Mechanical loss and thermal properties of materials for future interferometric gravitational wave detectors

Research in Jena and Kashiwa

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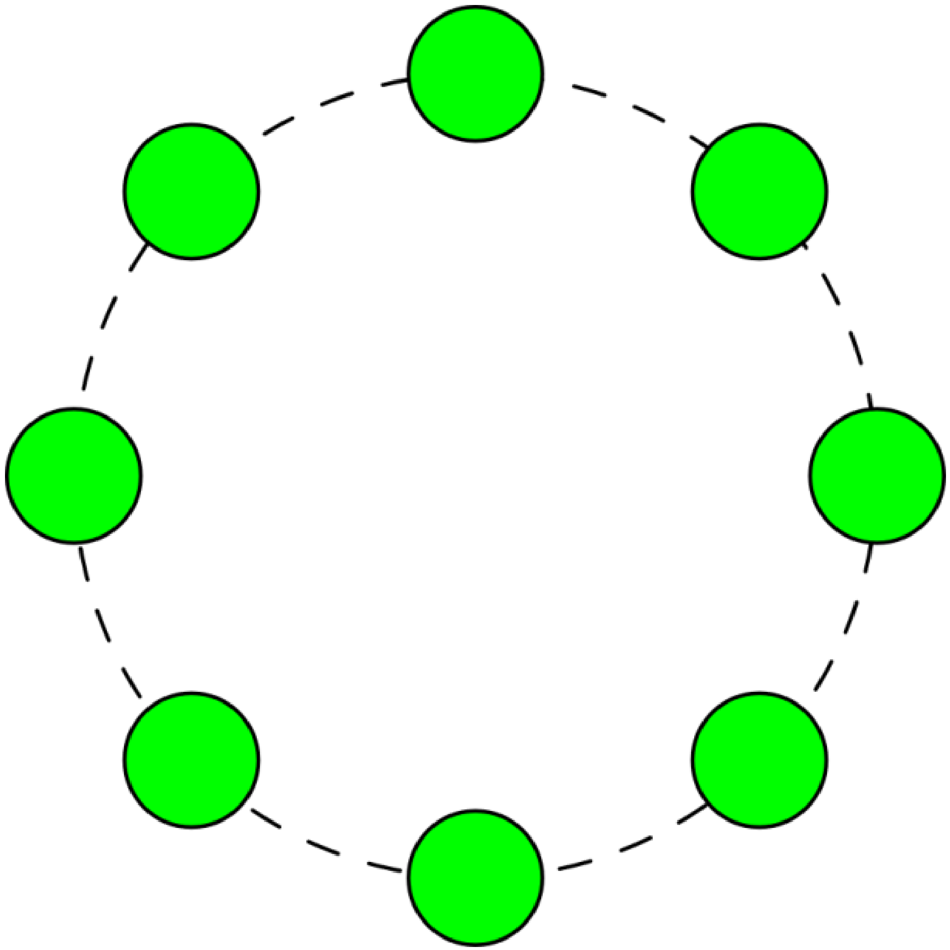
Outline

- Gravitational Wave Detectors (GWDs)
  - Detection principle
  - Noise and loss
- The standard anelastic solid
- Measurements of the mechanical loss
  - Measurement technique
  - Silicon
  - Sapphire
  - Sapphire fibres
  - Recent work at ICRR
- Thermal conductivity measurement on sapphire fibre
- Summary

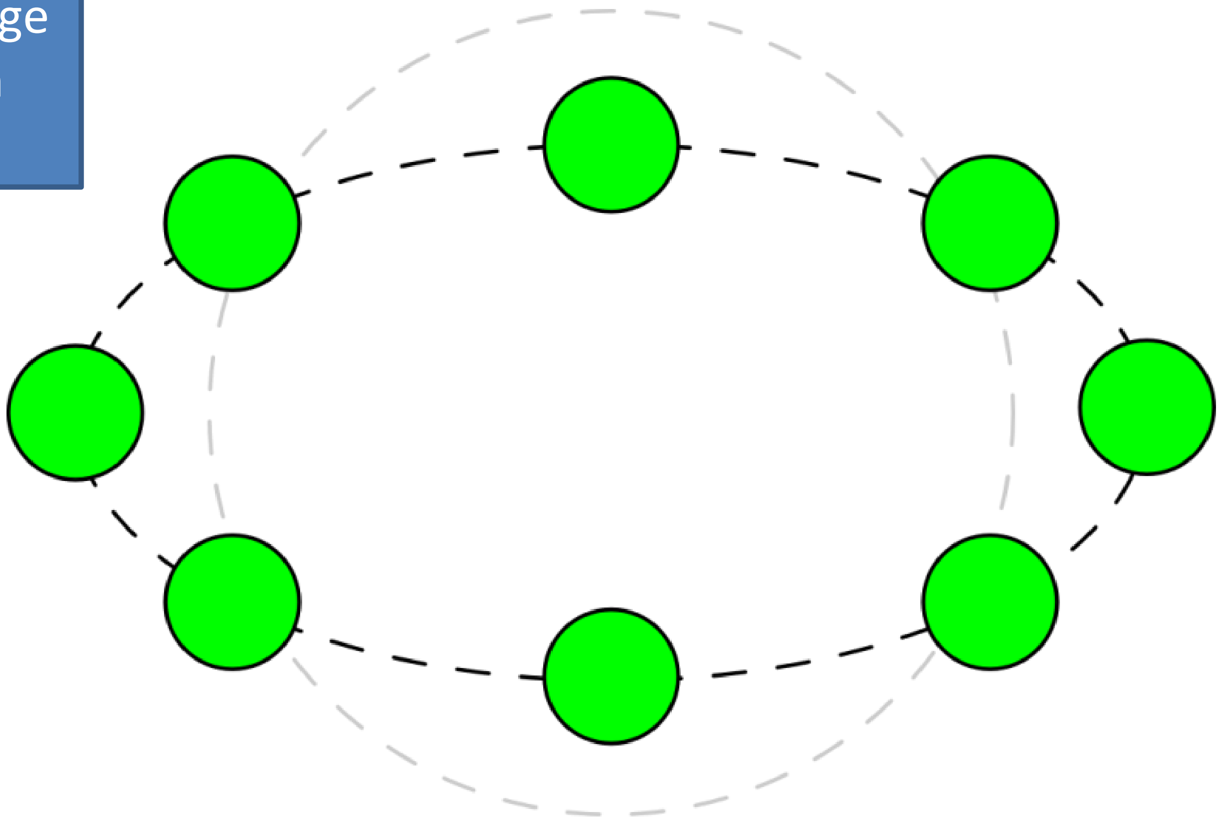
How to detect a gravitational wave?

Imagine a ring of free falling test masses:



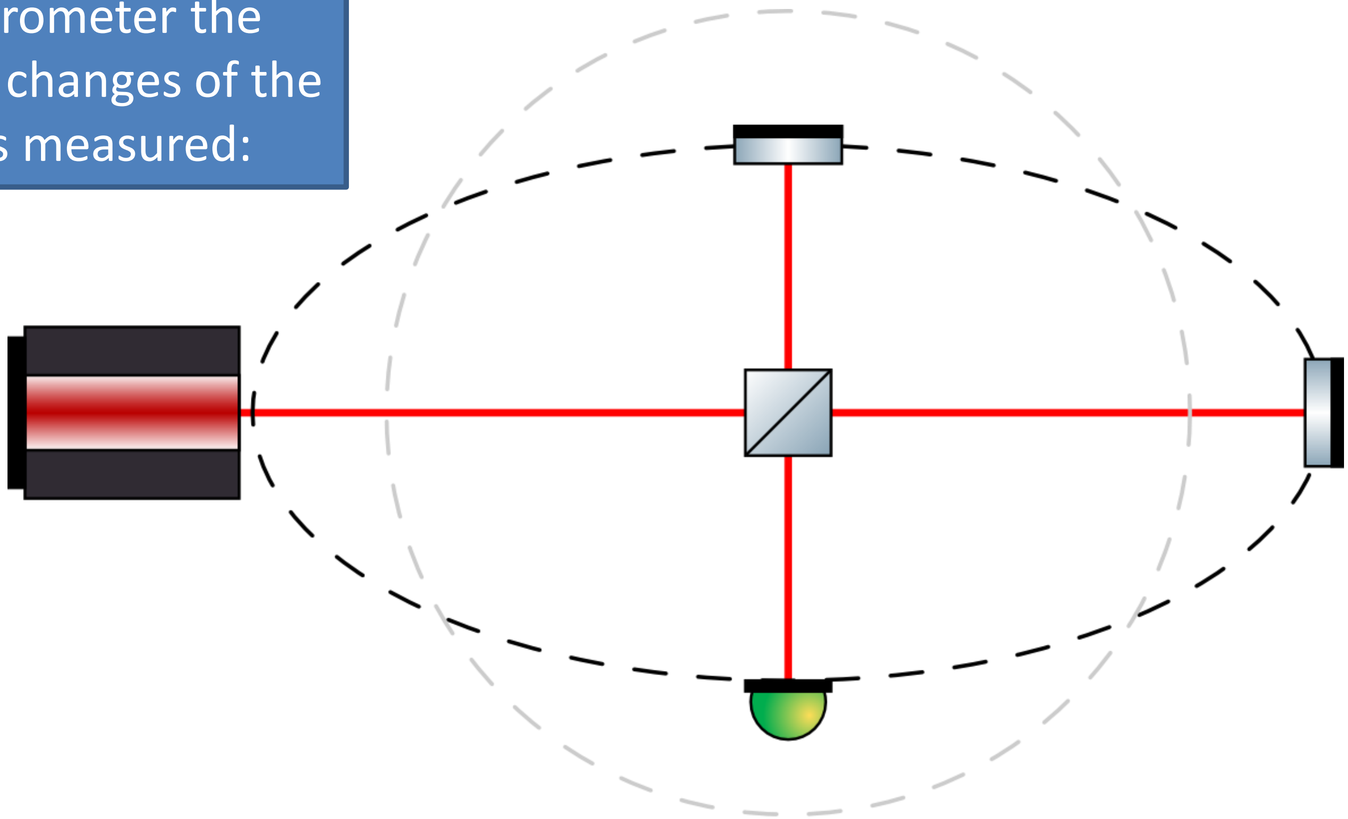
How to detect a gravitational wave?

A gravitational wave will change the circle to an ellipse:



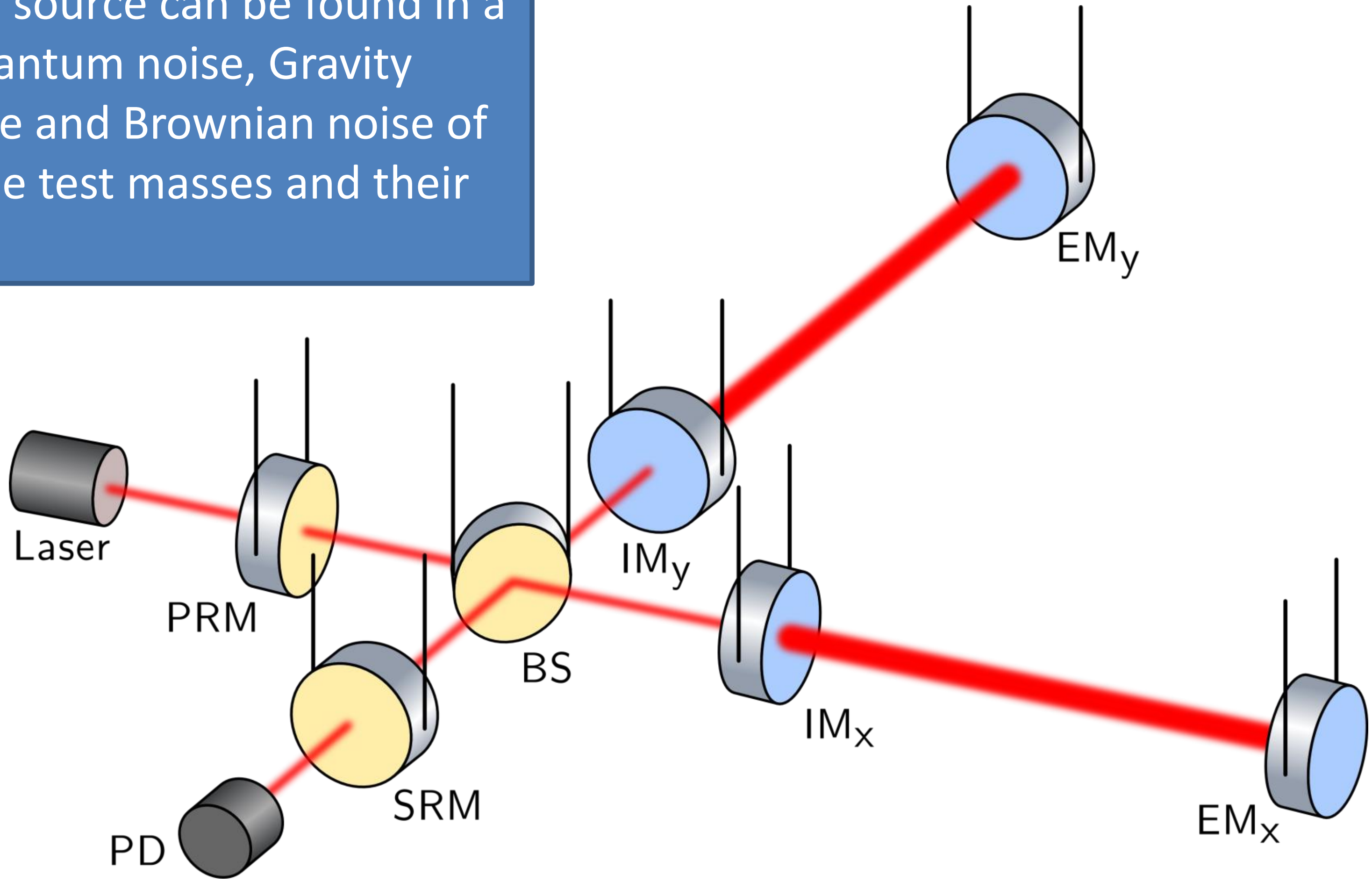
How to detect a gravitational wave?

Using an interferometer the length changes of the arms is measured:



Noise in interferometric GWDs

A lot of noise source can be found in a GWD like Quantum noise, Gravity gradient noise and Brownian noise of the optics, the test masses and their suspensions.



Relation of noise and loss

A simple example – Johnson-Nyquist-Noise

According to Johnson, *Thermal Agitation of Electricity in Conductors* (1928) and Nyquist, *Thermal Agitation of electric Charge in Conductors* (1928), we have the so called Johnson-Nyquist-Noise in a resistor.

The movement of a thermally driven charge Q_e (time derivation $\dot{Q}_e$ yields the current) is related to a thermal voltage U_{th} like:

$$U_{th} = R \times \dot{Q}_e$$

In an experiment one measures:

$$U_{th} = \sqrt{4k_B T R \Delta\omega}$$

and the power spectral density (PSD) S_U of the voltage U_R is given by:

$$S_U(\omega) = U_{th}^2/\omega^2 = 4k_B T R$$

k_B…Boltzmann's constant, R…resistance, $\Delta\omega$…bandwidth, ω…angular frequency

Relation of noise and loss

Fluctuation-Dissipation-Theorem (FDT)

PHYSICAL REVIEW VOLUME 83, NUMBER 1 JULY 1, 1951

Irreversibility and Generalized Noise*

HERBERT B. CALLEN AND THEODORE A. WELTON†
Randal Morgan Laboratory of Physics, University of Pennsylvania, Philadelphia, Pennsylvania

(Received January 11, 1951)

Callen and Welton deduced a theory based on thermodynamics and statistics to calculate fluctuations of a generalized force V in any dissipative linear system.

The PSD (i.e. fluctuations in a certain bandwidth) of this force is given by:

$$S_V(\omega) = 4k_B T \Re(Z)$$

The impedance Z is defined to link the generalized force V with a generalized displacement Q:

$$Z(\omega) = V/\dot{Q}$$

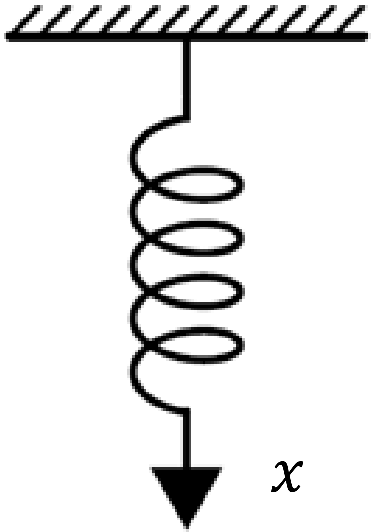
Relation of noise and loss

Applying the FDT to a mechanical system

In an oscillator we expect thermally driven fluctuations of the position x (Q) of the attended mass due to some force F_{th} (V) extending the spring.

Thus the mechanical impedance is:

$$Z(\omega) = F_{th}/\dot{x}$$



And the PSD of the force is again obtained using:

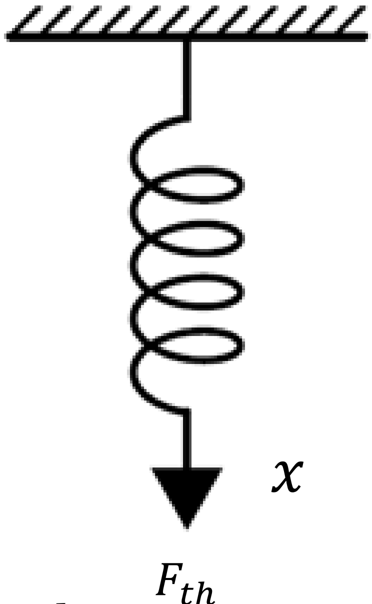
$$S_F(\omega) = F_{th}^2/\omega = 4k_B T\Re(Z)$$

Ok, but what about the fluctuations in position i.e. the power spectral density of the displacement in x?

Relation of noise and loss

Displacement noise of a oscillator I

Assuming a harmonic oscillator with a linear-elastic spring k and viscous damping f the equation of motion follows:



$$m\ddot{x} + f\dot{x} + kx = F_{th}$$

Using $x = x(\omega)e^{i\omega t}$ and $F_{th} = F_{th}(\omega)e^{i\omega t}$:

$$(-\omega^2 m + i\omega f + k)\, x = F_{th}$$

The impedance is then:

$$Z = \frac{F_{th}}{\dot{x}} = \frac{(-\omega^2 m + i\omega f + k)\, x}{i\omega\, x} = \mathrm{i}\omega m + f + \frac{k}{i\omega}$$

The real part of it is simply $\Re(Z) = f$.

Relation of noise and loss

Displacement noise of a oscillator II

Now we recalculate the PSD after Callen/Welton using our former results $(-\omega^2 m + i\omega f + k)\,x = F_{th}$ and $\Re(Z) = f$:

$$S_F(\omega) = \frac{F_{th}^2}{\omega} = 4k_B T \Re(Z)$$

Rearrange: $$\frac{x^2}{\omega} = 4k_B T \frac{f}{(-\omega^2 m + i\omega f + k)^2}$$

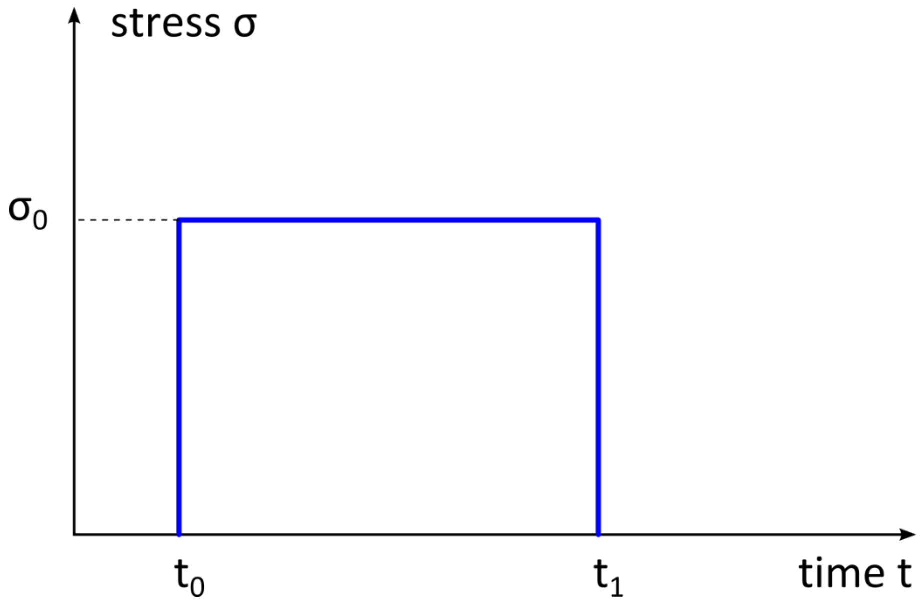
Taking the real part to get a physical result we finally obtain the displacement power spectral density of a harmonic oscillator with its resonance at $\omega_0 = \sqrt{k/m}$:

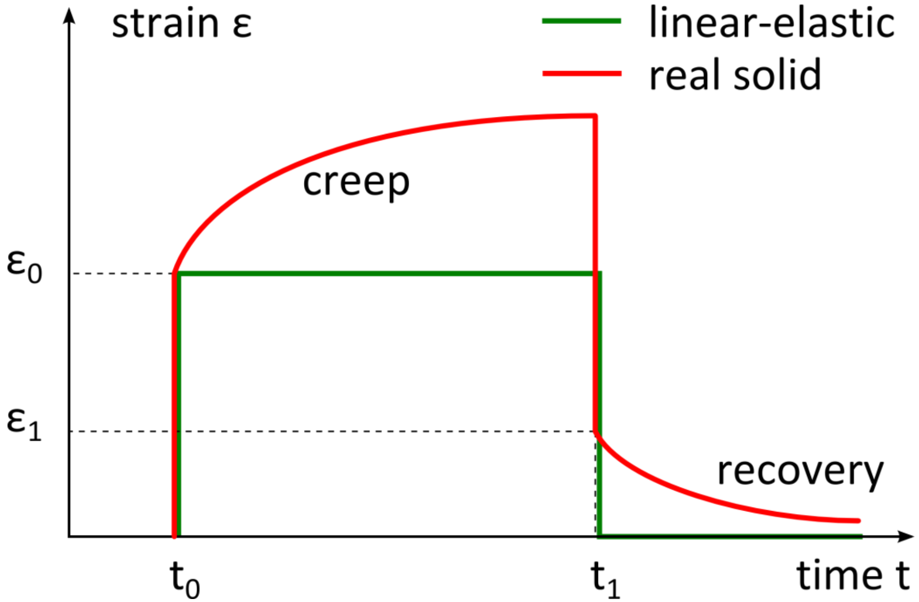
$$S_x(\omega) = 4k_B T \frac{f}{(-\omega^2 + \omega_0^2)^2 m^2 + \omega^2 f^2}$$

Elastic behaviour of solids

Ideal solid vs. real solid I

Compare the elastic behaviour of the linear-elastic spring to that of a real solid under some static stress applied and released. We observe creep and recovery of the strain:

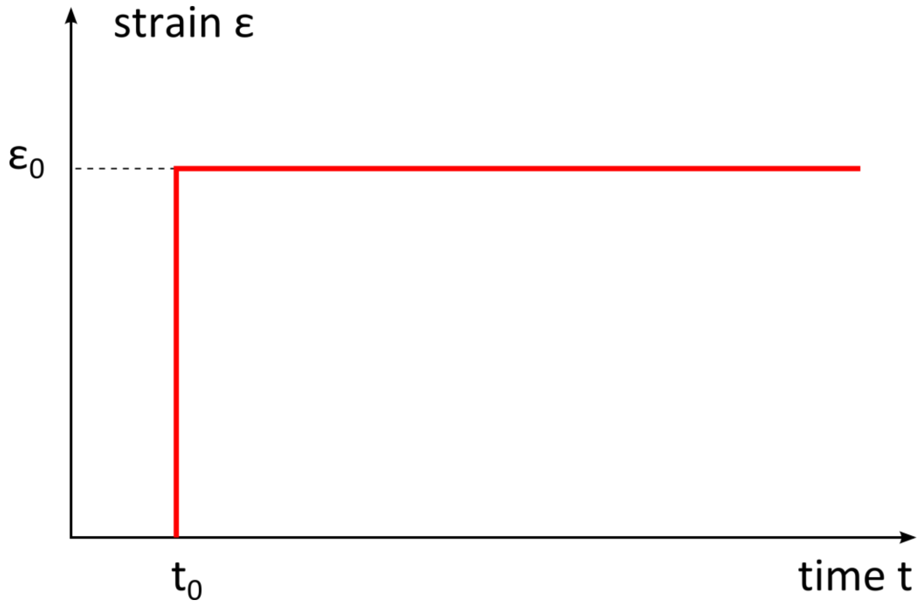


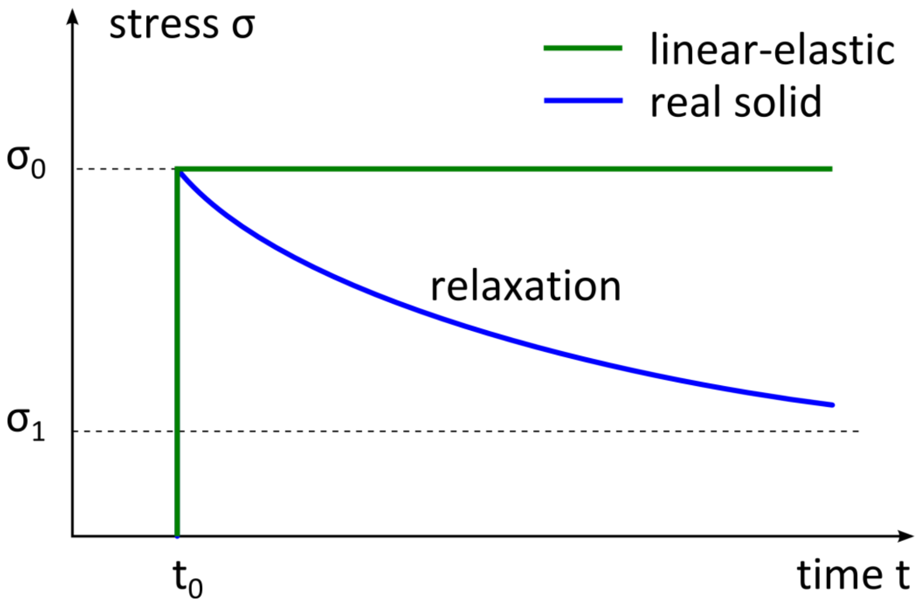


Elastic behaviour of solids

Ideal solid vs. real solid II

Now consider some static strain applied. After some initial stress the real solid relaxes during time while the linear-elastic keeps constant:

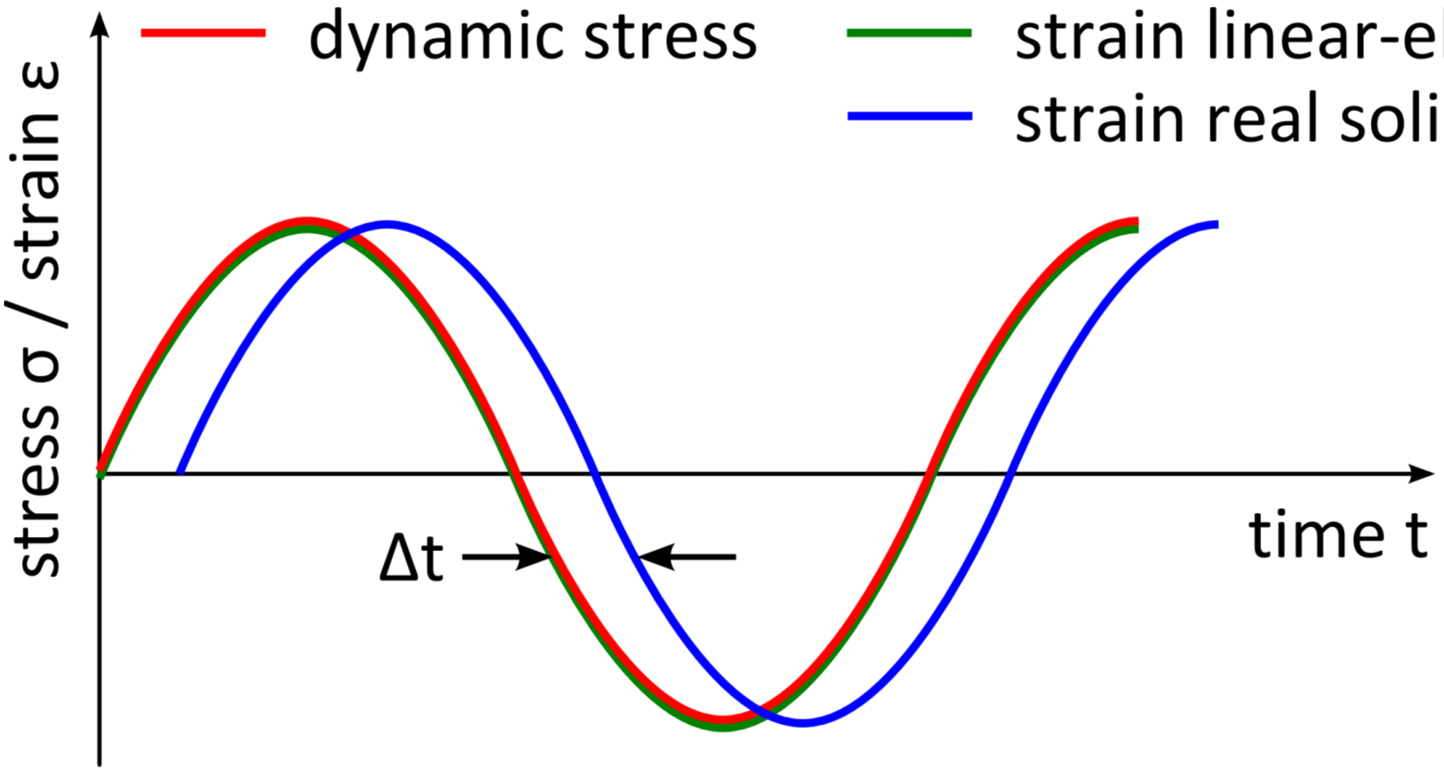




Elastic behaviour of solids

Ideal solid vs. real solid III

In case of a dynamic stress the strain of the real solid will follow with some time lag Δt.



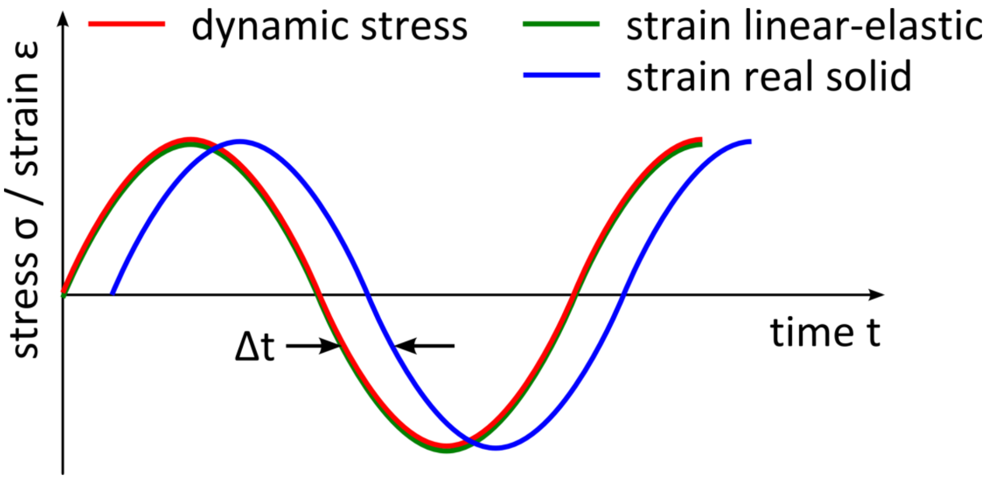
Elastic behaviour of solids

Dynamic stress-strain relation

The time lag is related to the phase lag like $\Delta t = \phi/2\pi f$. Thus the equations for stress and strain are given by:

$$\sigma = \sigma_0 e^{i\omega t}$$

$$\varepsilon = \varepsilon_0 e^{i(\omega t - \phi)}$$



The complex Young's modulus Y (likewise spring constant k) is obtained by dividing stress and strain:

$$Y = \frac{\sigma}{\varepsilon} = Y_0 e^{i\phi} \approx k\,(1 + i\phi) \qquad \phi \ll 1$$

Relation of noise and loss

Displacement noise of a structural damped oscillator

Again, we assume a harmonic oscillator. Instead of viscous damping we use a complex spring constant i.e. structural damping (also internal damping):

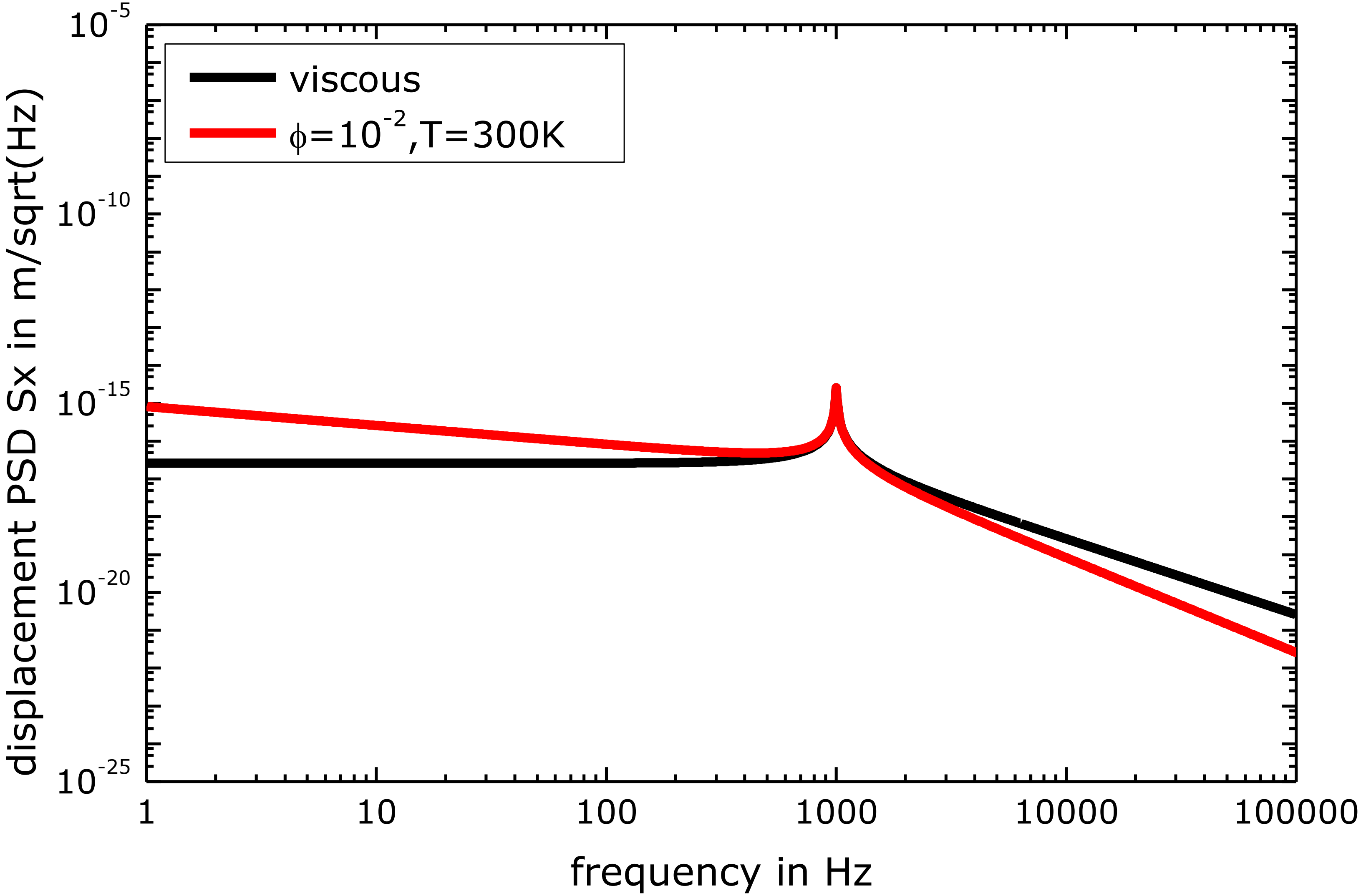
$$m\ddot{x} + k(1 + i\phi)x = F_{th}$$

Following the former calculation, this results in a displacement power spectral density for a structural damped harmonic oscillator :

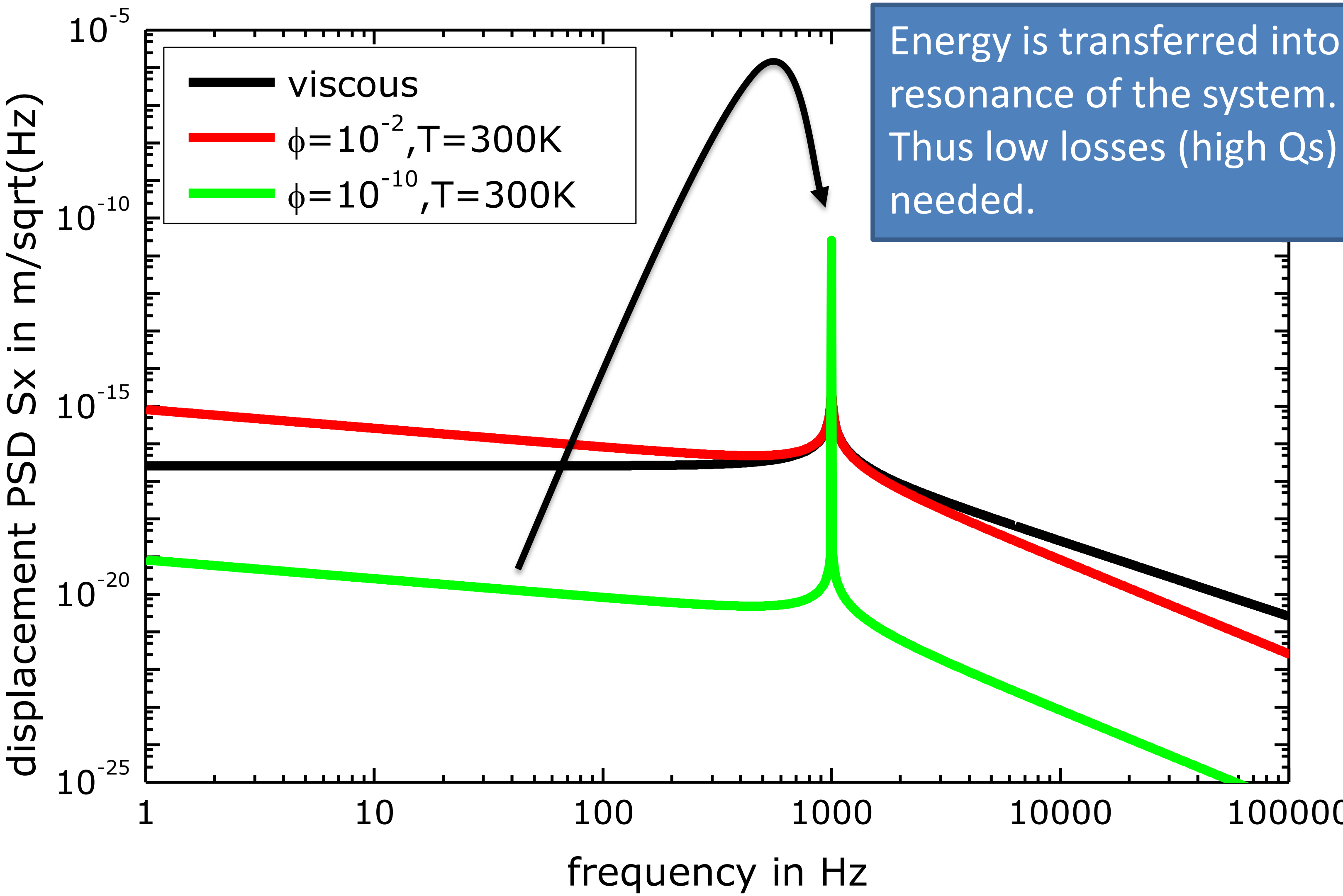
$$S_{x,struc}(\omega) = \frac{4k_BT}{m\omega}\frac{\phi\ \omega_0^2}{(-\omega^2 + \omega_0^2)^2 + \omega^4\phi^2}$$

Hereby ϕ is called the loss angle of the system, respectively ϕ^{-1} denotes the Q-factor which gives a measure of the dissipation of energy ($\phi \ll 1$).

Relation of noise and loss

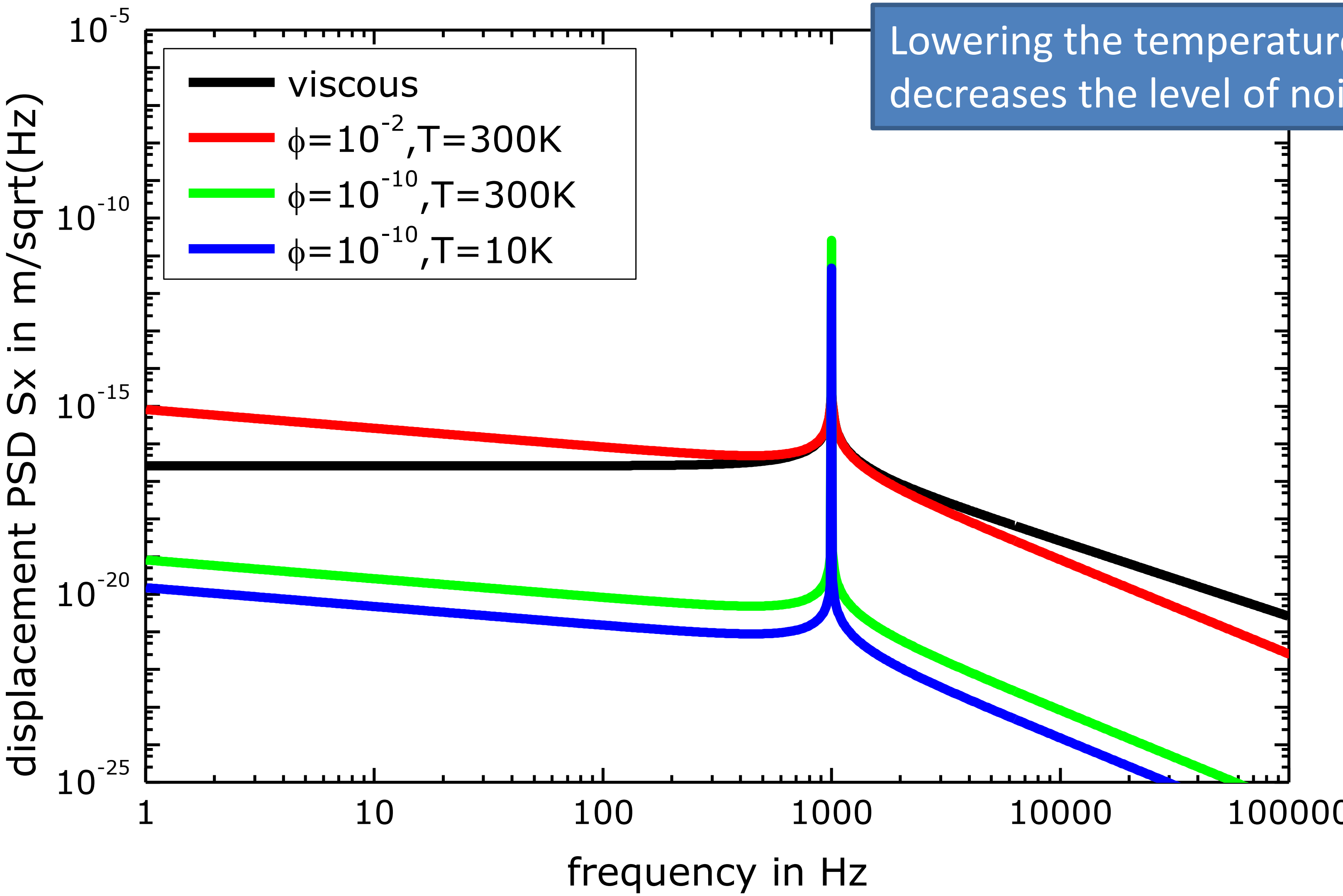


Relation of noise and loss



Energy is transferred into the resonance of the system. Thus low losses (high Qs) are needed.

Relation of noise and loss



Lowering the temperature decreases the level of noise.

So far …

What we have:

- Two ways to introduce loss in a 1D harmonic oscillator
  - Viscous damping (e.g. particle moving in a fluid or gas)
  - Structural damping (e.g. vibration of a solid)
- Noise is related to
  - Temperature (lower is better)
  - Loss in a system (lower is better)

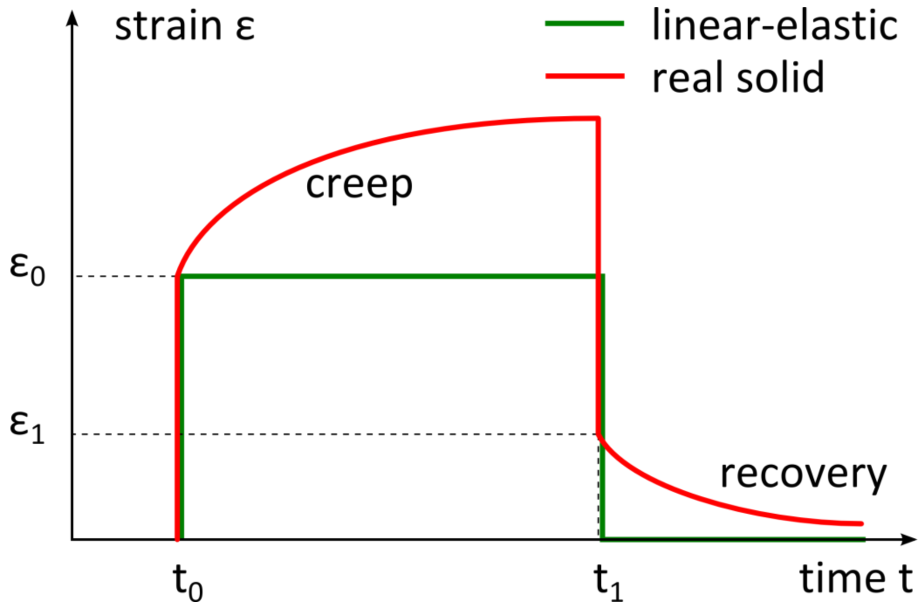
➔ To understand the mechanical loss of a solid, an appropriate model is needed.

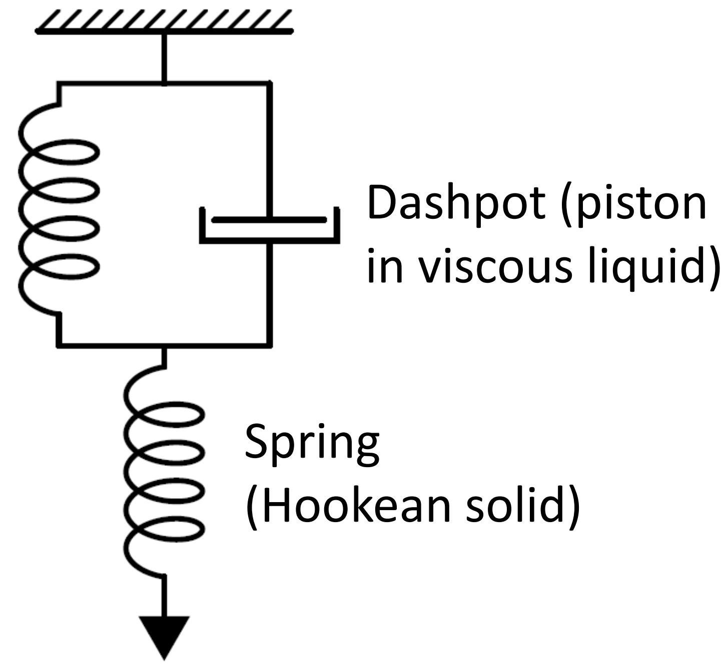
STANDARD ANELASTIC SOLID

Anelasticity of solids

A closer look to internal damping I

A lot of energy dissipating mechanisms can be described by some kind of stress-induced relaxation process of an internal order parameter. Zener was the first one to describe this behaviour using a combination of two springs and dashpot, called Maxwell-unit:



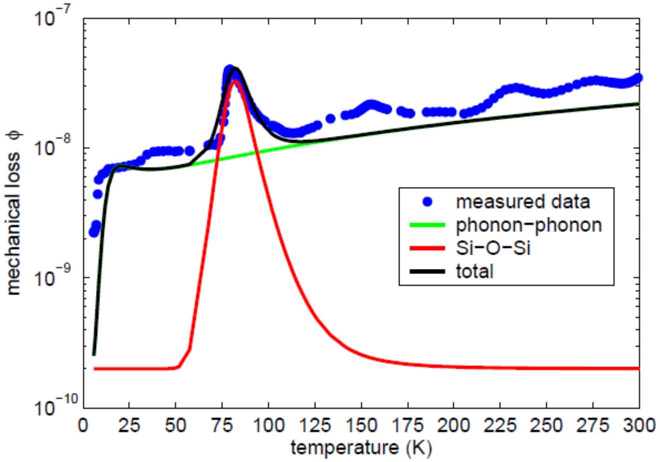


Anelasticity of solids

A closer look to internal damping II

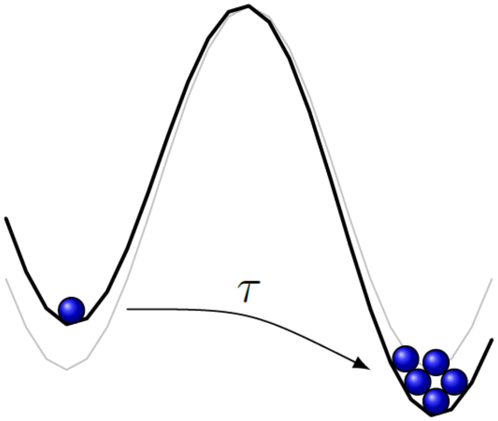
Nowick and Berry, *Anelastic Relaxation In Crystalline Solids* (1972), give a dependence of the loss angle as follows:

$$\phi = \Delta \frac{\omega\tau}{1+\omega^2\tau^2}$$



Δ...relaxation strength,
ω…angular frequency,
τ…time constant for a specific loss process

A maximum (Debye peak) occurs in case of $\omega\tau = 1$, resonant coupling of the relaxation process (time constant τ) and the disturbance (frequency ω, e.g. moving an atom out of its point of rest). Often τ is related to temperature and activation energy like: $\tau = \tau_0 e^{E_a/k_B T}$



Anelasticity of solids

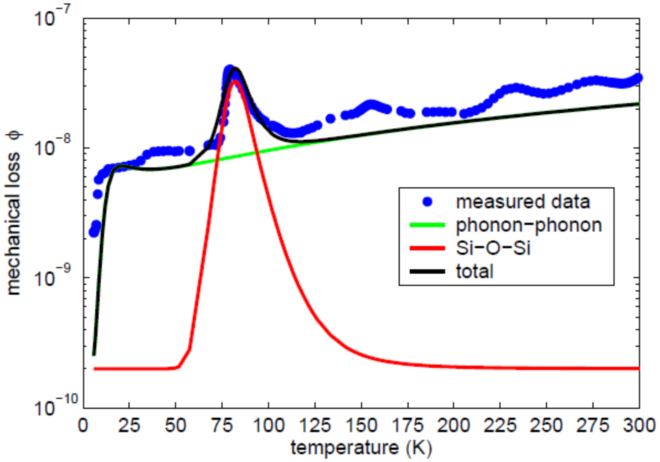
A closer look to internal damping III

In a real solid one may find more than one dissipating mechanism. The loss angle is related as follows:

$$\phi = Q^{-1} = \Delta E / 2\pi E_{tot}$$

ΔE...energy dissipation,

E_{tot}...total energy,



Thus we can sum up the loss of different mechanisms to the total loss of the system:

$$\phi_{tot} = \phi_{internal} + \phi_{surface} + \phi_{coating} + \phi_{external} + \cdots$$

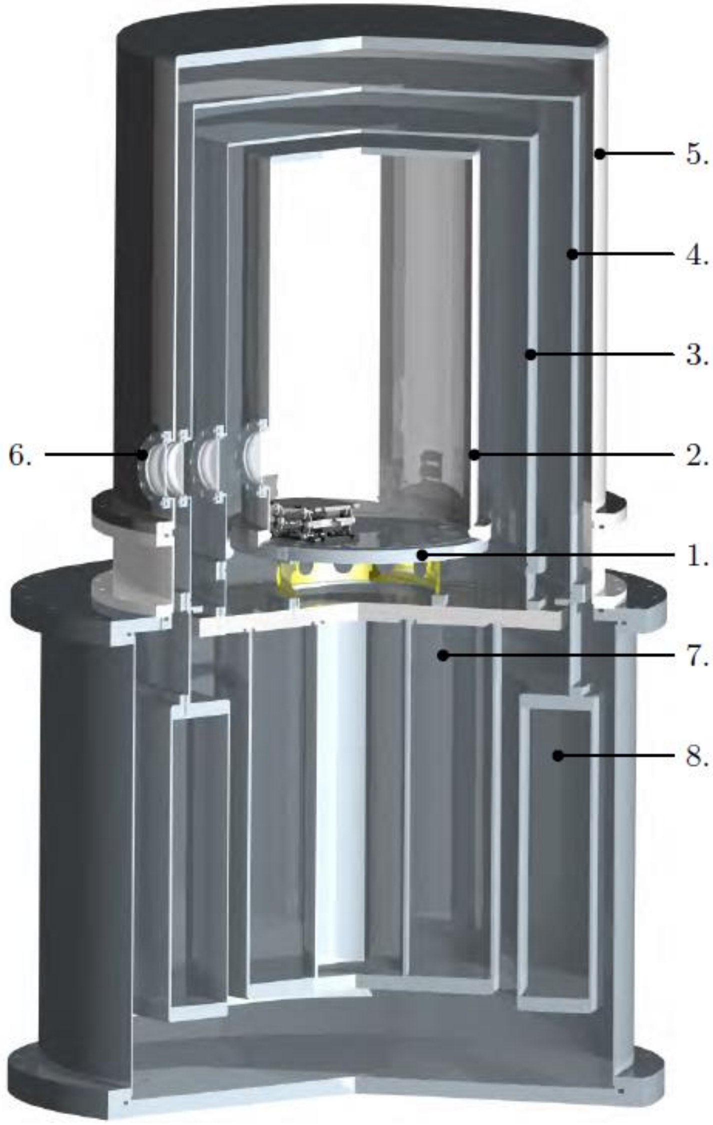
External losses result from e.g. gas damping or friction. To be minimized!

MEASUREMENT TECHNIQUES

Measuring mechanical loss

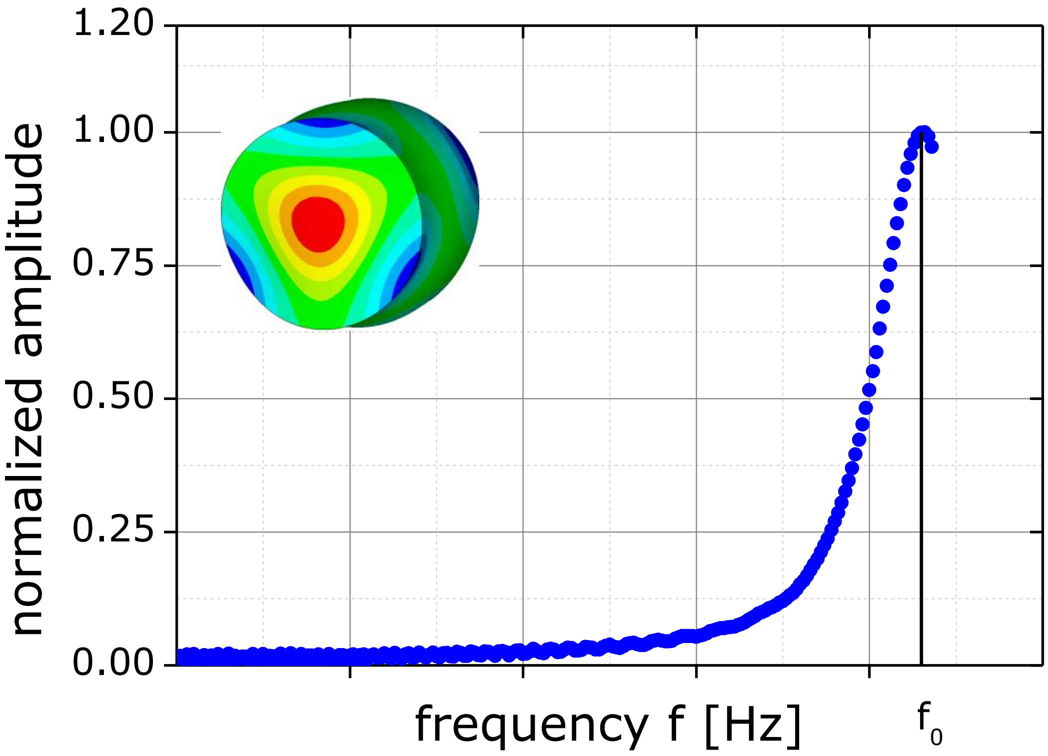
Liquid He cryostat in Jena:

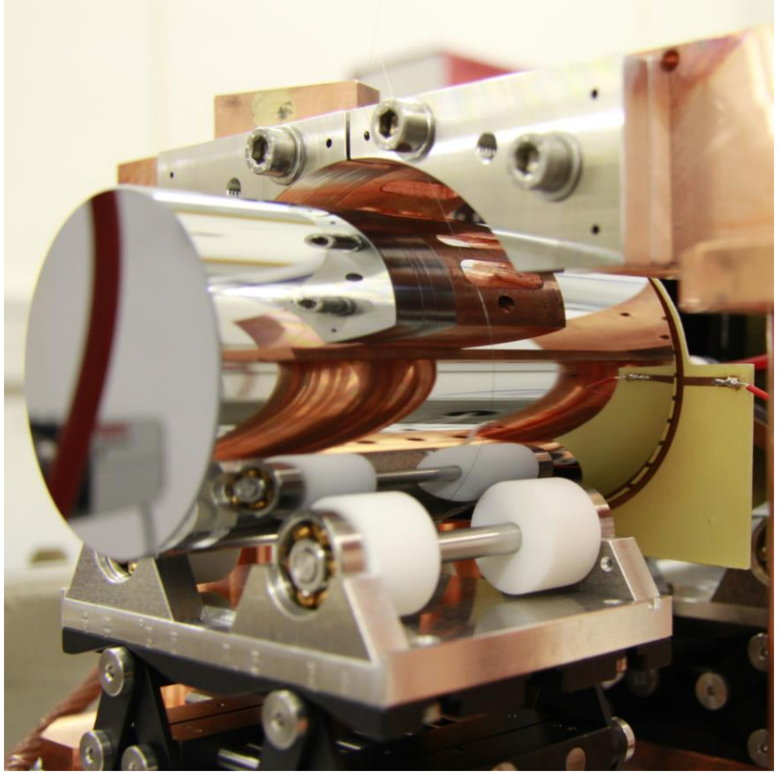
1. Experimental platform
2. Probe chamber
3. 4K-shield
4. 80K-shield
5. Isolation chamber
6. Optical windows
7. LHe-tank
8. LN_2-tank



Measuring mechanical loss

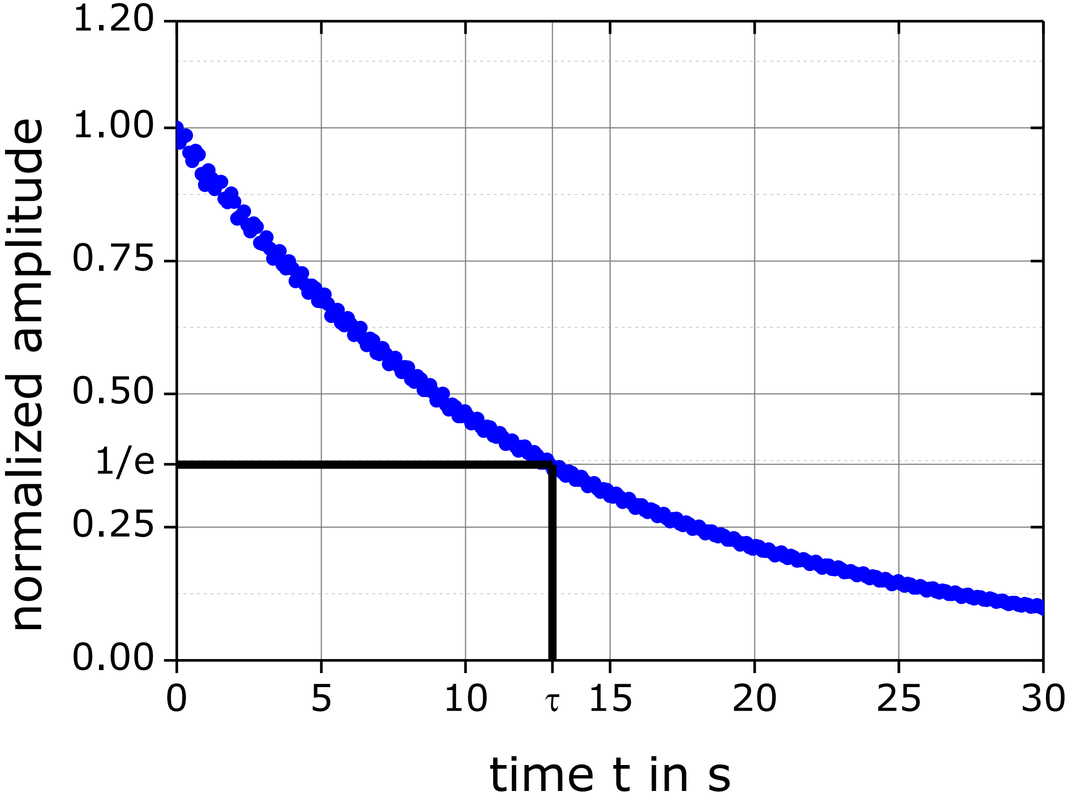
Excitation of resonant vibrations in the solid:





Measuring mechanical loss

Free ring down of the resonant vibration:



Free ring down of the oscillation at f_0:

$$x(t) = x_0 \exp\frac{-t}{\tau}\cos\omega_0 t$$

Ratio of dissipated to stored energy per cycle yields the mechanical loss:

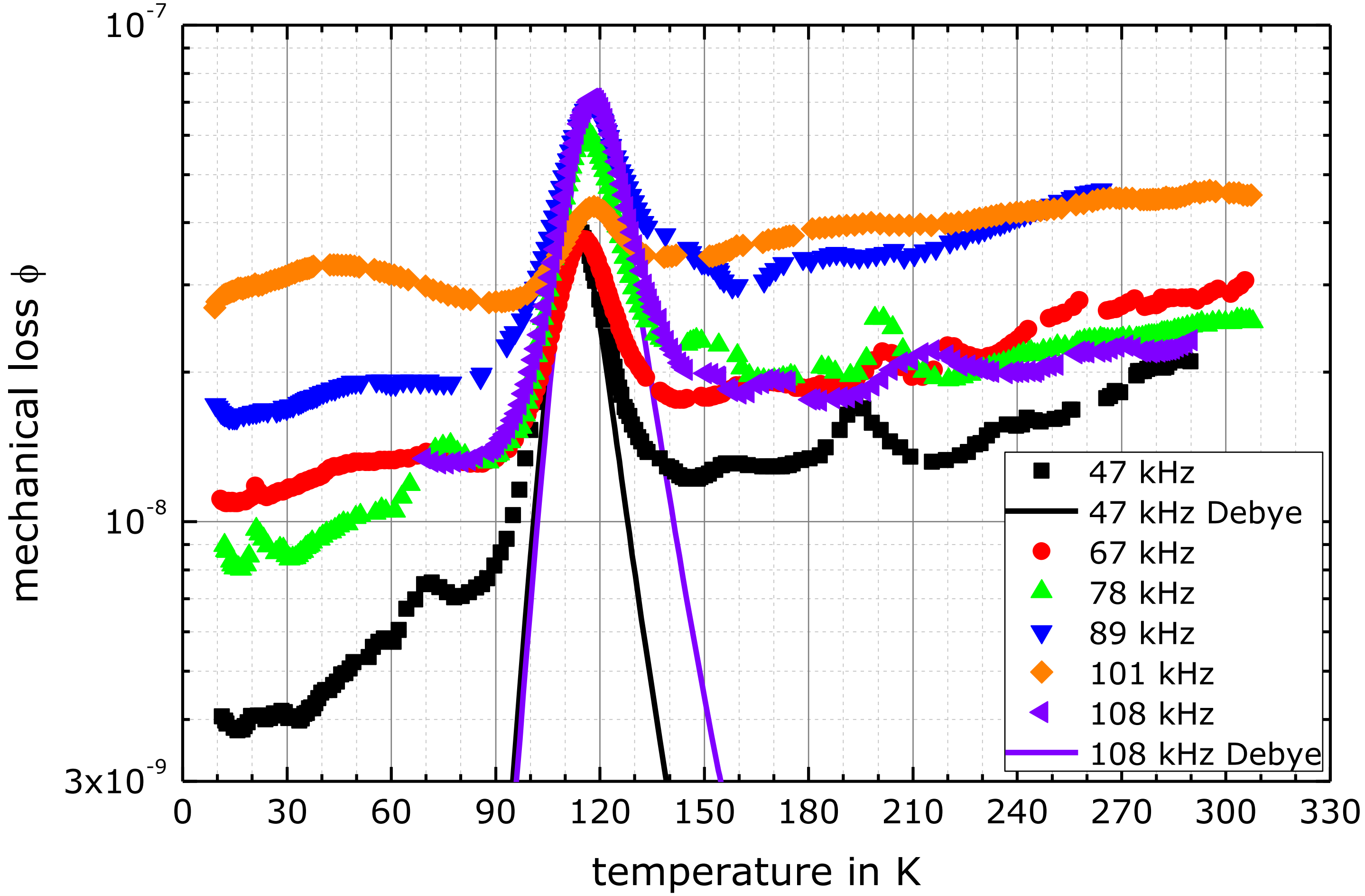
$$\phi = \frac{\Delta E}{2\pi E} = \frac{1}{\pi f \tau}$$

Mechanical loss measurement on

BULK SILICON

Silicon (111) Ø 65 mm x 50 mm

- Debye peak in all modes indicates a strong loss mechanism.
- Temperature dependence refers to thermal activation.

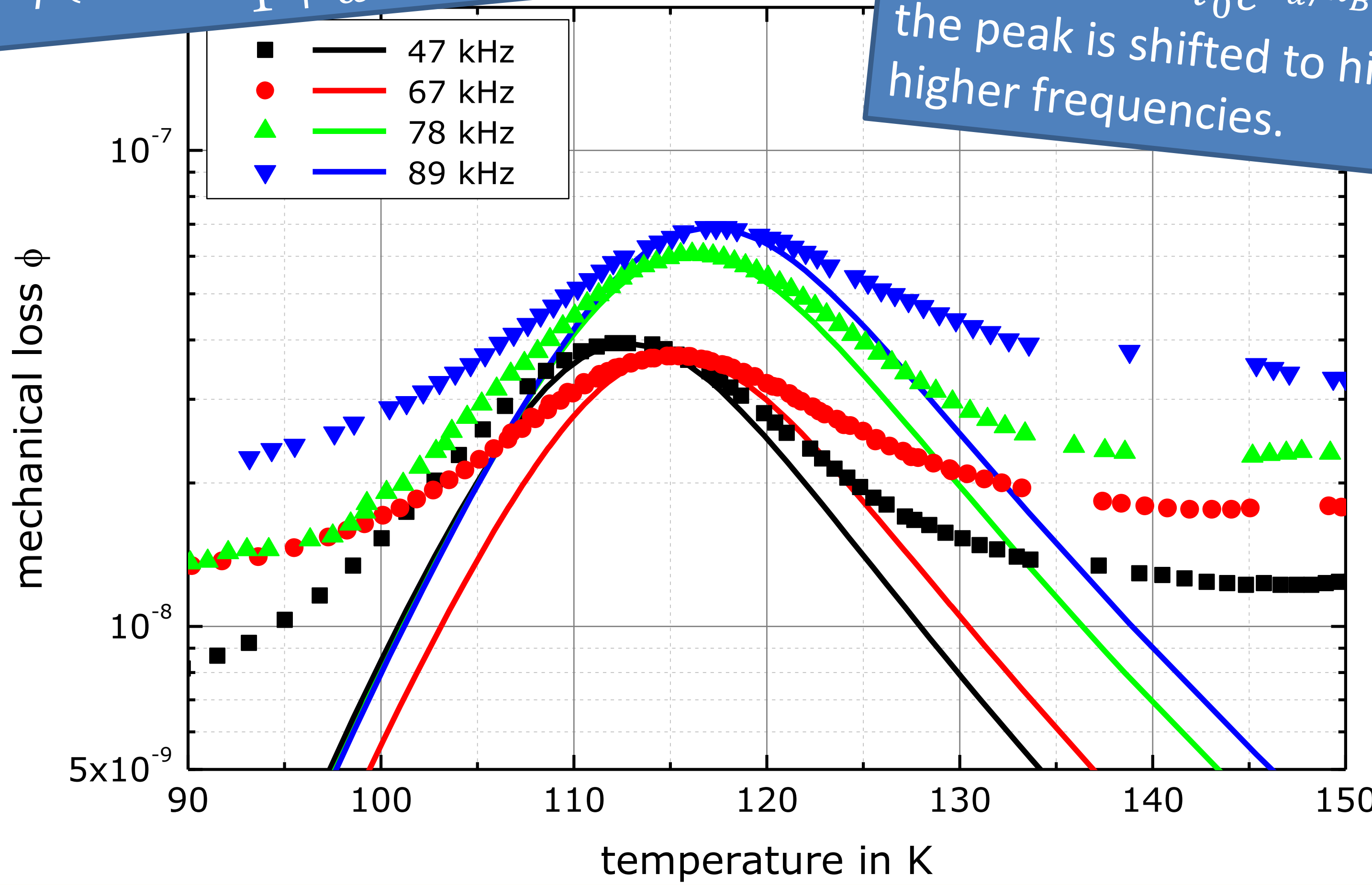


Silicon (111) Ø 65 mm x 50 mm

Debye peak fits well at the maximum

$$\phi(\omega) = \Delta \frac{\omega\tau}{1+\omega^2\tau^2}$$

Because of temperature depence $\tau = \tau_0 e^{E_a/k_B T}$ the peak is shifted to higher T for higher frequencies.



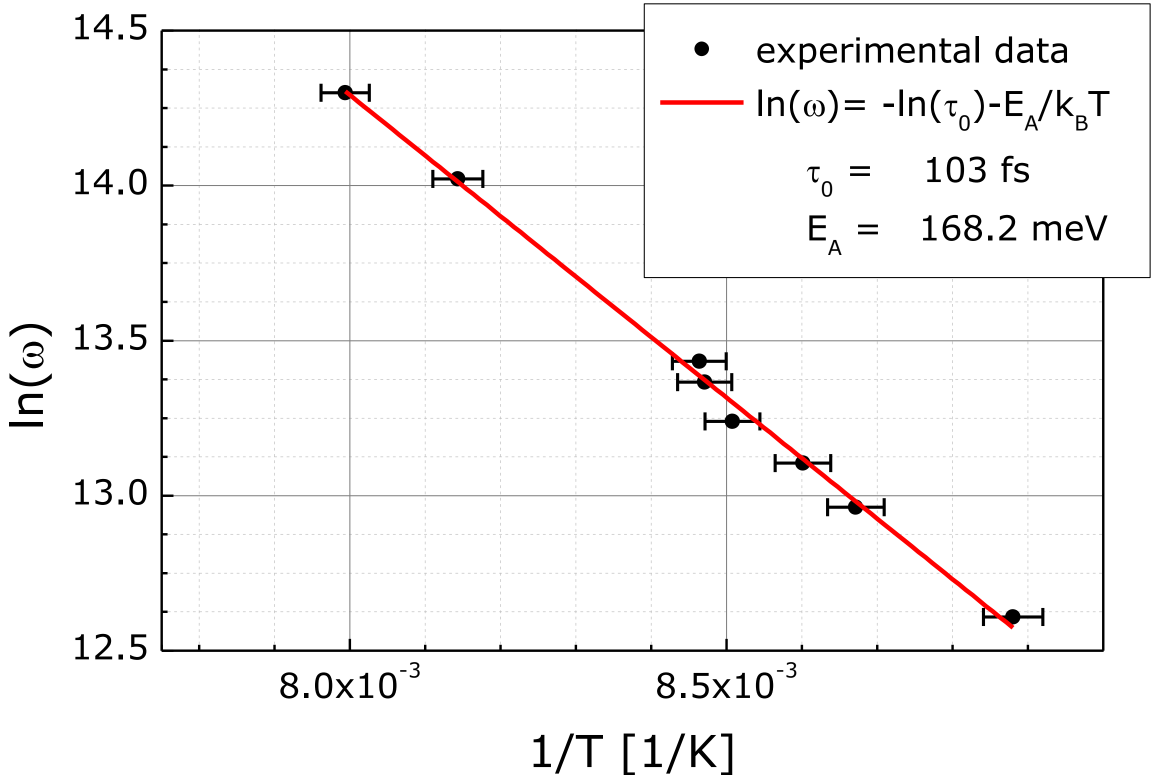
Mechanical loss and Arrhenius plot

The peak is reached at:

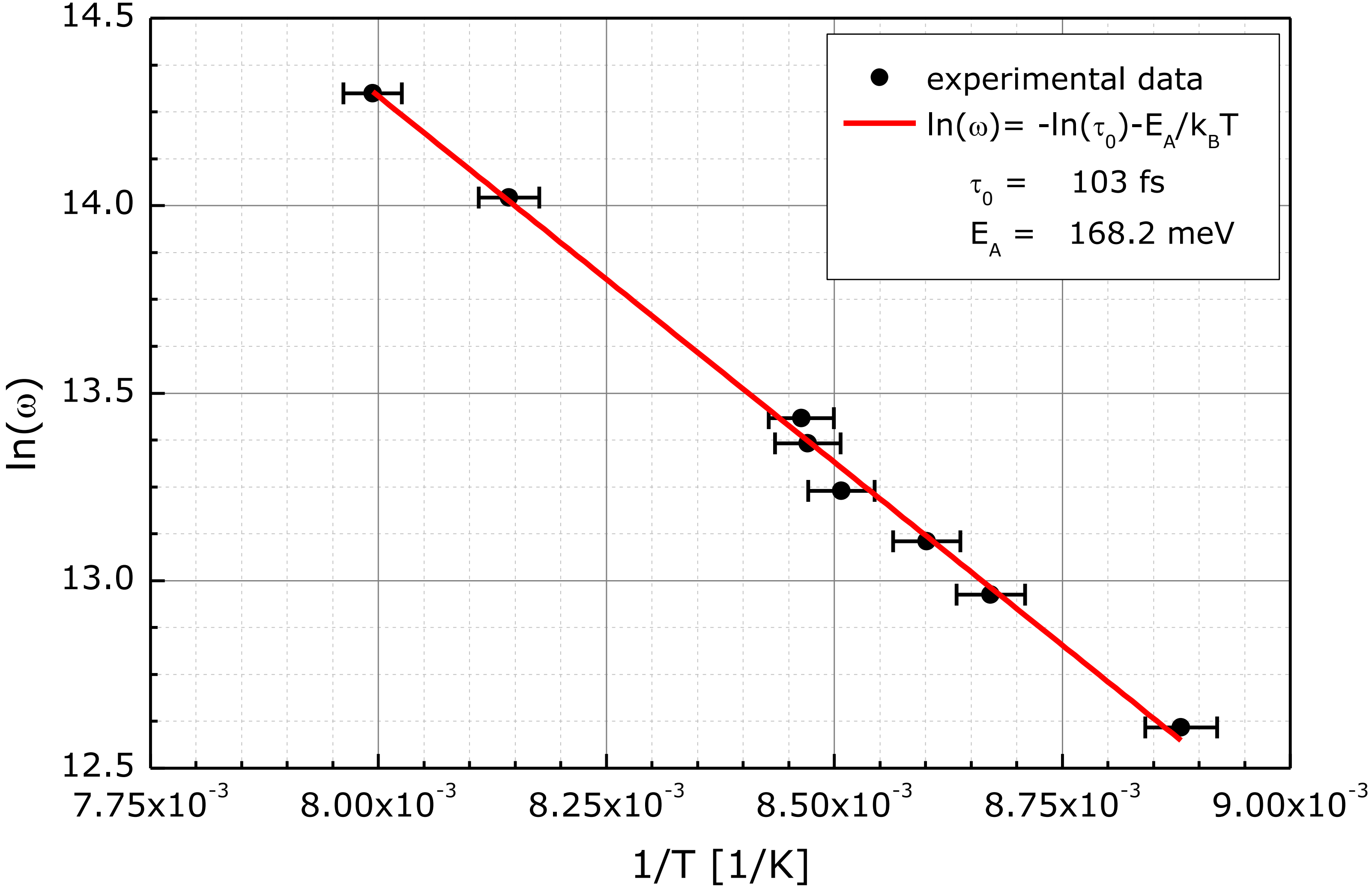
$$1 = \omega\tau = \omega\,\tau_0 \exp\frac{E_A}{k_B T}$$

Arrhenius plot:

$$\ln\omega = -\ln\tau_0 - \frac{E_A}{k_B}\times\frac{1}{T}$$

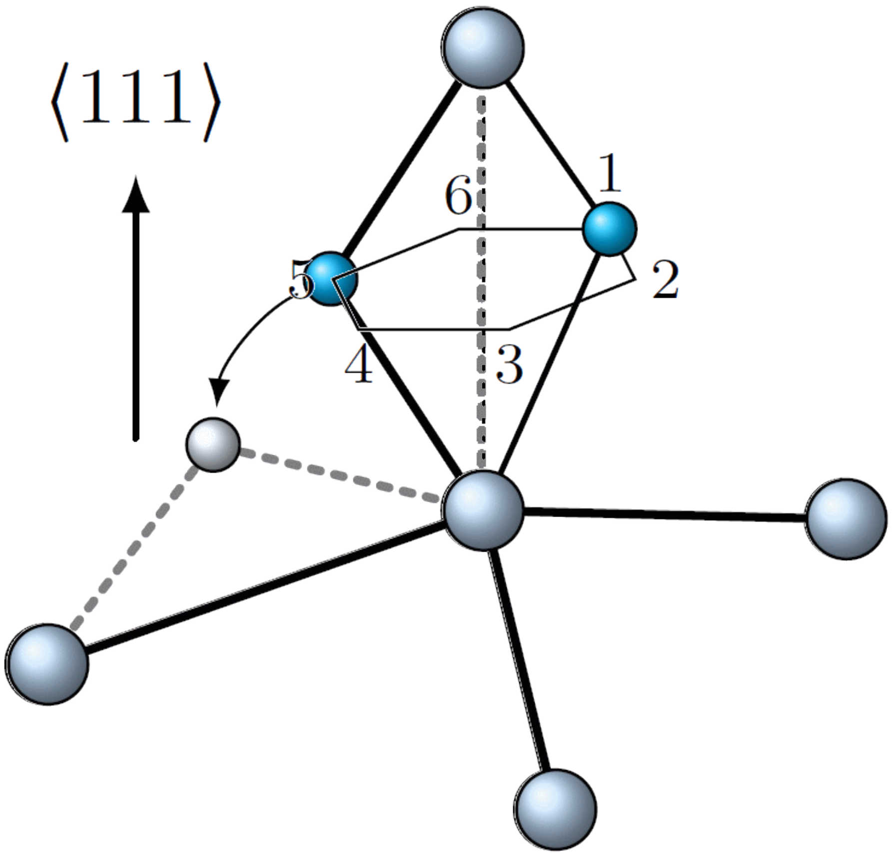


Arrhenius plot for Ø 65 x 50 mm sample



Interstitial oxygen in silicon

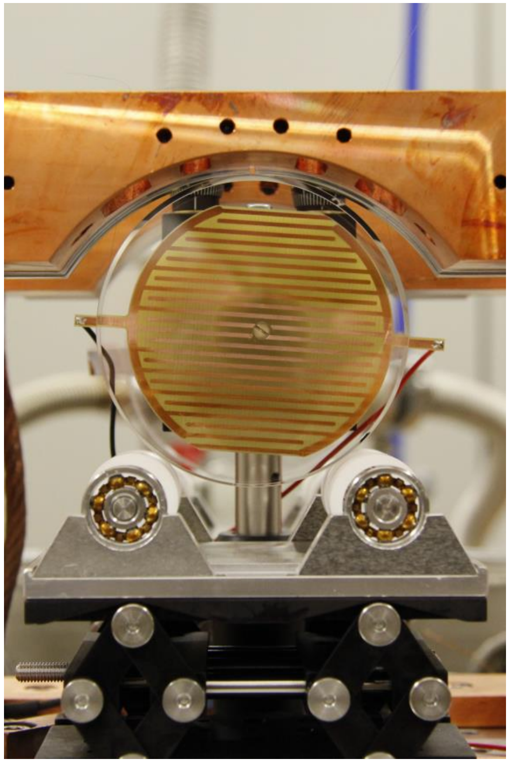
- Czochralski grown crystals with oxygen impurities
- Oxygen covalently bonded between two silicon atoms
- Potential loss mechanisms:
  - Rotation due to six-fold symmetry (E_A too high)
  - Diffusion by hoping (E_A too low)
- Annealing did not change the loss peak – exclusion of kinks and dislocations

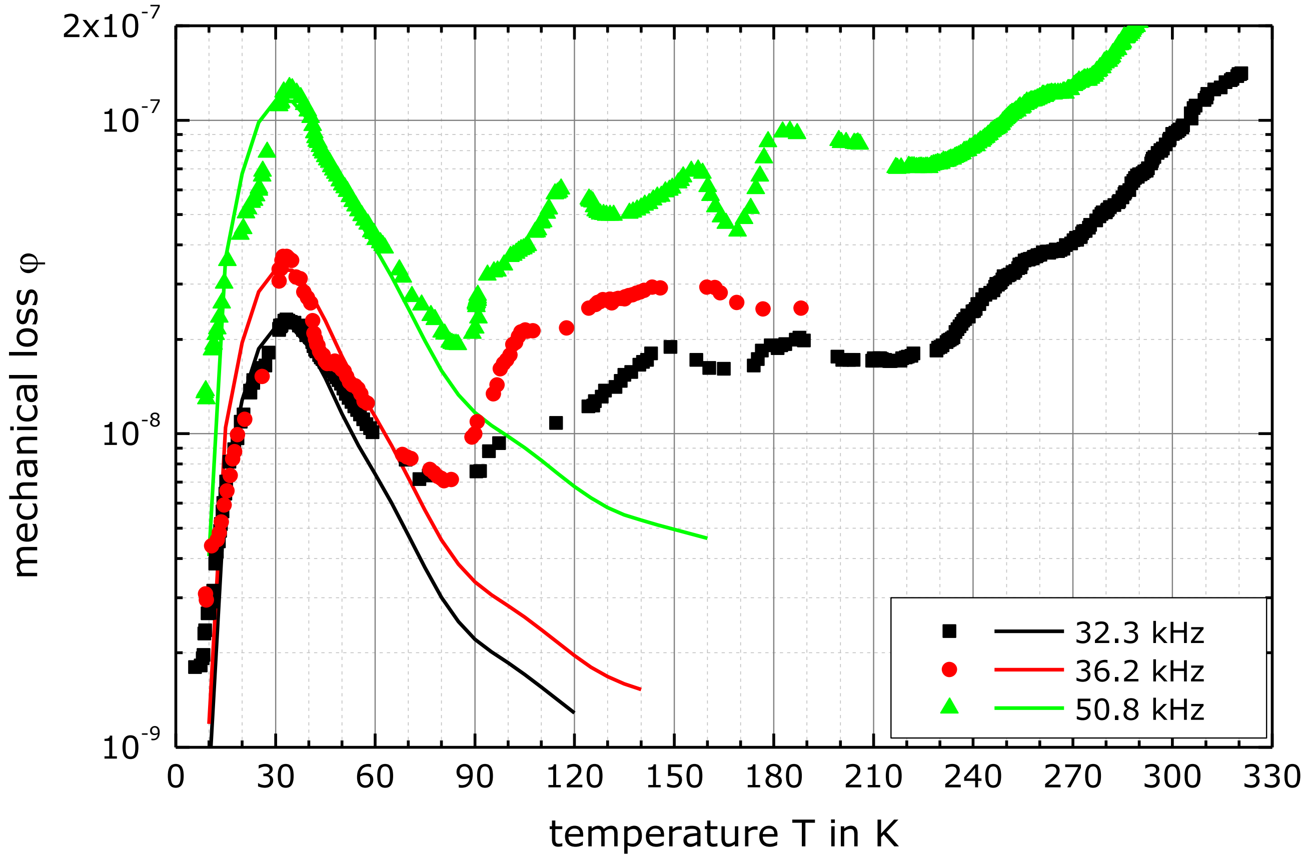


Mechanical loss measurement on

BULK SAPPHIRE

Measured mechanical loss of sapphire





A strong loss peak at 35 K was observed at all measured frequencies.
The continous line represents the so called Akhiezer-loss of the solid.

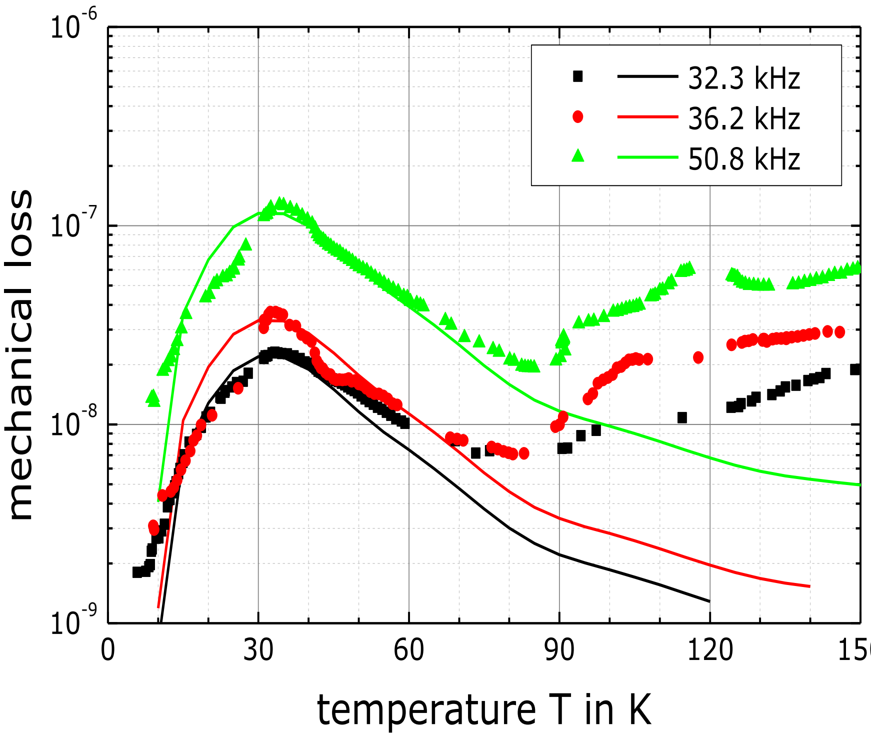
Akhiezer damping in bulk sapphire

Loss peak at 35 K is linked to Akhiezer loss (interaction of acoustic and thermal phonons) as follows:

$$\phi = \frac{TC\gamma^2}{\nu^2}\frac{\omega\tau_p}{1+(\omega\tau_p)^2} \text{ where } \tau_p = \frac{3\,\kappa}{C\rho\nu^2}.$$

[A. Akhieser: *On the absorption of sound in solids*. Journal of Physics (1939)]
[V. B. Braginskyet al.: *Systems with Small Dissipation*.The University of Chicago Press, Chicago and London (1985)]

C… heat capacity, γ… Grüneisen‘s constant,
ν… solid‘s speed of sound,
τ_p… lifetime of thermal phonons,
κ… heat conductivity, and ρ… density of material.



→ Akhiezer loss can not be overcome thus it is an intrinsic limit.

Mechanical loss measurement on

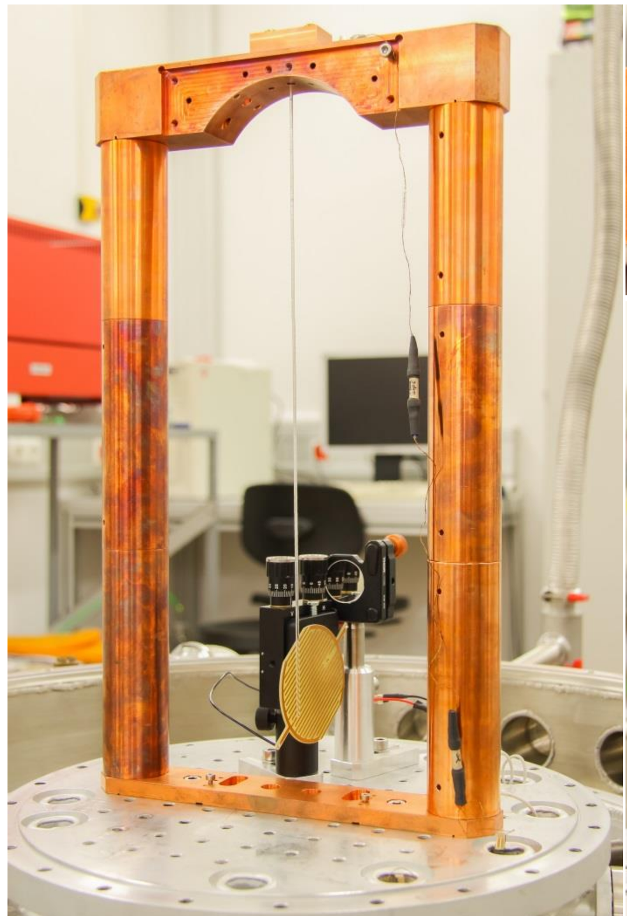
SAPPHIRE FIBERS

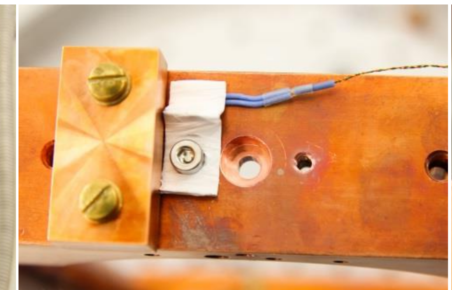
Sapphire fibers measured in Jena

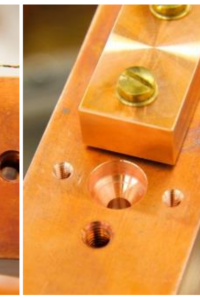
- MolTech fibers (4 in total)
  - single nail head with flat Ø 10 mm x 5 mm
  - fiber Ø 1.8 mm
  - 1 unbroken (350 mm)
  - 1 broken (86 mm & 264 mm)
- Impex fibers (5 in total)
  - double nail head Ø 10 mm x 5 mm
  - fiber Ø 1.6 mm
  - total lenght 100 mm

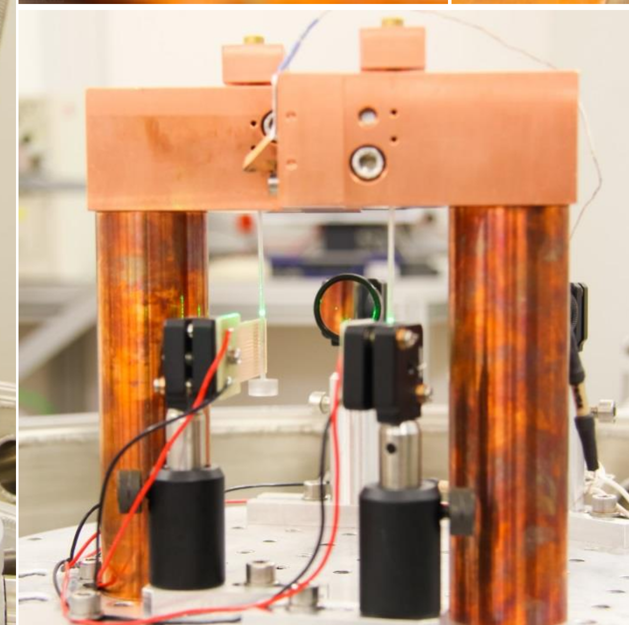
Measurement setup

- Use of massive cooper supports and clamps:
- Flat drill hole vs. Cone drill hole
- Electrostatic driving plates for excitation
- Optical readout by use of shaddow sensor
- Ring down technique
- Liquid helium cryostat $T = 5 \ldots 300\ \mathrm{K}$

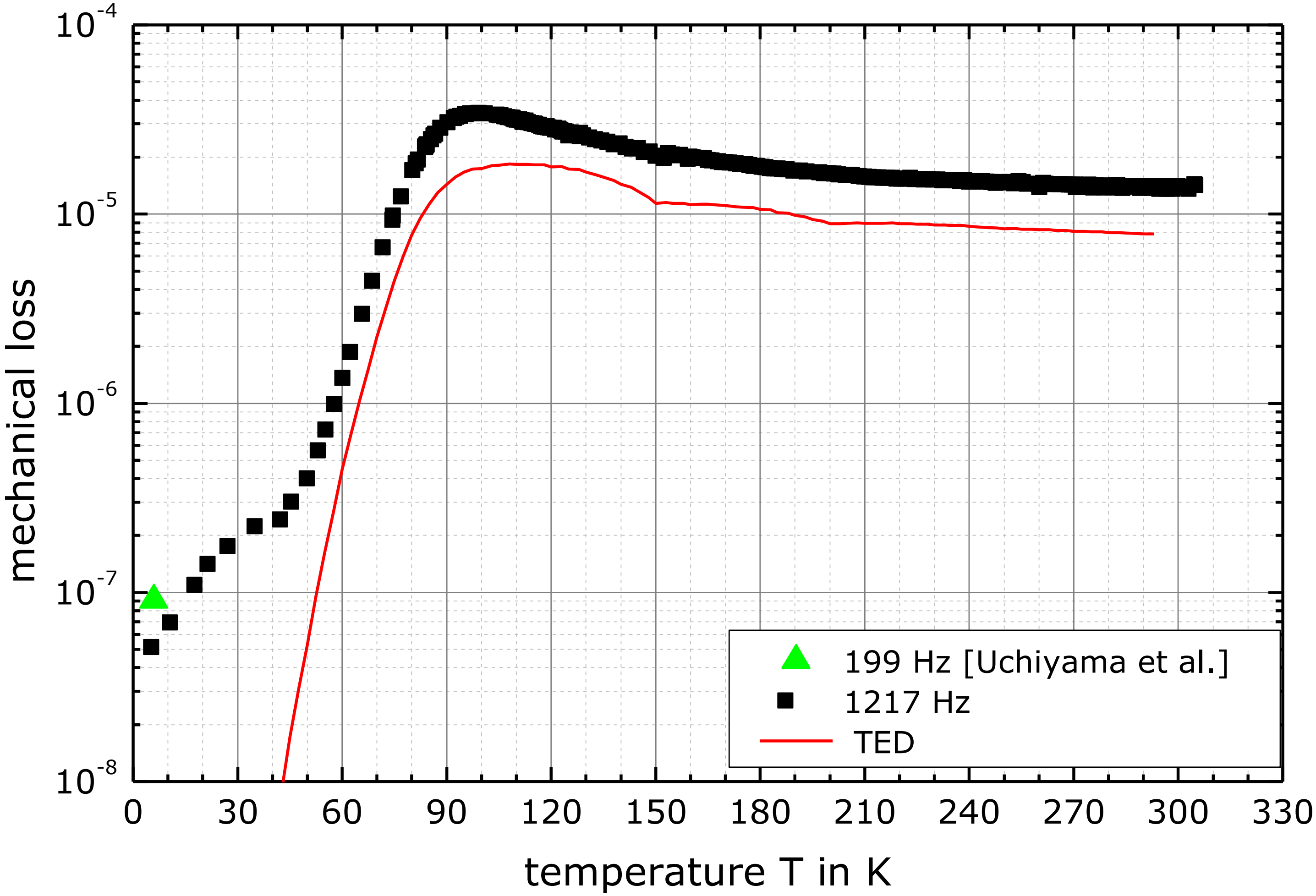


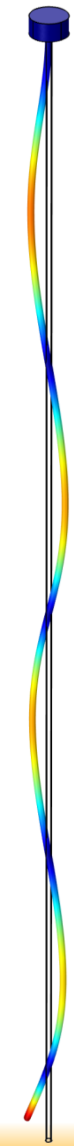






MolTech fiber Ø 1.8 mm x 350 mm, clamped in cone





[T. Uchiyama et al.: Mechanical quality factor of a sapphire fiber at cryogenic temperatures, in Physics Letters A (2000)]

- Lowest obtained loss on sapphire fiber so far: 5×10^{-8} at 5.2 K
- Measured loss dominated by hermo elastic damping (TED) above 60 K

Thermo elastic damping in sapphire fibers

Thermo elastic damping (TED) occurs from irreversible heat flow between compressed and strechted areas of the fiber. The loss is given by:

$$\phi = \frac{YT}{\rho C}\frac{\omega\tau_{TE}}{1+(\omega\tau)^2}$$

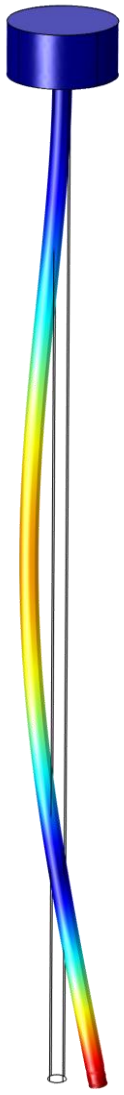
$$\tau_{TE} = \frac{1}{2.16\times 2\pi}\frac{\rho C d}{\kappa}$$

[C. Zener : *Internal Friction in Solids: I. Theory of Internal Friction in Reeds.* Physical Review 52 (1937)]
[C. Zener *: Internal Friction of Solids: II.General Theory of Thermoelastic Internal Friction*. Physical Review 53 (1938)]

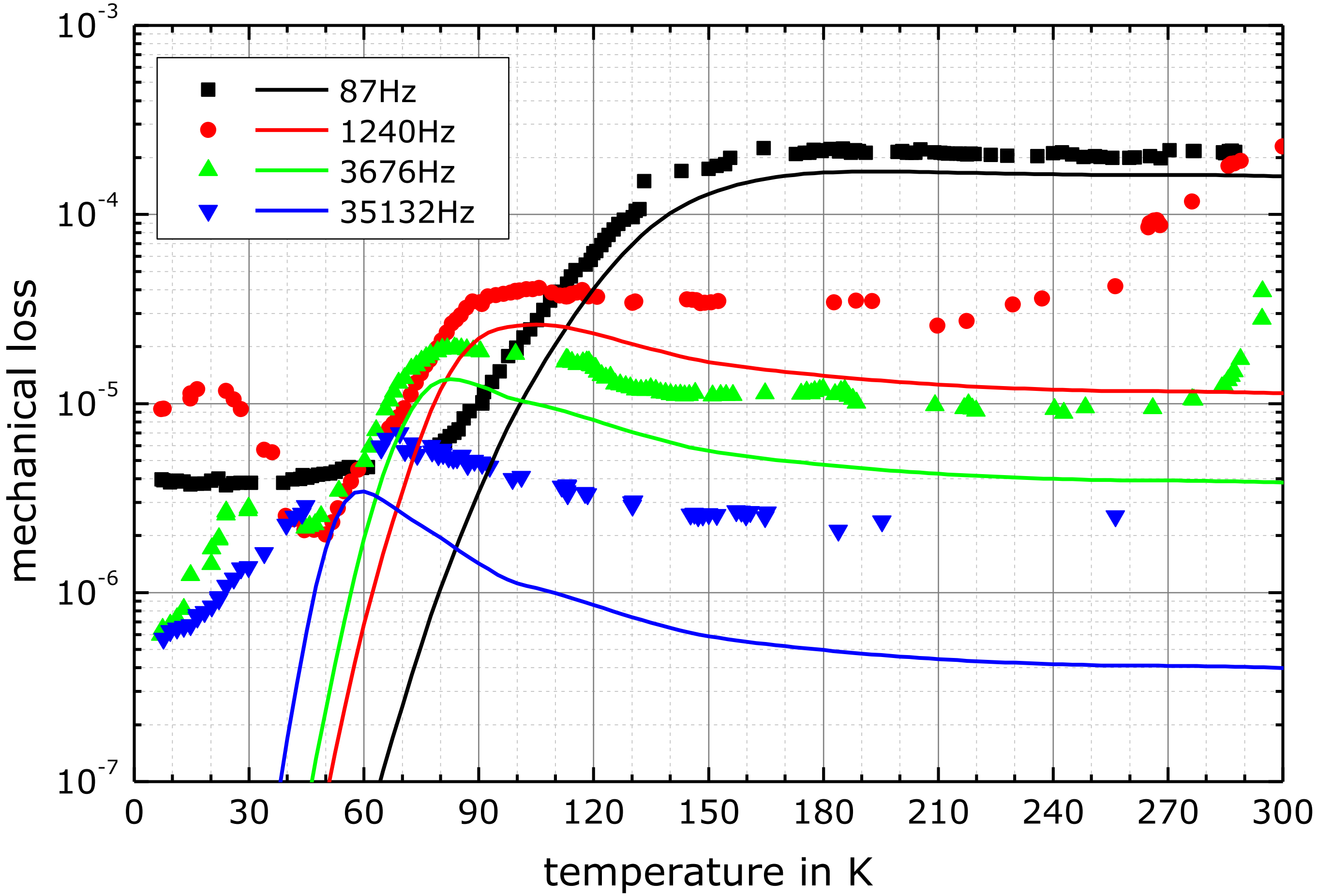
Y... Young's Modulus,

τ_{TE}...characteristic time,

d... diamter of the fiber



Impex fiber No.3, attached head clamped in cone

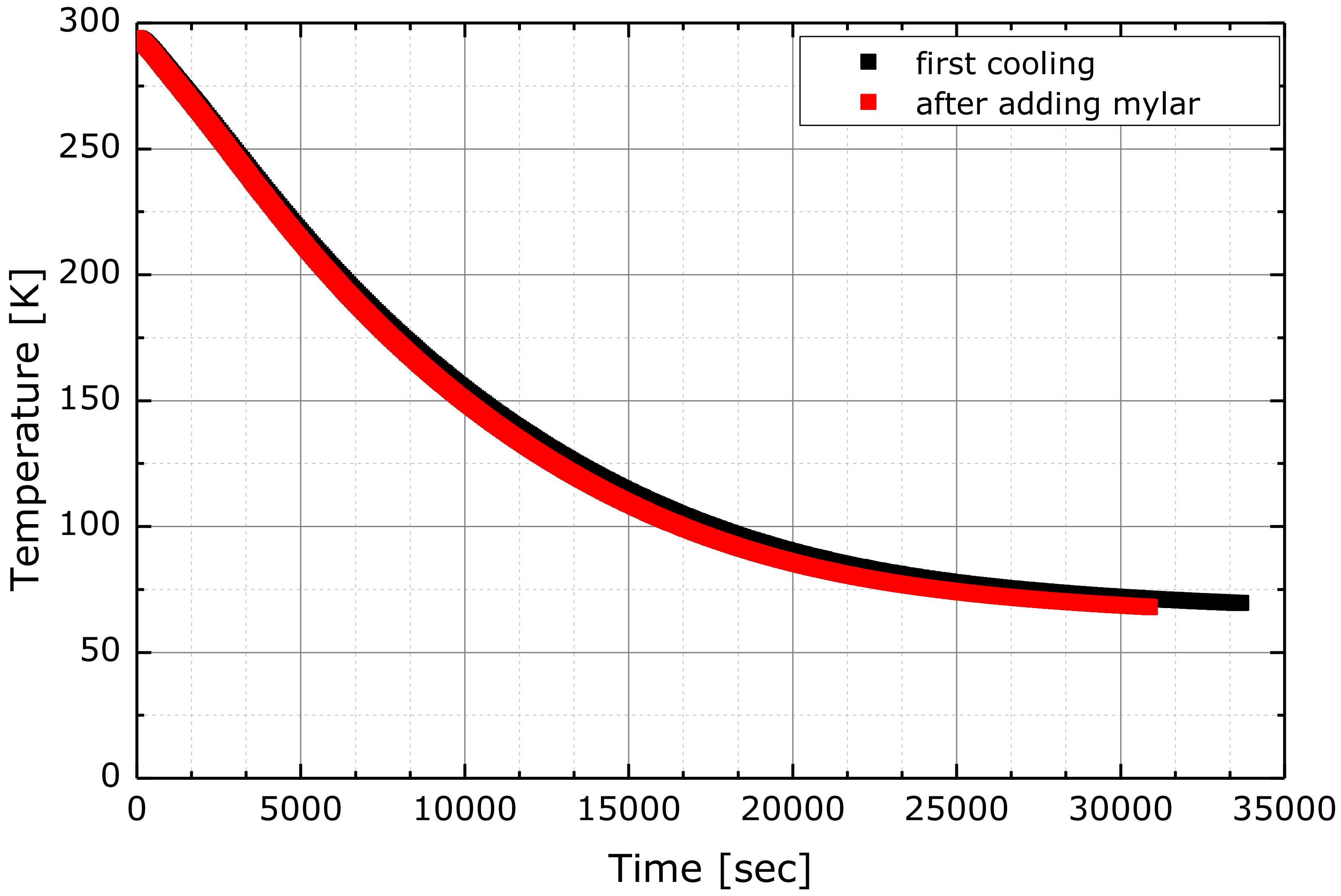


- Again: TED above 60 K seems to limit the loss
- Low temperature behaviour is affected by interactions between the fibre and the support (still under investigation)

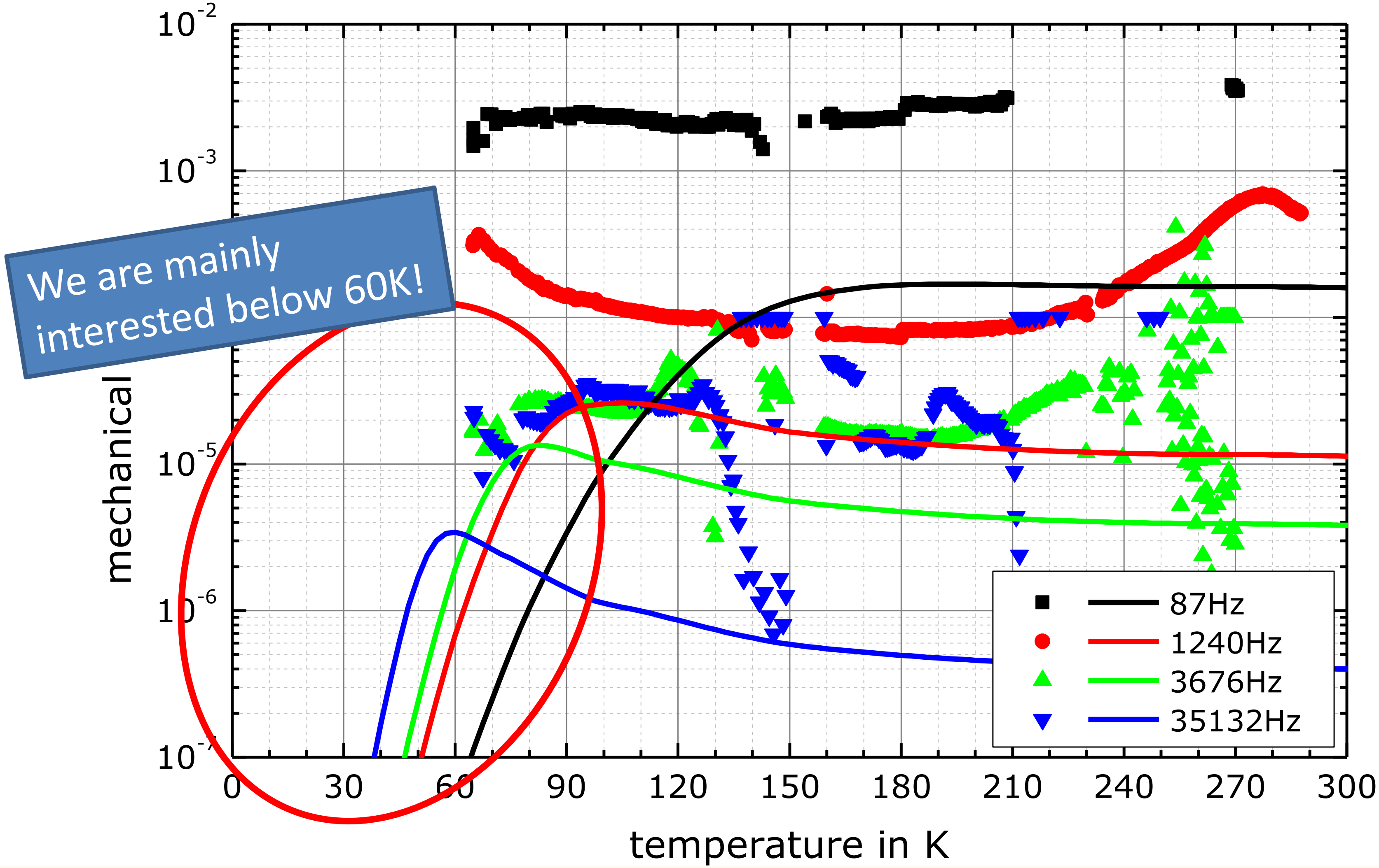
Mechanical loss of sapphire fibres

RECENT WORK AT ICRR

Cooling cycle of cryostat

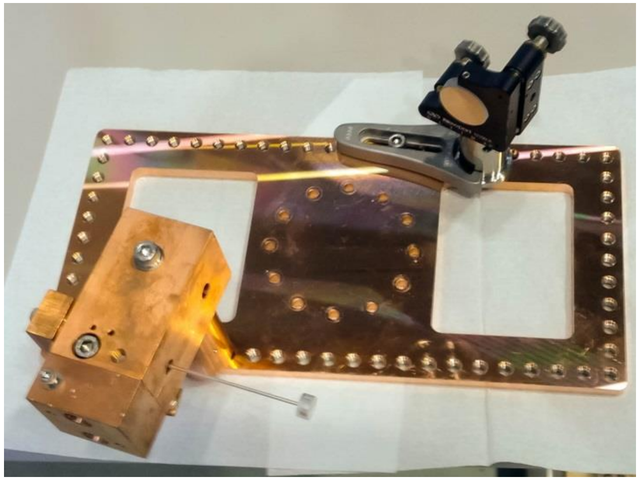


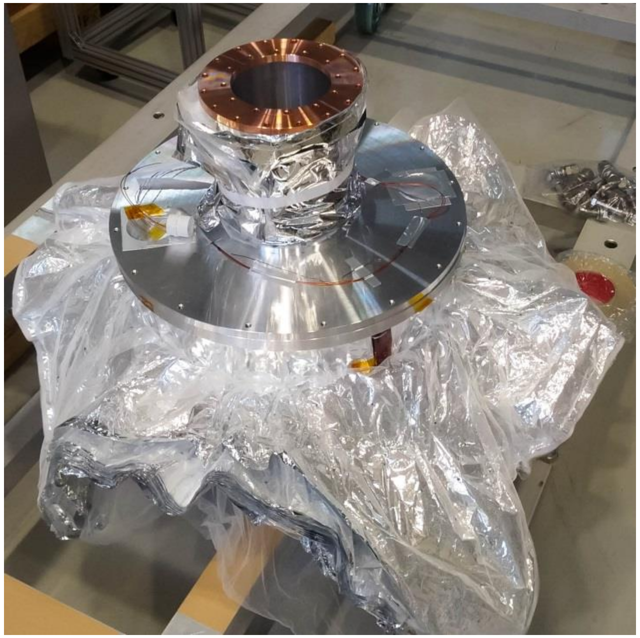
Results so far (preliminary):



Disassembling the cryostat for cryo cooler change

Removed the experiment …





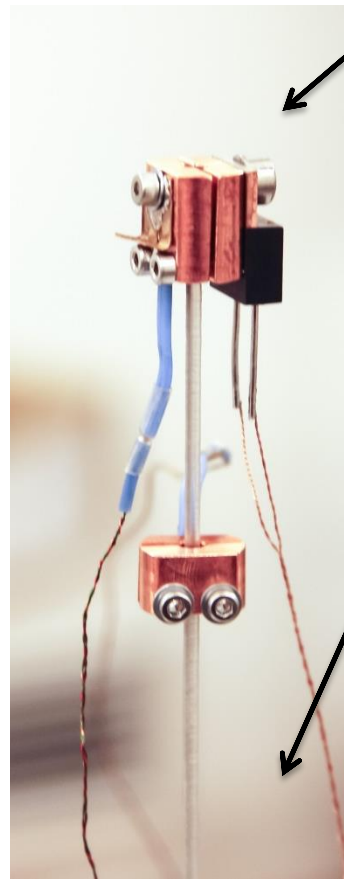
… and the shields already.

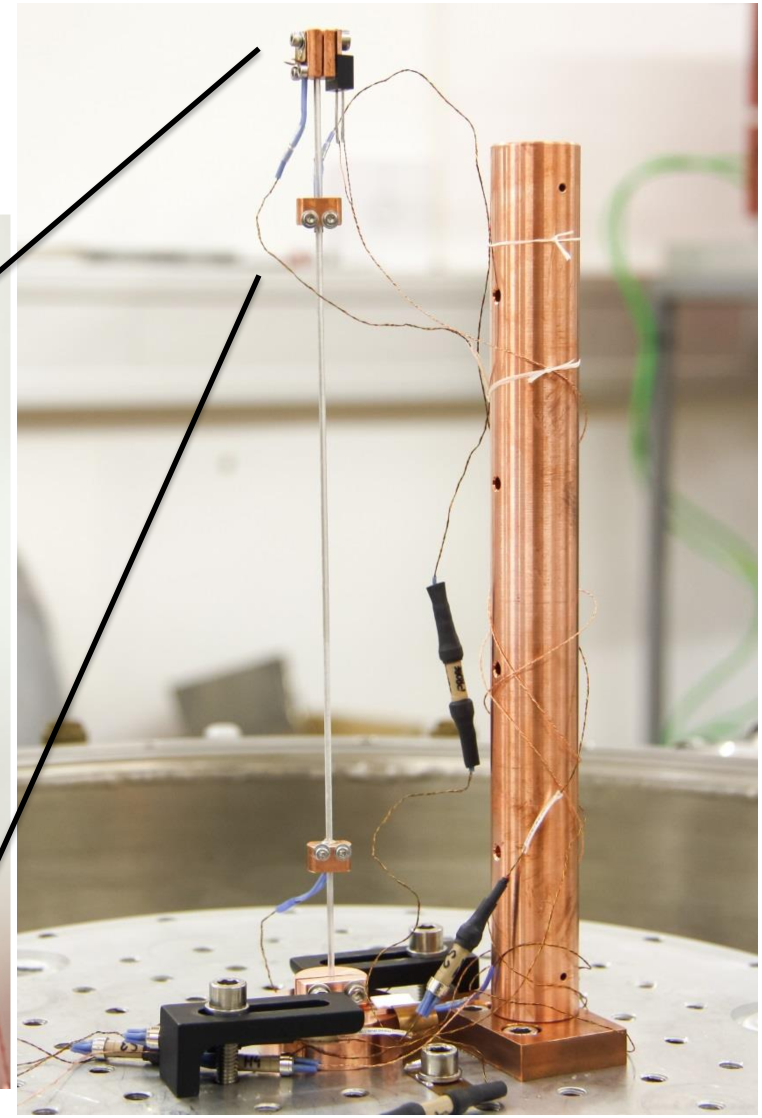
Measurements of the

THERMAL CONDUCTIVITY OF A SAPPHIRE FIBRE

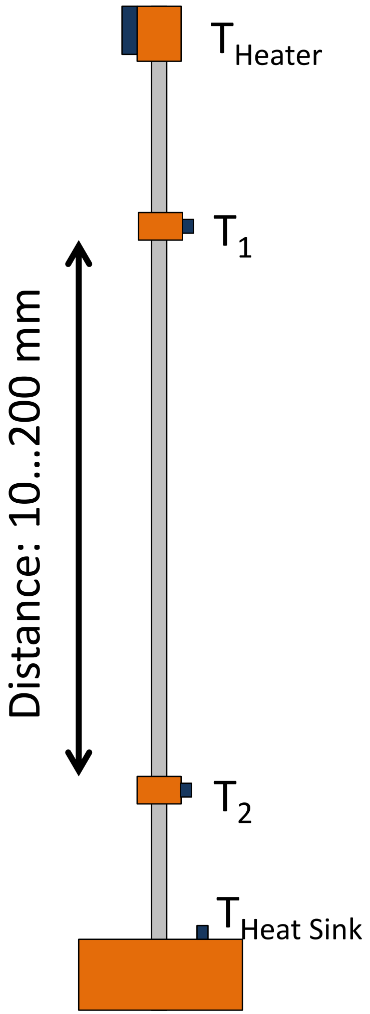
Thermal conductivity measurement

- Measured with the broken piece of MolTech fiber:
  - Ø 1.8 mm
  - 264 mm in length
- Copper clamps to attach
  - the heater
  - the sensors
  - the heat sink





Setup and measurement procedure



Measurement Procedure:

1. Set a heater power and wait until all sensors reach thermal equilibrium again (steady heat flow):

$$P_{Heater} = \frac{A}{L}\kappa(T_1 - T_2)$$

2. Repeat for different heater powers, $\Delta T = T_1 - T_2$ will change
3. Slope of ΔT over P_{Heater} yields κ after:

$$\kappa = \frac{L}{A}\frac{dP}{dT}$$

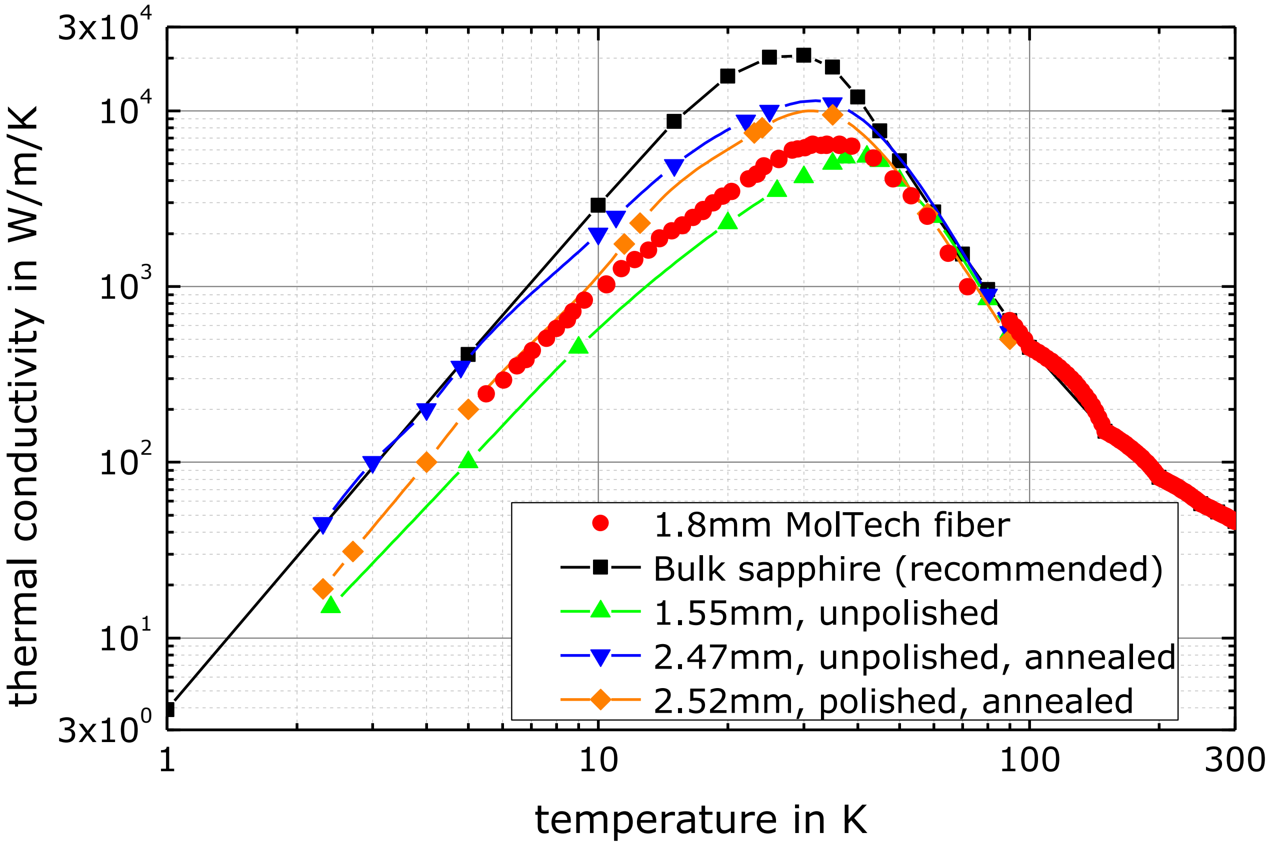
κ… therm. Conductivity, L… temp-sensor distance,
A… cross section, ΔT… temperature difference,
P_{Heater}… electric power

Thermal conductivity of sapphire

The fibres thermal conductivity is different from bulk sapphire below 50 K

At temperatures below 10 K thermal conductivity is limited by size effect: Ø 0.16 mm yields 100 W/m/K

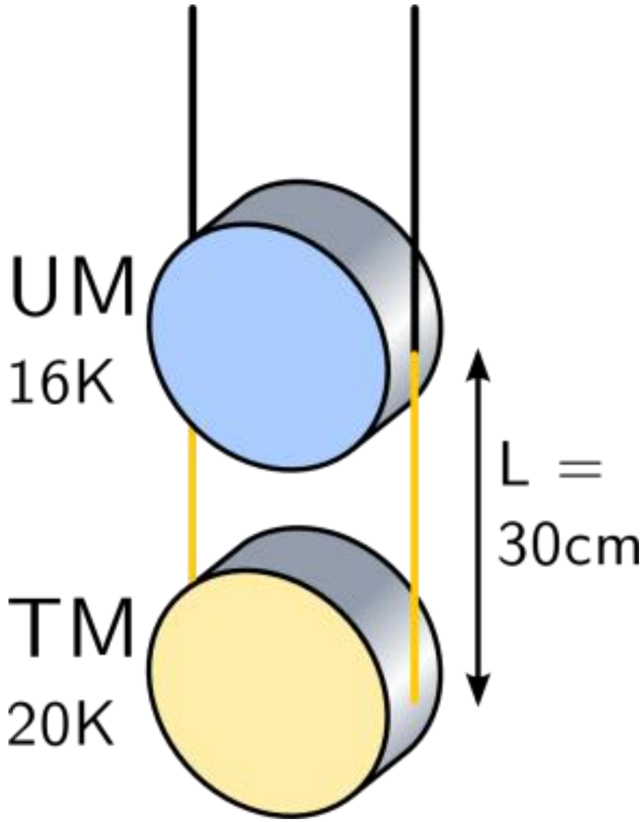
[Tomaru et al.: Maximum heat transfer along a sapphire suspension fiber for a cryogenic interferometric gravitational wave detector, in Physics Letter A (2002)]



[Y. S. Touloukian et al.: Thermophysical properties of matter, Volume 2. Thermal Conductivity - Nonmetallic Solids (1970)]

For the given fibre geometry a surface and heat treatment might change the thermal conductivity to slightly higer values (a few 10%)

Estimation of heat extraction from fibers



- Let's assume:
  - Length $L = 30$ cm
  - Cross section A_w (Ø 1.8 mm)
  - Upper mass UM at 16 K
  - Test mass TM at 20K
  - Thermal conductivity of $\kappa = 3 \times 10^3 \frac{W}{m \times K}$
- Calculate heat extraction of one fiber:

$$\dot{Q} = \frac{A_w}{L} k(T_{TM} - T_{UM}) \approx 100\ mW$$

- Around 1 W needs to be extracted for KAGRA

➔ Futher investigations and improvements are needed!

Summary

- Mechanical loss strongly affects high precision metrology like GWD in terms of noise
- The mechanical loss depends on intrinsic properties of solids and has to be studied well for further improvements
- Silicon and sapphire are very promising for future cryogenic GWD because of their low mechanical losses at low temperature
- Sapphire fibres will fulfil KAGRAs requirements in terms of mechanical loss and heat extraction with some improvements
- Exchange in ELiTES works quite well, technical issues will be solved

Thank you very much!