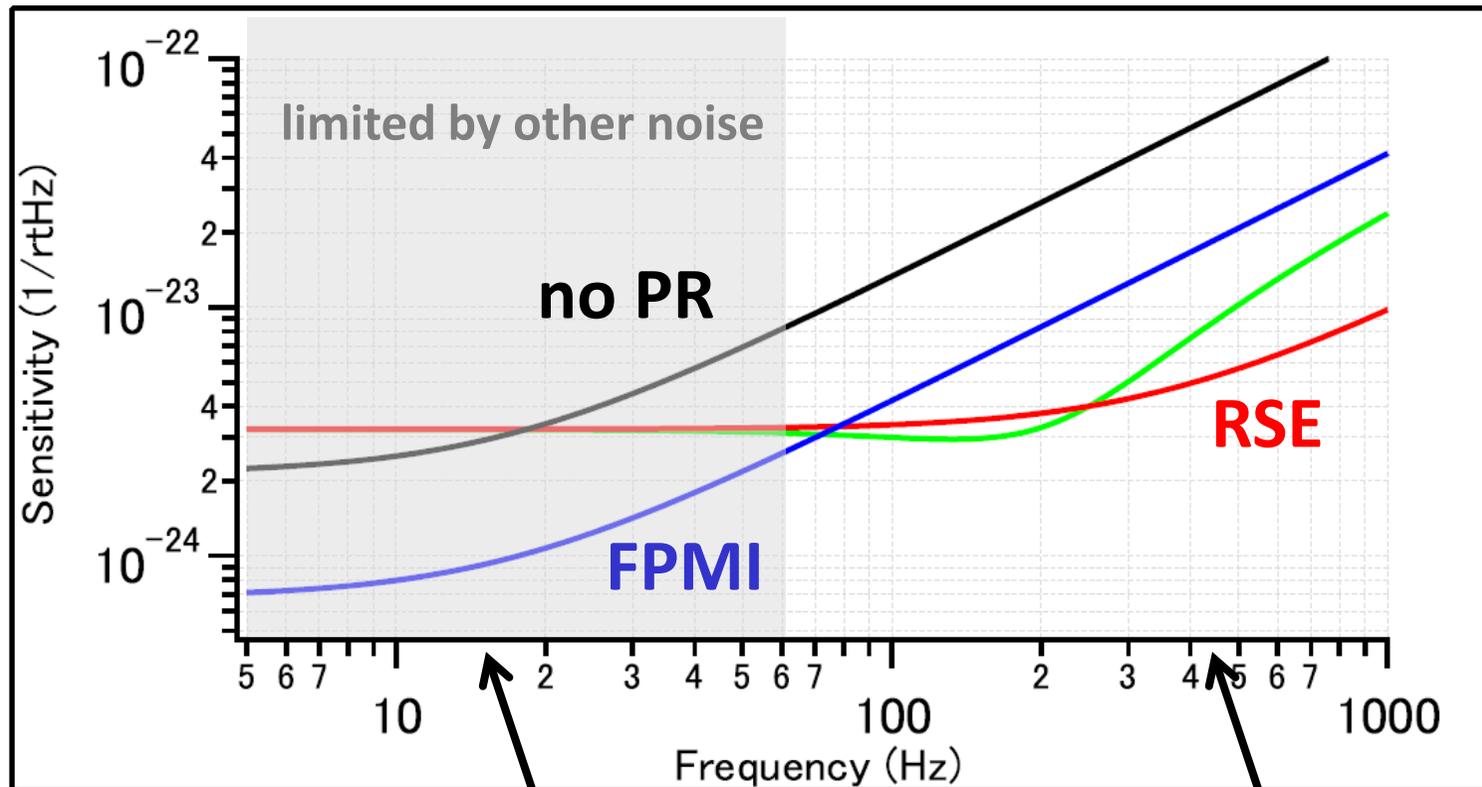
SRC: resonant

>>SR-Arm cavity

$$F = 65$$

PR: anti-resonant, SR: anti-resonant, **RSE**: resonant  
(RSE=Resonant Sideband Extraction)

# KAGRA's recycling cavities



cavity pole of the arm cavity

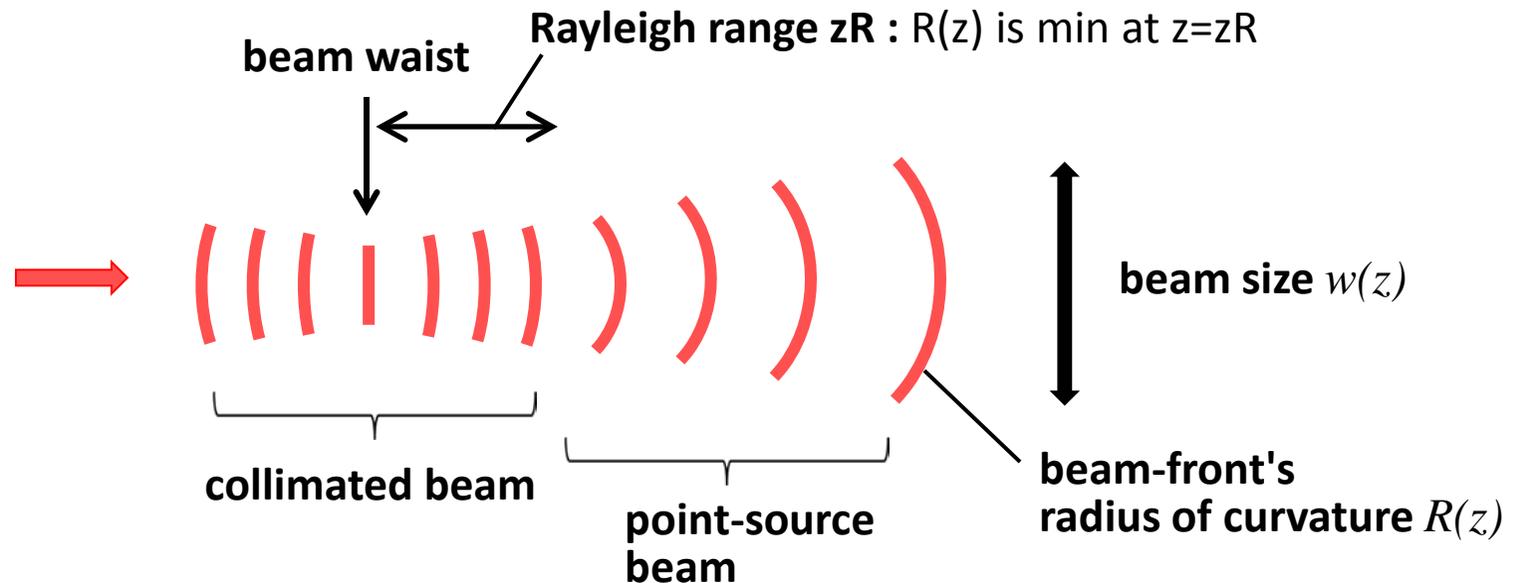
cavity pole of RSE

Appropriate bandwidth can be chosen  
Further optimization by **detuning** the SRC

# Gaussian beam

So far we've considered ray optics.

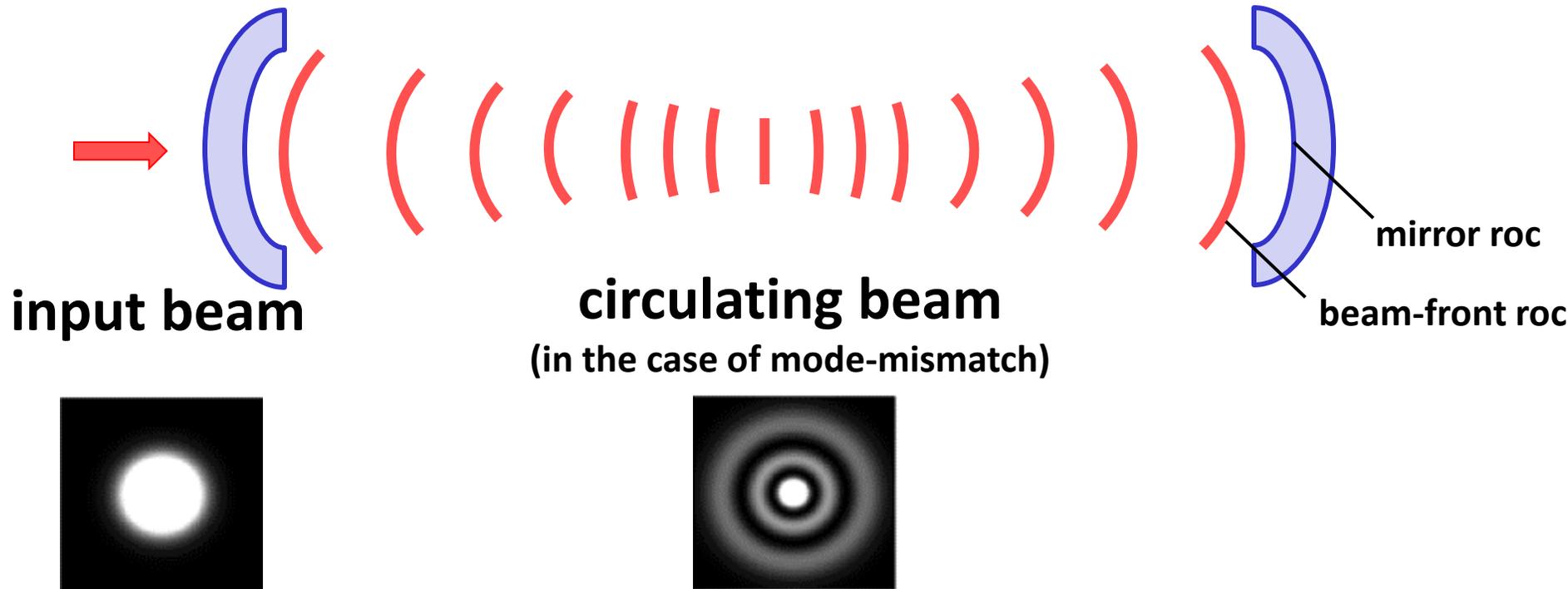
Actual laser beam has a finite beam size changing in space.



$$R(z) = \frac{z^2 + z_R^2}{z}, \quad w(z) = \sqrt{\frac{\lambda}{\pi} \frac{z^2 + z_R^2}{z_R}}$$

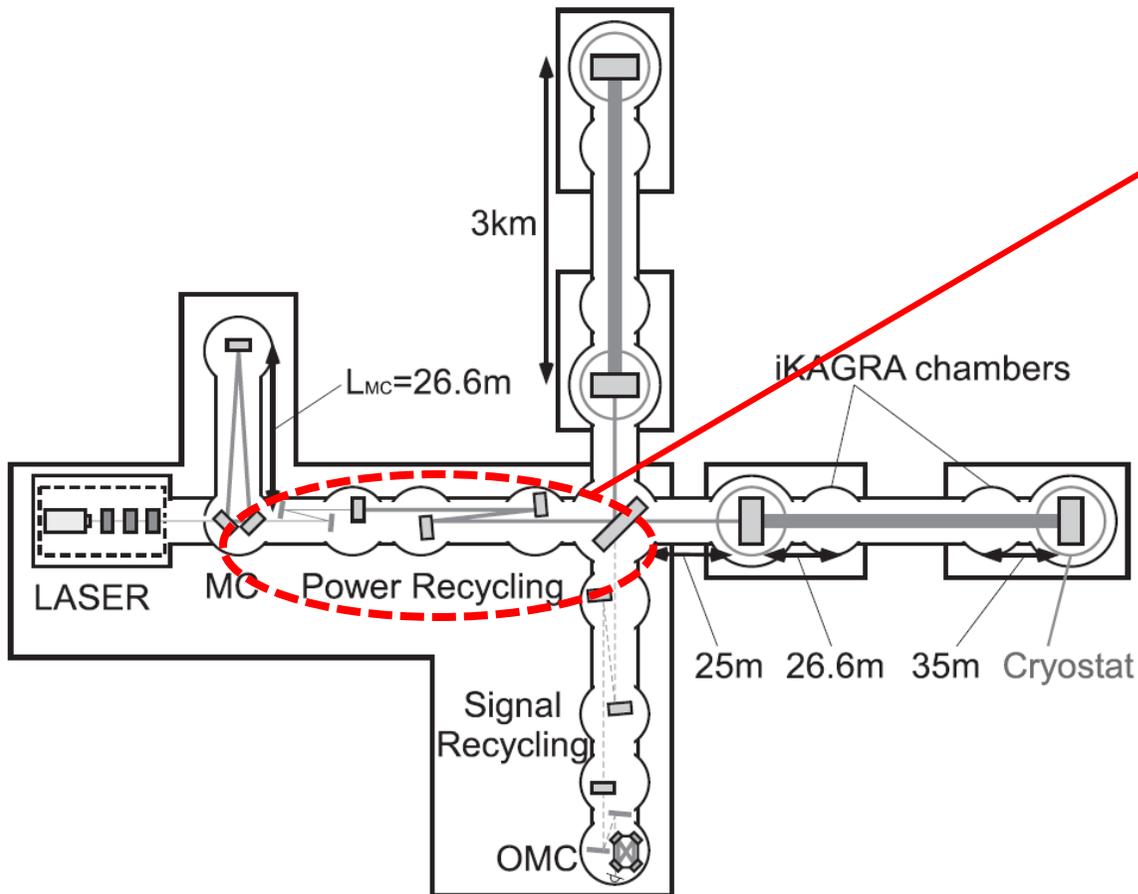
# Higher-order modes

Beam-front radius and the mirror radius of curvature should match, or spatial HOMs will be generated.



Roundtrip phase of the  $m$ -th HOM is shifted by  $2m\eta(L)$ ;  $\eta(z) = \arctan(z/zR)$  is called **Gouy phase**.

# Folded recycling cavities



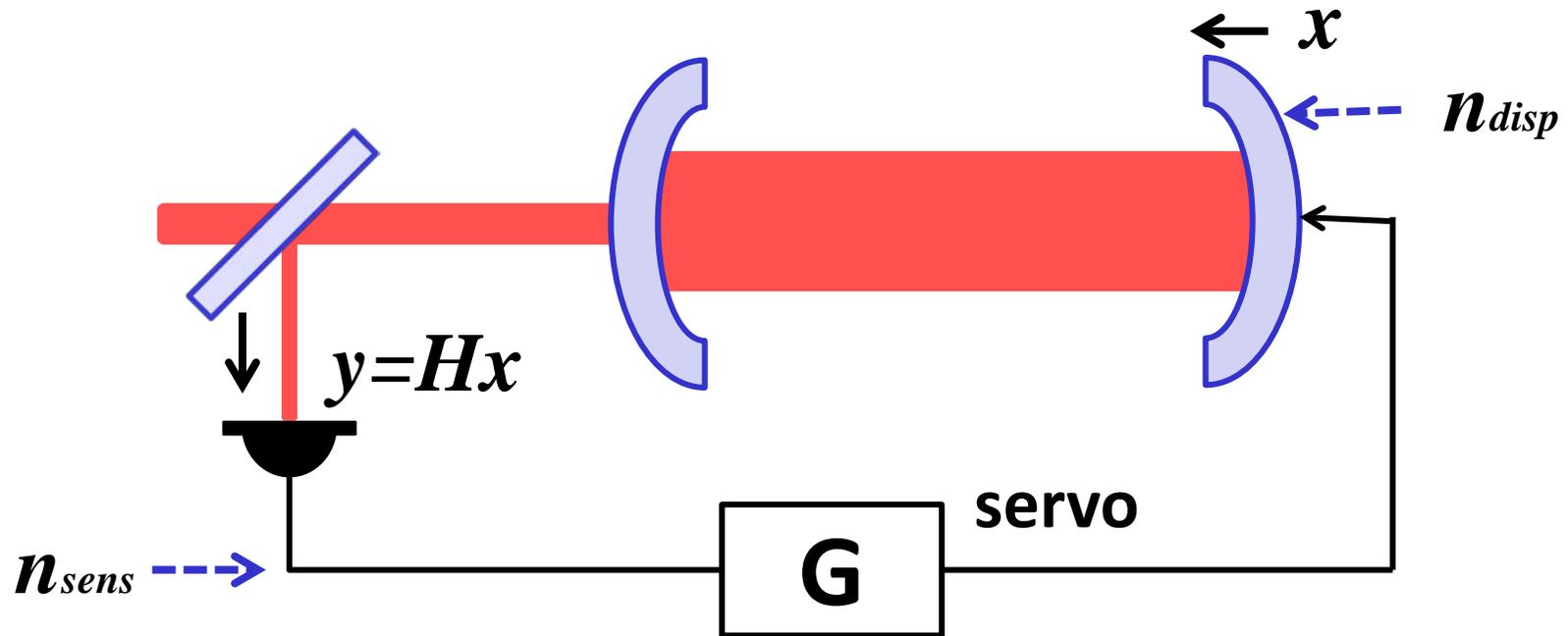
**Gouy phase shift of a straight recycling cavity is quite close to zero.  
>> HOMs nearly resonate.**



**aLIGO and KAGRA fold the recycling cavities to choose a good Gouy phase.**

**Recycling mirrors radii are determined for this reason.  
Test mass radii are determined for different reasons.  
(thermal noise, radiation pressure instability, etc.)**

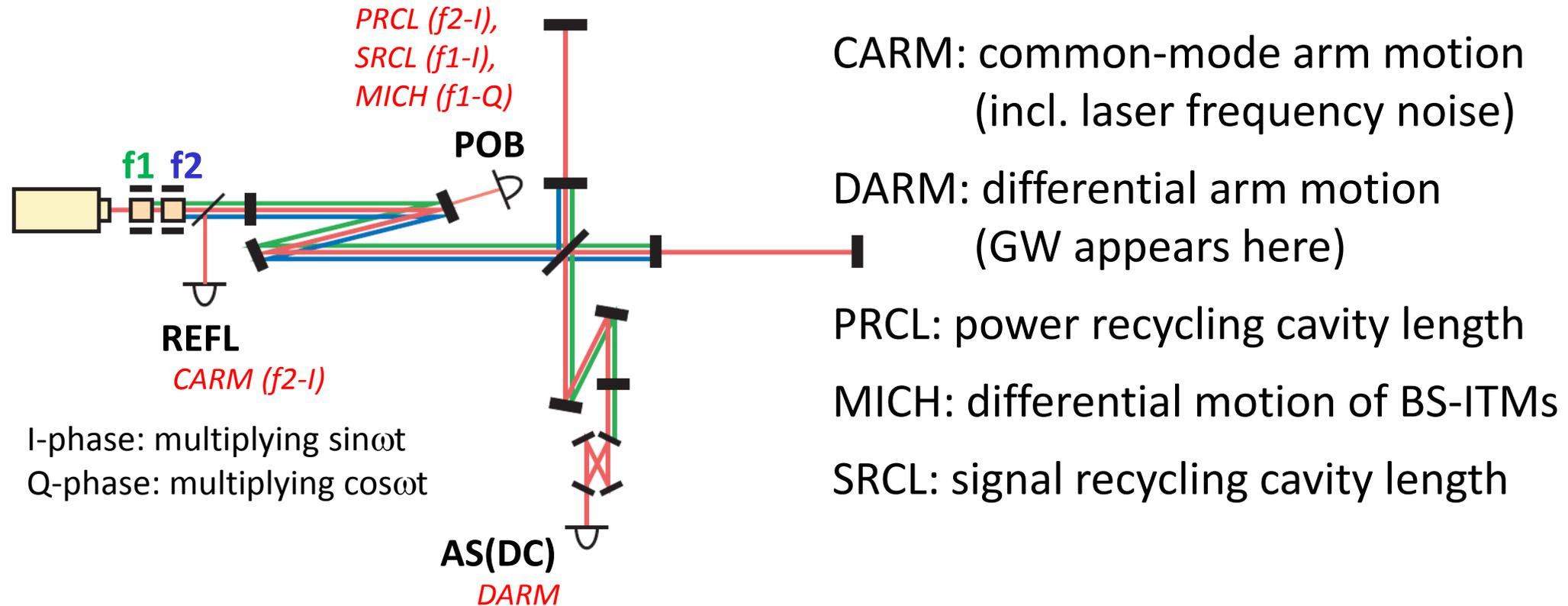
# Length sensing and control



$$\Delta x = \frac{n_{disp} + Gn_{sens}}{1 + GH} \Rightarrow \begin{cases} n_{disp} & (1 \ll GH) \\ \frac{n_{sens}}{H} & (GH \ll 1) \end{cases}$$

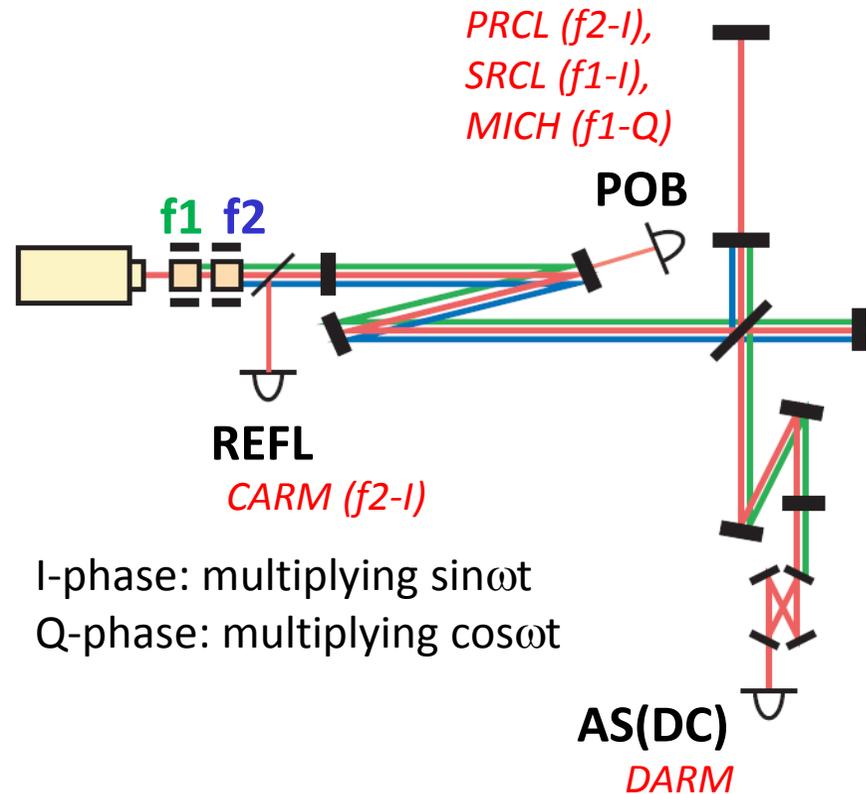
With servo, displacement noise is suppressed but sensing noise is imposed. Increasing the response  $H$  is important.

# Length sensing and control



**For the auxiliary DoFs (CARM, PRCL, MICH, SRCL),  
we should choose the right sensing scheme to  
increase the responses and to extract them independently.**

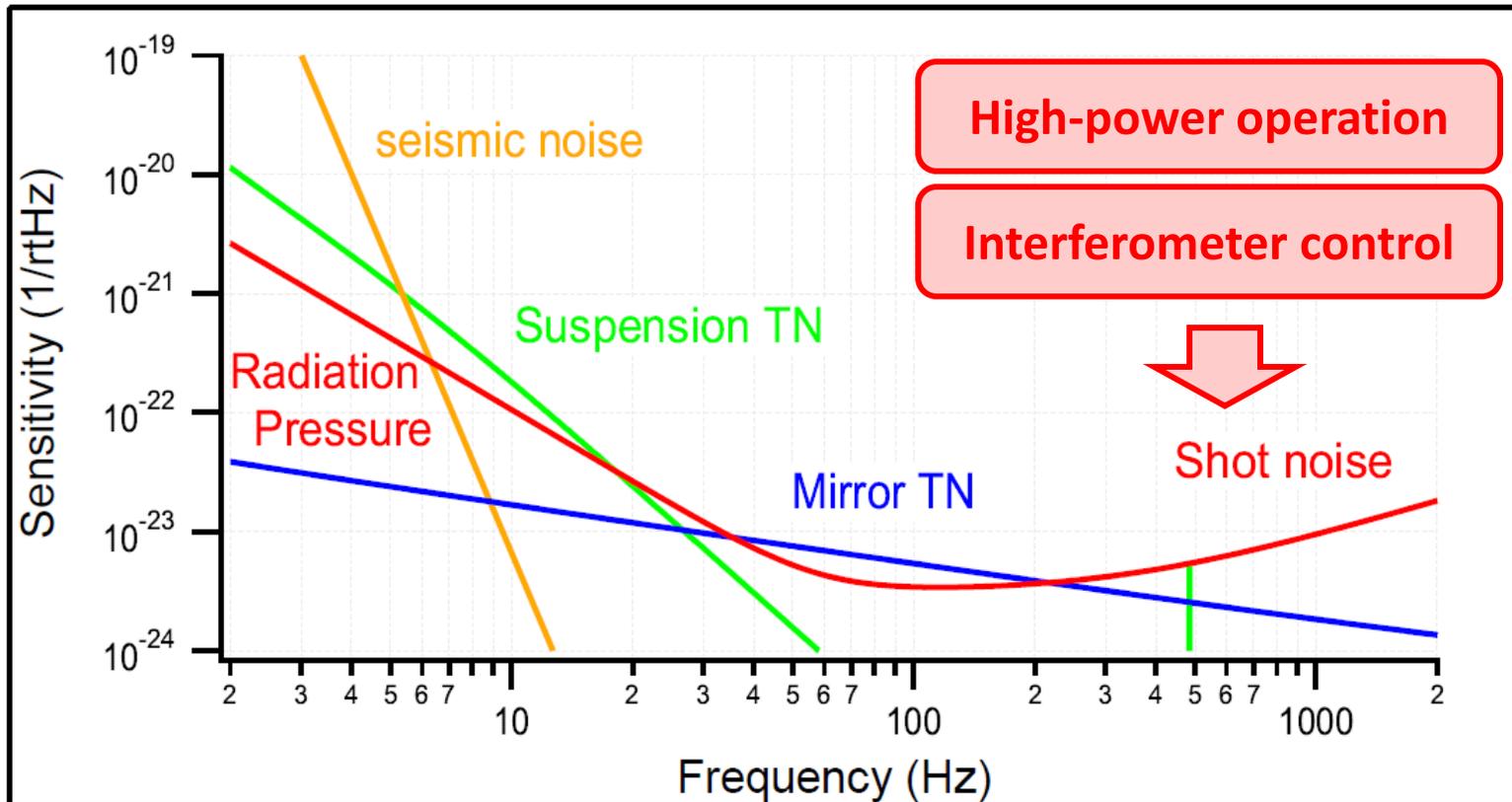
# Length sensing and control



- f1 and f2 are multiples of 5.625MHz, transmitting the MC with the carrier.
- f1, f2 < 45MHz is required for RF PD
- Schnupp asymmetry (BS-ITM1 - BS-ITM2) is selected to increase the contrast between f1 and f2 for PR-SRC

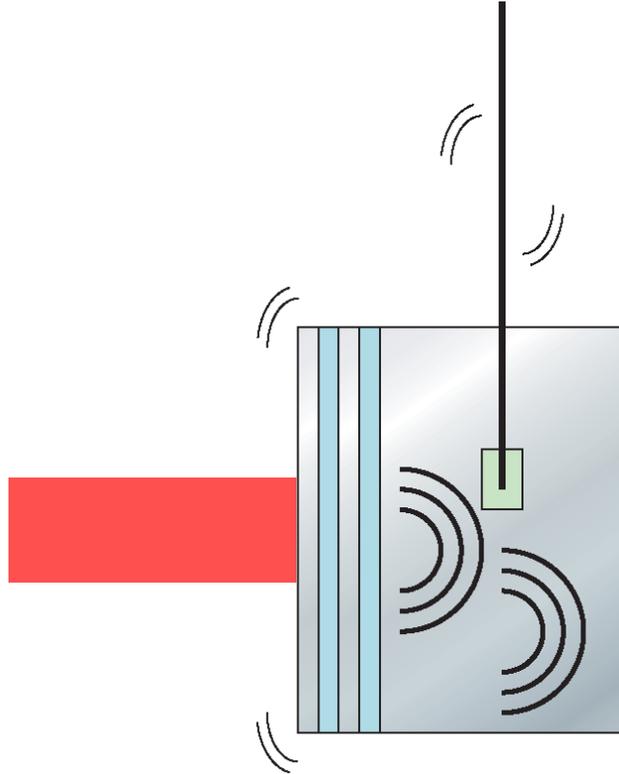
**SB frequencies and mirror locations are determined**

# Sensitivity spectrum



Proper IFO design -> high power operation -> low shot noise  
How can we reduce thermal noise? seismic noise? RP noise?

# Thermal noise



heat bath

- (1) **Dissipation** = decrease of elastic energy
- (2) Fluctuation = Brownian motion

Both are given by a coupling factor of the object to the heat bath = **mechanical loss** [Fluctuation-dissipation Theorem (FdT)]

- Mechanical loss ( $=1/Q$ ) can be measured easily
- TN spectrum can be estimated from the mechanical loss

# FdT and mirror TN

$k_B T$

$F_0 X_0 / 2$

$\frac{\text{"Internal energy of mass"}}{\text{"Thermal-noise kinetic energy"}} = \frac{\text{"Work done by external force"}}{\text{"Dissipated energy } W\text{"}}$

$mv^2/2$

$W/\Omega$

$$\Rightarrow S_x(\Omega) = \frac{8k_B T W}{\Omega^2 F_0^2}$$

$$W = U \Omega \phi$$

U: elastic energy,  $\phi$ : mechanical loss

Substrate thermal noise

$$S_x = \frac{4k_B T}{\Omega} \frac{1-\nu^2}{\sqrt{\pi} Y w_0^2} \phi$$

Coating thermal noise (simple form)

$$S_x = \frac{4k_B T}{\Omega} \frac{(1+\nu)(1-2\nu)2d_c}{\pi Y w_0^2} \phi_c$$

**Mirror TN can be reduced by cooling the mirror, increasing the beam radius  $w_0$ , or lowering the loss  $\phi$ .**

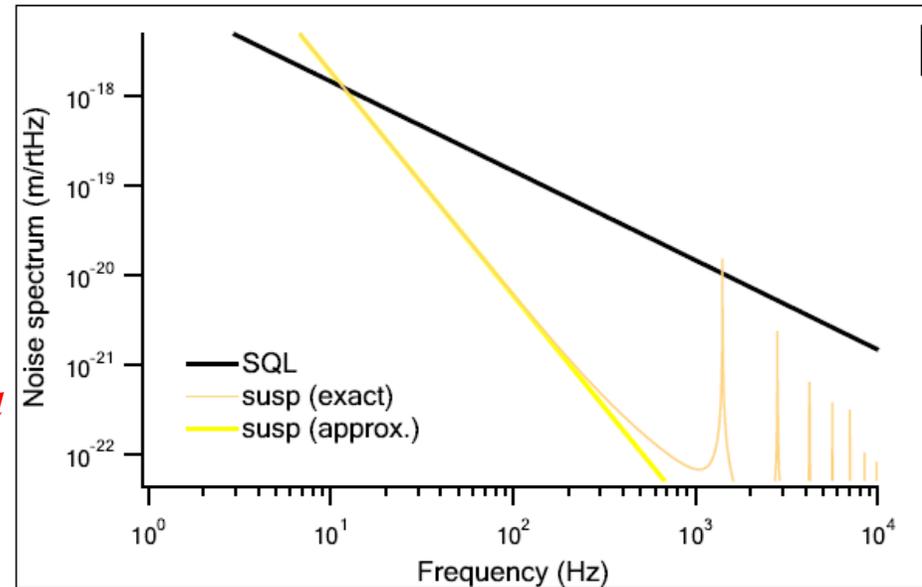
# Suspension TN

Suspension thermal noise (simple form)

$$S_x = \frac{4k_B T \omega_0^2}{m \Omega^5} \times \underbrace{\frac{k_e}{k_g} \phi_e}_{\text{dilution factor}} = \phi_{pend}$$

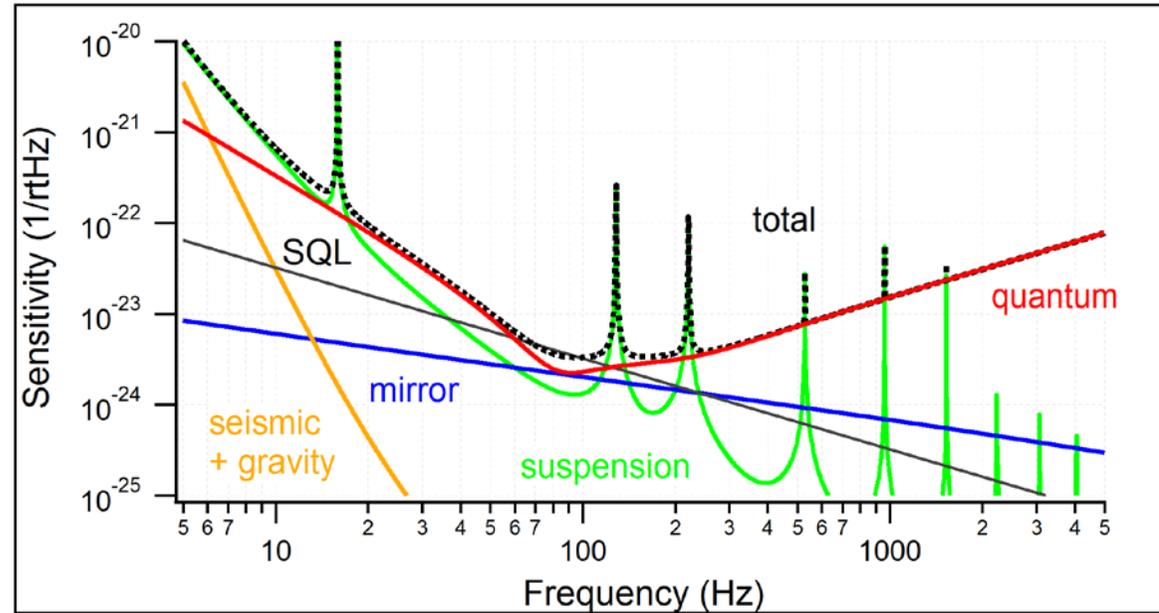
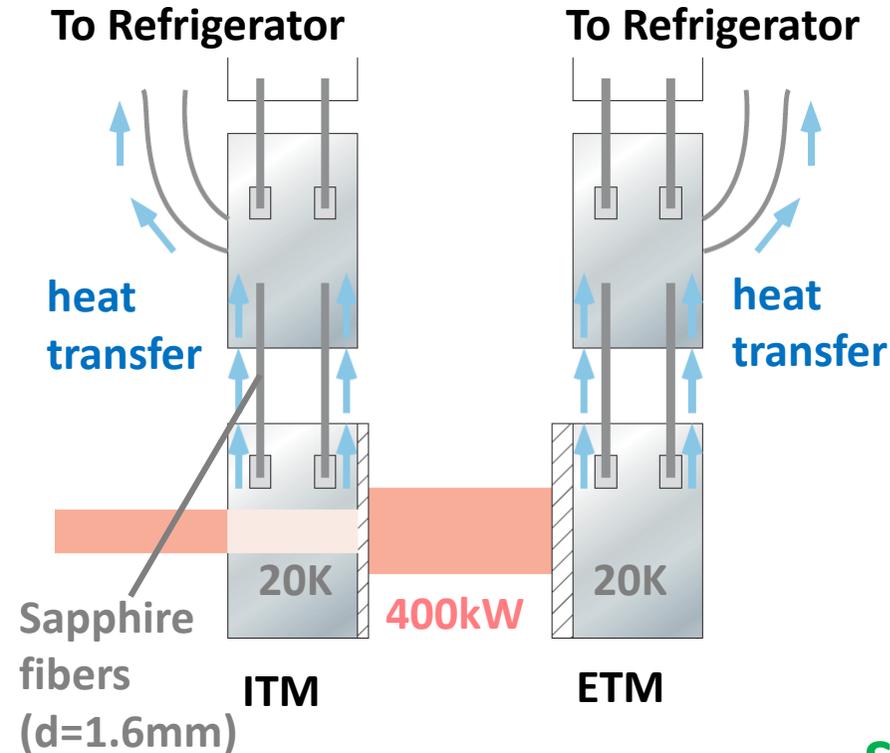
$$= \frac{4k_B T}{m \Omega^5} \sqrt{\frac{4\pi E g}{m}} \left( \frac{d}{4\ell} \right)^2 \phi_e$$

[E: Young's modulus, d: fiber thickness, l: fiber length]



**Suspension TN can be lowered by using thin fibers and increasing the mass.**

# Suspension TN in KAGRA



**Suspension TN**

$$S_x = \frac{4k_B T}{m\Omega^5} \sqrt{\frac{4\pi E g}{m}} \left(\frac{d}{4\ell}\right)^2 \phi_e$$

**Suspension TN is large in KAGRA as its suspension fibers have to be thick in order to transfer heat from the mirrors**

## Other noise sources

- **Seismic noise**
- **Gravity gradient noise**
- **Quantum radiation pressure noise**
- **Vertical/tilt suspension thermal noise**
- **Alignment control noise**
- **Scattering light noise**
- **Residual gas noise**
- **Electric noise**

**... etc.**

## Summary (Q&A)

Q1. How are **SR** and **RSE** different?

Q2. How are the mirror locations determined?

Q3. Why do we need the **folding recycling cavities**?

Q4. Why do we use **Sapphire**?

Q5. What is the **mechanical loss**?

Q6. What is the optical loss?

Q7. Why do we need such a big seismic isolation system?

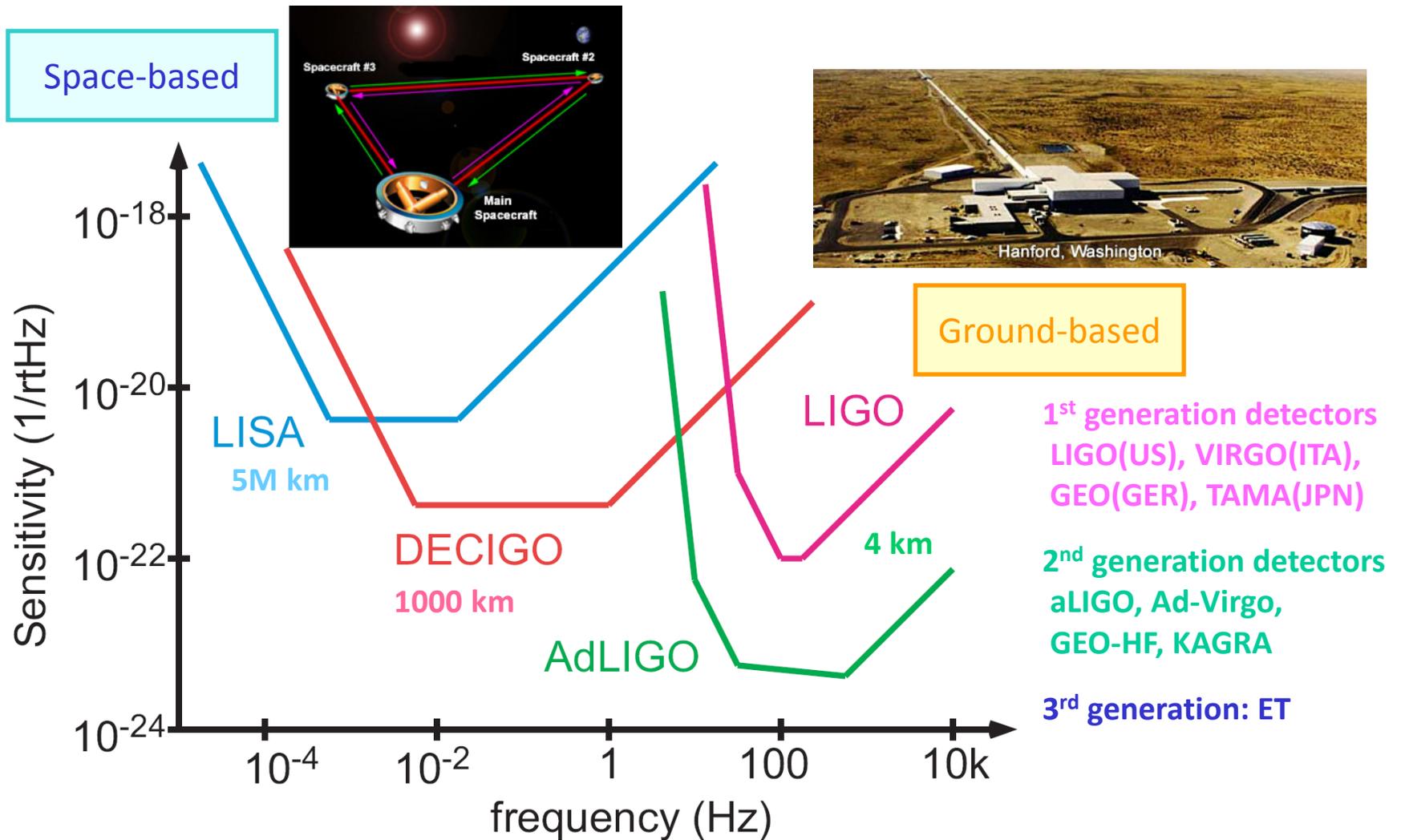
Q8. What is a benefit to build a detector in **underground**?

Q9. What is the difference between **KAGRA** and **aLIGO**?

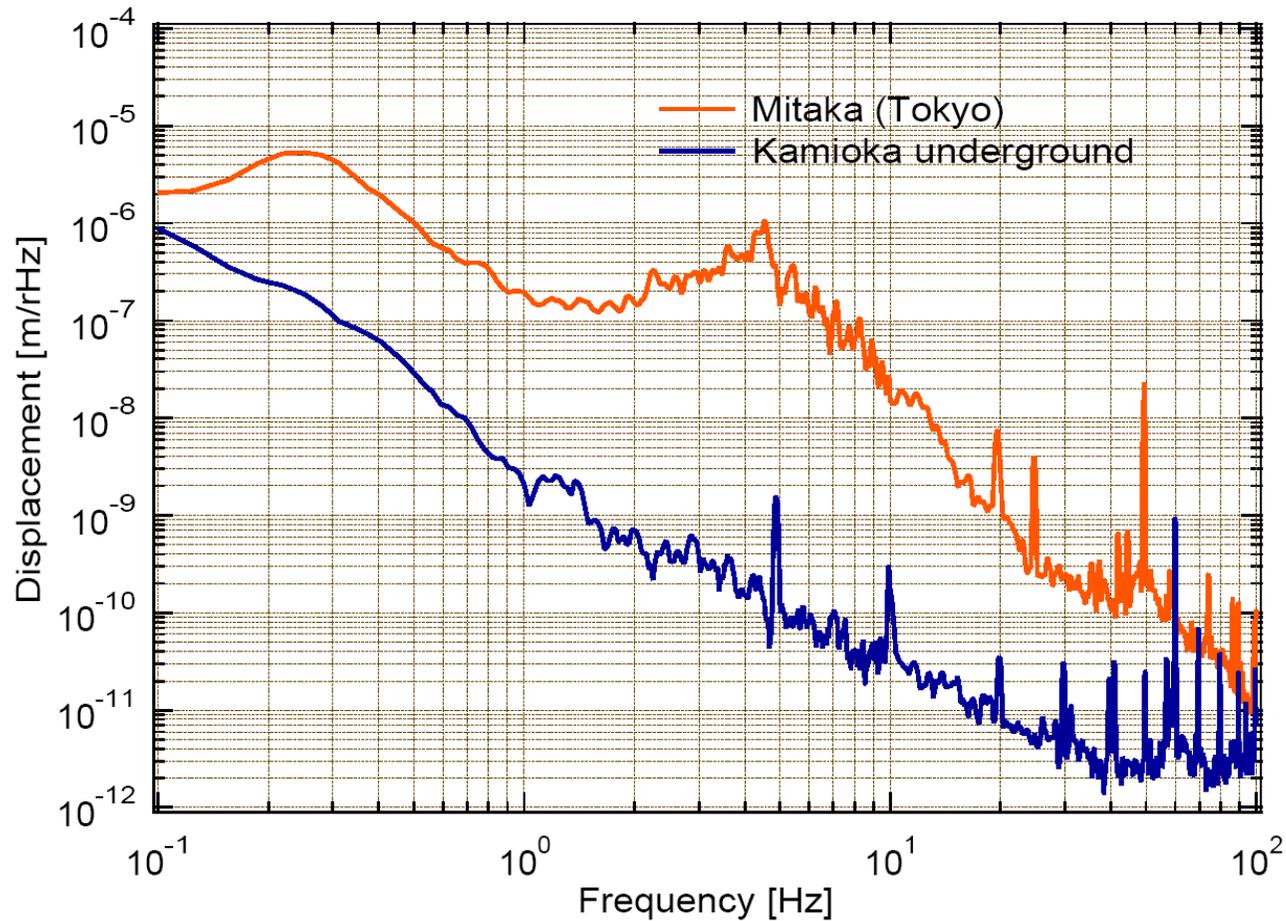
*End*



# Space detectors and ground-based detectors



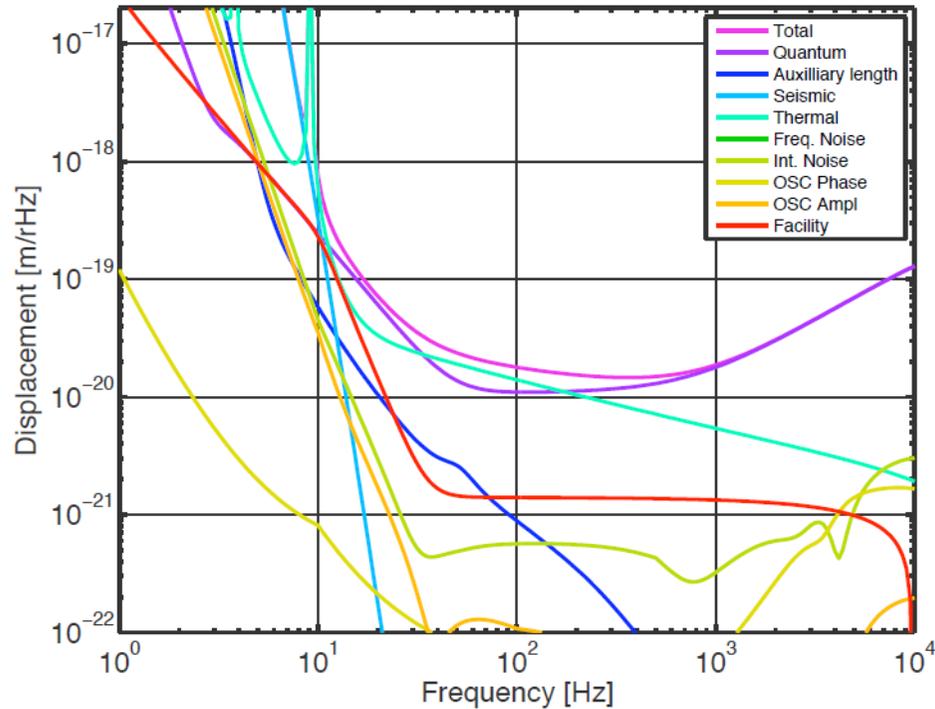
# Low Seismic noise in underground



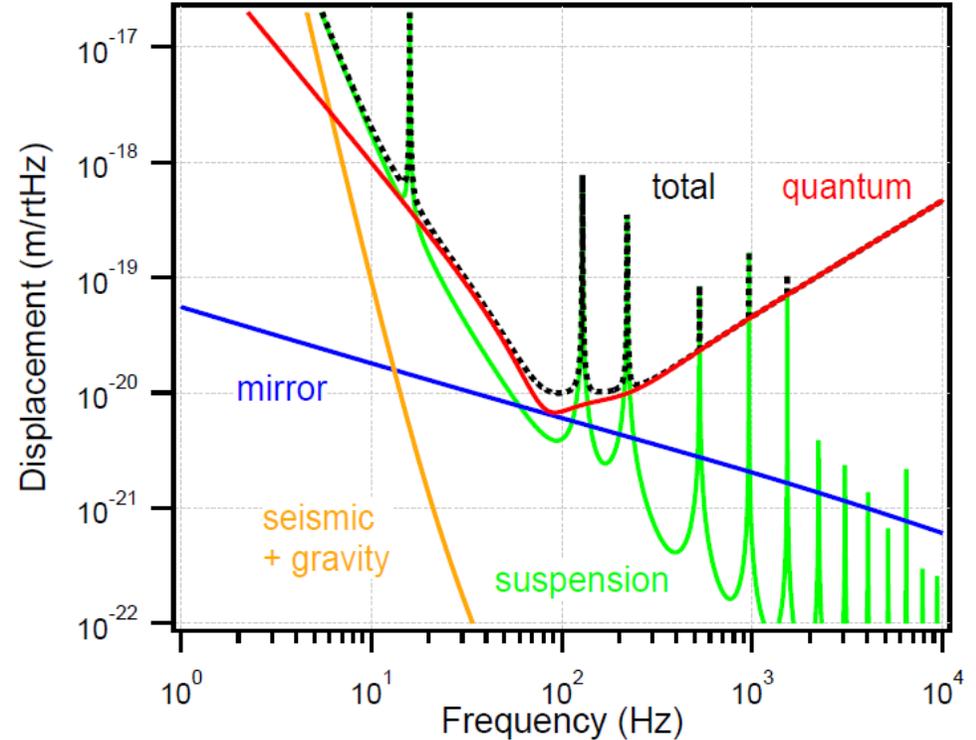
Japanese 2G detector LCGT will be built underground.  
About  $10^{-9}$  m/rtHz at 1Hz.

# Difference between aLIGO and KAGRA

aLIGO



KAGRA



Thermal noise is lower while quantum noise is higher in KAGRA, thus quantum noise reduction is important.

(SRC detuning and DC readout phase optimization)