Modeling beam and mirror distortions using modal models: **FINESSE V.1**

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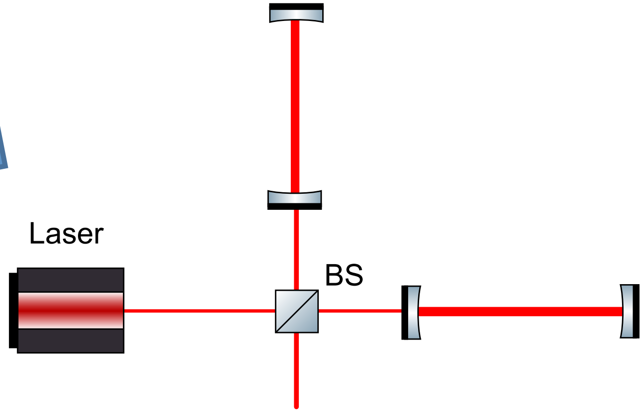
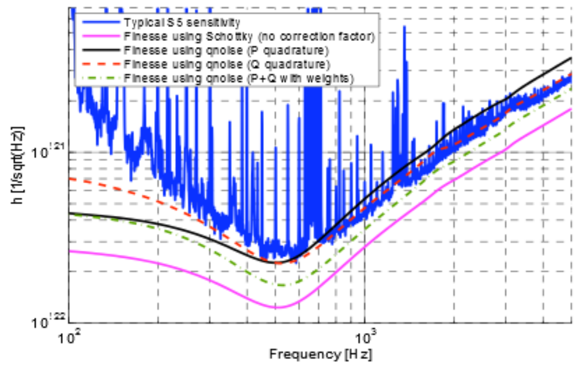
Overview

- FINESSE: what does it do?
- Modal models and mirror defects
- Introducing FINESSE version 1
- Testing FINESSE against FFT simulations
- FINESSE in action:
  - Higher order Laguerre-Gauss beams
  - Sagnac interferometer

FINESSE: What does it do?

```
%% INTERFEROMETER COMPONENTS

l L0 1 0 n1
s s0  1 n1 nbsp1
bs BSP 0.01 0.99 0 45 nbsp1 dump nbsp3 dump

s s01 1 nbsp3 n2

bs BS0 0.5 0.5 59.6 45 n2 n3 n4 n5          # Beam Splitter

const T_ITM 7e-3 # 7000ppm transmission from ET book
const T_ETM 0E-6 # 6ppm transmission from ET book

s sNin  1            n3 n6
m1 IMN $T_ITM 0 0    n6 n7
s sNarm 10000        n7 n8
m1 EMN $T_ETM 0 180 n8 dump

s sWin  1            n4 n9
m1 IMW $T_ITM 0 0    n9 n10
s sWarm 10000        n10 n11
m1 EMW $T_ETM 0 180 n11 dump
```





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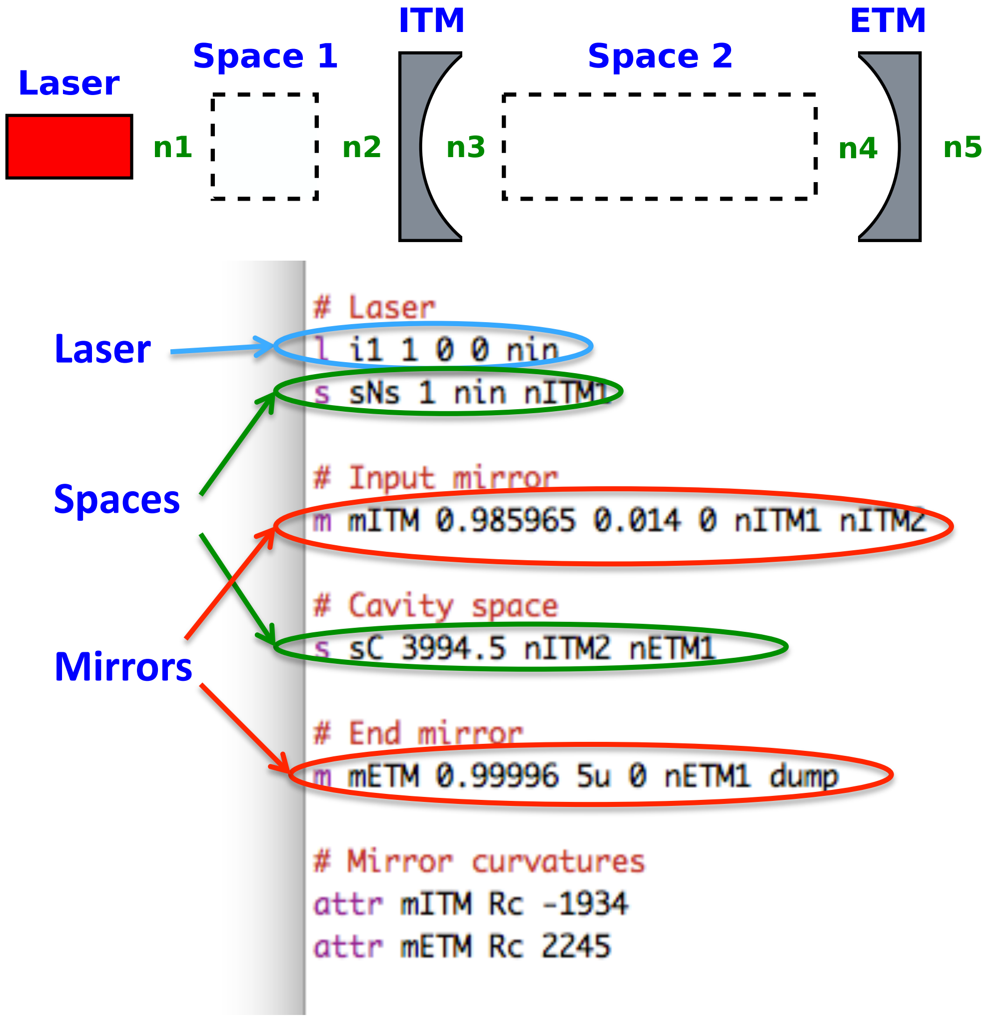
FINESSE

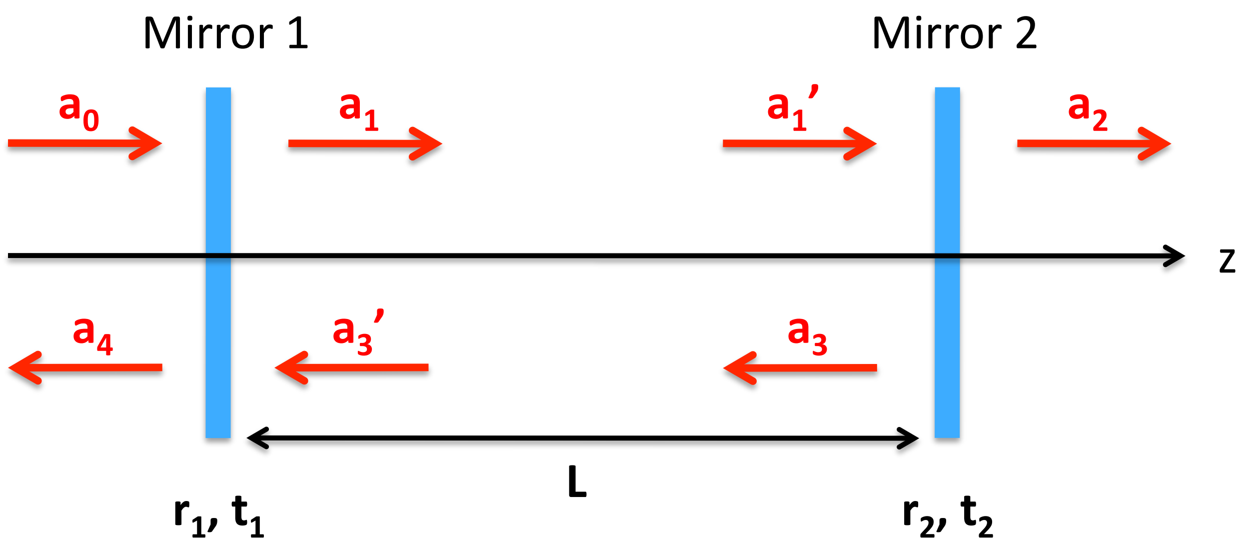
- Frequency domain interferometer simulation.
- User defined interferometer is specified by a series of components connected to each other via nodes.
- Used to compute error signals, noise couplings, misalignment, mode mismatch etc.
- Developed for use in GEO 10 years ago and used to model current gravitational wave detectors.



FINESSE: Plane waves

- To compute the interferometer Finesse generates a set of linear equations which represent the steady state light fields at each node of the interferometer and solves these numerically.

- Fast and effective tool for perfect optics using plane waves.

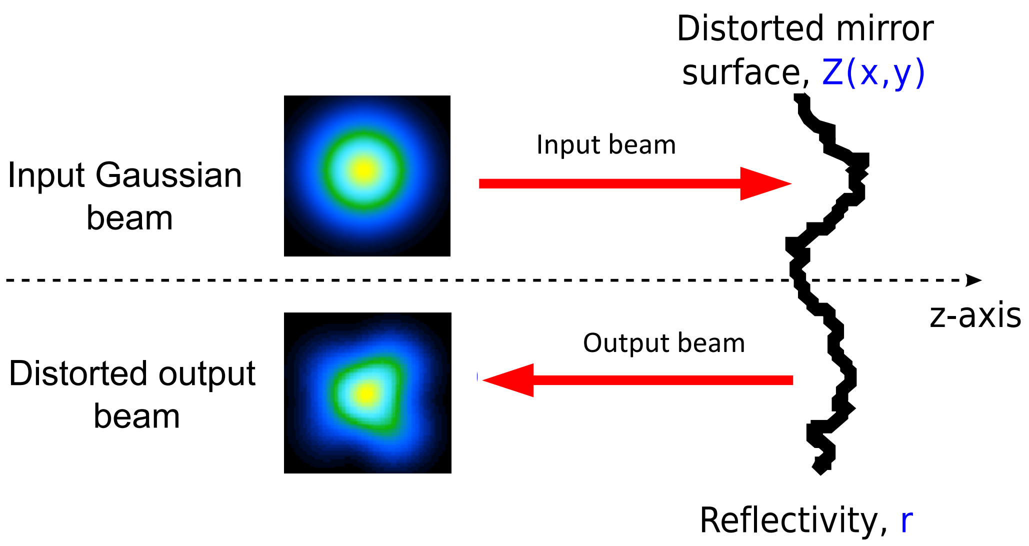


$$a_1 = it_1 a_0 + r_1 a_3'$$
$$a_1' = \exp(-ikL)\, a_1$$
$$a_2 = it_2 a_1'$$
$$a_3 = r_2 a_1'$$
$$a_3' = \exp(-ikL)\, a_3$$
$$a_4 = r_1 a_0 + it_1 a_3'$$

Moving beyond plane waves

- Advanced and future modeling tasks require information about the transverse properties of the light field (i.e. the shape of the beam or its position).

- Effects we want to model include:
  1. Alignment
  2. Mode mismatch
  3. Clipping
  4. **Surface effects**



- **Challenge:** modeling advanced interferometers to compute the effects of realistic optics.

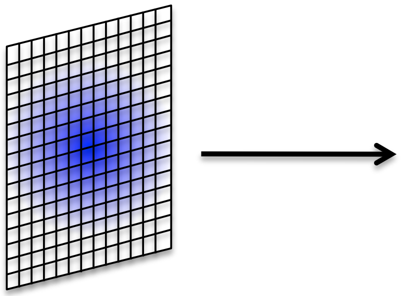
- **Conclusion:** Move from plane waves to a modal model to compute the effects of imperfect optics.

Modal models

- Modal models exploit the fact that a well behaved interferometer can be well described by cavity eigenmodes.

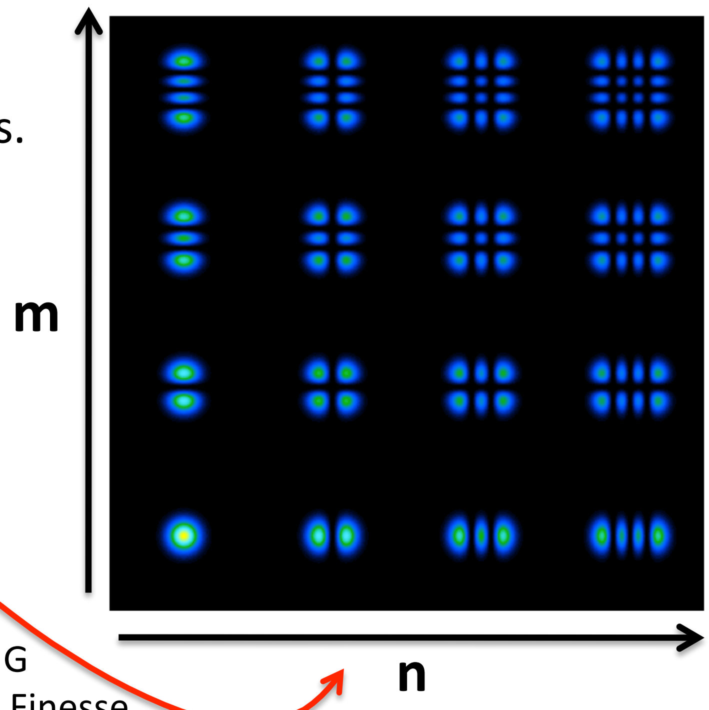
- Light fields with different spatial distortions can be described by a series expansion:

$$E(x,y,z) = \exp(-ikz)\sum_{n,m} a_{nm} u_{nm}(x,y,z)$$



$$\begin{pmatrix} a_{00} & a_{01} & a_{02} & \cdots \\ a_{10} & a_{11} & a_{12} & \cdots \\ a_{20} & a_{21} & a_{22} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

u_{nm} are HG modes in Finesse



- **Hermite-Gauss modes:** Complete set of functions which describe beam shapes with rectangular symmetry of cavity eigenmodes.

- **Maxtem:** Value in Finesse specifying the number of modes to be used.

$$\text{maxtem} \geq n + m \quad \text{(mode order)}$$

Laguerre-Gauss modes

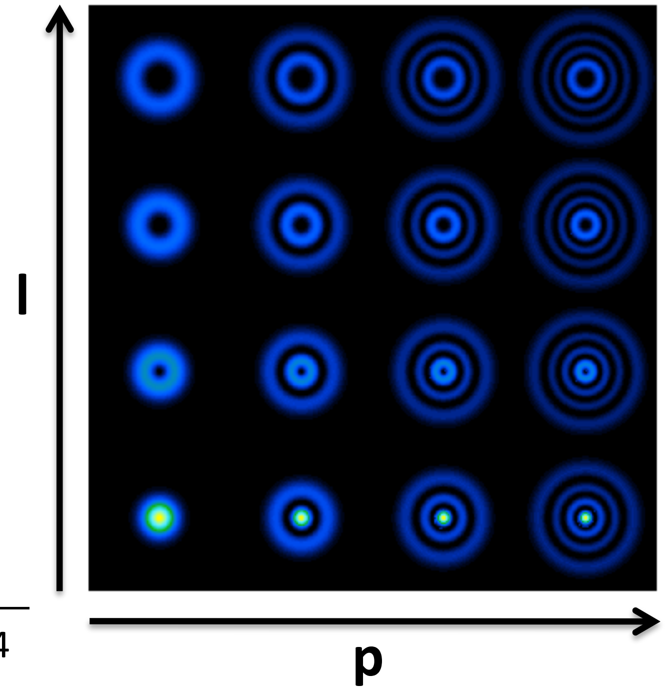
- **LG modes**: Complete set of cylindrically symmetric functions describing different beam shapes.

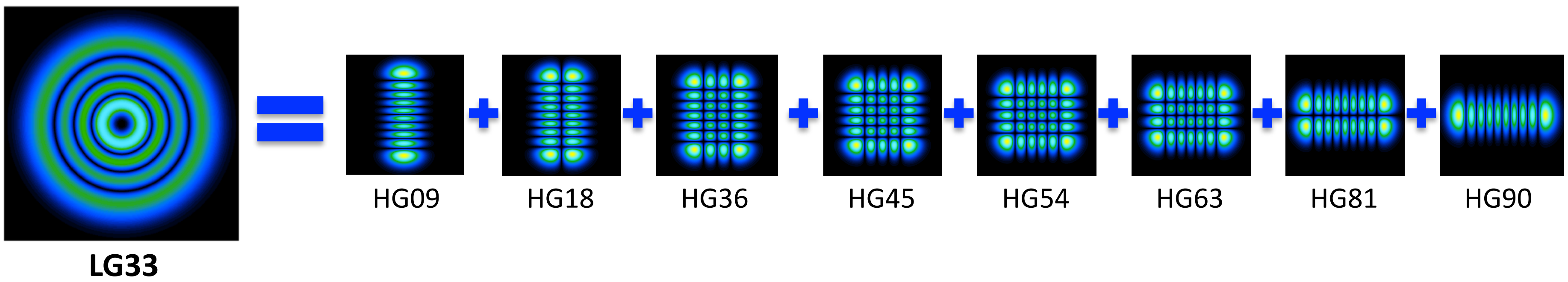
$$\text{order} = 2p + |l|$$

- LG modes can be created from a superposition of HG modes.

- Investigation into higher order LG modes used order 9 HG modes to represent LG33 in Finesse:

HG mode	0,9	1,8	3,6	4,5	5,4	6,3	8,1	9,0
$\lvert a_{nm} \rvert$	0.164	0.164	0.125	0.047	0.047	0.125	0.164	0.164





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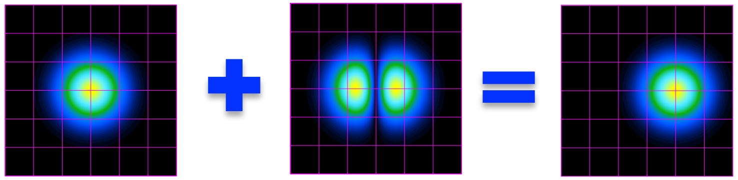
Describing effects with modes

1. Beam shift

small misalignments of the beam

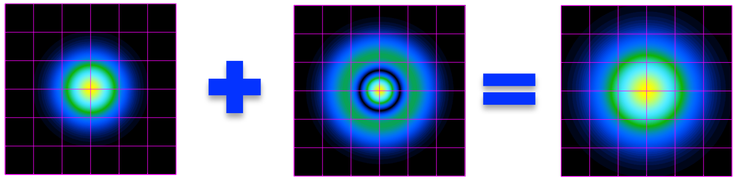


HG00 + HG10 = Shifted HG00

Order 1

2. Beam size

small curvature mismatch between beam and mirror

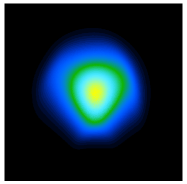


LG00 + LG10 = Large LG00

Order 2

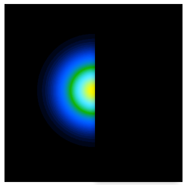
Simple, small effects can be described well with just one HOM.

3. Surface distortions



Order 1 - 10

4. Edge effects



Order 1 - 15+

More complicated effects require more modes.

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Mirror surface characteristics

- The deviation of a mirror surface from an ideal surface can be divided into different categories:

  1. **Measured deviation from perfect sphere.**
  2. **Environmental effects (thermal effects, gravity sag etc.).**
  3. Scratches and point defects.

- Our aim is to model realistic mirror surfaces and understand the effect these will have on the detectors.

  – **Simulations for Advanced LIGO commissioning.**

  – **Derivation of requirements for future detectors.**

- In order to do this we require some method of representing mirror surfaces in our models: we use **mirror maps**.

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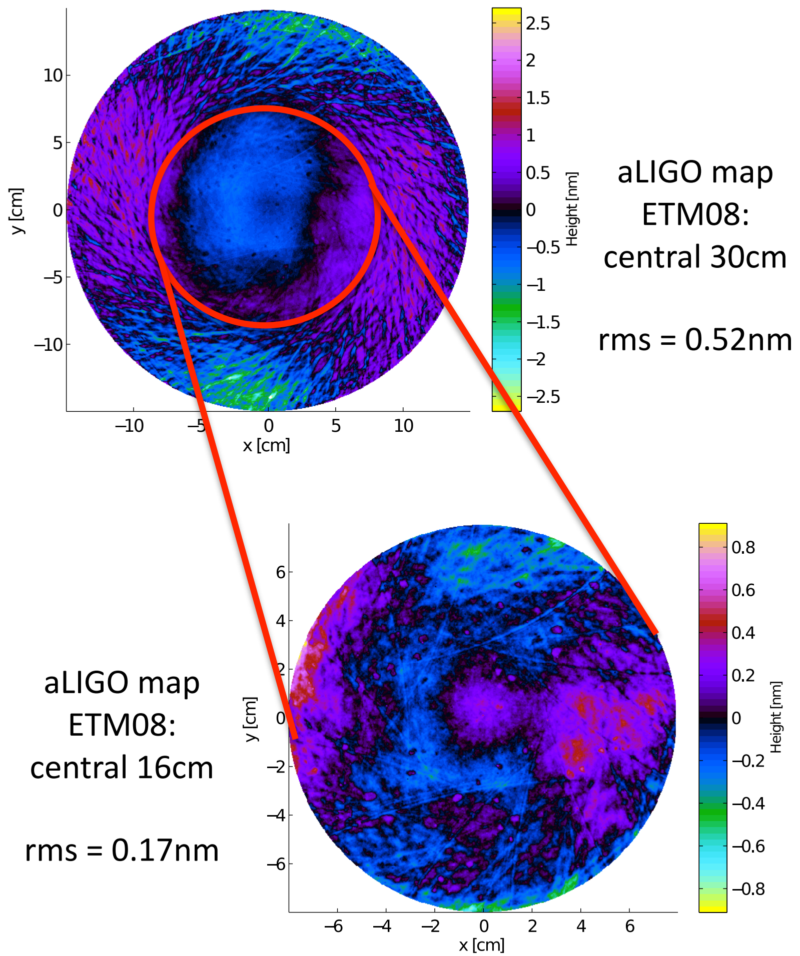
Advanced LIGO mirror maps

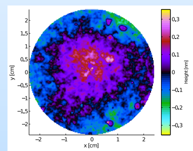


- Measured mirror maps representing mirrors from the Advanced LIGO project are now available (GariLynn Billingsley).
- High quality polishing results in extremely low rms figure error.
- The mirror maps can be used easily in simulations.

Mirror maps

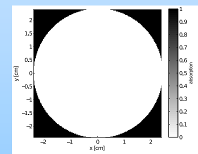
- The term 'mirror map` refers to a grid of data describing an optical property over the surface of a mirror.
- Different types of mirror map: phase (surface height) maps, absorption maps and reflectivity maps.

- **Phase maps**:
  - Surface height data
  - Optical center (x_0, y_0)
  - Scaling of x- and y-axis
  - Points defined outside the mirror



Phase map

- **Aperture maps**:
  - Absorption data (complete absorption outside aperture)
  - Optical center (x_0, y_0)
  - Scaling of x- and y-axis



Aperture map

Mirror maps in FINESSE

- Effect of mirror surface distortions can be described in terms of coupling from a mode $U_{n,m}$ in the incident beam to mode $U_{n',m'}$ in the reflected beam:

$$k_{n,m,n',m'} = \int_{surface} U_{n,m} \exp(2ikZ) U^{*}_{n',m'}$$

- Extensive work has been carried out to include maps in Finesse, using fast, adaptive integration routines.

- We can combine the effects of mirror maps with other effects, such as misalignments and mode mismatch.

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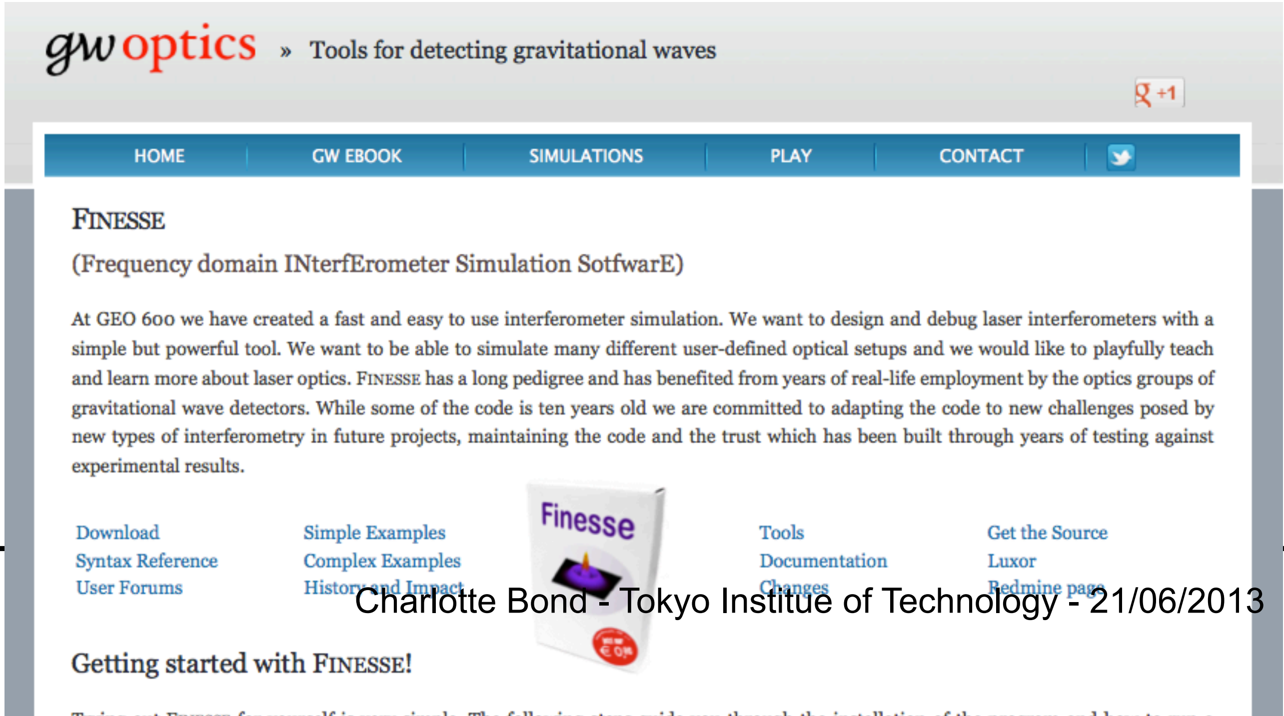
Introducing …

… FINESSE version 1!

http://www.gwoptics.org/finesse/


gw optics » Tools for detecting gravitational waves

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FINESSE

(Frequency domain INterfErometer Simulation SotfwarE)

At GEO 600 we have created a fast and easy to use interferometer simulation. We want to design and debug laser interferometers with a simple but powerful tool. We want to be able to simulate many different user-defined optical setups and we would like to playfully teach and learn more about laser optics. FINESSE has a long pedigree and has benefited from years of real-life employment by the optics groups of gravitational wave detectors. While some of the code is ten years old we are committed to adapting the code to new challenges posed by new types of interferometry in future projects, maintaining the code and the trust which has been built through years of testing against experimental results.

Download
Syntax Reference
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History and Impact
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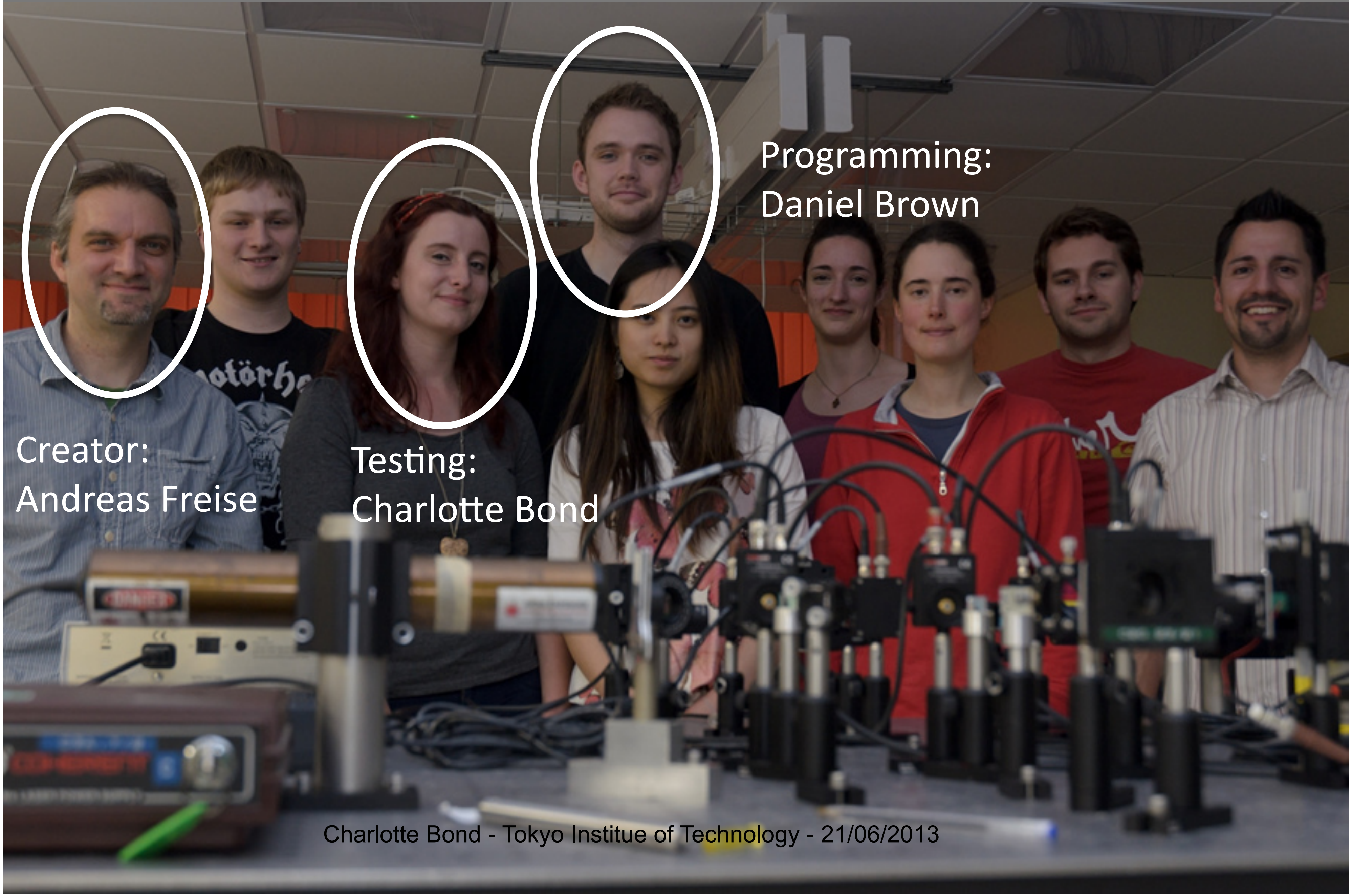
Getting started with FINESSE!

FINESSE team: University of Birmingham



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What's new?

- **Mirror maps:**
  - Bugs fixed
  - Extensive testing against analytics and other simulation methods
  - Combination effects properly implemented

- **New features:**
  - Maps applied to beam-splitters (beta)
  - LG modes

- **Manual V.1:**
  - Added examples
  - How to use mirror maps

- **To-do:**
  - Radiation pressure (GWADW 2014!)
  - 'Dynamic' thermal effects

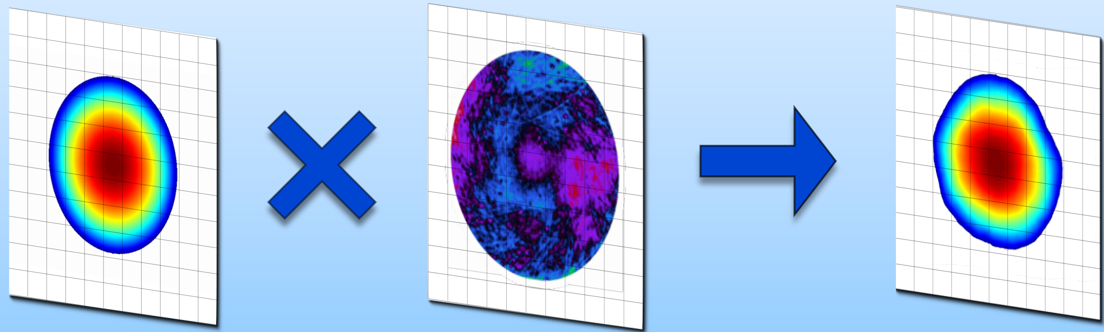
Checking modal results against FFT

Using mirror maps in simulations

FFT codes

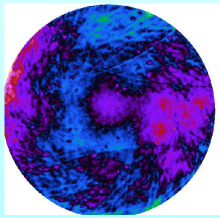
The effect of a mirror surface is computed by multiplying a grid of complex numbers describing the input field by grid describing a function of the mirror surface.

input field | mirror surface | output field



Modal models

mirror map



$$\begin{pmatrix} k_{00,00} & k_{00,01} & k_{00,10} & \cdots \\ k_{01,00} & k_{01,01} & k_{01,10} & \cdots \\ k_{10,00} & k_{10,01} & k_{10,10} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Coupling coefficients

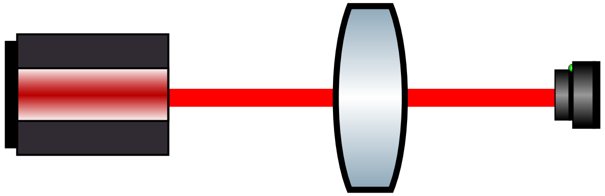
input field | output field

$$k_{nm,n'm'} \times \begin{pmatrix} a_{00} \\ a_{01} \\ a_{10} \\ \vdots \end{pmatrix} \rightarrow \begin{pmatrix} b_{00} \\ b_{01} \\ b_{10} \\ \vdots \end{pmatrix}$$

Matrix of coupling coefficients calculated from the mirror map transform the input field described by HG amplitudes into the output field described by HG amplitudes.

Lens: maps in transmission

f = 200 m

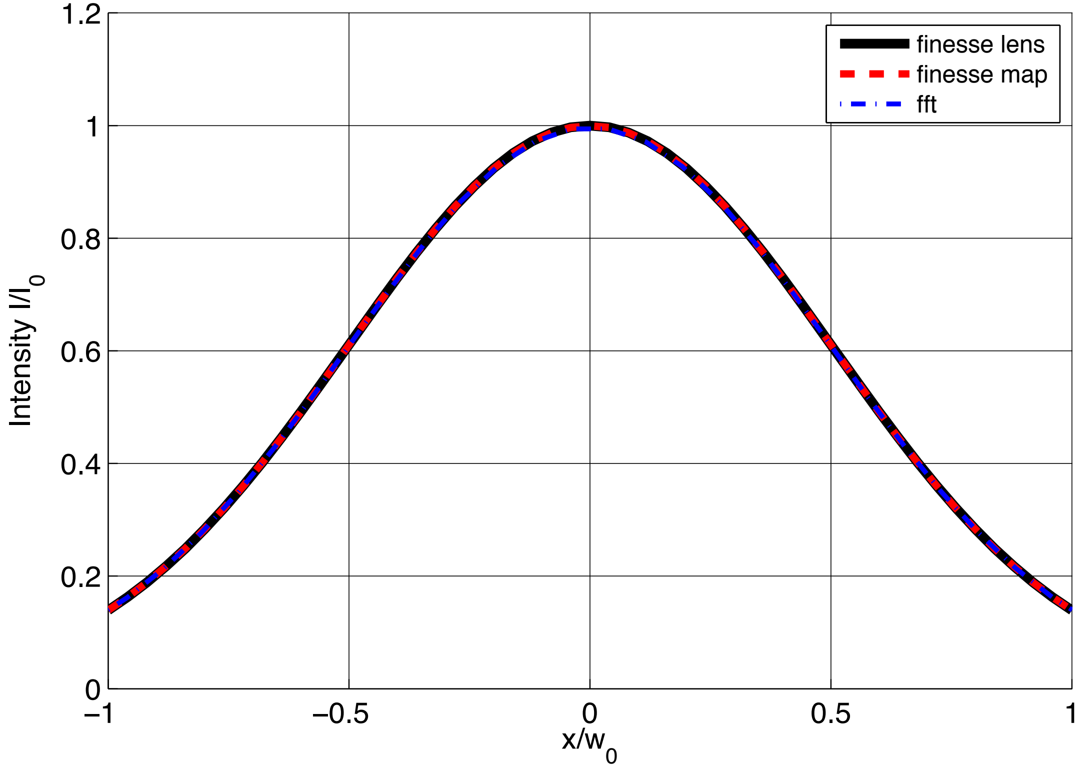


Simulated with 3 different methods:

1. FINESSE `lens` component
   Calculate effect using ABCD matrix.

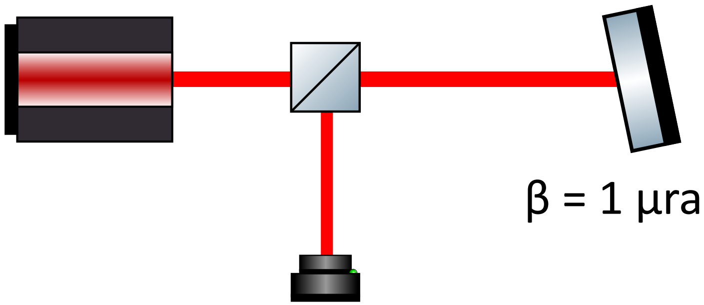
2. FINESSE with map
   Map describing the optical path length of lens.

3. FFT



Transmitted beam, cross-section

Tilted mirror: maps in reflection

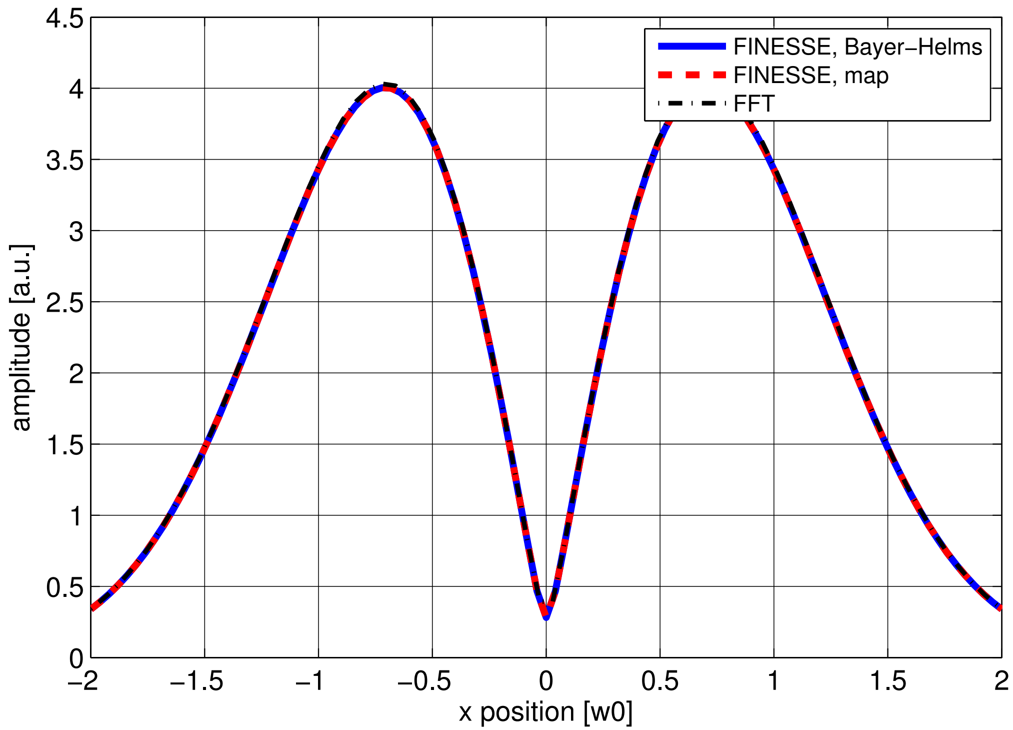


Simulated with 3 different methods:

1. FINESSE `attr xbeta` command
   Calculate effect using analytic coupling coefficients (Bayer-Helms) .

2. FINESSE with map
   Map describing tilted surface.

3. FFT

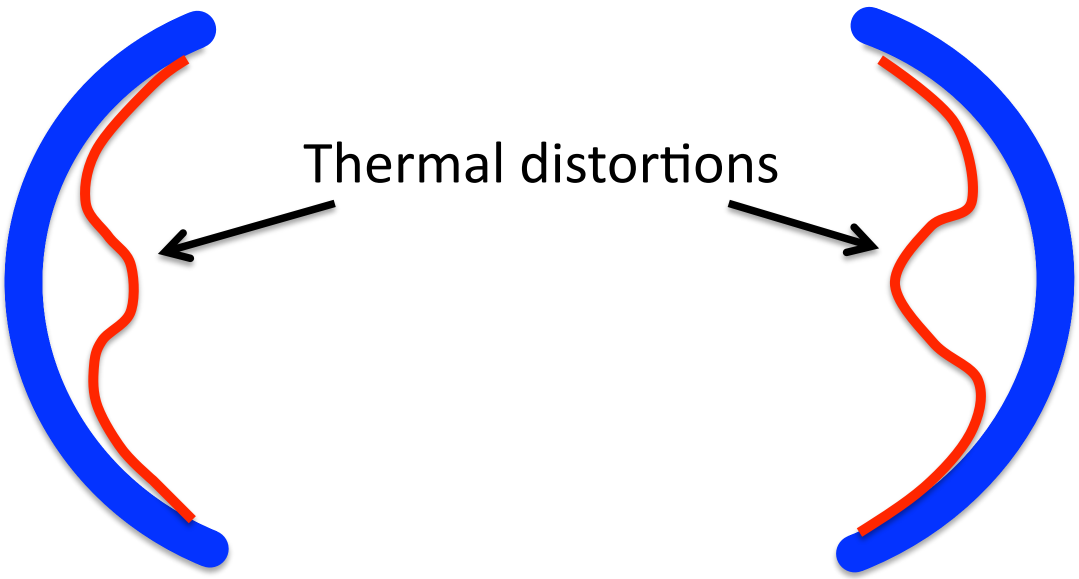


Reflected field with HG00 component removed, cross-section

Thermal challenge

- High circulating beam power leads to thermoelastic distortion of aLIGO cavity mirrors due to absorption in the coatings.

- We consider three cases: no absorption on either mirror; 1ppm absorption on the ETM and 1ppm absorption on both the ETM and ITM.

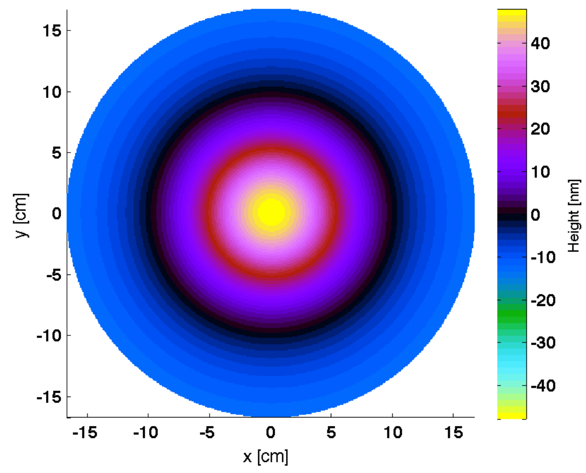


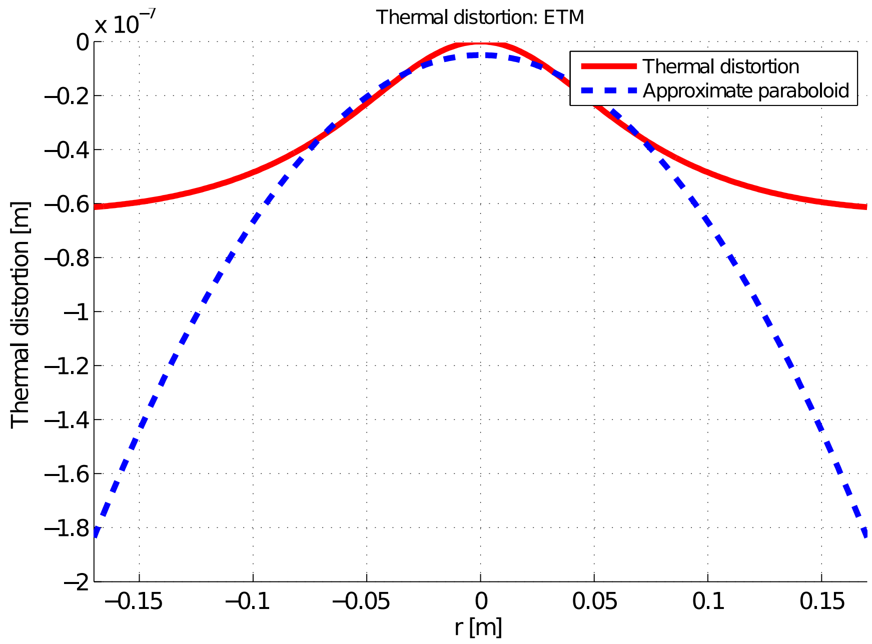
Thermal challenge: Mirror maps

- To represent the thermal distortions in Finesse mirror maps were created using the Hello-Vinet formula.

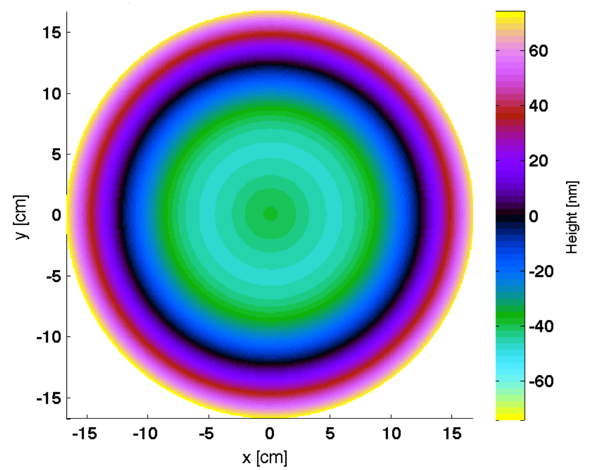
- To reduce the number of modes required in the Finesse simulations the curvature, weighted by the Gaussian beam, is removed.

ETM thermal map





ETM thermal map with curvature removed



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Thermal challenge: Results

Original results (Hiro Yamamoto)

Thermal distortion		Roundtrip loss [ppm]	
ITM	ETM	Hankel	SIS
-	-	0.8	0.8
-	✓	228	220
✓	✓	9.3	9.6

Current investigation

Thermal distortion		Roundtrip loss [ppm]		
ITM	ETM	Finesse (maxtem 10)	Finesse (maxtem 15)	FFT (OSCAR)
-	-	0.8	0.9	0.9
-	✓	203	238	234
✓	✓	32	26	27

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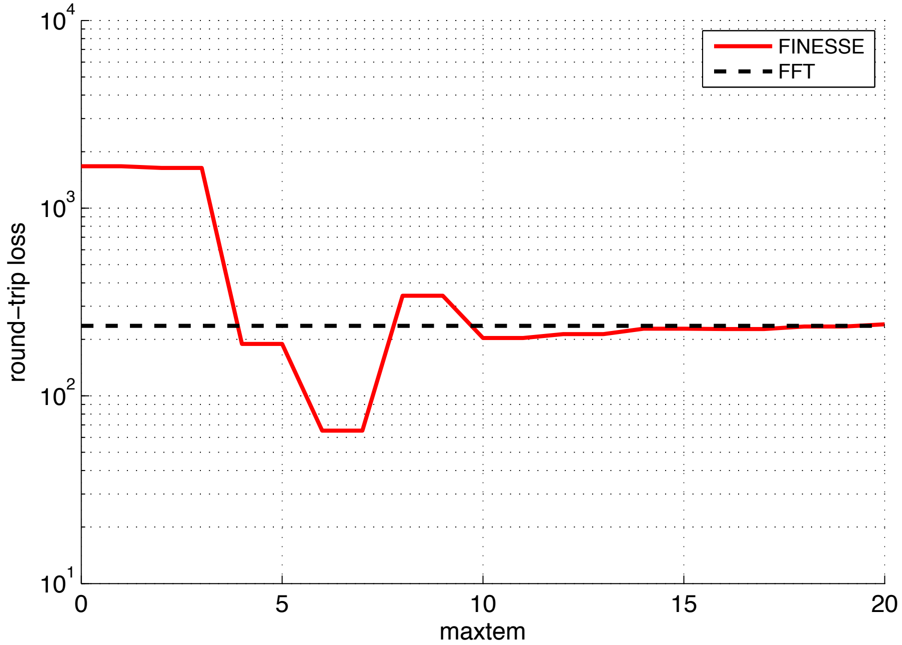
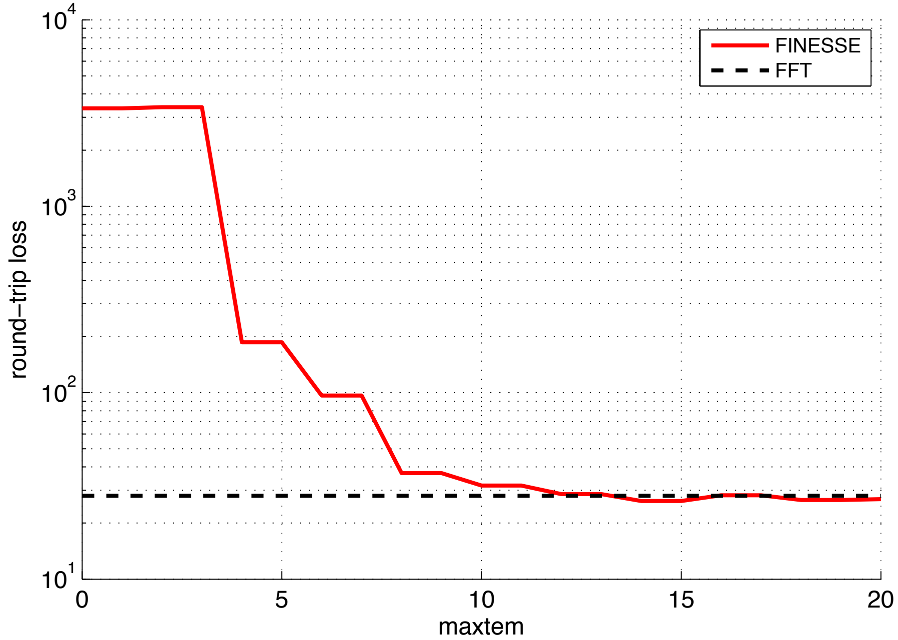
Thermal challenge: Results

- Results converge with high enough maxtem (10 – 15).



Absorption on ETM



Absorption on ITM and ETM

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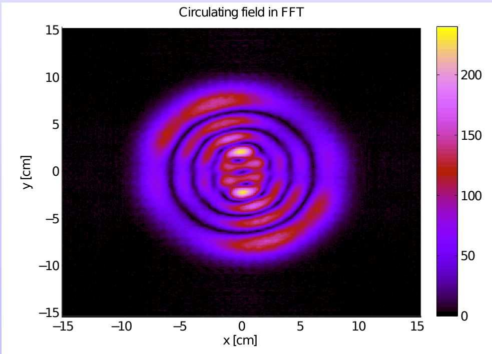
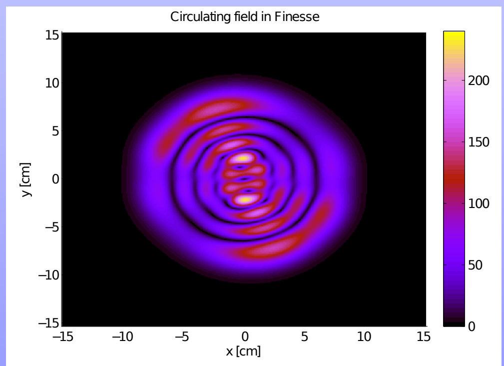
Other examples

LG$_{33}$ with mirror maps

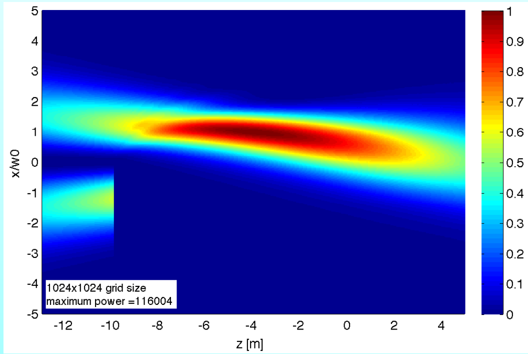


FFT

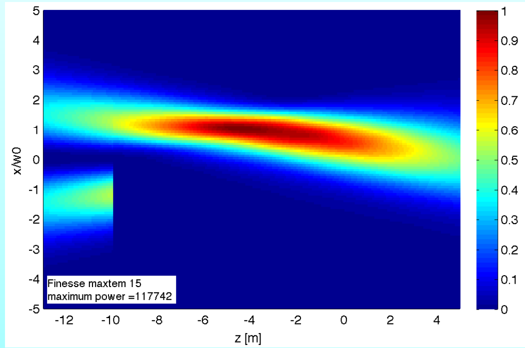


Finesse

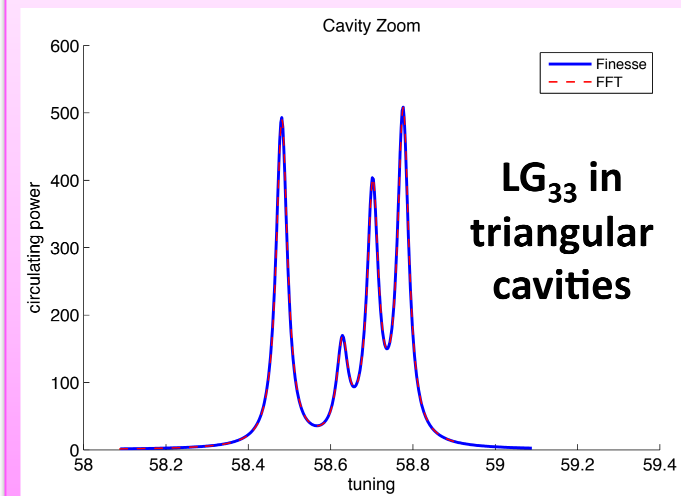
Edge effects



FFT



Finesse



LG$_{33}$ in triangular cavities

Why use modal models?

- Advantages:
  - Can be faster for complex tasks (i.e. noise couplings and combinations of maps, misalignments and mode mismatch).
  - Provides efficient and intuitive models of well behaved interferometers with `small' distortions (i.e. GW interferometers).

- Disadvantages:
  - Cannot model optics with significant power near the edge.
  - Can be slower at specific tasks, in particular calculation of optical losses from strong distortions.

- Carefully setup FFT simulations and model simulations should provide the same answer in the majority of cases.

Example: Bond et al., 2011

Higher order Laguerre-Gauss mode degeneracy in realistic, high finesse cavities

Phys. Rev. D 84, 102002 (2011)

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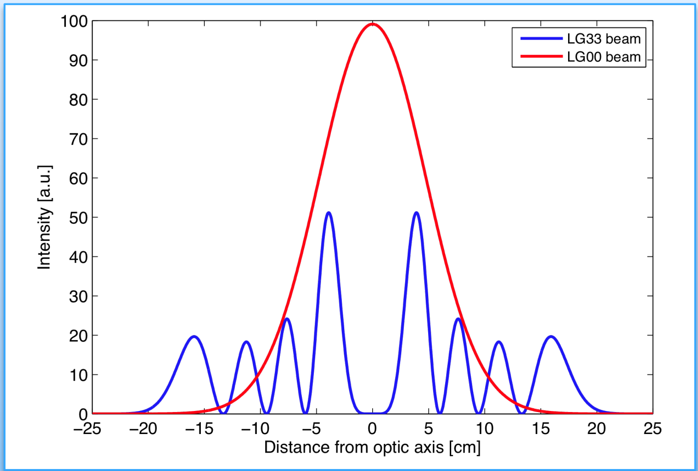
LG_{33} investigation

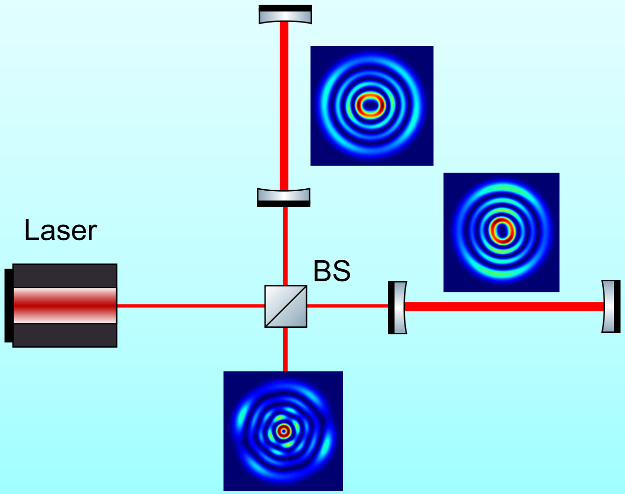


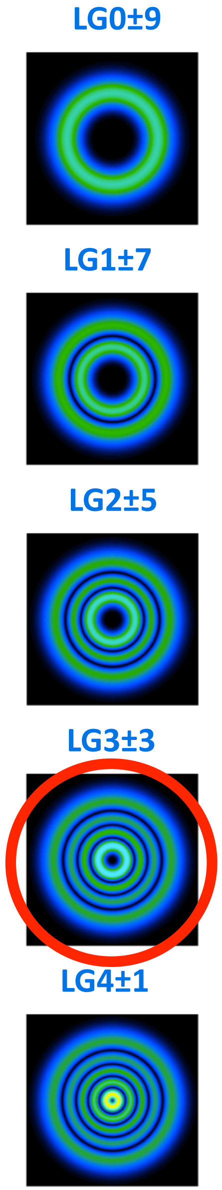
Higher order mode degeneracy

- LG_{33} mode proposed for its potential to reduce the thermal noise of the test masses.
- 10 modes of order 9 which will all be resonant in the arm cavities.

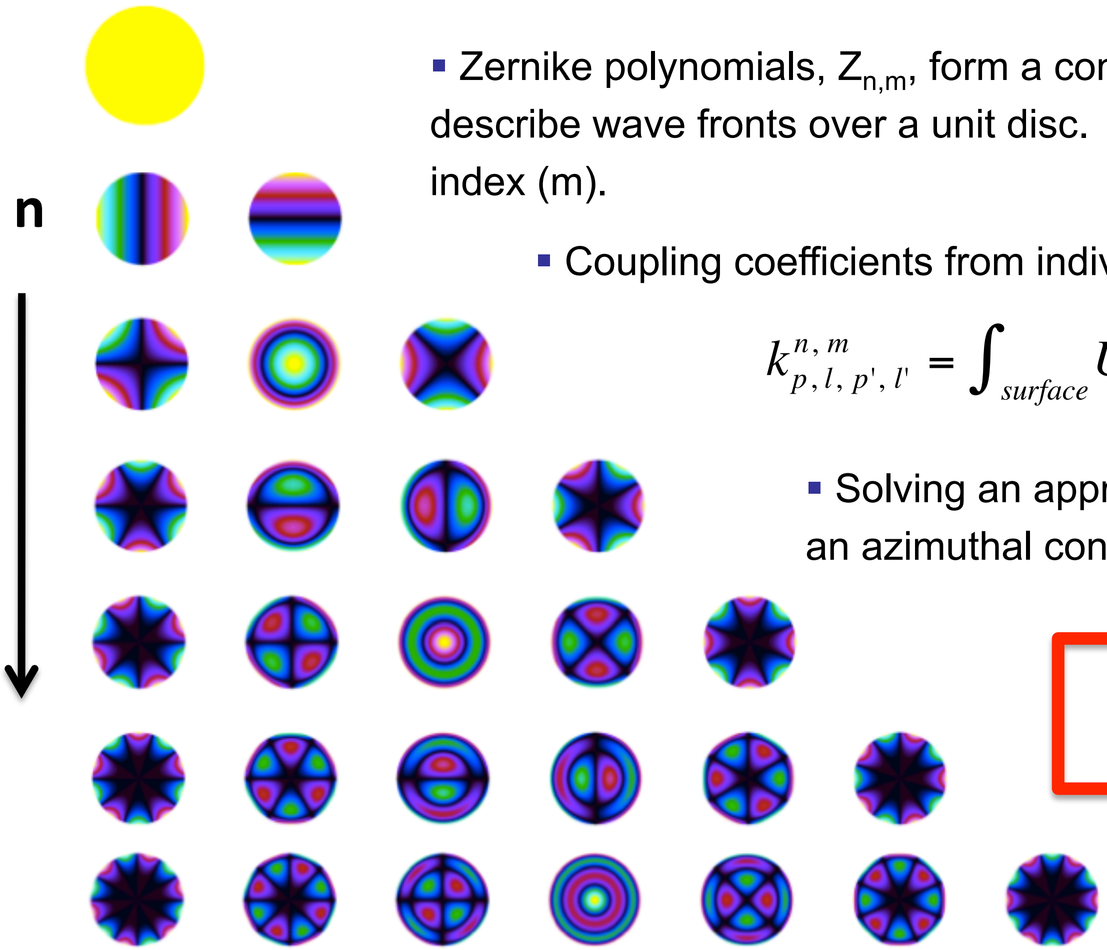
Potential problem

- Coupling into the order 9 modes leads to highly distorted circulating beams.
- Surface distortions and modes will be different in each arm, potentially resulting in an unacceptably high contrast defect at the output.





Coupling approximation: Zernike polynomials



- Zernike polynomials, $Z_{n,m}$, form a complete set of orthogonal functions which describe wave fronts over a unit disc. Defined by radial index (n) and azimuthal index (m).
  - Coupling coefficients from individual polynomials are calculated:

$$k^{n,m}_{p,l,p',l'} = \int_{surface} U_{p,l} \exp(2ikZ^m_n) U^*_{p',l'}$$

- Solving an approximation of k we get an azimuthal condition for coupling:

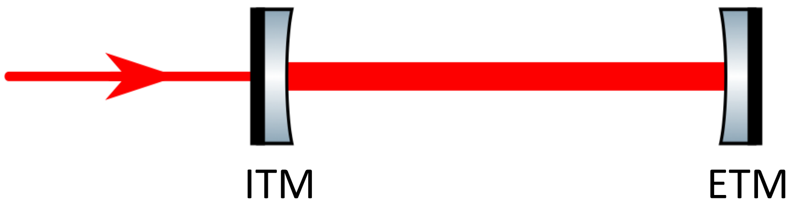
$$m = |l - l'|$$

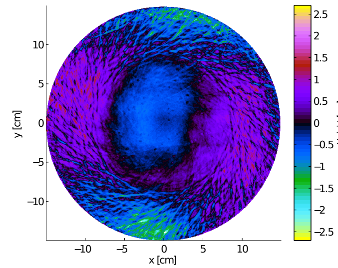
Mode	m
2, 5	2
4, 1	2
1, 7	4
4, -1	4
0, 9	6
3, -3	6
2, -5	8
1, -7	10
0, -9	12

m required for coupling from LG_{33} to the other order 9 LG modes

LG_{33} investigation: results

- High finesse cavity simulated with an LG_{33} input beam and aLIGO map applied to ETM.





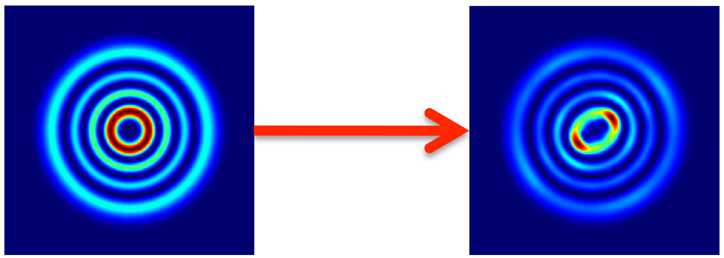
Mode	Power [%]
3, 3	88.6
4, 1	**5.70**
2, 5	**5.02**
4, -1	0.333
1, 7	0.313
other	< 0.05

- Direct coupling from etm08 calculated using Zernike approximation.

Polynomial	2, 2	4, 2	4, 4	other
Amplitude [nm]	0.908	0.202	0.231	-
Power [ppm]	4.66	0.331	0.0431	< 0.01

We expect **LG_{41}** and **LG_{25}** to have large amplitudes in the cavity

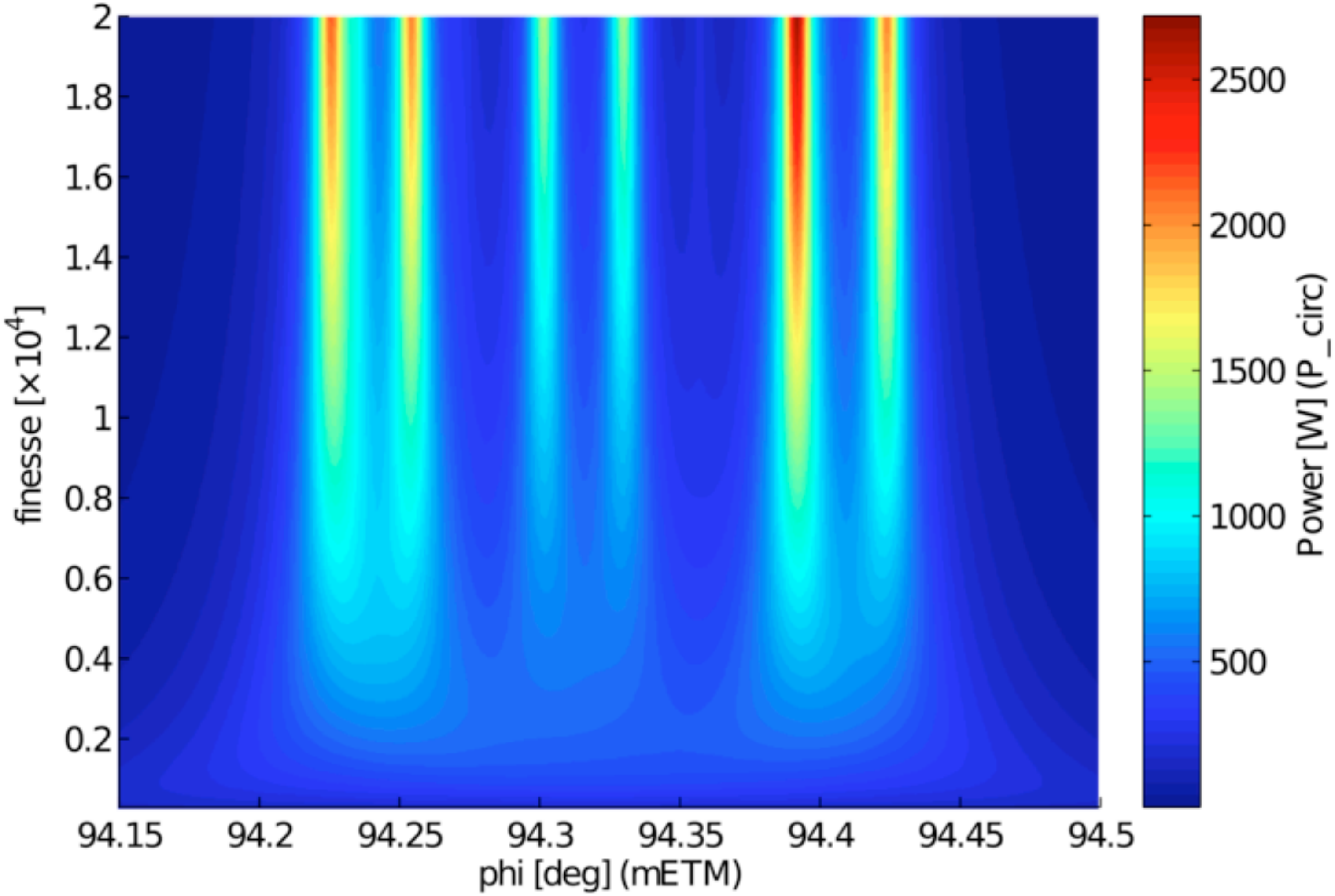
- Large presence of LG_{41} and LG_{25} in the circulating beam in Finesse model agrees with the prediction by the coupling approximation.



Pure LG33 input beam

Circulating field with ETM08

LG_{33} investigation: frequency splitting



LG_{33} investigation: mirror requirements

- Coupling approximation is used to derive mirror requirements.
- **Requirement:** power coupled from individual polynomials < 0.01 ppm.
- For the ETM08 example:

Polynomial	2, 2	4, 2	4, 4
Amplitude [nm]	0.042	0.035	0.10

	Original ETM08	Adapted ETM08
Direct coupling into order 9 [ppm]	6.8	0.043
Circulating field impurity [ppm]	114,000	815

[1] Bond et al. Phys. Rev. D, 2011, 84, 102002

Example: Carbone et al., 2013 [1]

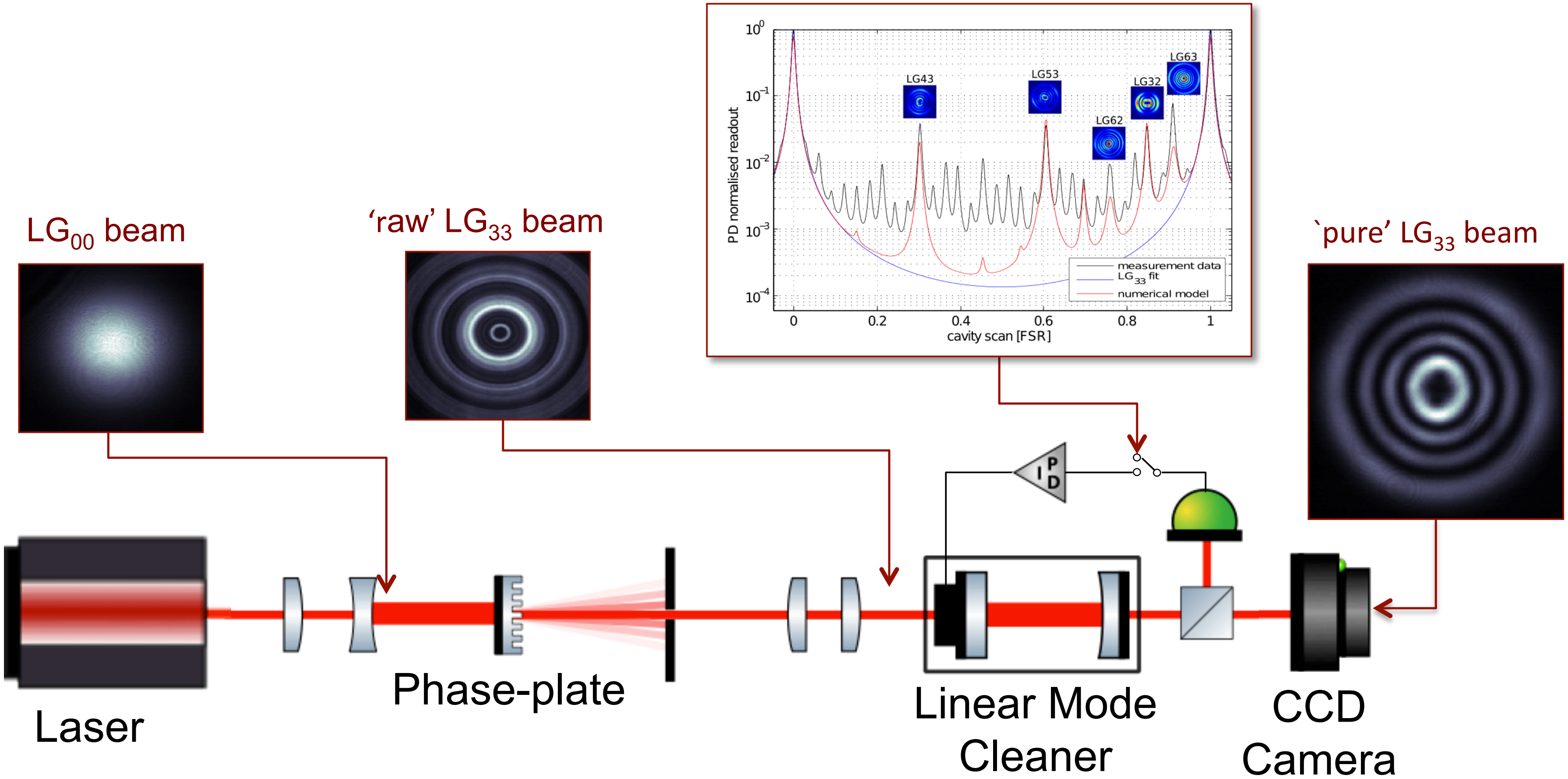
The generation of higher-order Laguerre-Gauss optical beams for high-precision interferometry

Carbone et al., 2013 [2]

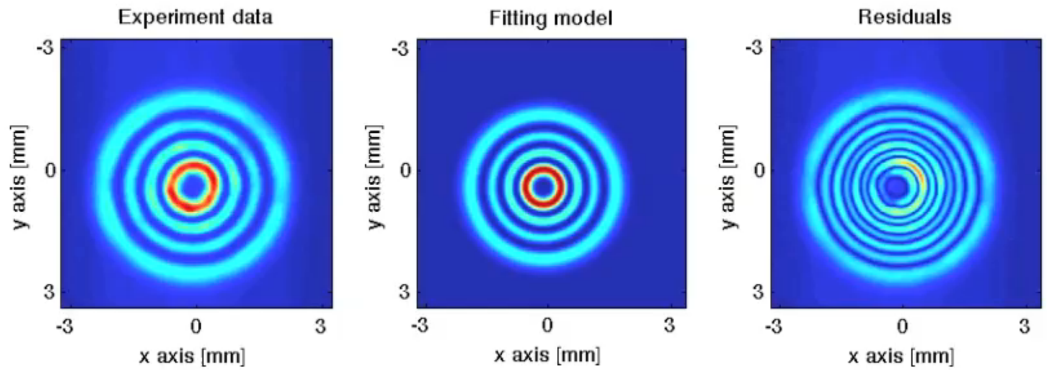
Generation of High-Purity Higher-Order Laguerre-Gauss Beams at High Laser Power

[1] Journal of Visualized Experiments, accepted for publication (2013)
[2] Phys. Rev. Lett. 110, 251101 (2013)

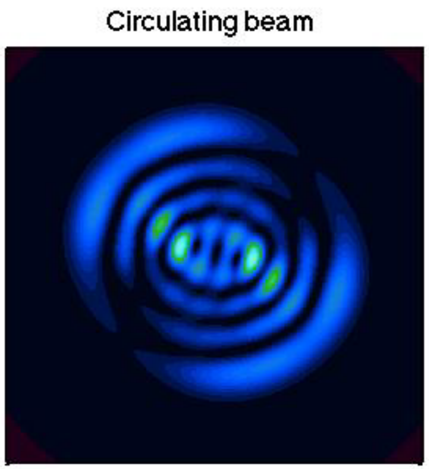
Generating an LG_{33} beam



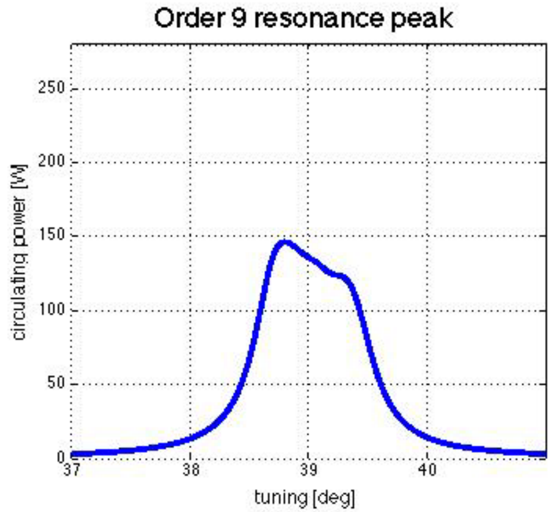
Analyzing output beam

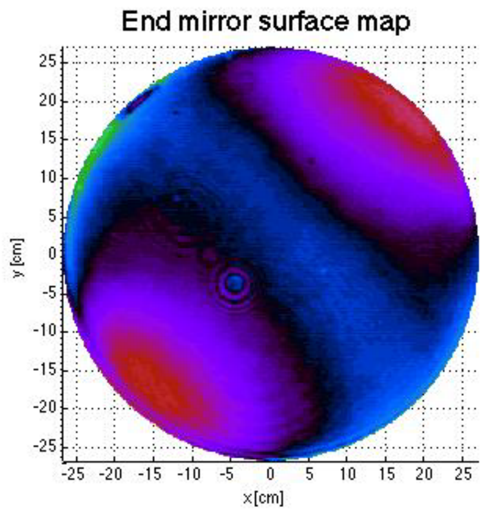


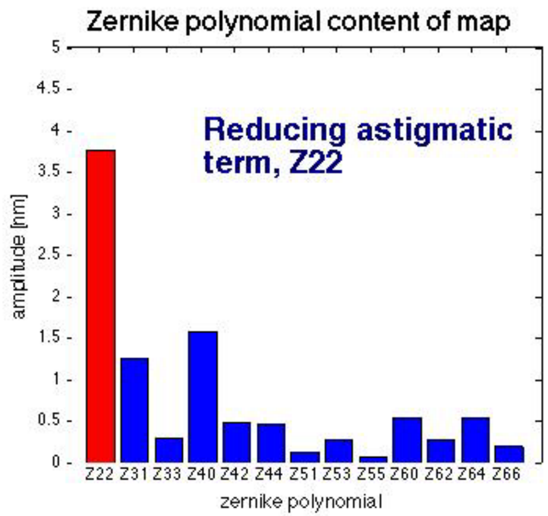
Designing mirrors for LG_{33}



LG33 purity = 55.8 %







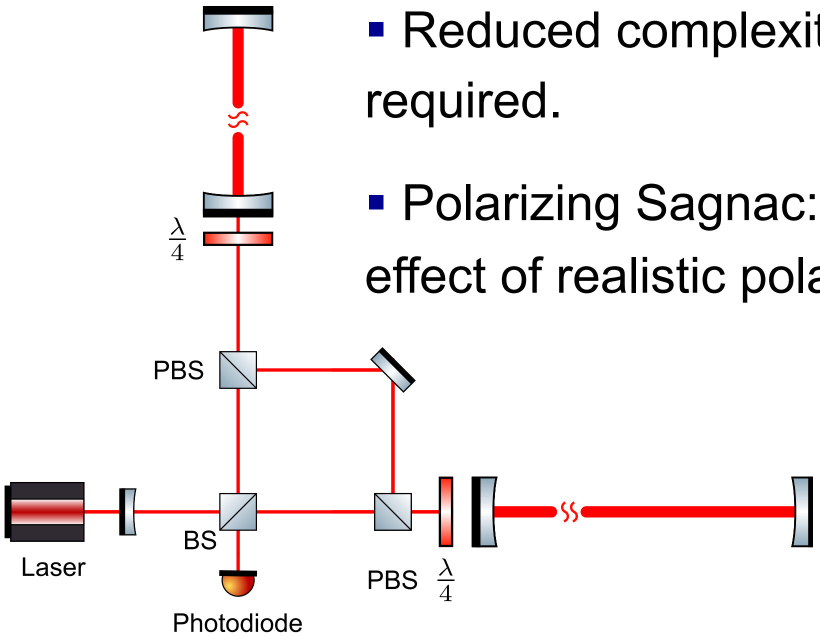
Example: Wang et al., 2013

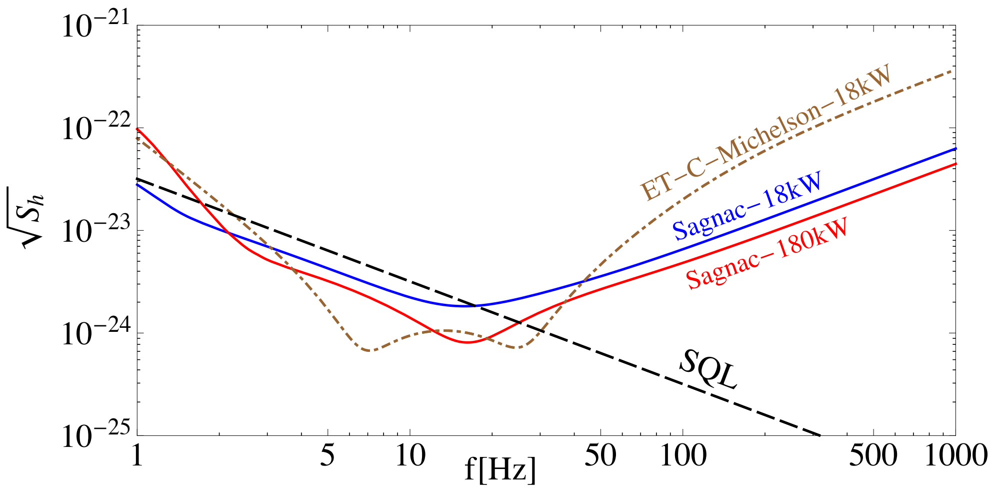
A realistic polarizing Sagnac topology with DC readout for the Einstein Telescope

Phys. Rev. D 87, 096008 (2013)

ET-LF as Sagnac interferometer

- A Sagnac interferometer has the potential for a similar sensitivity as the current ET-LF Michelson design.
- Reduced complexity: no filter cavities or signal recycling required.
- Polarizing Sagnac: no ring cavities but we need to consider effect of realistic polarizing optics





[1] Wang et al. Phys. Rev. D, 2013, 87, 096008
[2] Mengyao Wang, GWADW 2013

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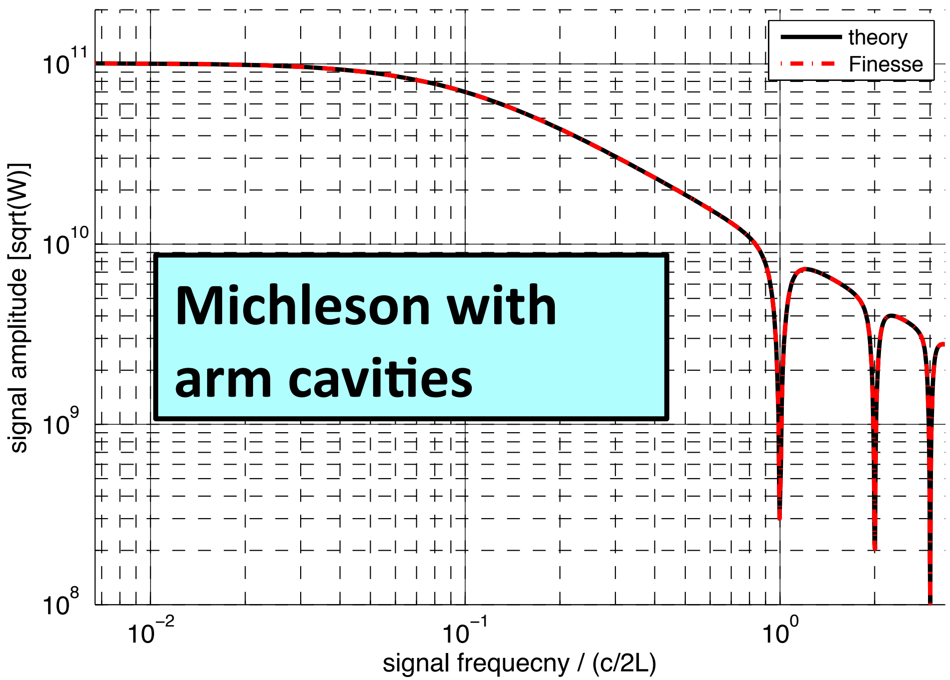
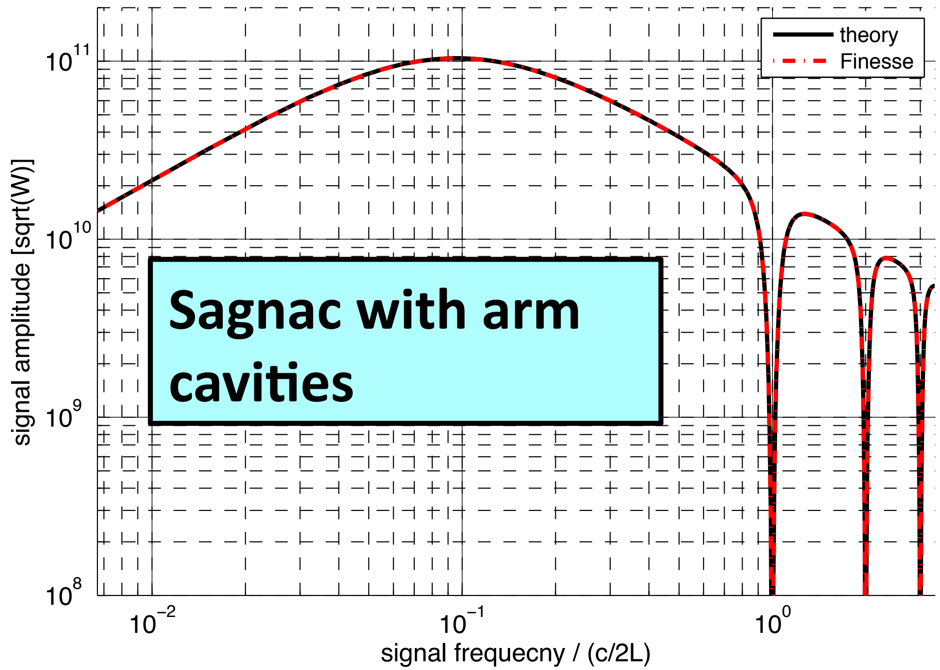
FINESSE testing: Sagnac and Michelson transfer functions



- Get a basic idea of how a Sagnac interferometer works, compared to a Michelson.



- Further testing of FINESSE against theoretical calculations.

[1] C. Bond et al., LIGO DCC: T1300190

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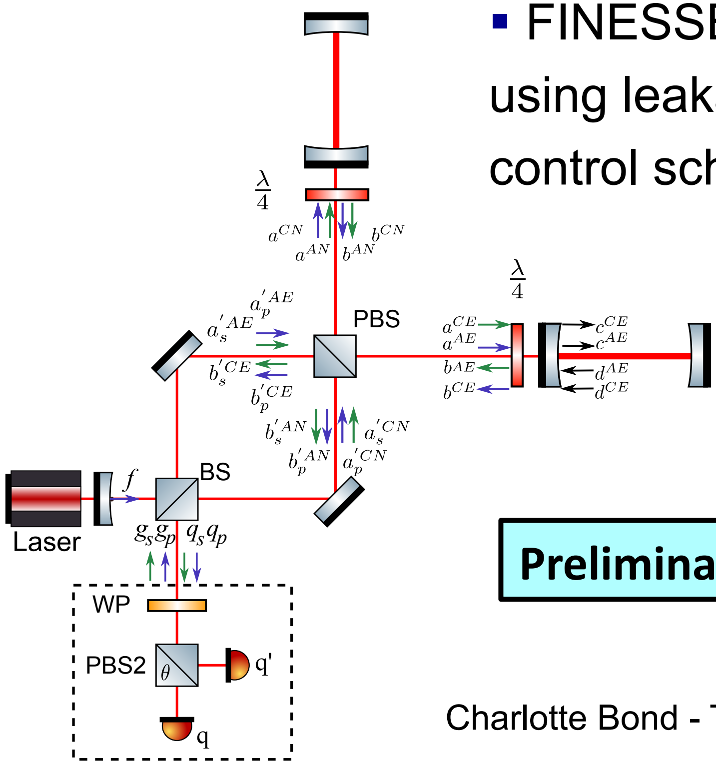
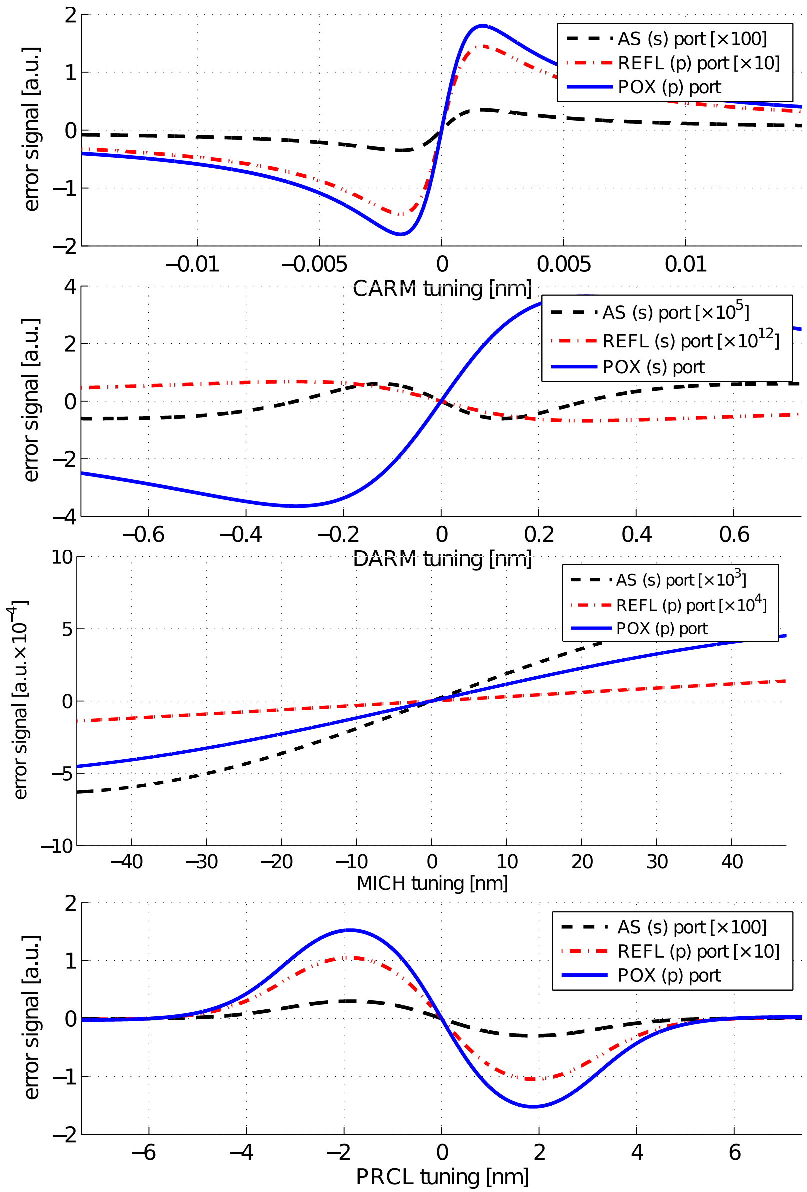
Preliminary investigation into Sagnac control

- Consideration of realistic polarizing components: leakage with Michelson-like response at the output.

- FINESSE simulations using leakage for potential control scheme.



Preliminary investigation



FINESSE available as open source

- Main page: http://www.gwoptics.org/finesse/
  - Executable
  - Documentation
  - Finesse examples
  - Simtools
  - Links to . . .

- Source code: http://kvasir.sr.bham.ac.uk/redmine/projects/finesse
  - Source code
  - Forum
  - Bug reports

Thank you for listening

FINESSE: What's the proper reference?

The best way to reference FINESSE (e.g. in a paper) is by citing

A Freise and G Heinzel and H Luck and R Schilling and B Willke and K Danzmann
Frequency-domain interferometer simulation with higher-order spatial modes
Classical and Quantum Gravity, 2004

…and our website http://www.gwoptics.org/finesse

Thanks!