Stochastic gravitational wave and its spectral property

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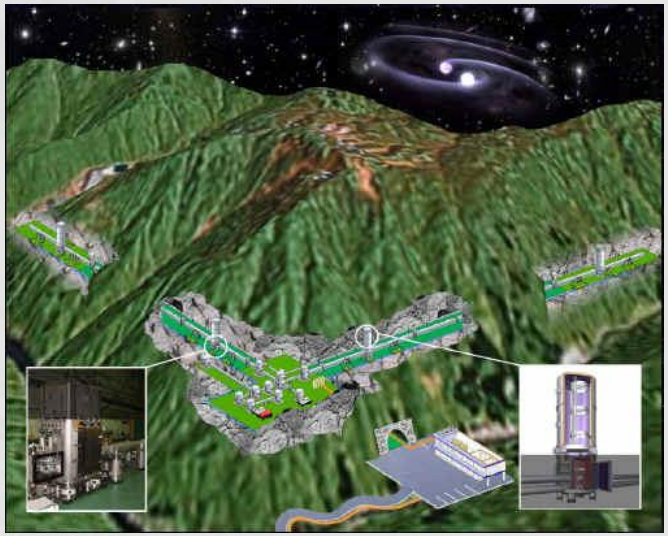
GW Observatories



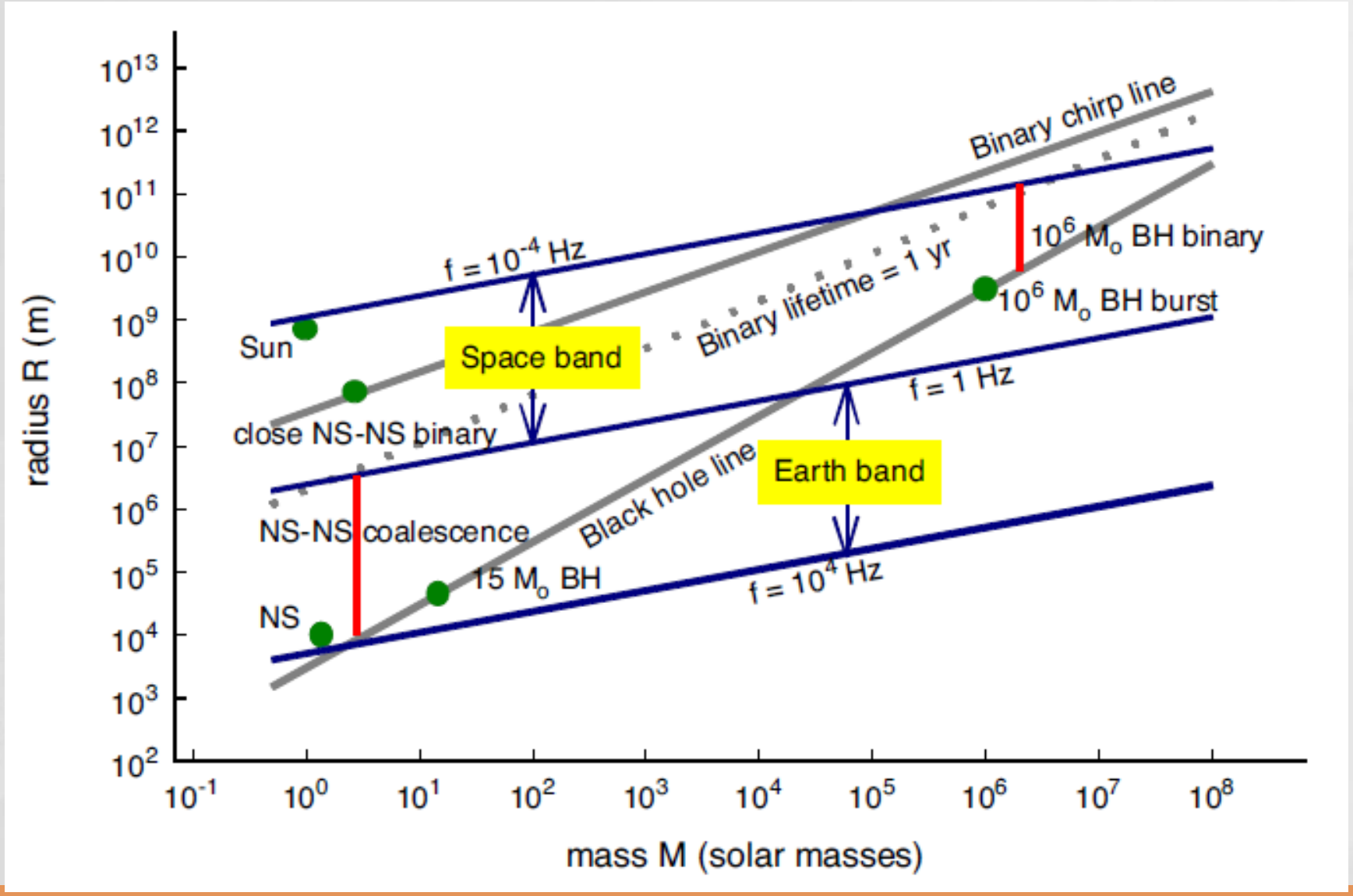
Introduction

- New window onto the Universe
- Large scale Laser interferometer detector for Gravitational Wave(GW)
  - LIGO, EGO-Virgo, GEO600, KAGRA
  - LISA, ET, BBO
  - Detecting known astrophysical sources
  - Expect real detection around 2020
- Possibly detect stochastic GW of cosmological origin
  - A new window to study very early universe

Gravitational wave generation

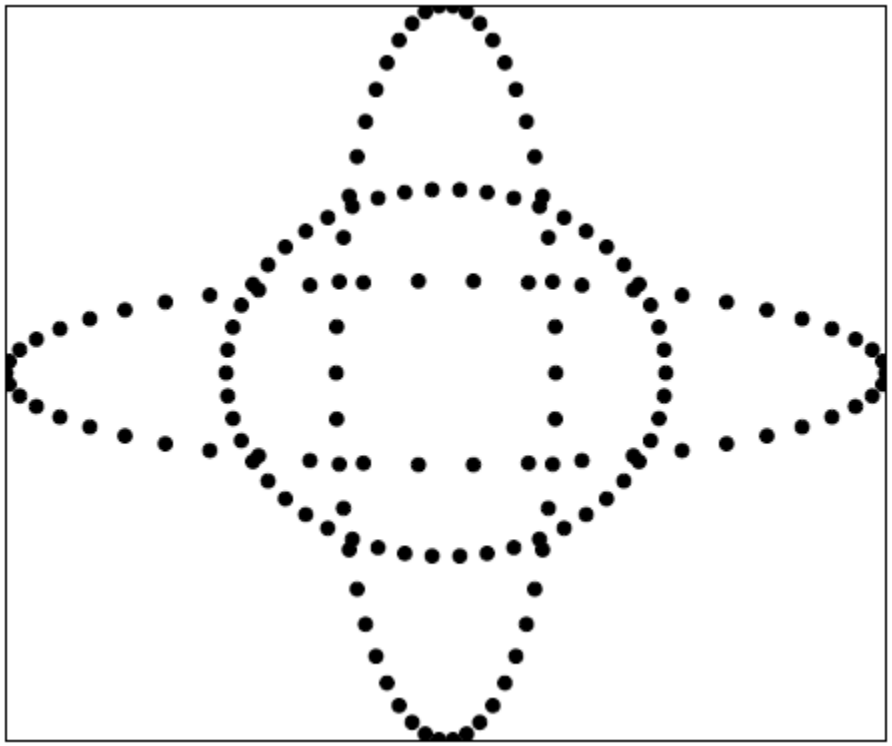
- Triple time variation of quadruple moment
- Sources
  - Man-made source (not possible to detect)
  - Astrophysical Objects
    - Burst from collapse
    - Pulsar
    - Binary system
    - Quasinormal mode of black hole
    - Stochastic background
  - Cosmological background

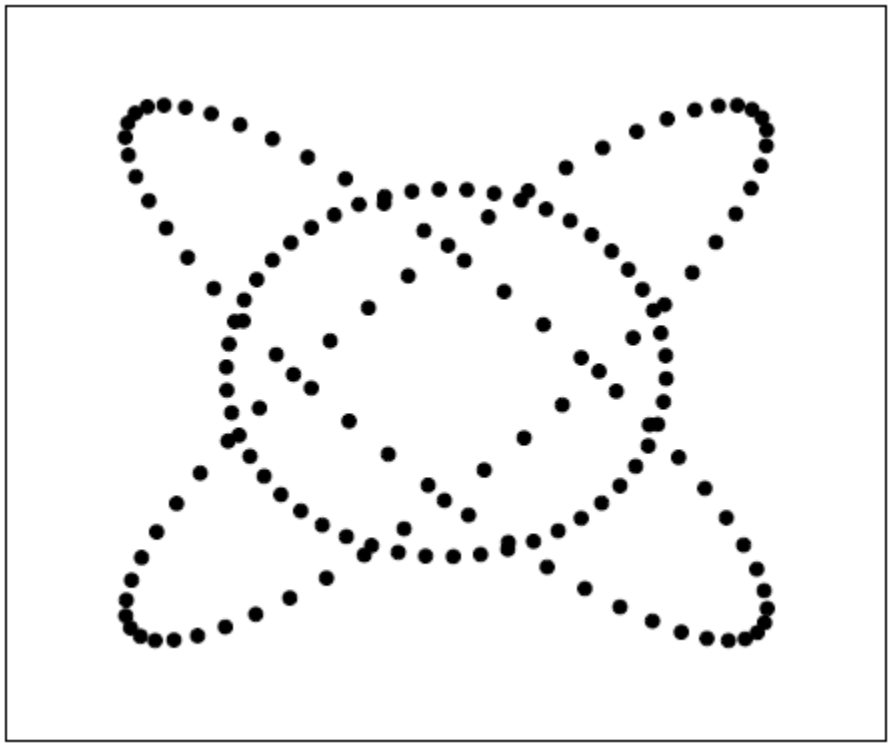
Astrophysical Sources



Gravitational wave

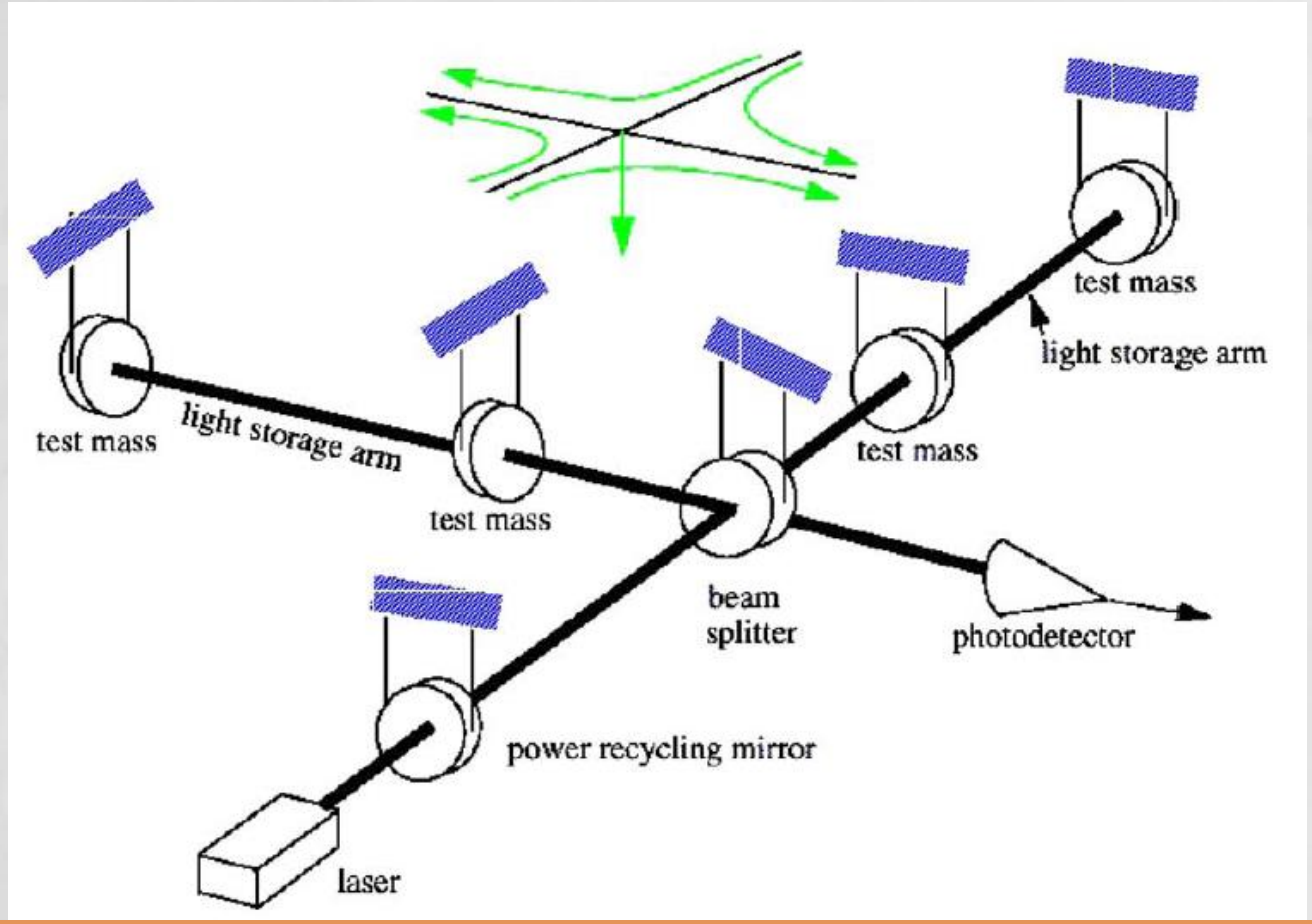
- Two polarization mode



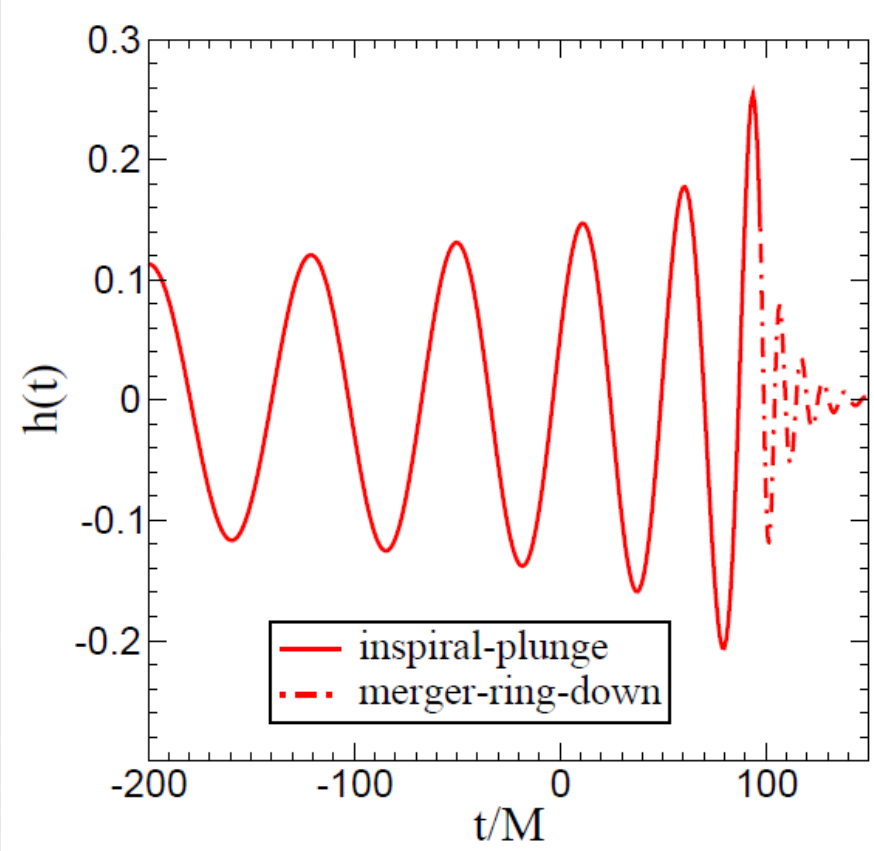


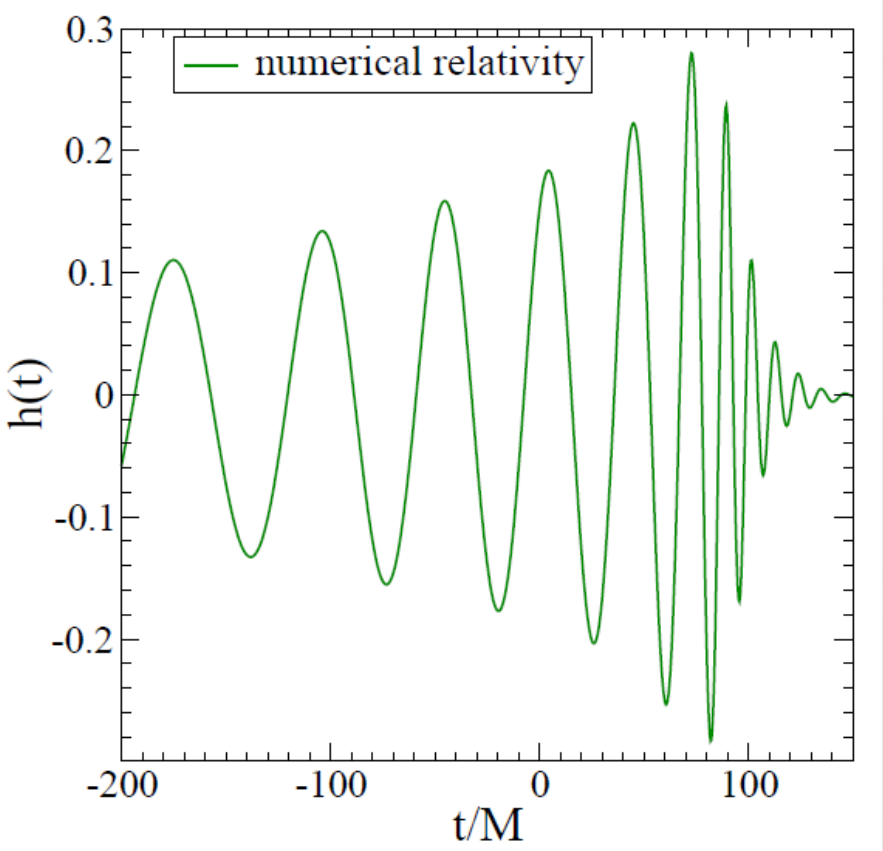
Detection of GW

- Large scale interferometer

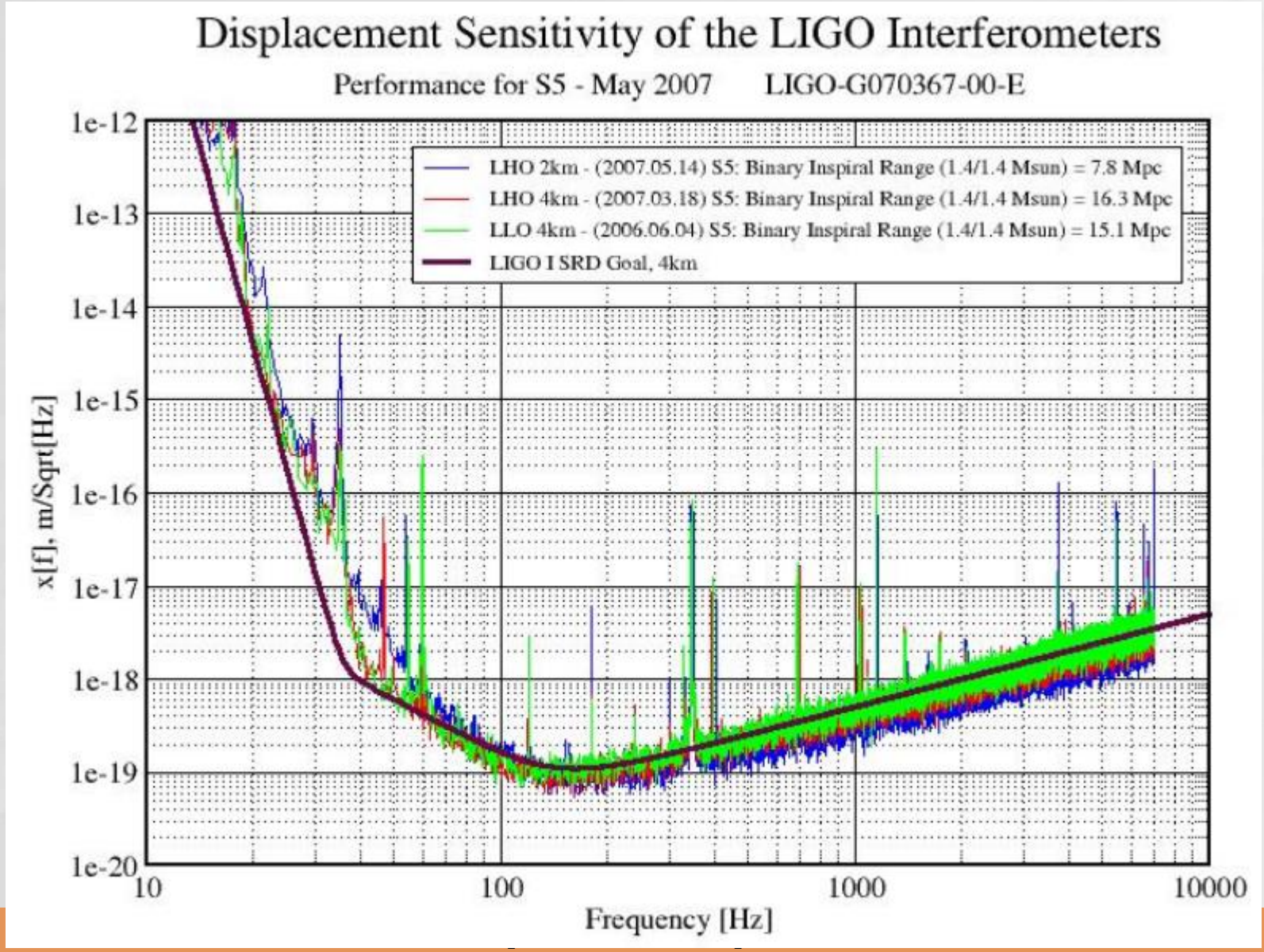


Matched filter

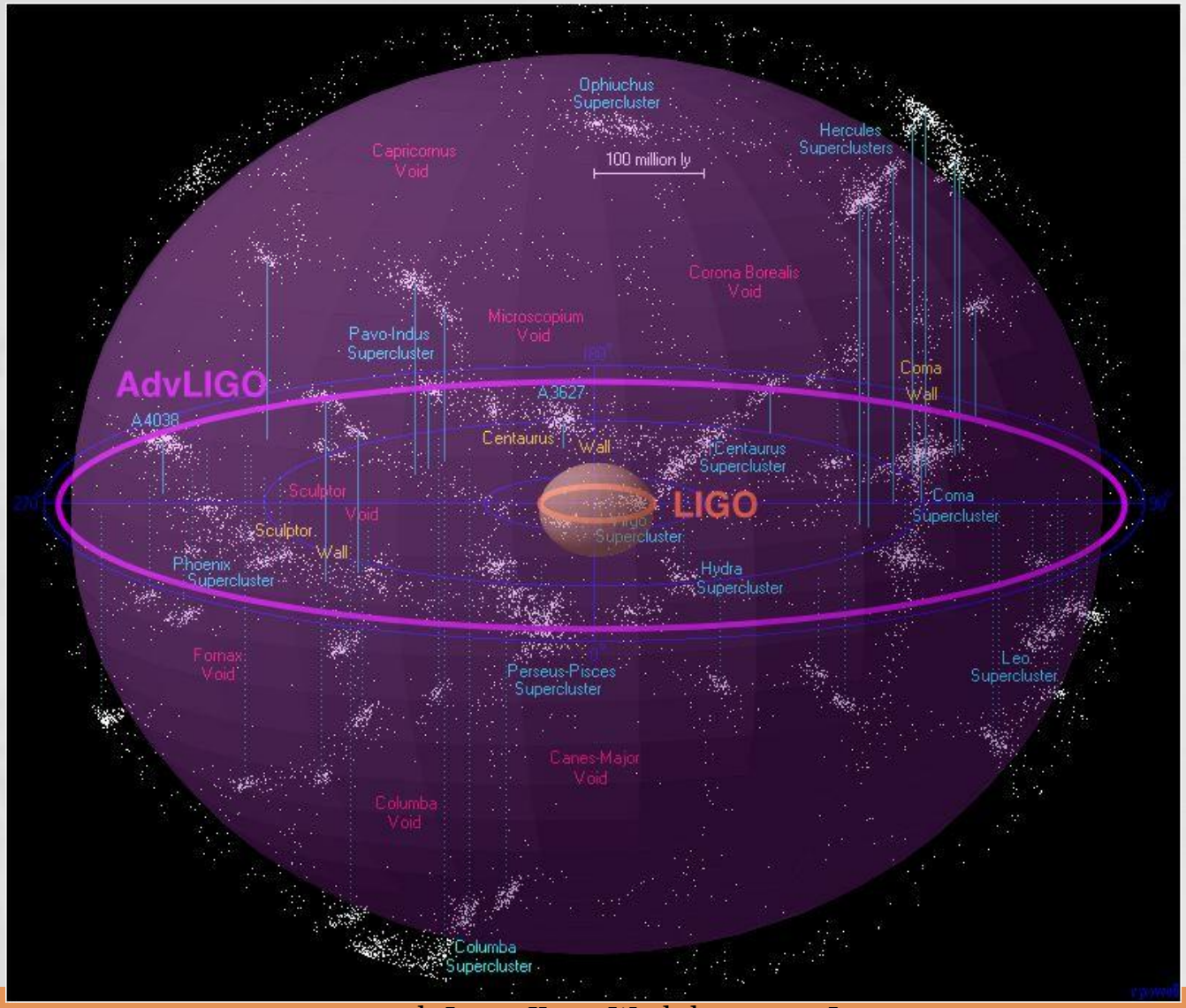




Sensitivity

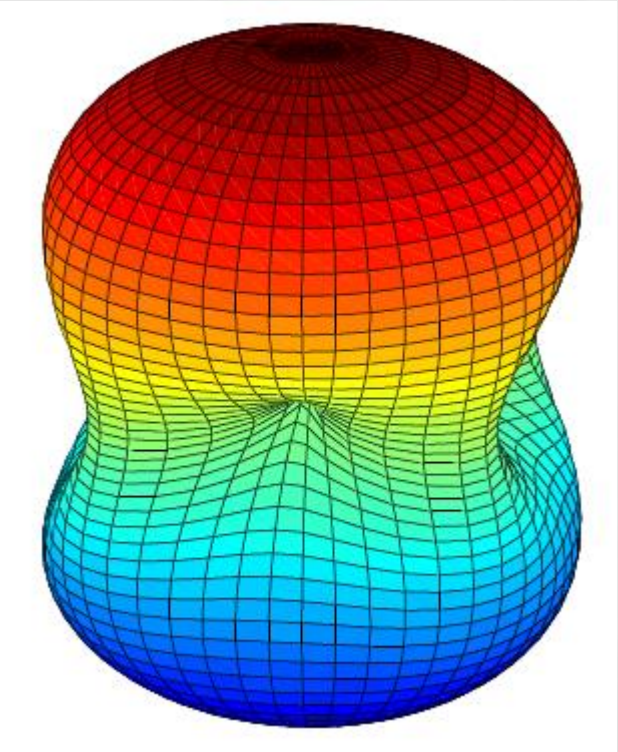


Detection range



Data Analysis

- GW detector is Omni-directional : all-sky search
- Interferometer is broad band
- GW is polarized : correlation measurement required
- Measure phase of GW
- Computationally intense



Characterization of stochastic backgrounds of GWs

- Isotropic, stationary, unpolarized
- Frequency spectrum is main property
  - Energy density per unit log frequency, $h_0^2 \Omega_{GW}(f)$
  - Spectral density of ensemble average the Fourier component of the metric, $S_h(f)$
  - Characteristic amplitude of the stochastic background, $h_c(f)$

Energy density parameter

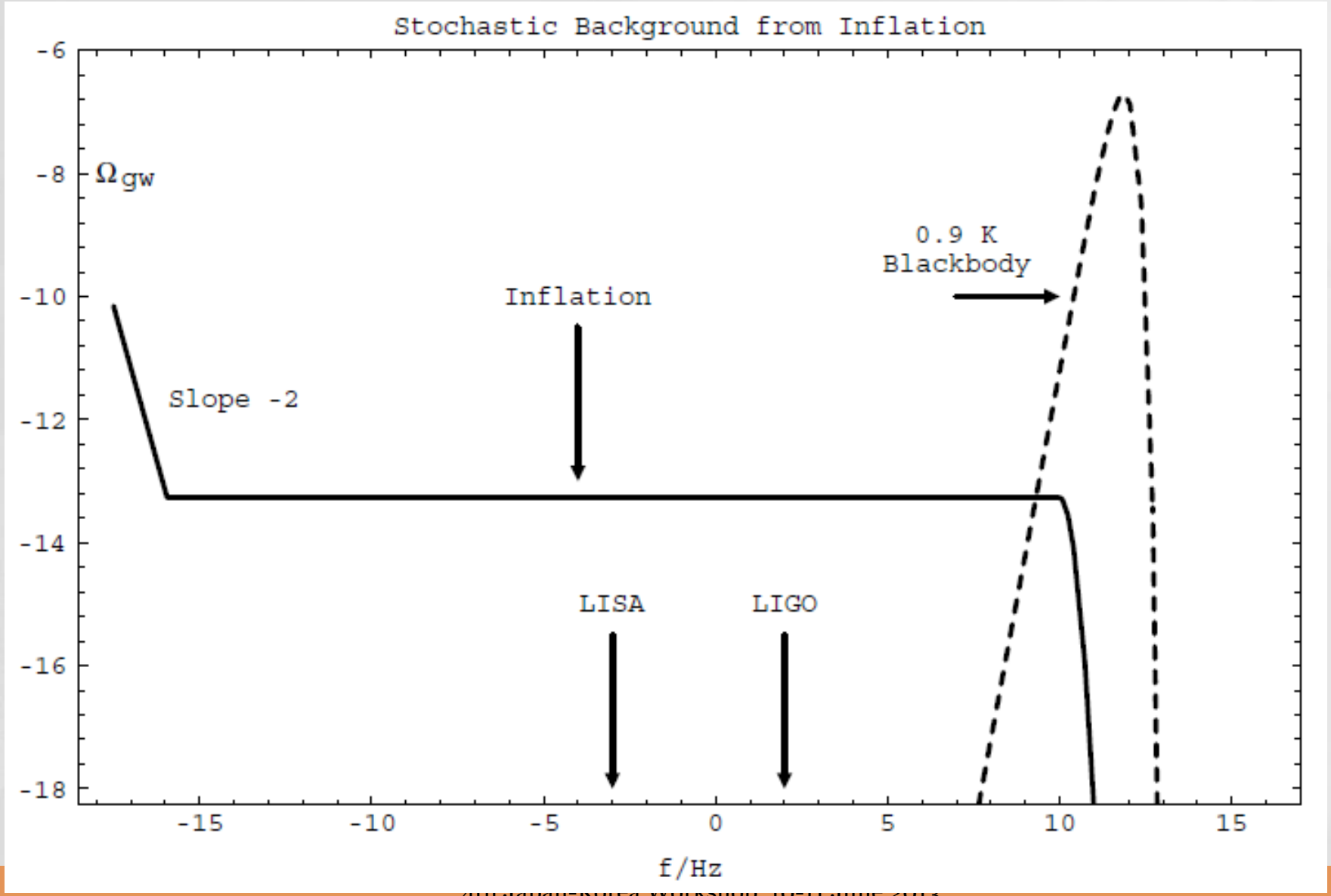
- Definition $\Omega_{GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\ln f}$
- $\rho_c = \frac{3H_0^2}{8\pi G}$, $H^2 = \frac{8\pi G}{3}\rho$ (1st Hubble equation)
- $H_0 = h_0 \times 100\text{km/s}\cdot\text{Mpc}$, $0.5 < h_0 < 0.65$
- $h_0^2\Omega_{GW}(f)$ independent of h_0
- Total density parameter for GW

$$\Omega_{GW} = \int_0^\infty \Omega_{GW}(f) d\ln f$$

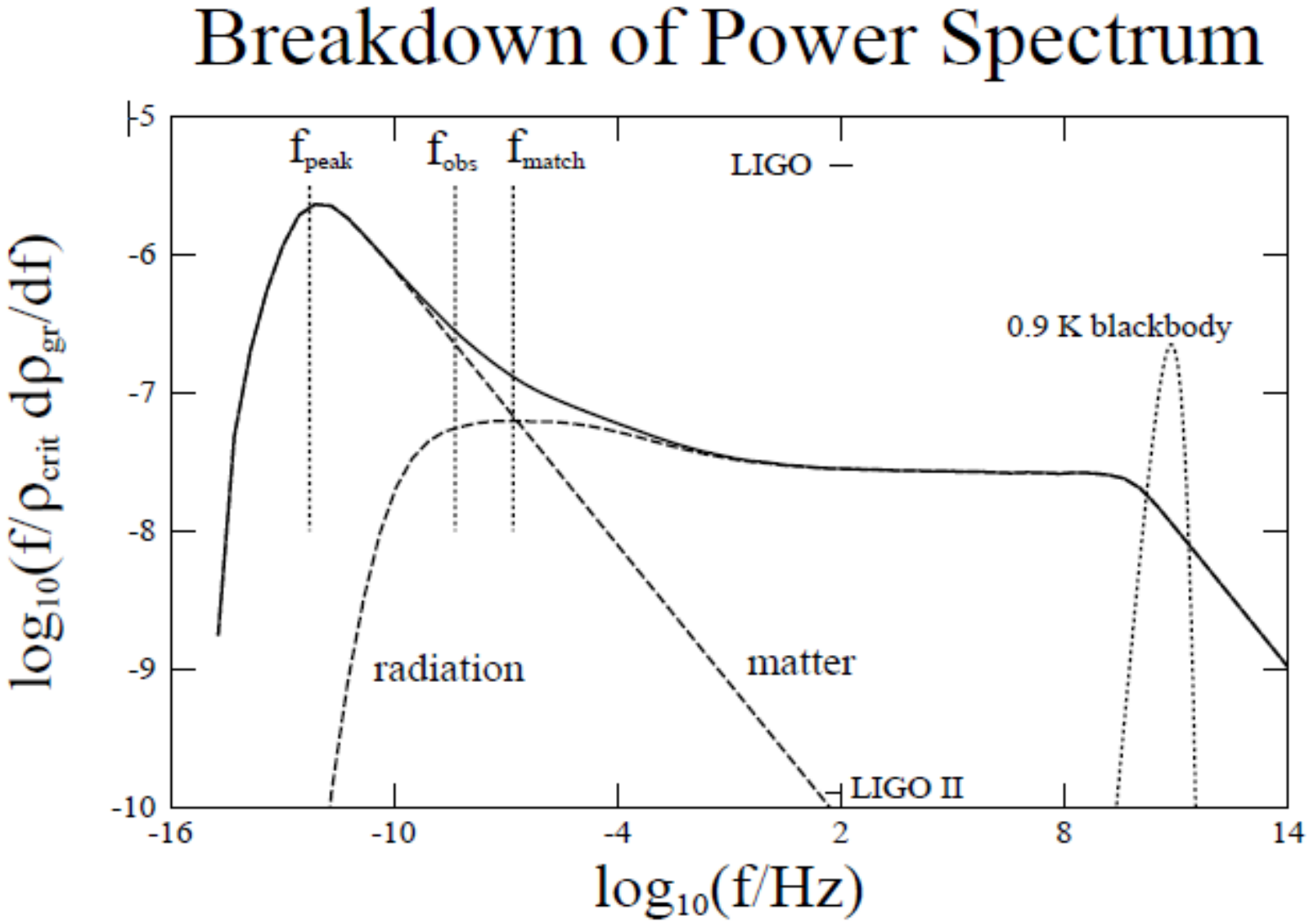
Sources of stochastic background of GWs(Allen, 1996)

- Inflation : Allen, et al.
- Cosmic String network : Vilenkin, et al.
- Bubbles from Phase transition : Kamionkowski, et al.
- Modified f(R) gravity : Capozziello, et al.
- Modified Chaplygyn Gas : Bouhmadi-Lopez, et al.
- New Idea?

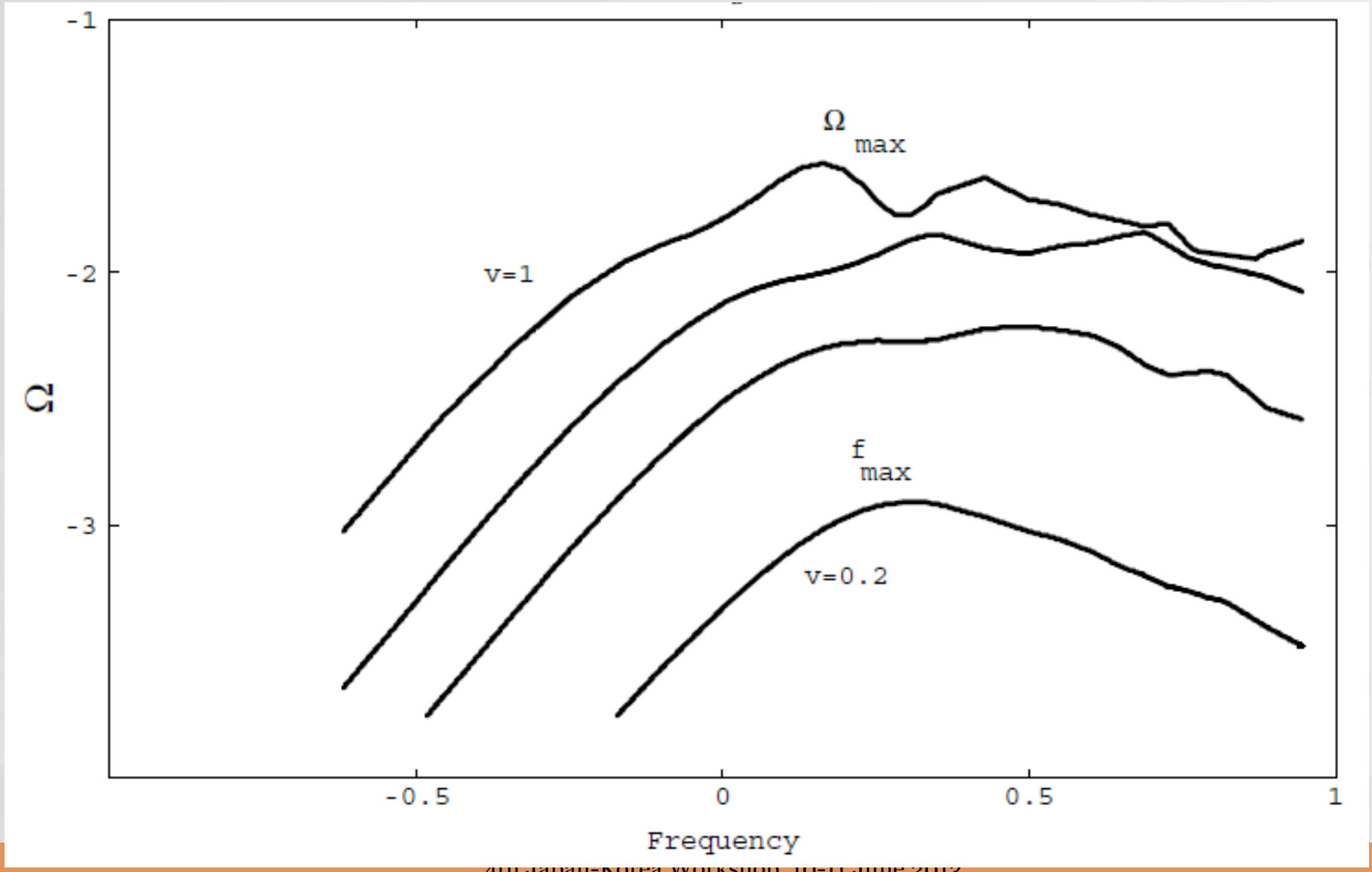
Spectrum by Inflation



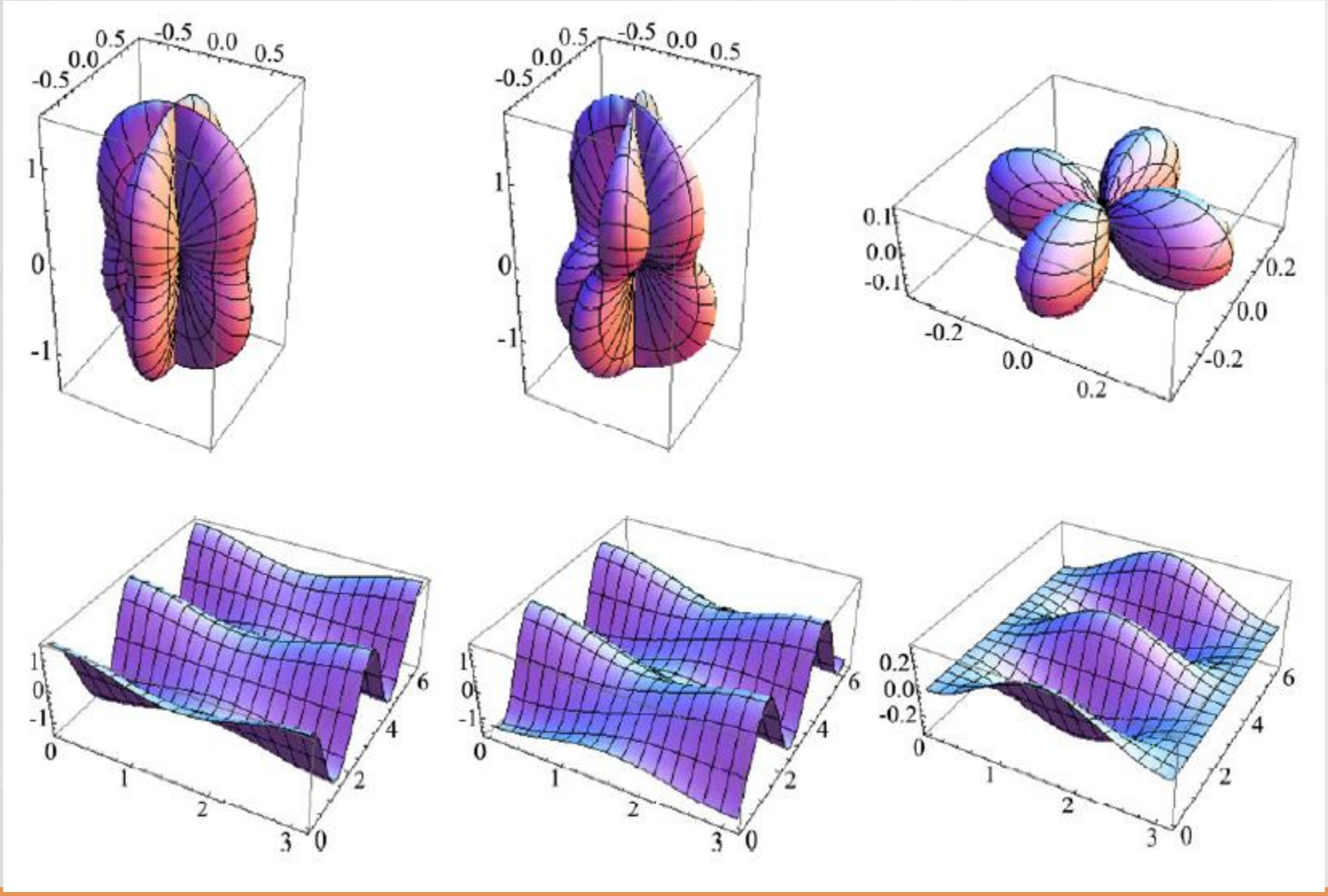
Spectrum by Cosmic String



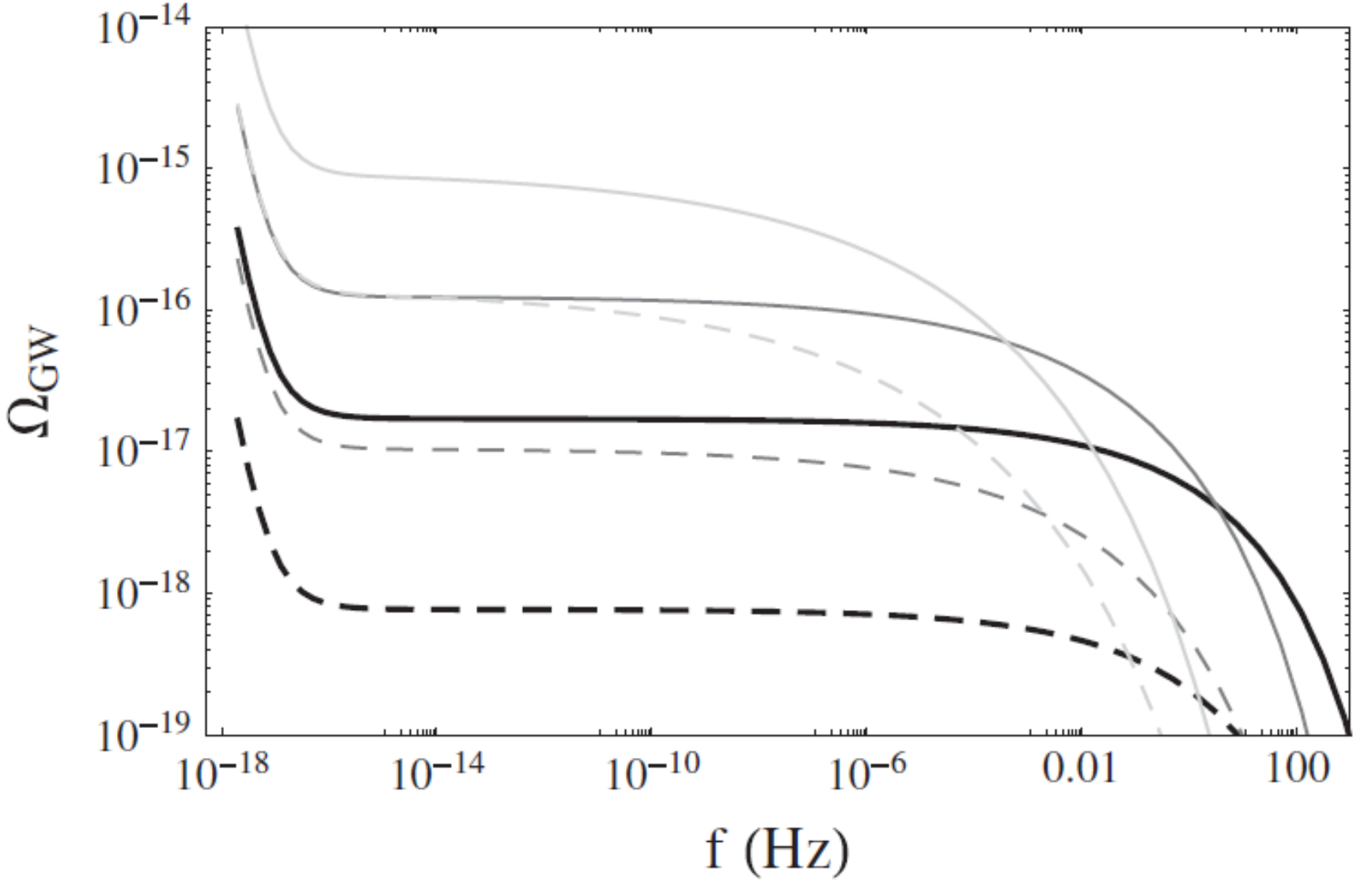
Spectrum by Bubbles from Phase tr.



Modified f(R) gravity



Modified Chaplygin Gas



New Idea?

- Transplakian mechanism
- Quantum gravity effect
- Black body like spectrum for GWs

Detectability with Single Detector

- Noise spectrum

$$\langle \tilde{n}^*(f)\tilde{n}(f')\rangle = \delta(f-f')\tfrac{1}{2}S_n(f)$$

$$\langle n^2(t)\rangle = \int_0^\infty \mathrm{d}f\, S_n(f) \qquad \tilde{h}_f \equiv \sqrt{S_n(f)}$$

- Detectable if

$$S_h(f) > (1/F)S_n(f)$$

Detection with Correlation

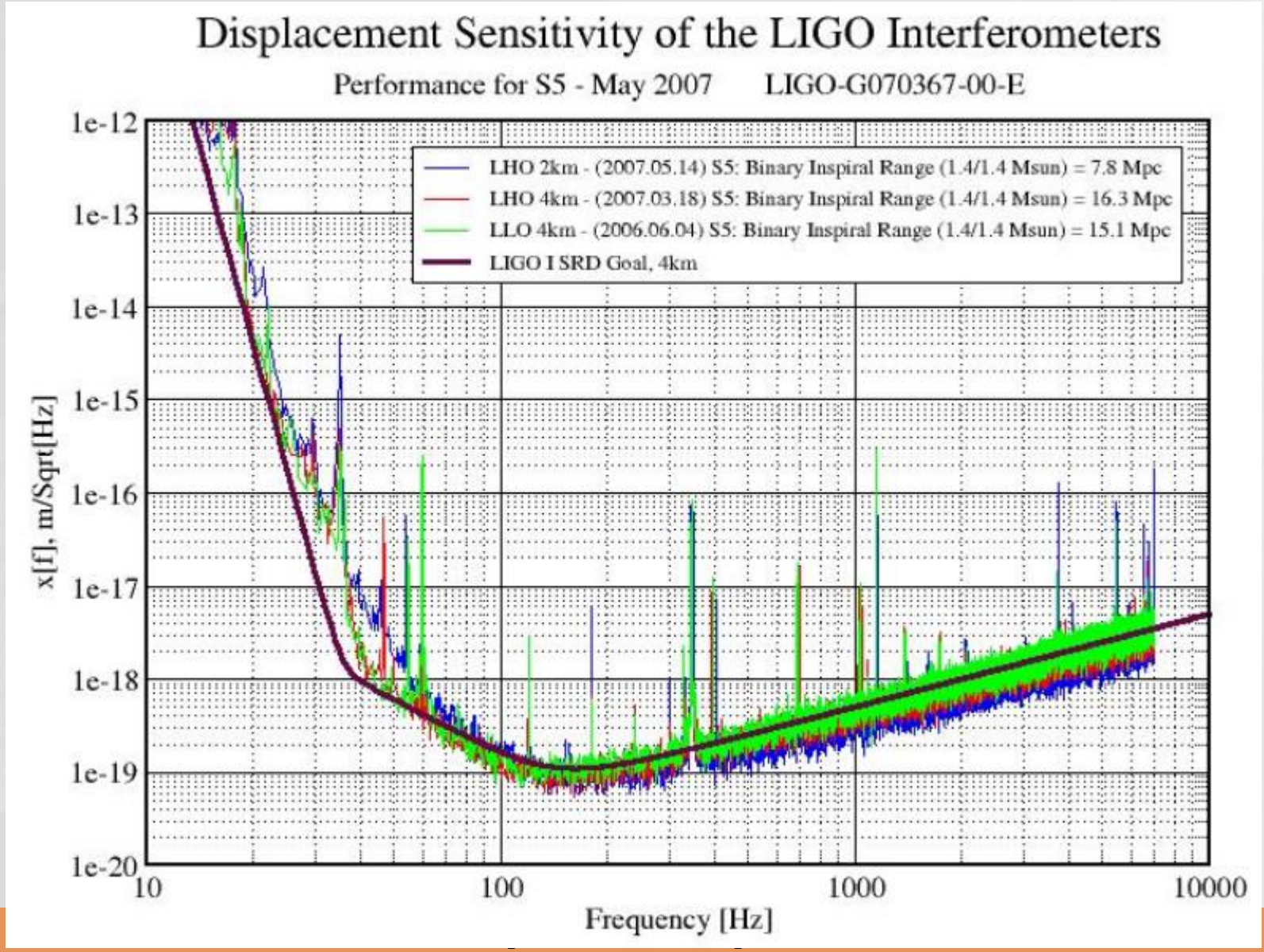
$$\langle s_{12} \rangle \equiv \int_{-\infty}^{+\infty} \mathrm{d}f \langle \tilde{s}_1^*(f)\tilde{s}_2(f) \rangle \, \tilde{Q}(f)$$

$$= \int_{-\infty}^{+\infty} \mathrm{d}f \int \mathrm{d}\hat{\Omega}\, \mathrm{d}\hat{\Omega}' \mathrm{e}^{(2\pi \mathrm{i} f/c)(\hat{\Omega}\cdot x_1 - \hat{\Omega}'\cdot x_2)} \sum_{A,A'} F_1^A(\hat{\Omega}) F_2^{A'}(\hat{\Omega}') \langle \tilde{h}_A^*(f,\hat{\Omega}) \tilde{h}_{A'}(f,\hat{\Omega}') \rangle \, \tilde{Q}(f)$$

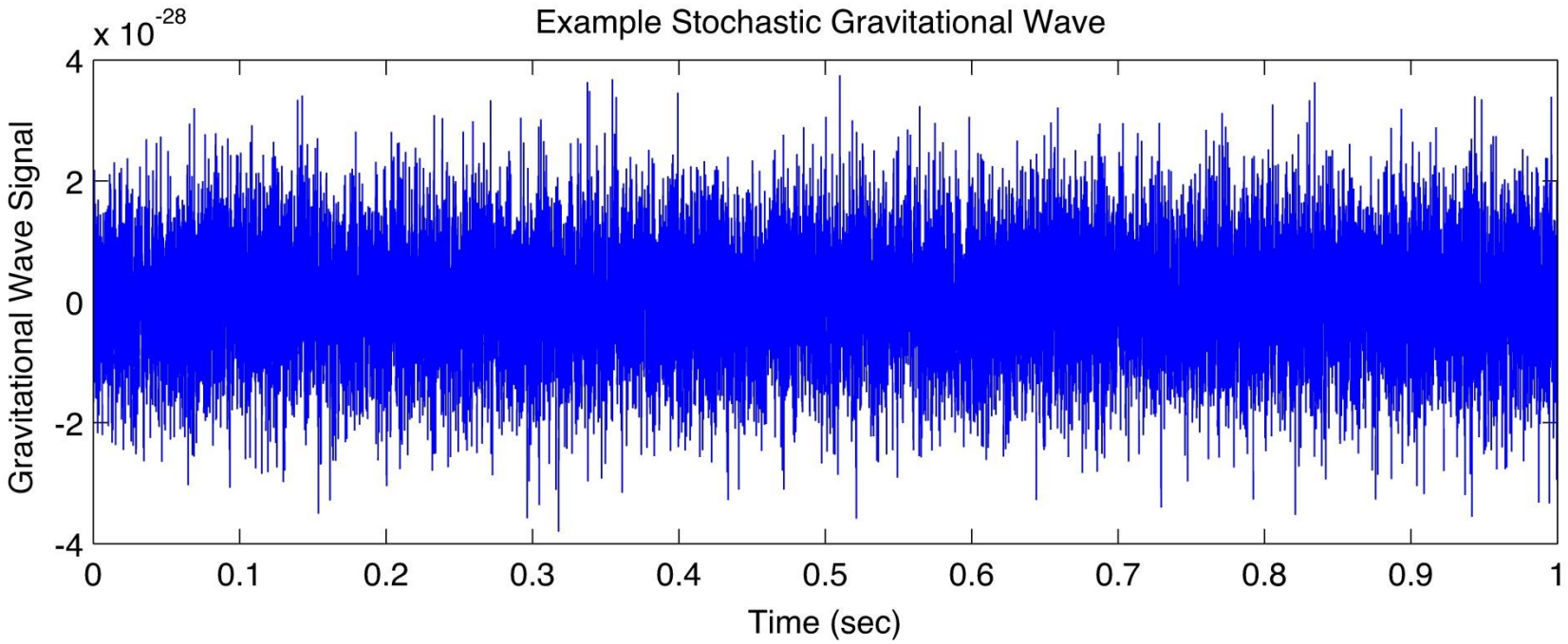
$$= T \int_{-\infty}^{\infty} \mathrm{d}f \tfrac{1}{2} S_h(f) \Gamma(f) \tilde{Q}(f) \ ,$$

$$\Gamma(f) \equiv \int \frac{\mathrm{d}\hat{\Omega}}{4\pi} \left[\sum_A F_1^A(\hat{\Omega}) F_2^A(\hat{\Omega}) \right] \exp\left\{ 2\pi \mathrm{i} f \hat{\Omega} \cdot \frac{\Delta \boldsymbol{x}}{c} \right\}$$

Sensitivity curve



Stochastic signal



From LIGO homepage

Remarks

- GW open a new window onto the Universe
- Hard to detect
- Specialized treatment for data
- Cross correlation between detectors
- Thermalisation like black-body radiation?
- Model dependent
- We need to see real GW data